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MASS AND SPECTROSCOPIC MEASUREMENTS IN THE COMPLETED
MASS 21 AND 37 ISOSPIN QUARTETS

Gilbert W. Butler, Joseph Cerny, S. W. Cosper, and Robert L. McGrath

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ABSTRACT

Low-lying $T = 3/2$ levels of the mass 21 isospin quartet ($^{21}$F, $^{21}$Ne, $^{21}$Na, and $^{21}$Mg) and of three members of the mass 37 quartet ($^{37}$Ar, $^{37}$K, and $^{37}$Ca) have been investigated. An improved particle-identification system which employs two transmission ($\Delta E$) and one stopping ($E$) detector and permits the measurement of low-yield nuclear reactions was used to determine the masses of the $T_z = -3/2$ nuclei $^{21}$Mg and $^{37}$Ca via the ($^3$He, $^6$He) reaction on $^{24}$Mg and $^{40}$Ca at $E_{3He} = 56$ MeV. Complete experimental procedures followed in these three-neutron transfer investigations are discussed. The first three $T = 3/2$ levels in $^{21}$Ne were established by a simultaneous study of the $^{22}$Ne(d,t)$^{21}$Ne and $^{22}$Ne(d,$^3$He)$^{21}$F reactions at 39.6 MeV, while the $^{23}$Na(p,t)$^{21}$Na reaction at 42 MeV was utilized to observe the lowest $T = 3/2$ level in $^{21}$Na. Excitation energies of the lowest $T = 3/2$ levels in $^{37}$K and $^{37}$Ar were determined via the $^{39}$K(p,t)$^{37}$K and $^{39}$K(p,$^3$He)$^{37}$Ar reactions induced by 45-MeV protons. Data from the complete quartets were used to test the isobaric multiplet mass equation and the various predictions of masses of neutron-deficient nuclei.
I. INTRODUCTION

The measurement of masses of nuclei far from stability enables one to evaluate the various theoretical approaches toward prediction of the limits of particle stability; recent theories of interest include those of Goldansky, Jänecke, and Kelson and Garvey. In addition, mass measurements, in conjunction with other data, permit a further study of the isobaric multiplet mass equation (IMME): 

\[ M(A,T,T_z) = a(A,T) + b(A,T)T_z + c(A,T)T_z^2 \]

Since this equation relating the masses of members of an isospin multiplet possesses three coefficients, its validity can be independently checked only if four or more members of such a multiplet with \( T \geq 3/2 \) are known.

We wish to report the completion of the mass 21 and mass 37 isospin quartets. Although the previous investigations of complete isospin quartets (masses 9 and 13) have shown very good first order agreement with the IMME, a test among isobars possessing significantly greater Coulomb energy seemed desirable.

Although one- and two-nucleon transfer reaction investigations quite readily locate the \( T = 3/2 \) states in \( T_z = -1/2 \) and +1/2 nuclei, and the masses of the \( T_z = +3/2 \) nuclei are generally known, in order to complete the quartets it has been necessary to employ the three-neutron transfer reaction \((^3\text{He},^6\text{He})\) to measure the masses of \( T_z = -3/2 \) nuclei. The earlier reported measurements of the \( ^{12}\text{C}(^3\text{He},^6\text{He})^9\text{C} \) and \( ^{16}\text{O}(^3\text{He},^6\text{He})^{13}\text{O} \) reactions utilized a standard particle identifier, but due to the low \((^3\text{He},^6\text{He})\) cross sections (<1 \( \mu \text{b/sr (lab)} \) for \( A>16 \)), an improved particle-identification system employing two transmission \( \Delta E \) detectors and an \( E \) detector was developed. This new system permitted the measurement of the masses of \( ^{21}\text{Mg} \) and \( ^{37}\text{Ca} \) via the \( ^{24}\text{Mg}(^3\text{He},^6\text{He})^{21}\text{Mg} \) and \( ^{40}\text{Ca}(^3\text{He},^6\text{He})^{37}\text{Ca} \) reactions. Excited
levels of both $^{21}\text{Mg}$ and $^{37}\text{Ca}$ were also observed in these experiments. The mass 21 ground-state analogue levels were studied through the $^{22}\text{Ne}(d,^3\text{He})^{21}\text{F}$, $^{22}\text{Ne}(d,t)^{21}\text{Ne}$ and $^{23}\text{Na}(p,t)^{21}\text{Na}$ reactions, while the lowest $T = 3/2$ levels in $^{37}\text{K}$ and $^{37}\text{Ar}$ were established by the $^{39}\text{K}(p,t)^{37}\text{K}$ and $^{39}\text{K}(p,^3\text{He})^{37}\text{Ar}$ reactions.

II. EXPERIMENTAL PROCEDURE

These experiments were carried out using the external beam facilities of the Berkeley 88-inch, variable-energy cyclotron.

A. Beam Optics and General Cave Layout

Figure 1 shows the layout of the 88-inch cyclotron experimental area that was used in these studies. The beam was extracted from the cyclotron at a radius of 39.3 inches by means of a tungsten septum and an electrostatic deflector. After the beam passed through a radial collimator, a quadrupole doublet and a vertical collimator, it entered a beam analysis magnet where it was deflected toward the experimental cave. Another quadrupole lens was required to produce a further radial focus at the analyzing slit. The expected energy resolution after analysis was 0.14%. The beam was then focused to a spot typically 0.06 inch wide by 0.10 inch high in the center of a 20-inch scattering chamber by means of two additional quadrupole doublets; essentially only clean-up collimation was employed between the analysis slit and the target. Beam intensities used in these experiments varied from 50 – 800 nA, depending on the scattering angle and the reaction being studied. The accumulated beam was measured with a Faraday cup and an integrating electrometer.

The angle at which the beam passed through the center of the scattering chamber was determined by observing via remote television its impact on
luminous foils placed at the center of the chamber and 28 inches downstream. Further downstream was a foil-wheel assembly consisting of five independently rotating, 10-position metal wheels that contained varying thicknesses of aluminum. These foil wheels were used in conjunction with the Faraday cup and electrometer to measure the beam range.

The scattering chamber had two independently moveable turrets with counter axes that were either 10 deg above or below the median plane, so that scattering angles from 10 - 170 deg were observable. Counter assemblies were located just outside the scattering chamber and could be isolated from it by a vacuum valve. Rectangular tantalum collimators with approximate dimensions of 0.085 inch wide by 0.200 inch high were used and were located approximately 18.9 inches from the chamber center, giving an angular resolution (radial) of 0.26 deg and a solid angle of $5 \times 10^{-5}$ sr. The counter assembly could be rotated symmetrically about its axis so that the vertical collimator height was always perpendicular to the scattering plane. When a gas target was used, an additional set of collimators (radial only) was attached to the counter assembly at a distance of 4.3 inches from the chamber center. A counter which monitored the beam energy was located in one of the viewing ports of the center stationary section of the scattering chamber. This counter was in the median plane at a scattering angle of 27.5 deg and was collimated so that it subtended approximately the same solid angle as the counter telescope systems.

A target gas cell 3.0 inches in diameter by 1.25 inches in height and with a 315 deg continuous window made of 0.1 mil Havar foil $^{10}$ (2.11 mg/cm$^2$) was employed when necessary. Neon gas enriched $^{11}$ to 91.3% $^{22}$Ne (8.2% $^{20}$Ne and 0.5% $^{21}$Ne) for the study of the $^{22}$Ne(d,t)$^{21}$Ne and $^{22}$Ne(d,$^{3}$He)$^{21}$F reactions was contained in this cell at a pressure of 21 cm of Hg as measured
by a mercury manometer. A Toepler pump was used to recover the enriched neon after the experiment had been completed.

B. Procedure For High-Yield Nuclear Reactions

High-yield nuclear reactions \(\frac{d\sigma}{d\Omega} > 1 \mu b/sr\) were investigated utilizing two power-law type particle identifiers\(^8\) fed by two independent \(\Delta E-E\) silicon detector telescopes; a schematic diagram of one of these two systems is shown in Fig. 2. The \(\Delta E\) detectors were 5.6 mil phosphorus-diffused silicon transmission counters, whereas the 120 mil E detectors, the 20 mil rejection detectors, and the various monitor detectors were all lithium-drifted silicon counters. Signals from the rejection detectors were used in anti-coincidence with those from the \(\Delta E - E\) telescope to eliminate particles traversing the first two counters. Each detector was connected by a short length of low-capacity cable to the input of a fast rise-time (<10 nsec) charge-sensitive preamplifier, and via a 10 M\(\Omega\) resistor to the bias supply. The preamplifiers were connected by about 150 feet of 125 \(\Omega\) cable to the counting area where the signals were fed into delay-line shaped linear amplifiers. After coincidence requirements in the respective \(\Delta E\) and E amplifiers were met, the amplified signals were sent to the particle identifiers. The function of these particle identifiers has been discussed in detail in Ref. 8; they work on the basis of an empirical range-energy relationship for light particles, namely \(R = aE^{1.73}\), where \(R\) is the range of the particle, \(E\) is its energy, and \(a\) is a proportionality constant characteristic of the particle. A typical identifier spectrum is shown in Fig. 3. Particle-identifier pulses were fed into two four-channel routers where single-channel analyzers were set around the \(t, ^3\)He, and \(\alpha\) peaks (the fourth channel was a narrow one centered about the deuteron-triton valley to make certain no tritons were lost). The energy
spectrum of a given particle was then routed into one of the four 1024-channel memory units of a 4096-channel pulse-height analyzer; data from the second counter telescope were recorded in an on-line computer which served as a second 4096-channel analyzer.

The $^{22}\text{Ne}(d,t)^{21}\text{Ne}$ and $^{22}\text{Ne}(d,^3\text{He})^{21}\text{F}$ reactions were carried out with a 39.6-MeV deuteron beam. Angular distributions of reaction products from 11.6 to 41.0 deg. were obtained. The $^{23}\text{Na}(p,t)^{21}\text{Na}$ and $^{23}\text{Na}(p,^3\text{He})^{21}\text{Ne}$ reactions were studied at 42 MeV over an angular range of 11.4 to 40.0 deg. An evaporated, self-supporting sodium foil approximately 1 mg/cm$^2$ thick was transferred to the scattering chamber in an inert atmosphere, but this target oxidized to such an extent that energy levels over 11.0 MeV excitation in $^{21}\text{Na}$ and over 6 MeV excitation in $^{21}\text{Ne}$ were not observed due to very strong contaminant peaks.

The $^{39}\text{K}(p,t)^{37}\text{K}$ and $^{39}\text{K}(p,^3\text{He})^{37}\text{Ar}$ reactions were studied at a proton energy of 45 MeV. The target was an evaporated, self-supporting natural potassium foil 0.8 mg/cm$^2$ thick that was allowed to oxidize slowly before being transferred to the scattering chamber. A small carbon impurity was also found in the target; the impurities provided convenient internal calibration points. An angular range from 14.1 to 28.0 deg was studied.

C. Procedure For Low-Yield Nuclear Reactions

Although the earlier ($^3\text{He},^6\text{He}$) experiments$^{5,7}$ ($d\sigma/d\Omega_{(\text{lab})} \sim 1 - 4 \mu\text{b/sr}$) employed the particle identifier described above, the presence of a relatively high background in the $^6\text{He}$ region of the identifier spectra complicated the measurements. This background arose primarily from rare events in which a few of the large number of alpha-particles stopping in the counter telescope dropped an abnormally large amount of energy in the $\Delta E$ detector (due to blocking,
Landau tail, "nuclear reactions," etc.), causing their identification signal to be significantly higher than normal. When survey experiments on the $^3\text{He}, ^6\text{He}$ reaction on heavier targets showed a decreasing cross section, it became necessary to develop an identification system which eliminated this background. A schematic diagram of a typical counter telescope and the electronics for this improved particle-identification system is shown in Fig. 4. The counter telescope was augmented to incorporate two $\Delta E$ detectors (denoted $\Delta E_2$ and $\Delta E_1$) and an $E$ detector; pulses in these detectors were amplified and fed to a fast-slow coincidence system with a fast-coincidence resolving time of 50 nsec. If the coincidence requirements were met, these three pulses were sent to the particle identifier along with timing pulses.

The rejection detector performed the same function as described in the preceding section. The triple-counter identifier produced three identification pulses for each particle as schematically shown in Fig. 5; note that these identification signals were proportional to the $\Delta E_2$, $\Delta E_1$, and $(\Delta E_2 + \Delta E_1)$ detector thicknesses. The first two (A, B) of these signals (which were both generated in Ident. 1 on Fig. 4) were then compared; if their ratio agreed within preset limits, a linear gate was opened allowing the third identification pulse (generated in Ident. 2 on Fig. 4) to emerge as the output of the particle identifier. Since there was only a very small chance that both $\Delta E$ signals for a particular event would deviate markedly from normal in the same direction by about the same amount, very few incorrect identifications resulted after the comparison-rejection process. Tests had shown that those particles which lost an abnormal amount of energy in a $\Delta E$ detector and were identified improperly (predominantly filling the valleys of the identifier spectrum) could be eliminated while 95 to 99% of the total counts were retained.
The following data contrast the performance of the standard and the triple-counter identifier systems. First, Fig. 6 shows standard particle-identifier spectra from the two-neutron pick-up reaction $^{12}\text{C}(\alpha, ^6\text{He})^{10}\text{C}$ at 70 MeV with $\text{d}\sigma/\text{d}\Omega$ $\approx$ 30 - 40 $\mu$b/sr and from the three-neutron pick-up reaction $^{12}\text{C}(^3\text{He}, ^6\text{He})^9\text{C}$ at 65 MeV with $\text{d}\sigma/\text{d}\Omega$ $\approx$ 1 - 4 $\mu$b/sr. Although a $^6\text{He}$ identification peak was resolved in the former experiment, the lower cross section of the latter reaction was insufficient to cause a peak above the general background in this region of the identifier spectrum. Figure 7 is a $^6\text{He}$ energy spectrum from the $^{12}\text{C}(^3\text{He}, ^6\text{He})^9\text{C}$ reaction using the standard particle identifier and showing the presence of a large background. The improvement acquired with a triple-counter particle-identification system is apparent in Fig. 8, which presents an identifier spectrum from $^{12}\text{C} + 65$-MeV $^3\text{He}$—note particularly the greatly improved peak-to-valley ratios. Finally, Fig. 9 is a $^6\text{He}$ energy spectrum from the $^{12}\text{C}(^3\text{He}, ^6\text{He})^9\text{C}$ reaction obtained using the triple-counter particle-identification system and showing essentially no background.

Previous experiments had shown that chance coincidences between two particles "simultaneously" traversing the counter telescope were a significant problem in the study of low-yield nuclear reactions (see Refs. 12 and 13) in that some of these coincidences produced identification signals near those of the particles of interest. Even though a 50 nsec fast coincidence was required between the cross-over pickup signals from the three detectors, it was found that two light particles of appropriate energies traversing the detector telescope within a time $\Delta t \leq 400$ nsec could satisfy the fast coincidence requirements because the cross-over points of all three (pile-up) energy signals were shifted in the same direction by approximately the same amount. Note Fig. 10(c), which shows a pulser-simulated triple-counter particle-identifier
spectrum for all the particle-stable nuclides from $^3\text{He}$ to $^7\text{Li}$, while Fig. 10(b) shows the pulser-simulated particle-identifier positions of various $^3\text{He}$ + light particle ($p$, $d$, $t$, $^3\text{He}$, or $^4\text{He}$) chance coincidences, and Fig. 10(a) shows the same for various $^4\text{He}$ + light particle chance coincidences. The different particles shown in Fig. 10 were restricted to the same narrow limits of energy that were allowed experimentally in the ($^3\text{He}$, $^6\text{He}$) investigations using the particular three-counter telescope shown in Fig. 4. As under our normal experimental conditions, a minimum energy loss of 5 MeV in the $E$ detector was required. The silicon range-energy relations that were necessary for these calculations and for the analysis of the experimental data were coded at this laboratory and agreed very well with available experimental data and prior computations. It is apparent from Fig. 10 that a $^3\text{He}$-$d$ chance coincidence can very nearly simulate a $^6\text{He}$-particle and an $\alpha$-$d$ chance coincidence can simulate a $^8\text{He}$-particle.

Table I is a list of the calculated energy losses of various particles in the three-counter telescope indicated, assuming no detector dead layers. Energy losses for a $^6\text{He}$-particle and a typical $^3\text{He}$-$d$ chance coincidence are listed in the upper portion of the table; although the $^3\text{He}$-$d$ coincidence event would be identified just above the $^6\text{He}$ region of the particle-identifier spectrum (see Fig. 10), an analysis of the $\Delta E_2$ and $\Delta E_1$ energy losses would distinguish this chance coincidence event from a $^6\text{He}$-particle. (See Sec. D for a discussion of this procedure.) Unfortunately, not all $^3\text{He}$-$d$ chance coincidences can be positively distinguished from $^6\text{He}$ events, and these non-removable events appear as a background in the $^6\text{He}$ energy spectra. Also listed in the lower portion of Table I are the energy losses for a $^8\text{He}$-particle and two sets of $\alpha$-$d$ chance coincidences—all three events having the same
total energy. The first $\alpha$-$d$ chance coincidence has $\Delta E_2$ and $\Delta E_1$ energy losses such that it could not be distinguished from the $^8He$-particle (see Fig. 10 and Refs. 12 and 13), whereas the second $\alpha$-$d$ chance coincidence, for which only the summed energy losses are given, could be distinguished from the $^8He$-particle by an analysis of the $\Delta E_2$ and $\Delta E_1$ energy losses.

Two approaches were followed in order to reduce these chance coincidences. First, the pile-up rejector shown schematically in Fig. 4 was designed to decrease the background due to chance coincidences. Using it, events which consisted of two particles traversing the counter telescope spaced in time by $\geq 47$ nsec were rejected—thus restricting all chance coincidence events to those occurring in a single cyclotron beam burst. The details of this pile-up rejector are described in Ref. 9. Second, the $\Delta E_2$, $\Delta E_1$, ($\Delta E_2 + \Delta E_1 + E = E_{\text{total}}$), and particle identifier pulses for each event in any desired region of the identifier spectrum were recorded in an on-line PDP-5 computer via an eight-channel pulse multiplexer and a 4096 channel ADC and were later individually analyzed (see following section) using the known detector thicknesses and range-energy relations in silicon. This procedure both rejected some coincidence events and enabled us to attach a higher degree of reliability to the "good" events. Accurate analysis of these rare events of interest required two further procedures. First, a monitor detector independently measured the beam energy variation with time by recording the $^3He$ elastic scattering peak; Fig. 11(a) is a time plot of the $^3He$ beam energy as followed by the monitor detector. Second, the computer stored a pulser-simulated $^6He$ event every 12 minutes to check the entire system and provide an accurate measure of electronic drifts during the long data-collection times necessary.
in the \(^3\text{He},^6\text{He}\) experiments; Fig. 11(b) shows the electronic stability of the \(E_{\text{total}}\) pulse from one triple-counter particle-identification system during a 45-hour cyclotron run.

Detector thicknesses in the triple-counter telescopes were selected to provide optimum operation only for the \(^6\text{He}\)-particles and a few \(^6\text{Li}\) groups from the \(^3\text{He},^6\text{Li}\) reaction, which served as a calibration reaction. Diffused-junction silicon transmission detectors were used (including the rejection detectors) to obtain minimum detector window thicknesses. Each detector was calibrated by comparing a precision pulse with the energy losses of \(\alpha\)-particles from a \(^{210}\text{Po} - ^{212}\text{Bi}\) natural \(\alpha\)-source.

A 1.1 mg/cm\(^2\) separated-isotope \(^{24}\text{Mg}\) foil was the target for the \(^{24}\text{Mg}(^3\text{He},^6\text{He})^{21}\text{Mg}\) reaction, which was observed in two independent systems at identical scattering angles of 14.1 deg; a 12-hour run at a beam intensity of 400 nA was required. An evaporated natural calcium foil 0.75 mg/cm\(^2\) thick served as the \(^{40}\text{Ca}\) target. It was transferred to the scattering chamber in an inert atmosphere but was found to contain small amounts of \(^{12}\text{C}\) and \(^{16}\text{O}\) impurities. The \(^{40}\text{Ca}(^3\text{He},^6\text{He})^{37}\text{Ca}\) reaction was studied simultaneously by two independent triple-counter telescopes at identical scattering angles of 11.2 deg during a 45-hour period at a beam intensity of 400 nA (some data were also taken at scattering angles of 14.1 and 18.0 deg).
D. Data Analysis Procedures For \(^{3}\text{He},^{6}\text{He}\) Reactions

As an example of our analysis procedures for these low-yield reactions, we will utilize the data from the \(^{40}\text{Ca}(^{3}\text{He},^{6}\text{He})^{37}\text{Ca}\) experiment since it had a considerably lower cross section than the \(^{24}\text{Mg}(^{3}\text{He},^{6}\text{He})^{21}\text{Mg}\) reaction. Figure 12 presents a triple-counter particle-identifier spectrum from \(^{40}\text{Ca} + 56\text{MeV}^{3}\text{He}\)-particles; the positions of \(^{3}\text{He}\)-p, \(^{3}\text{He}\)-d, and \(^{3}\text{He}\)-\(^{3}\text{He}\) chance coincidences are indicated on the spectrum. The \(^{3}\text{He}\) and \(^{4}\text{He}\) peak heights in this spectrum are not representative since over 98% of these particles were rejected either by the rejection detector or by single channel analyzers which were set to pass only the \(^{6}\text{He}\) and \(^{6}\text{Li}\) ions of interest (thus recording only \(^{3}\text{He}\)-particles between 20.0 and 24.7 MeV and \(\alpha\)-particles between 22.3 and 27.8 MeV). Each event in the \(^{6}\text{He}\) region of the identifier spectrum (and including the \(^{3}\text{He}\)-p and \(^{3}\text{He}\)-d chance coincidence regions) was routed to the on-line computer where its \(\Delta E_2\), \(\Delta E_1\), \(E_{\text{total}}\) and particle-identifier signals were stored on magnetic tape.

In order to discriminate between good and bad events, a plot of the \(\Delta E_2\) and \(\Delta E_1\) energy losses versus \(E_{\text{total}}\) for each event in the \(^{6}\text{He}\) region of the identifier spectrum was made; such a plot for all "\(^{6}\text{He}\)" events observed in the System Two counter telescope during the \(^{40}\text{Ca}(^{3}\text{He},^{6}\text{He})^{37}\text{Ca}\) experiment is shown in Fig. 13. Events in which the \(\Delta E_2\) and/or the \(\Delta E_1\) energy loss deviated more than \(\pm 5\%\) from the smooth curves, which were calculated from the average detector thicknesses, were rejected (these are the circled events and they amount to about 20% of all the events) and were attributed primarily to chance coincidences. Some residual background due to non-rejectable coincidences plus other rare events remains. The peak near \(E_{\text{total}}\) channel 2160 corresponds to the \(^{40}\text{Ca}(^{3}\text{He},^{6}\text{He})^{37}\text{Ca}\) ground-state transition and the peak near channel 2040 corresponds to a transition to an excited state of \(^{37}\text{Ca}\);
some of the data in this figure were obtained at a scattering angle of 14.1 deg and some at a scattering angle of 11.4 deg so that these two peaks are really sharper than they appear to be. All of the valid $^6$He events were then individually corrected for beam energy and electronic drifts (see Fig. 11) as well as for target and detector dead layer losses; the energy of each event was determined from the $^6$He energy scale discussed below. Data from the $^{40}$Ca($^3$He, $^6$Li)$^{37}$K calibration reaction was collected at intervals throughout the run to observe whether any energy shifts occurred arising from radiation damage to the counters.

The problem of establishing an absolute beam energy was treated as follows: calibration curves for $^6$He and $^6$Li ions were obtained by observing highly negative Q-value ($\alpha$, $^6$He) and ($\alpha$, $^6$Li) reactions on suitable targets (here $^{12}$C) at angles chosen so that the ground-state groups stopped in the E detector. Since each detector was previously calibrated via a precision pulser against an $\alpha$-source, an absolute $\alpha$-particle beam energy of 71.0 MeV could be established from these calibrations and the known Q-value and kinematic relations, after dead layer and target losses were taken into account. (Since stopping $^6$He-particles possessed ~30 MeV and $^6$Li-particles ~45 MeV and since these energies were distributed among the three counters, the calibrations were such that minor extrapolations were required for the former and reasonable ones for the latter.) This $\alpha$-particle beam energy agreed within 0.5% with that obtained from range-energy measurements in aluminum. The incident beam was then changed to $^3$He-particles at the appropriate energy; this energy was again measured via aluminum range-energy measurements. More importantly, its energy was also determined from the $^6$Li-particles from a known Q-value calibration reaction (here $^{40}$Ca($^3$He, $^6$Li)$^{37}$K, see Fig. 14(b)) to be 55.92 MeV;
this was again obtained from the earlier $\alpha$-source calibration and agreed with the range-energy measurements to 0.6%. This is equivalent to forcing the $^6$Li ions from the $(^3\text{He},^6\text{Li})$ reaction to be on the same energy scale as those from the earlier $(\alpha,^6\text{Li})$ reaction, so that the $^6$He ions from the $(^3\text{He},^6\text{He})$ reaction should be on the $(\alpha,^6\text{He})$ line established earlier. Using this procedure, an error of 1 MeV in the $^4$He beam energy would make a difference of about 40 keV in the $^{37}$Ca mass.

III. RESULTS

The results of the $^{21}$Mg and $^{37}$Ca mass measurements will be presented first, followed by the rest of the spectroscopic studies in the mass 21 and mass 37 isospin quartets.

A. $^{21}$Mg and $^{37}$Ca Mass Measurements

The mass of $^{21}$Mg, which is the $T_z = -3/2$ member of the mass 21 isospin quartet, was determined by the $^{24}\text{Mg}(^3\text{He},^6\text{He})^{21}\text{Mg}$ reaction at 56 MeV, and Fig. 15 shows a particle-identifier spectrum obtained during this experiment. Figure 14(a) is an energy spectrum from the $^{24}\text{Mg}(^3\text{He},^6\text{Li})^{21}\text{Na}$ reaction, which was recorded simultaneously and used as a calibration reaction. Two independent $^6$He energy spectra from the $^{24}\text{Mg}(^3\text{He},^6\text{He})^{21}\text{Mg}$ reaction are shown in Fig. 16; each event in both spectra was corrected for beam energy variations, electronic drifts, and for energy losses in the target and detector dead layers. Each block represents a single event and the block width was chosen as 100 keV. There appear to be five peaks common to the two spectra which correspond to the ground state and four excited states of $^{21}$Mg. The average differential
cross section for the $^{21}$Mg ground-state transition at 14.1 deg (lab) was 0.84 μb/sr and the average Q-value was determined to be -27.22±0.12 MeV, which corresponds to a $^{21}$Mg mass excess of 10.62±0.12 MeV ($^{12}$C = 0); therefore $^{21}$Mg is stable to proton emission by 3.65±0.12 MeV. Average excitations of the observed $^{21}$Mg excited levels were 0.22±0.03, 1.27±0.06, 1.62±0.04, and 1.89±0.04 MeV; these are consistent with the known level structure of the $^{21}$Mg. 

The mass of $^{37}$Ca ($T_z = -3/2$) was determined via the $^{40}$Ca($^3$He, $^6$He)$^{37}$Ca reaction at 56 MeV. Corrected $^6$He energy spectra from two independent observations of this reaction are shown in Fig. 17; each block represents a single event and the block width is 100 keV. Although several angles were observed in each system, all the data have been kinematically corrected to 11.2 deg. Each $^6$He spectrum contains two sharp states which correspond to the ground and first excited states of $^{37}$Ca. The average cross section for the $^{37}$Ca ground-state transition at 11.2 deg (lab) was 0.175 μb/sr and the Q-value was determined to be -24.27±0.05 MeV, which corresponds to a $^{37}$Ca mass excess of -13.24±0.05 MeV ($^{12}$C = 0). Hence $^{37}$Ca is stable to proton emission by 3.2±0.2 MeV (the mass excess of $^{36}$K calculated in Ref. 17 is probably good to ±0.2 MeV). The first excited state of $^{37}$Ca appears at an excitation energy of 1.62±0.03 MeV, which is consistent with the known first excited state of the $T_z = +3/2$ nucleus $^{37}$Cl at 1.72 MeV. 

Table II presents the forward angle center-of-mass differential cross sections for the four ($^3$He, $^6$He) reactions that have been utilized at this laboratory for mass determinations of $T_z = -3/2$ nuclides. As can be noted from this table, there appears to be a marked decrease in cross section with increasing mass number A. Unfortunately, the general behavior of these
(\(^3\text{He}, \(^6\text{He}\)) cross sections with angle and with bombarding energy has not been determined, so that this apparent pattern could easily be a property of our experimental conditions.

B. \(T = 3/2\) States in \(^{21}\text{Ne}\) and \(^{21}\text{Na}\)

The \(^{22}\text{Ne}(d,t)^{21}\text{Ne}\) and \(^{22}\text{Ne}(d,\(^3\text{He}\)^{21}\text{F}\) reactions were studied simultaneously to characterize the \(T = 3/2\) states in \(^{21}\text{Ne}\). Triton and \(^3\text{He}\) energy spectra from these reactions are shown in Fig. 18; energy resolutions (FWHM) were 160 keV for tritons and 182 keV for \(^3\text{He}\). The ground-state analogue level in \(^{21}\text{Ne}\) was expected to lie near 8.9 MeV excitation since its analogue level in the mirror nucleus \(^{21}\text{Na}\) appears at 8.90±0.04 MeV excitation. \(^{21}\text{Na}\) Three sharp states at 8.92±0.025, 9.21±0.04, and 10.04±0.03 MeV were observed in the \(^{22}\text{Ne}(d,t)^{21}\text{Ne}\) spectra; they are indicated as \(T = 3/2\) levels in Fig. 18 and are the presumed analogues of the \(^{21}\text{F}\) ground-, 0.28- and 1.10-MeV levels, respectively.

Angular distributions of the \(^{22}\text{Ne}(d,t)^{21}\text{Ne}\) transitions to the ground state (\(3/2^+\)), 0.35-MeV (\(5/2^+\)), and 8.92-MeV levels, together with the distribution of the \(^{22}\text{Ne}(d,\(^3\text{He}\)^{21}\text{F}\) ground-state (\(5/2^+\)) transition, are shown in Fig. 19; all these transitions are expected to involve \(L = 2\) angular momentum transfers. The abscissa in Figs. 19 - 21 is the dimensionless quantity \(QR\), where \(Q\) is the center-of-mass linear momentum transfer and \(R\) is the radius of interaction calculated from the formula \(R(\text{fm}) = 1.5(A_{\text{target}})^{1/3} + 1.2\) for incident deuterons. The error bars on the data are based on counting statistics only and the absolute cross sections are expected to be accurate to ±10%. It is of particular interest to compare the \(^{22}\text{Ne}(d,\(^3\text{He}\)^{21}\text{F}\) ground-state transition with the \(^{22}\text{Ne}(d,t)^{21}\text{Ne}\) 8.92-MeV transition since these two
states are expected to be analogues. Assuming the charge independence of nuclear forces, transitions populating these analogue states proceed from identical initial to identical final states and hence should possess identical cross sections after phase space and isospin coupling corrections are made. These corrections involve (compare Bayman$^{21}$):

\[
\frac{\sigma(d,t)}{\sigma(d,^3\text{He})} = \frac{1}{2T_a + 1} \times \frac{k(t)}{k(^3\text{He})};
\]

where $T_a$ is the isobaric spin of the target nucleus and $k(t)$, $k(^3\text{He})$ are the center-of-mass momenta of the particles. Noting Fig. 19 one sees that the angular distributions of the $^{22}\text{Ne}(d,t)^{21}\text{Ne}^*$ 8.92-MeV state and the $^{22}\text{Ne}(d,^3\text{He})^{21}\text{F}$ ground state have similar $L = 2$ shapes and that their adjusted cross sections agree quite well; therefore the 8.92-MeV level of $^{21}\text{Ne}$ has been assigned $T = 3/2$ and $J^\pi = 5/2^+$ to agree with its analogue.

Angular distributions of the $^{22}\text{Ne}(d,^3\text{He})^{21}\text{F}^*$ 0.28-MeV (1/2$^+$) $^{22}$ transition and the $^{22}\text{Ne}(d,t)^{21}\text{Ne}^*$ 9.21-MeV transition are presented in Fig. 20. The $(d,^3\text{He})$ cross sections have again been divided by 2.80 to correct for isospin coupling and phase space factors. Upon comparison of the magnitudes (and $L = 0$ shapes) of these two angular distributions, the 9.21-MeV level of $^{21}\text{Ne}$ can be assigned as a 1/2$^+$, $T = 3/2$ state. Angular distributions of the $^{16}\text{O}(d,^3\text{He})^{15}\text{N}$ ground-(1/2$^-$) and 6.33-MeV (3/2$^-$) states,$^{23}$ together with the distributions of the $^{22}\text{Ne}(d,^3\text{He})^{21}\text{F}^*$ 1.10-MeV level and the $^{22}\text{Ne}(d,t)^{21}\text{Ne}^*$ 10.04-MeV level, are shown in Fig. 21. The spin and parity of the 1.10-MeV level of $^{21}\text{F}$ are not known, but when the angular distribution of this level is compared to the known $L = 1$ transitions to $^{15}\text{N}$, it appears to possess an $L = 1$ shape, so that it may be tentatively assigned as $J^\pi = (1/2, 3/2)^+$. The
angular distributions of the $^{21}$Ne* 10.04-MeV and $^{21}$F* 1.10-MeV levels (the latter cross section has been divided by the appropriate phase space and isospin coupling factor in Fig. 21) are similar in shape and have the proper relative magnitudes, so that the $^{21}$Ne* 10.04-MeV state is the third $T = 3/2$ level in $^{21}$Ne and it is also tentatively assigned spin and parity (1/2, 3/2). The $^{23}$Na(p,t)$^{21}$Na and the $^{23}$Na(p,3He)$^{21}$Ne reactions were studied in order to locate $T = 3/2$ levels in $^{21}$Na through comparison of (p,t) transitions with (p,3He) transitions to the now established $T = 3/2$ levels in $^{21}$Ne. An energy spectrum from the $^{23}$Na(p,t)$^{21}$Na reaction is shown in Fig. 22 with typical triton energy resolution (FWHM) of 150 keV. Peak 19 in the spectrum possessed an average excitation energy of 8.92±0.03 MeV and was presumed to be the lowest $T = 3/2$ level since this excitation agreed with the value of 8.90±0.04 MeV reported earlier. Due to the large amount of oxygen in the target, the (p,3He) transitions to the $^{21}$Ne analogues of the 8.92-MeV and higher levels of $^{21}$Na were not observed. Several previously unreported energy levels of $^{21}$Na were observed in this experiment and the results are presented in Table III along with a summary of previously published data on states of this nucleus. No new spin and parity assignments have been made by us; and, in fact, some of the excited levels may be unresolved doublets. In general, the results of this experiment are in good agreement with the published data on the energy levels of $^{21}$Na.

These new data on the mass 21 isospin quartet are summarized in Fig. 23, which shows energy level diagrams for the members of the quartet. For clarity, ground-state energies of mirror nuclei have been equated and many of the energy levels of $^{21}$Ne and $^{21}$Na below 10 MeV excitation have been
C. $T = 3/2$ States in $^{37}K$ and $^{37}Ar$

The $^{39}K(p,t)^{37}K$ and $^{39}K(p,^3He)^{37}Ar$ reactions at 45 MeV were used to locate the lowest $T = 3/2$ levels in $^{37}K(T_z = -1/2)$ and $^{37}Ar(T_z = +1/2)$. Triton and $^3He$ energy spectra from these reactions at a laboratory angle of 26 deg are shown in Fig. 24; the energy resolutions (FWHM) were about 180 keV for tritons and 280 keV for $^3He$-particles. Sharp states were observed in these spectra in the expected regions for the $T = 3/2$ levels, as predicted from Coulomb calculations and the $^{37}Cl$ mass, at excitations of $5.010 \pm 0.030$ and $5.035 \pm 0.025$ MeV in $^{37}Ar$ and $^{37}K$, respectively. This excitation for the lowest $T = 3/2$ level of $^{37}K$ agrees with that of $5.030 \pm 0.018$ MeV obtained previously. Angular distributions to these two states and to the $^{37}K$ ground-(3/2$^+$) and 1.37-MeV (1/2$^+$) states are shown in Fig. 25. Since a spin transfer of $S = 0$ is required to first order for (p,t) reactions, the $^{37}K \rightarrow 1.37$-MeV transition is expected to be a pure $L = 2$ transfer. Although the $^{37}K$ ground-state transition is allowed by angular momentum and parity selection rules to proceed via either $L = 0$ or $L = 2$ angular momentum transfers, previous (p,t) experiments have shown that such transitions appear to proceed by relatively pure $L = 0$ transfers. Assuming the charge independence of nuclear forces, (p,t) and (p,$^3He$) transitions to $T = 3/2$ analogue states proceed from identical initial to identical final states primarily through only $S = 0$, $T = 1$ pick-up of two nucleons. The lowest $T = 3/2$ states in $^{37}K$ and $^{37}Ar$, which are the analogues of the known $^{37}Cl$ ground state (3/2$^+$), should therefore have identical cross sections after isospin coupling and
phase space corrections have been made; $L = 0$ behavior is again expected. Indeed, Fig. 25 shows that the angular distributions to these supposed analogue states possess $L = 0$ and are essentially identical, thereby confirming that they are $T = 3/2$ analogue levels with expected spins and parities of $3/2^+$. Figure 26 summarizes the mass 37 isospin quartet data. Ground-state energies of mirror nuclei have been equated for the sake of clarity and only a few of the levels of $^{37}$Ar and $^{37}$K below 5 MeV excitation are shown; the level structure of $^{37}$K is from Goosman and Kavanagh.  

IV. DISCUSSION

If the wave functions of the members of an isospin multiplet can be regarded as identical and if any charge-dependent forces are of a two-body character that can be treated as perturbations, then the masses of the members are given by the isobaric multiplet mass equation (IMME) $^4$:

$$M(A,T,T_z) = a(A,T) + b(A,T)T_z + c(A,T)T_z^2.$$  

The completion of the mass 21 and mass 37 isospin quartets permits two further independent tests of the parabolic character of the IMME since the coefficients can be determined from three of the members of a quartet and then used to predict a mass for the fourth member; we have chosen to use these data to predict the mass of the $T_z = -3/2$ nucleus in each quartet. A list of the $T_z = -3/2$ nuclei whose mass excesses have been measured, together with the coefficients and the predictions of the IMME for the four
isospin quartets which have been completed to date, is presented in Table IV. In all four of the completed isospin quartets there is very good first order agreement between the predicted and experimental mass excesses within the generally relatively large experimental errors, which indicates that the IMME is accurate over a fairly large variation in atomic number Z. An apparent discrepancy in the much more accurate mass 9 quartet data\(^6\) was interpreted as indicating a possible (and ultimately to be expected\(^7\)) need for the inclusion of a higher order term \(\tilde{d}(A,T)T_z^3\) in the IMME, but unfortunately no explicit theoretical estimate of the magnitude of such a coefficient is available. These \(\tilde{d}(A,T)\) coefficients of an IMME containing a \(\tilde{d}(A,T)T_z^3\) term were calculated from the four experimental masses in each quartet and the results are listed in Table IV; clearly such a coefficient is no greater than \(\bar{Z}\alpha\), where \(\bar{Z}\) is the average atomic number of an isospin quartet and \(\alpha\) is the fine structure constant. Theoretical predictions of the magnitude of the expected second order corrections would be quite valuable.

Various systematic mass predictions for \(^9\)C, \(^{13}\)O, \(^{21}\)Mg, and \(^{37}\)Ca, together with the experimentally measured masses, are shown in Table V. Only the Kelson-Garvey predictions (Ref. 3) cover all four nuclei and their predictions appear to be in excellent agreement with the experimental masses except for \(^{13}\)O. In general, though, all three sets of predictions are in reasonable agreement with the data and a more stringent test of such predictions would appear to require investigations of nuclei nearer proton-instability, such as \(^{12}\)O and \(^{36}\)Ca, which could be reached through the \((\alpha,^8\text{He})\) reaction.
ACKNOWLEDGEMENTS

The authors are indebted to F. S. Goulding and D. A. Landis for designing the pile-up rejector and other electronic equipment used in these experiments, to C. C. Maples, Jr., for developing several data analysis programs, to R. P. Lothrop and M. D. Roach for providing the transmission detectors, and to the several graduate students who aided during the long hours of data collection.
FOOTNOTES AND REFERENCES

*This work was done under the auspices of the U.S. Atomic Energy Commission.

†Present address: Physics Department, University of Southwestern Louisiana, Lafayette, Louisiana.

4. E. P. Wigner and E. Feenberg, Rept. Progr. Phys. 8, 274 (1941);
11. Monsanto Research Corporation, Mound Laboratory, Miamisburg, Ohio.


23. The \(^{16}\text{O}(d,^{3}\text{He})^{15}\text{N}\) angular distributions at 39.6 MeV are from unpublished data of J. Cerny, et al.


Table I. Energy losses of various particles in a triple-counter telescope.

<table>
<thead>
<tr>
<th>Particle</th>
<th>$E_{\text{total}}$ (MeV)</th>
<th>5.2 mil $\Delta E$ ctr</th>
<th>4.2 mil $\Delta E$ ctr</th>
<th>6.5 mil E ctr</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^6\text{He}$</td>
<td>29.00</td>
<td>8.27</td>
<td>9.17</td>
<td>11.56</td>
</tr>
<tr>
<td>$^3\text{He}$</td>
<td>21.25</td>
<td>6.25</td>
<td>7.15</td>
<td>7.85</td>
</tr>
<tr>
<td>$^3\text{He} + d$ (sum)</td>
<td>29.00</td>
<td>8.78</td>
<td>10.09</td>
<td>10.13</td>
</tr>
<tr>
<td>$^8\text{He}$</td>
<td>35.50</td>
<td>8.67</td>
<td>8.93</td>
<td>17.90</td>
</tr>
<tr>
<td>$^4\text{He}$</td>
<td>27.50</td>
<td>6.18</td>
<td>6.24</td>
<td>15.08</td>
</tr>
<tr>
<td>$^4\text{He} + d$ (sum)</td>
<td>35.50</td>
<td>8.37</td>
<td>8.93</td>
<td>18.00</td>
</tr>
<tr>
<td>$^4\text{He} + d$ (sum)</td>
<td>35.50</td>
<td>8.66</td>
<td>9.45</td>
<td>17.39</td>
</tr>
</tbody>
</table>
### Table II. Forward angle differential cross sections for four \( ^3\text{He}, \text{He} \) reactions.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Bombarding Energy (MeV)</th>
<th>( \theta_{\text{CM}} ) (deg)</th>
<th>( \frac{d\sigma}{d\Omega} ) (( \mu\text{b/sr} ))_{\text{CM}}</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^{12}\text{C}( ^3\text{He}, \text{He})^9\text{C} ) g.s.</td>
<td>65.5</td>
<td>15.8</td>
<td>1.6 ± 0.4</td>
</tr>
<tr>
<td>( ^{16}\text{O}( ^3\text{He}, \text{He})^{13}\text{O} ) g.s.</td>
<td>65.3</td>
<td>15.3</td>
<td>0.76 ± 0.12</td>
</tr>
<tr>
<td>( ^{24}\text{Mg}( ^3\text{He}, \text{He})^{21}\text{Mg} ) g.s.</td>
<td>55.9</td>
<td>18.0</td>
<td>0.52 ± 0.12</td>
</tr>
<tr>
<td>( ^{40}\text{Ca}( ^3\text{He}, \text{He})^{37}\text{Ca} ) g.s.</td>
<td>55.9</td>
<td>12.9</td>
<td>0.13 ± 0.03</td>
</tr>
</tbody>
</table>
Table III. Levels in $^{21}$Na observed in the $^{23}$Na(p,t)$^{21}$Na reaction at 42 MeV. The $J^\pi$ assignments were all taken from the references cited.

<table>
<thead>
<tr>
<th>Level Number</th>
<th>Excitation This Work (MeV ± keV)</th>
<th>Excitation Other Work (MeV ± keV)</th>
<th>$J^\pi$</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0</td>
<td></td>
<td>$3/2^+$</td>
<td>18</td>
</tr>
<tr>
<td>1</td>
<td>0.335 ± 10</td>
<td>0.338 ± 3</td>
<td>$5/2^+$</td>
<td>24</td>
</tr>
<tr>
<td>2</td>
<td>1.71 ± 10</td>
<td>1.72</td>
<td>$(7/2^+)$</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>2.42 ± 30</td>
<td>2.430 ± 3</td>
<td>$1/2^+$</td>
<td>25</td>
</tr>
<tr>
<td>4</td>
<td>2.81 ± 20</td>
<td>2.81 ± 40</td>
<td></td>
<td>18</td>
</tr>
<tr>
<td>5</td>
<td>3.54 ± 20</td>
<td>3.540 ± 2</td>
<td>$5/2^+$</td>
<td>24</td>
</tr>
<tr>
<td>6</td>
<td>3.64 ± 30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>3.83 ± 20</td>
<td>3.88, 3.82</td>
<td>4.15</td>
<td>18, 24</td>
</tr>
<tr>
<td>8</td>
<td>4.28 ± 30</td>
<td>4.29 ± 4</td>
<td>$5/2^+$</td>
<td>24</td>
</tr>
<tr>
<td>9</td>
<td>4.41 ± 30</td>
<td>4.41 ± 20</td>
<td>4.46 ± 5</td>
<td>24</td>
</tr>
<tr>
<td>10</td>
<td>4.99 ± 30</td>
<td>5.00 ± 50</td>
<td>$3/2^+$</td>
<td>18</td>
</tr>
<tr>
<td>11</td>
<td>5.34 ± 30</td>
<td></td>
<td>5.47</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5.70</td>
<td>18</td>
</tr>
<tr>
<td>12</td>
<td>5.78 ± 30</td>
<td>5.83</td>
<td></td>
<td>18</td>
</tr>
<tr>
<td>13</td>
<td>6.16 ± 30</td>
<td></td>
<td>6.26</td>
<td>18</td>
</tr>
<tr>
<td>14</td>
<td>6.54 ± 30</td>
<td>6.52 ± 50</td>
<td>$3/2^+$</td>
<td>18, 26</td>
</tr>
<tr>
<td>15</td>
<td>7.06 ± 30</td>
<td>7.21 ± 50</td>
<td>$1/2^+$</td>
<td>26</td>
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</table>
Table III. (continued)

<table>
<thead>
<tr>
<th>Level Number</th>
<th>Excitation This Work (MeV ± keV)</th>
<th>Excitation Other Work (MeV ± keV)</th>
<th>$^\pi$</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>7.60 ± 60</td>
<td>7.45 ± 50</td>
<td>5/2$^+$</td>
<td>18, 26</td>
</tr>
<tr>
<td>17</td>
<td>8.11 ± 30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>8.31 ± 30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>8.92 ± 30</td>
<td>8.90 ± 40</td>
<td>(5/2$^+$), T=3/2</td>
<td>19</td>
</tr>
<tr>
<td>20</td>
<td>9.19 ± 30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>10.05 ± 40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>11.00 ± 30</td>
<td></td>
<td></td>
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</tr>
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</table>
Table IV. Isobaric multiplet mass equation predictions and calculated coefficients.

Isobaric Multiplet Mass Equation

\[
M(A,T,T'_{\pi}) = a(A,T) + b(A,T)T'_{\pi} + c(A,T)T'_{\pi}^2 + d(A,T)T'_{\pi}^3
\]

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Experimental Mass Excess (MeV)</th>
<th>Predicted Mass Excess (MeV)</th>
<th>(b(A,T)) (MeV)</th>
<th>(c(A,T)) (keV)</th>
<th>(d(A,T)) (keV)</th>
<th>(\Sigma\alpha) (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{9}\text{C})</td>
<td>28.99 (70)(^{a,b})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(^{9}\text{C})</td>
<td>28.916 (5)(^{c})</td>
<td>28.961 (29)</td>
<td>-1.332 (7)</td>
<td>278 (11)</td>
<td>7 (5)</td>
<td>9</td>
</tr>
<tr>
<td>(^{13}\text{O})</td>
<td>23.11 (70)(^{d})</td>
<td>23.10 (49)</td>
<td>-2.180 (16)</td>
<td>254 (11)</td>
<td>-1 (14)</td>
<td>12</td>
</tr>
<tr>
<td>(^{21}\text{Mg})</td>
<td>10.62 (120)</td>
<td>10.59 (120)</td>
<td>-3.545 (40)</td>
<td>154 (24)</td>
<td>-5 (28)</td>
<td>12</td>
</tr>
<tr>
<td>(^{37}\text{Ca})</td>
<td>-13.24 (50)</td>
<td>-13.24 (118)</td>
<td>-6.176 (39)</td>
<td>176 (25)</td>
<td>1 (21)</td>
<td>24</td>
</tr>
</tbody>
</table>

\(^a\) All errors are in parentheses and are in keV.

\(^b\) See Ref. 5.

\(^c\) See Ref. 6.

\(^d\) See Ref. 7.
Table V. Comparison between various systematic mass predictions and the experimental masses.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$^9$C</td>
<td>28.916 (5)$^e$</td>
<td>-</td>
<td>29.3</td>
<td>28.88</td>
</tr>
<tr>
<td>$^{13}$O</td>
<td>23.11 (70)$^f$</td>
<td>23.2</td>
<td>23.4</td>
<td>23.52</td>
</tr>
<tr>
<td>$^{21}$Mg</td>
<td>10.62 (120)</td>
<td>10.9</td>
<td>10.8</td>
<td>10.79</td>
</tr>
<tr>
<td>$^{37}$Ca</td>
<td>-13.24 (50)</td>
<td>-12.9</td>
<td>-</td>
<td>-13.17</td>
</tr>
</tbody>
</table>

$^a$Values in parentheses are the mass excess errors in keV.

$^b$See Ref. 1.

$^c$See Ref. 2.

$^d$See Ref. 3.

$^e$See Ref. 6.

$^f$See Ref. 7.
FIGURE CAPTIONS

Fig. 1. Schematic diagram of the cyclotron facility and the relevant beam optics and equipment.

Fig. 2. Schematic diagram of the electronic setup when employing the standard particle-identifier system; it presents one of the two independent counter telescopes.

Fig. 3. Standard particle-identifier spectrum from the bombardment of a sodium target with 42-MeV protons. The \( \Delta E \) counter thickness was 5.6 mils.

Fig. 4. Schematic diagram of the electronics for the triple-counter particle-identification system.

Fig. 5. Simplified illustration of the function of the triple-counter particle identifier.

Fig. 6. Standard particle-identifier spectra from the bombardment of a $^{12}$C target with 70-MeV $^4$He ions (open circles) and with 65-MeV $^3$He ions (filled circles). The $\Delta E$ and E detector thicknesses were 8.5 mils and 120 mils, respectively.

Fig. 7. A $^6$He energy spectrum from the $^{12}$C($^3$He,$^6$He)$^9$C reaction at 65 MeV. This reaction was investigated using a standard particle-identifier system.

Fig. 8. Triple-counter particle-identifier spectrum from $^{12}$C + 65-MeV $^3$He-particles. The $\Delta E_2$, $\Delta E_1$, and E detector thicknesses were 5.2 mils, 4.2 mils, and 5.6 mils, respectively.

Fig. 9. Energy spectrum of the $^{12}$C($^3$He,$^6$He)$^9$C reaction at 65 MeV; the counts due to the $^{16}$O($^3$He,$^6$He)$^{13}$O reaction resulted from the oxygen in the Mylar target. These results were obtained using the triple-counter particle-identification system.
Fig. 10. Pulser-simulated particle-identifier spectra for the triple-counter system using the detector thicknesses shown in Fig. 4; (c) shows the normal positions of the particle-stable nuclei from $^3\text{He}$ to $^7\text{Li}$, while (a) and (b) show the positions of $^4\text{He} + \text{X}$ and $^3\text{He} + \text{X}$ (X denotes p, d, t, $^3\text{He}$, or $^4\text{He}$) chance coincidences, respectively.

Fig. 11. Time plots of (a) the $^3\text{He}$ beam energy as determined by elastic scattering in the monitor detector and (b) the electronic drifts in the $E_{\text{total}}$ pulse of one triple-counter $^6\text{He}$ detection system during a study of the $^{40}\text{Ca}(^3\text{He},^6\text{He})^3\text{He}$ reaction.

Fig. 12. Triple-counter particle-identifier spectrum from 55.9-MeV $^3\text{He}$ ions incident on a $^{40}\text{Ca}$ target.

Fig. 13. Plots of $\Delta E_{22}$ and $\Delta E_{21}$ versus $E_{\text{total}}$ for the events in the $^6\text{He}$ region of the particle-identifier spectrum during the $^{40}\text{Ca}(^3\text{He},^6\text{He})^3\text{He}$ experiment.

Fig. 14. Energy spectra from (a) the $^{24}\text{Mg}(^3\text{He},^6\text{Li})^{21}\text{Ne}$ reaction and (b) the $^{40}\text{Ca}(^3\text{He},^6\text{Li})^{37}\text{K}$ reaction at 55.9 MeV.

Fig. 15. Triple-counter particle-identifier spectrum resulting from the bombardment of $^{24}\text{Mg}$ with 55.9-MeV $^3\text{He}$ ions.

Fig. 16. Two independent $^6\text{He}$ energy spectra from the $^{24}\text{Mg}(^3\text{He},^6\text{He})^{21}\text{Mg}$ reaction at 55.9 MeV and a scattering angle of 14.1 deg. Each block represents one count and the block width is 100 keV.

Fig. 17. Two independent $^6\text{He}$ energy spectra from the $^{40}\text{Ca}(^3\text{He},^6\text{He})^{37}\text{Ca}$ reaction at 55.9 MeV; the data from both systems were kinematically corrected to 11.2 deg. Each block is one count and the block width is 100 keV.

Fig. 18. Energy spectra from the $^{22}\text{Ne}(d,t)^{21}\text{Ne}$ and $^{22}\text{Ne}(d,^3\text{He})^{21}\text{F}$ reactions at 20.5 deg. The $^{21}\text{Ne}$ spectrum has been adjusted to align the first three $T = 3/2$ levels in $^{21}\text{Ne}$ with their analogues in the $^{21}\text{F}$ spectrum.
Fig. 19. Angular distributions of the $^{22}\text{Ne}(d,t)^{21}\text{Ne}$ transitions to the ground state ($3/2^+$), 0.35-MeV ($5/2^+$) and 8.92-MeV levels and of the $^{22}\text{Ne}(d,^3\text{He})^{21}\text{F}$ ground-state ($5/2^+$) transition. In all cases smooth curves have been drawn through the experimental points to guide the eye; the abscissa is the dimensionless quantity $QR$, where $Q$ is the momentum transfer and $R$ is the interaction radius.

Fig. 20. Angular distributions for the $^{22}\text{Ne}(d,^3\text{He})^{21}\text{F}^*$ 0.28-MeV ($1/2^+$) and the $^{22}\text{Ne}(d,t)^{21}\text{Ne}^*$ 9.21-MeV transitions.

Fig. 21. Angular distributions of the $^{16}\text{O}(d,^3\text{He})^{15}\text{N}$ ground-state ($1/2^-$) and 6.33-MeV ($3/2^-$) transitions, the $^{22}\text{Ne}(d,^3\text{He})^{21}\text{F}^*$ 1.10-MeV transition and the $^{22}\text{Ne}(d,t)^{21}\text{Ne}^*$ 10.04-MeV transition.

Fig. 22. Triton energy spectrum from the $^{23}\text{Na}(p,t)^{21}\text{Na}$ reaction at 42 MeV. The numbered levels are $^{21}\text{Na}$ levels whose average excitation energies are listed in Table III.

Fig. 23. Energy level diagrams for the members of the mass 21 isospin quartet showing the positions of the $T = 3/2$ levels in each nucleus. For the sake of clarity the ground-state energies of mirror nuclei have been equated and many of the excited levels of $^{21}\text{Ne}$ and $^{21}\text{Na}$ below 10 MeV excitation have been deleted.

Fig. 24. Energy spectra from the $^{39}\text{K}(p,^3\text{He})^{37}\text{Ar}$ and $^{39}\text{K}(p,t)^{37}\text{K}$ reactions at 45 MeV and a scattering angle of 26 deg. The spectra have been adjusted to align the positions of the ground-state peaks.

Fig. 25. Relative angular distributions for the $^{39}\text{K}(p,t)^{37}\text{K}$ ground- ($3/2^+$), 1.37-MeV ($1/2^+$), and 5.035-MeV ($T = 3/2$) states and for the $^{39}\text{K}(p,^3\text{He})^{37}\text{Ar}^*$ 5.010-MeV ($T = 3/2$) state. The cross section of the last transition has been corrected for phase space and isospin coupling factors.
Fig. 26. Energy level diagrams of the members of the mass 37 isospin quartet showing the positions of the $T = 3/2$ levels in each nucleus.

For clarity the ground-state masses of mirror nuclei have been equated and several of the $^{37}$Ar and $^{37}$K energy levels have been deleted.
Fig. 1.
Fig. 2.
Particle identifier spectrum
$^{23}$Na + 42 MeV protons
$\theta_{\text{lab}} = 25$ deg

Fig. 3.
Incident particles

$\Delta E_2$ (5.2 mils)
$\Delta E_1$ (4.1 mils)
$E$ (5.6 mils)
$E_{rej}$ (8.5 mils)

Silicon detector telescope
Fast rise-time preamps
Delay-line shaped linear amplifiers with cross-over pick-off

To on-line computer

Valid event signal

Master gate generator

Amp
$\Delta E_2$ signal
$\Delta E_1$ signal
$E$ signal
Inhibit signal

Amp

Linear gate

ADC and buffer

$\Delta E_2$ shape
$\Delta E_1$ shape

Ident 1
Ident 2

Gate
$E_{total}$
$E_{rej}$

Mixer

$E_{1.73}$ function generator

Comparator

Allow signal

Sequence generator

All timing signals

Fig. 4.
Three counter particle identifier

Reaction products

$\Delta E_2$ $\Delta E_1$ $E$

Counters

Different identification modes of the particle identifier

<table>
<thead>
<tr>
<th>ctr(s) used as $\Delta E$ ctr</th>
<th>ctr(s) used as $E$ ctr</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. $\Delta E_2$</td>
<td>$\Delta E_1 + E$</td>
</tr>
<tr>
<td>B. $\Delta E_1$</td>
<td>$E$</td>
</tr>
</tbody>
</table>

If the ratio $A/B$ is within a chosen percentage, then the final output is $C$

C. $\Delta E_2 + \Delta E_1$ $E$

Fig. 5.
Fig. 6.
$^{12}$C($^3$He,$^6$He)$^9$C

12 deg

Fig. 7.
Figure 8.
Fig. 9.
Fig. 10.
Fig. 11.
Particle identifier spectrum

$^{40}\text{Ca} + 55.9 \text{ MeV} \quad ^3\text{He}$

$\theta_{\text{lab}} = 11.2 \text{ deg}$

27,600 $\mu$C

Fig. 12.
Fig. 13.
Fig. 14.
Particle identifier spectrum

$^{24}\text{Mg} + 55.9 \text{ MeV} \ 3\text{He}$

$\theta_{lab} = 14.1 \text{ deg} - \text{System One}$

5100 $\mu$C

Fig. 15.
Fig. 16.

\[ \text{Counts} \]

\[ \text{Energy (MeV)} \]

\[ \begin{align*}
&24\text{Mg} (^3\text{He}, ^6\text{He}) 21\text{Mg} \\
&M-A = 10.66 \text{MeV} \\
&\theta_{\text{lab}} = 14.1 \text{ deg} - \text{System One} \\
&5150 \mu\text{C}
\end{align*} \]

\[ \begin{align*}
&24\text{Mg} (^3\text{He}, ^6\text{He}) 21\text{Mg} \\
&M-A = 10.56 \text{MeV} \\
&\theta_{\text{lab}} = 14.1 \text{ deg} - \text{System Two} \\
&5150 \mu\text{C}
\end{align*} \]
$^{40}\text{Ca}(^{3}\text{He}, ^{6}\text{He})^{37}\text{Ca}$ Composite energy spectrum

$\theta_{\text{lab}} = 11.2 \text{ deg}$ - System Two

55,500 \(\mu\)C

$^{40}\text{Ca}(^{3}\text{He}, ^{6}\text{He})^{37}\text{Ca}$ Composite energy spectrum

$\theta_{\text{lab}} = 11.2 \text{ deg}$ - System One

46,250 \(\mu\)C

Fig. 17.
Fig. 18.
Fig. 19.
Fig. 20.
Fig. 21.
Fig. 22.

\[ ^{23}\text{Na}(p, t)\, ^{21}\text{Na} \]

15.6 deg
Fig. 23.
Fig. 24.
\[ {^{39}\text{K}}(p,t)^{^{37}\text{K}} \text{ g.s.} \] (x10)

\[ {^{39}\text{K}}(p,t)^{^{37}\text{K}}^* 1.37 \text{ MeV} \] (x10)

\[ {^{39}\text{K}}(p,t)^{^{37}\text{K}}^* [5.035 \text{ MeV}, T=\frac{3}{2}] \]

\[ {^{39}\text{K}}(p, ^3\text{He})^{^{37}\text{Ar}}^* [5.010 \text{ MeV}, T=\frac{3}{2}] \]

Fig. 25.
Fig. 26.
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