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ON THE ABSENCE OF THE GOLDSTONE MODE IN THE VECTOR GLUON MODEL

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ABSTRACT

We show, contrary to common belief, that the self-consistent gap equation for the fermion mass is not compatible with the presence of a Goldstone boson in the vector gluon model. The only physically realizable situation consists of a $\gamma_5$ degenerate vacuum accompanied by a nonconserved axial current. We discuss how this happens in finite quantum electrodynamics. A possible implication for the pion is briefly indicated.

One of the most attractive ideas in particle physics is that fermion masses arise dynamically. The initial stimulus to this idea was provided by the work of Nambu and Jona-Lasinio who proceeded by analogy with the B.C.S. theory of superconductivity. It was quickly pointed out by Baker, Johnson and Lee that renormalizability was an essential factor in determining whether or not the Goldstone boson found by Nambu and Jona-Lasinio in their nonrenormalizable model would be present in other models. Indeed in Johnson, Baker and Willey's version of quantum electrodynamics $^5$ (finite Q.E.D.) the electron mass arises dynamically without an accompanying Goldstone boson. Johnson $^4$ noted that in this case the axial-vector Ward identity was satisfied simply by the lack of conservation of the axial current. On the other hand Johnson, $^4$ and also Fagels, $^5$ pointed out that the renormalizable vector gluon model could display the Goldstone mode as well; and recently Jackiw and Johnson $^6$ tried to generate a dynamical Higgs-Englert-Brout mechanism $^7,^8$ by gauging this Goldstone particle away.

Clearly some criterion is necessary for deciding which of the above two alternatives a theory chooses. In this paper we point out that, independent of the value of the coupling constant and of the mass of the gluon (if we study a massive neutral gluon coupled to a conserved fermion current), the Goldstone mode is never realizable, and we give the necessary and sufficient conditions under which the finite Q.E.D. solution is obtained.

We begin by recalling the analysis of Ref. 2. Suppose there is a massless pseudoscalar bound state in the fermion anti-fermion scattering amplitude. The Bethe-Salpeter wave function, $\tilde{\mathcal{G}}_p(p,p)$, for the coupling of the bound state to a particle anti-particle pair carrying equal and opposite momentum, $p$, satisfies the homogeneous Bethe-Salpeter equation

$$\tilde{\mathcal{G}}_p(p,p) = \int d^4k \tilde{\mathcal{G}}_p(k, k) \mathcal{S}(k) \tilde{\mathcal{G}}(p, k, 0) \mathcal{S}(k)$$

expressed in terms of the renormalized kernel and propagators. Let the purely dynamical mass term in the fermion propagator be $\tilde{\mathcal{E}}(p)$. Then $\tilde{\mathcal{E}}(p)$ satisfies an identical homogeneous equation
\[ \tilde{\Sigma}(p) = \int d^4k \tilde{\Sigma}(k) \tilde{\Sigma}(k) \tilde{\Sigma}(p,k,0) \tilde{\Sigma}(k). \]  

(2)

This is of course a nonlinear equation for \( \tilde{\Sigma}(p) \) which may have a nontrivial solution, the gap equation. [Curiously Eq. (2) is not usually solved in the literature but rather the Schwinger-Dyson equation is discussed instead. It is very difficult to go beyond the ladder approximation to the Schwinger-Dyson equation, whereas Eq. (2) can be handled more easily in higher orders, and in particular at the finite Q.E.D. eigenvalue.] Though the compatibility of Eqs. (1) and (2) leads to the proportionality of \( \tilde{\Sigma}_p(p,p) \) and \( \tilde{\Sigma}(p) \), the existence of a nontrivial solution to Eq. (2) does not necessarily imply a Goldstone pol~ since the Fredholm determinant can have a noncompact kernel causing the usual Fredholm analysis to break down.

To determine when there would be a pole we can ask what the equation is that \( \tilde{\Sigma}_p(p,p) \) satisfies, i.e. what the off-shell coupling of the bound state is. If there is to be a bound state then it would have to appear as a pole in \( \tilde{\Sigma}_p(p,p+q) \), the vertex obtained by inserting \( \tilde{\psi}\psi \) into the fermion propagator carrying a momentum \( q_\mu \). This vertex satisfies the inhomogeneous equation

\[ m \tilde{\Sigma}_p(p,p+q) = m_0 \partial_2 \gamma_5 + m \int d^4k \tilde{\Sigma}_p(k,k+q) \tilde{\Sigma}(k) \tilde{\Sigma}(p,k,q) \tilde{\Sigma}(k+q). \]

(3)

Thus the off-shell \( \tilde{\Sigma}_p(p,p+q) \) satisfies an equation which contains an inhomogeneous term of the form \( q^2 m_0 \partial_2 \gamma_5 \). [It is important to note that the pseudoscalar particle appears as a pole in the \( \tilde{\psi}\psi \) vertex, rather than that \( \tilde{\psi}\psi \) is the source of \( \partial \phi \).] Now in the case in which the mass is dynamical, \( m_0 \) is zero (and \( \partial_2 \) is made finite by a clever choice of gauge) so that \( \tilde{\Sigma}_p(p,p+q) \) satisfies a homogeneous equation for arbitrary momentum transfer, which is satisfied nontrivially once it is satisfied nontrivially at \( q_\mu = 0 \). [The explicit solution to Eq. (3) for asymptotic \( p \) and arbitrary \( q \) is then given by conformal invariance and depends on the anomalous dimension, \( \gamma_6 \), of \( :\bar{\psi}\gamma_5\psi: \).] Thus the off-shell bound state would have to satisfy a homogeneous equation also, meaning poles at all \( q^2 \).

So finally we must conclude that once \( m_0 \) is zero there must be no bound state at all.

Having excluded the Goldstone mode we now discuss the finite Q.E.D. solution in more detail. Though we can no longer relate \( \tilde{\Sigma}(p) \) to \( \tilde{\Sigma}_p(p,p) \) we note that since Eq. (3) is now homogenous we have

\[ \gamma_5 \tilde{\Sigma}(p) = m \tilde{\Sigma}_p(p,p) \]

(4)

up to an overall coupling constant dependent normalization. (This identification implicitly requires \( \gamma_6(\alpha) \) to be negative so that the integrals exist.) It is more convenient however to introduce \( \tilde{\Sigma}_S(p,p) \), the insertion of \( :\bar{\psi}\psi: \) into the fermion propagator. It satisfies an equation identical to Eq. (3) so that

\[ \tilde{\Sigma}(p) = \Gamma \tilde{\Sigma}_S(p,p). \]

(5)

However in the Callan-Symanzik approach \( \tilde{\Sigma}_S(p,p) \) is the mass insertion on the right-hand side of the equation

\[ \left[ m \frac{\partial}{\partial m} + \beta(\alpha) \frac{\partial}{\partial \alpha} - 2\gamma_6(\alpha) \right] \tilde{\Sigma}_S(p) = -m(1 - \gamma_6(\alpha)) \tilde{\Sigma}_S(p,p) \]

(6)

where \( \gamma_6(\alpha) \), the anomalous dimension of \( :\bar{\psi}\psi: \), is equal to that of \( :\bar{\psi}\gamma_5\psi: \) since the theory is asymptotically chiral invariant. Thus from Eqs. (5) and (6) we deduce that both \( \tilde{\Sigma}(p) \) and \( \tilde{\Sigma}_S(p,p) \) behave like \( (-p^2/m^2)^{\gamma_6(\alpha)} \) at the finite Q.E.D. eigenvalue. This is of course the
original Johnson-Baker-Willey result, whose self-consistency then required that \( \gamma_0(\alpha) < 0 \) so that \( m_0 \) indeed vanishes in the limit of infinite cut-off. However we have made a substantial gain in that our derivation gives the power solution as an exact result for both asymptotic and nonasymptotic momenta. Thus our use of Eq. (5) eliminates the need to appeal to the Weinberg bounds and enables us to obtain low energy information from a scaling equation. We thus see the power of dilatation invariance. The mass arises dynamically through a bootstrap equation for \( \tilde{F}_B(p,p) \) and Eq. (6) simply expresses the arbitrariness of the scale in which it is measured, i.e. all nonzero values of \( m \) are equivalent. This then gives us the complete solution to Eq. (2) for all momenta without any need to explicitly construct the Bethe-Salpeter kernel. Thus the same coupling constant controls both the ultraviolet and infrared regions at once. This appears to be a reflection in the fermion sector of a result which Adler \(^{12}\) obtained by looking at the photon sector. There a loopwise summation permits the physical (low energy) coupling constant to control the asymptotic region. This suggests that study of the infrared structure of the fermion sector may shed some light on the nature of the essential singularity possessed by \( \beta(\alpha) \).

It is instructive to see how the solution satisfies the axial-vector Ward identity

\[
\frac{Z_A}{Z_\mu} \tilde{F}_\mu^5(p, p+q) = S^{-1}(p+q)\gamma_5 + \gamma_5 S^{-1}(p) + 2m \frac{Z_A}{Z_\mu} \tilde{F}_\mu^5(p, p+q) .
\]  

We have recently calculated \( \tilde{F}_\mu^5 \) at the eigenvalue using conformal invariance \(^{13}\) and find that

\[
\tilde{F}_\mu^5(p, p) = Z_A \gamma_\mu \gamma_5
\]  

at \( q_\mu = 0 \) and \( p \) asymptotic. \( Z_A(\gamma_5) \) is finite so that the axial-vector vertex, while satisfying an inhomogeneous Bethe-Salpeter equation, is still well-defined at \( q_\mu = 0 \) (how this happens is explained in Ref. 13.), to confirm that it contains no pole. Thus in finite Q.E.D. we have chiral symmetry with \( Z_5 = Z_\mu \) so that Eq. (7) becomes Eq. (4) at \( q_\mu = 0 \) and the current is no longer conserved.

This nonconservation is usually described as being due to a non-perturbative short-distance anomaly. This is somewhat misleading since the parameter \( m_0 \) of Eq. (3) is not the bare mass of the original input Lagrangian taken in the normal vacuum, but rather it parametrizes the deep Euclidean quantum fluctuations about the degenerate vacuum to be described below in which \( m \) is nonzero. The nonconservation is an infrared effect and this will be explained in Ref. 14. One further remark about the Ward identity is in order. Adler's essential singularity \(^{12}\) allows us to sum the theory loopwise. There is thus no need to consider closed fermion loops in the Bethe-Salpeter kernel, so that there is no two photon intermediate state in the axial-vector Ward identity, but just a continuous fermion line dressed to all orders with photon lines. Hence Adler's triangle anomaly plays no role at all in the analysis of this paper.

Up to this point we have followed Johnson, Baker and Willey and have obtained a necessary condition \( (\gamma_0(\alpha) < 0) \) that the mass be dynamical. To investigate whether the massive solution is energetically favorable we need now to study the infrared structure of the theory.

Suppose we have a massless theory \((H_0)\) in which a mass is induced dynamically to give us a massive theory \((H_m)\) with a new vacuum. For stability we require the vacuum energy difference...
To be negative. Now\(^{14}\)

\[
\epsilon(m) = \sum_{n} \frac{1}{n!} \zeta^{(n)}(p_i = 0) m^n
\]

(10)

where the \(\zeta^{(n)}\) are the \(\bar{\psi} \psi\) connected \(n\)-point Green's functions.

In Refs. 10 and 14 we have used Eq. (10) to show that the infrared divergences generated self-consistently by the \(\left(-\frac{p^2}{m^2}\right)^{\frac{3}{2}} \gamma_s(\alpha)\) solution to the gap equation indeed nonperturbatively cause \(\epsilon(m)\) to become negative provided \(\gamma_s(\alpha) = -1\), with \(\bar{\psi}\) acquiring an expectation value to set the scale, and with the new vacuum, \(\bar{\zeta}_m\), being \(\gamma_5\) degenerate. This is then the sufficient condition that the fermion acquire a mass via dynamical symmetry breaking. The compatibility of this new eigenvalue condition with that for \(\beta(\alpha)\) is currently being investigated.

Should the absence of bound state poles be welcome or not?

The conventional picture of spontaneous breakdown is that we start off with a degenerate vacuum and a conserved current and of course a massless pion. Then some soft operators of unknown origin appear, the linear terms of the Gell-Mann, Oakes and Renner Hamiltonian say, which remove the degeneracy analogously to an external field applied to a ferromagnet, with the real world now possessing a unique vacuum and a massive pion. In finite QED however the tadpole mass term arises through the agency which makes the vacuum degenerate (i.e., despite the absence of the Goldstone pole we still have long range order, a feature typical to theories with an infinite number of degrees of freedom; and it is this property rather than the pole itself which we regard as the essential feature of particle physics), so that the real world has a completely degenerate vacuum. (The one we live in is presumably labeled by the lack of parity conservation in the weak interaction.) Since there is no massless particle there is no need to then find a mechanism to give it a mass, a major advantage of course. However the axial current is still partially conserved, in the sense that its divergence has dimension \(3 + \frac{3}{2} = 3\frac{3}{2}\), \(^{15}\) and there is an \(f_{\pi}\)-like scale, \(\langle \bar{\psi}\psi \rangle\), in the theory. Thus we would still have most of the main features required for FCAC. However, whether some such \(SU(3)\) type tadpoles would indeed arise from the infrared structure of a weak interaction with a nontrivial eigenvalue \(^{16}\) (a natural way to unify the abelian and nonabelian cases would be to give them the same infinite order zero), and whether \(2m \bar{\psi} \gamma_\beta \gamma_5 \psi\) would act as an interpolating field for the pion remain to be explored.

FOOTNOTES AND REFERENCES

9. An exactly similar analysis can be applied to the ultraviolet conformal bootstrap discussed by A. A. Migdal in Phys. Lett. 37B, 386 (1971). The condition $Z_1 = 0$ converts the pion-nucleon vertex equation for coupling constant renormalization into a homogeneous equation which has a conformal solution for all momenta. Hence the pion has to be a fundamental field (with $0 \beta = g: \bar{\psi} \psi:$), so that the condition $Z_1 = 0$ is not a bound state condition at all. Hence only the coupling is bootstrapped and not the meson. The work described in Ref. 10 is complementary to that of Migdal since it is an infrared bootstrap for the mass.
15. A rather intriguing fact is that a four Fermi interaction will now have dynamical dimension 4 (since the $: \bar{\psi} \psi:$ composite has dimension 2), so that it may be nonperturbatively renormalizable.
16. Such a mechanism has been suggested as a solution to the $\eta \rightarrow 3\pi$ problem, P. D. Mannheim, Phys. Rev. D5, 3438 (1974).
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