Title
THE K* SPIN AND ISOVECTOR KAON CHARGE

Permalink
https://escholarship.org/uc/item/4f37p07k

Author
Pignotti, Alberto.

Publication Date
1962-09-12
DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
UNIVERSITY OF CALIFORNIA
Lawrence Radiation Laboratory
Berkeley, California
Contract No. W-7405-eng-48

THE K* SPIN AND THE ISOVECTOR KAON CHARGE
Alberto Pignotti

September 12, 1962
THE K* SPIN AND THE ISOVECTOR KAON CHARGE*

Alberto Pignotti†

Lawrence Radiation Laboratory
University of California
Berkeley, California

September 12, 1962

There has been considerable discussion recently on the spin assignment
to the K* resonance based on the analysis of various experiments.1,2,3 The
purpose of this note is to show that if the spin of the K* is assumed to be one,
good agreement is obtained for the isovector charge of the kaon,4 while no such
agreement can be obtained if the K* spin is zero. Throughout this work the
approximation of retaining only the ρ meson contribution in the I = 1, J = 1
channel will be performed, and the K* will be assumed to be the only effective
πK resonance.

The isovector kaon form factor satisfies the dispersion relation5

\[
F_K(t) = \frac{1}{\pi} \int_4^{\infty} \frac{2q'^3 F_\pi^*(t') B_1^-(t')}{\sqrt{t'(t' - t)}} dt',
\]

where \( q' = [(t'/4) - 1]^{1/2} \), \( B_1^-(t) \) is the \( I = 1, J = 1 \) amplitude for the
\( \pi\pi - K\bar{K} \) process as defined by Lee,6 and the pion mass is taken to be one. Here
\( F_\pi(t) \) is the pion form factor normalized to one at \( t = 0 \) and is given by

\[
F_\pi(t) = \frac{1}{D_\rho(t)} = \frac{t_\rho}{t_\rho - t - iy [(t - 4)^3/t]^{1/2} \Theta(t - 4)},
\]
where \( t \) and \( \gamma \) are the energy squared and reduced width of the \( \rho \) meson, respectively, and \( \Theta(x) \) is the usual step function.

In order to calculate \( B_1(-)(t) \), it is convenient to define the function

\[
\Gamma(t) = B_1(-)(t) D_{\rho}(t),
\]

which has the same singularities as \( B_1(-)(t) \) except for the right-hand cut due to the \( \rho \) intermediate state. The left-hand cut in \( \Gamma(t) \) starts at \( t \approx -16 \) if the lowest-energy intermediate state in the crossed channels (\( nK \) channels) is the \( K^* \) resonance at 885 MeV, and the right-hand cut starts effectively at the value of the square of the energy of the first significant state in the \( J = 1, I = 1 \) channel above the \( \rho \) meson. The dispersion relation for \( \Gamma(t) \) can be evaluated in the kernel approximation first used by Balázs,\(^7\) which provides a means of taking into account with reasonable accuracy the contribution from the left-hand cut and even some contribution from the inelastic cut. The result is a two-pole expression for \( \Gamma(t) \):

\[
\Gamma(t) = \frac{\alpha_1}{t - p_1} + \frac{\alpha_2}{t - p_2}. \tag{4}
\]

The positions of the poles in Eq. \((4)\) are essentially determined by the approximation procedure, and the values \( p_1 = -21 \) and \( p_2 = -200 \) will be used, in agreement with the criterion used in references \(^7c\) and \(^8\). The residues \( \alpha_1 \) and \( \alpha_2 \) will be determined by analytic continuation from the crossed channels in which only the \( K^* \) is retained in the \( S \)-function approximation. For this purpose the \( P \) wave in the \( t \) channel is projected from a fixed-\( t \) dispersion relation. This gives
where $\Gamma^*$ is the half width of the $K^*$, $l^*$ is the spin of the $K^*$, $M^*$ is the $K^*$ energy, $M$ is the kaon mass, $p^2 = t/4 - M^2$, $q^2 = t/4 - 1$, and

$$k^* = \left[ M^*^2 - (M-1)^2 \right] \frac{M^*^2 - (M-1)^2}{4M^*^2}.$$ 

Equations (3), (4), and (5) evaluated at $t = -1$ and $t = -2$ are used to calculate the residues $\alpha_1$ and $\alpha_2$. At these values of $t$ the improved expression of Singh and Udgaonkar for $D_\rho (t)$ is used.

With the above results one can compute the integral in Eq. (1), which, according to the normalization used, should yield $1/2$ for $t = 0$. Integrating with the help of the usual $\delta$-function approximation and using $25$ MeV and $50$ MeV for the half widths of the $K^*$ and $\rho$, one obtains

$$F_k (0) \approx \frac{1}{2} \times 1.6 \text{ for } l^* = 1$$

$$\approx \frac{1}{2} \times 0.06 \text{ for } l^* = 0.$$ 

These results are directly proportional to the $K^*$ width and inversely proportional to the $\rho$ width. Now the experimental values for these widths are not well established, and further, one does not expect the entire vector charge of the kaon to be due to the two-pion contribution. However, within these approximations it is clear from Eq. (6) that one can obtain the kaon charge if the spin of the $K^*$ is one but not if it is zero.

This work originated from a suggestion by Professor G. F. Chew to whom I am indebted. I also wish to express my gratitude to L. Balázs, B. M. Udgaonkar,
and other members of the theoretical group at Berkeley for encouragement and helpful discussions. It is a pleasure to thank Dr. David L. Judd for his kind hospitality at the Lawrence Radiation Laboratory.
FOOTNOTES AND REFERENCES

* This work was done under the auspices of the U. S. Atomic Energy Commission and was performed during a fellowship sponsored by the Consejo Nacional de Investigaciones Científicas y Técnicas of Argentina. This does not imply that this institution either approves or assumes any liabilities for the information contained in this report.

† On leave from Universidad de Buenos Aires, Buenos Aires, Argentina.


4. A qualitative argument in this direction was given by G. Frye [Nuovo Cimento 15, 282 (1960)]. S. K. Bose recently studied the isovector kaon form factor, using a subtracted dispersion relation [Nuovo Cimento 21, 970 (1962)]. The method used involves a divergency in the case of a spin-one K*, so that a cutoff is needed. No conclusion is drawn on the spin of the K*. 


9. In order to see how sensitive these results are to a variation in the positions of the poles $p_1$ and $p_2$, the extreme case of $p_1 = -13.5$ and $p_2 = -186$ has been considered. Although this corresponds to placing a pole outside the left-hand cut (which starts at $t = -16$) the value for the $s^* = 1$ case is only decreased by 15%, while the result for $s^* = 0$ is increased by a factor of two. On the other hand a displacement of the left pole to $-2 \times 10^6$ increases the result for the P-wave case by 10% and decreases the S-wave result by 22%. 
This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.