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BOOTSTRAP AND A UNIFORM FORMALISM OF THE FOUR FORCE FIELDS

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October 6, 1970

ABSTRACT

The concept of bootstrap and co-identification are presented in terms of universal constants and Planck units or "quantal units." Physical variables are uniquely expressed in terms of universal constants in an analogous manner to that of Wheeler's "wormhole" length, $l = (\hbar G/c^3)^{1/2}$. Other physical variables and such as time, mass, energy, momentum and power are also expressed in this manner. Several implications result in the uniform formalism of the four force fields when they are expressed in terms of universal constants.
I. INTRODUCTION

Previously we introduced a set of quantities terms "quantal units" which represent a unique expression of physical variables in terms of universal constants in an analogous "wormhole" length, 
\[ \ell = \left( \frac{\hbar}{c^3} \right)^{\frac{1}{3}}. \]
Wheeler also introduced a "quantum of mass," 
\[ m = \left( \frac{\hbar}{c} \right)^{\frac{1}{2}}, \]
a "quantum of energy," 
\[ E = \left( \frac{\hbar^5}{c^3} \right)^{\frac{1}{3}}, \]
and a "quantum of density," 
\[ \rho = \frac{c^5}{G^2 \hbar}. \]
Earlier M. Planck introduced the "Planck units" of length, mass, and time, 
\[ t = \left( \frac{\hbar}{c^5} \right)^{\frac{1}{3}}. \]
We term physical variables, formed in this manner, "quantal units." (See Table I) Wheeler proposed that these quantities represent a geometric structure of the space-time manifold.

There has been much interest in the recent work of B. A. Taylor, W. H. Parker, and D. A. Langenberg on the theoretical implications of the universal constants. They demonstrate the manner in which the universal constants, on a fundamental theoretical framework, may possibly unify various diverse branches of physics. This is what is proposed and may be possible by use of the quantal unit form of physical variables which are manifest in atomic, nuclear, and cosmological physics. It is suggested that the quantal unit form of all physical variables may represent a more complete geometric description of the space-time manifold than that proposed by Wheeler. For a more detailed discussion, see Ref. 1. Presented are some quantum theoretical aspects of the theory and in Refs. 1 and 2 and some of the cosmological aspects of the theory are also presented.

We define two distinct quantization procedures, primary and secondary, where primary or maximal quantization is that quantization
procedure in terms of the quantal units and secondary is that quantization procedure which is the ordinary or standard form of quantization.

The concept of symmetry and constancy is presented with particular attention to that aspect of symmetry such as partial symmetry or symmetry breaking mechanisms. A generalized concept of bootstrap in terms of universal constants and a uniform formalism of the four force field coupling constants, expressed uniquely in terms of universal constants is also presented.
II. THE CONCEPT OF PARTIAL SYMMETRY AND CONSTANCY

We have previously discussed the role of primary quantization in nuclear quantum mechanics and the generalized Heisenberg relations. Elementary particle physics is a specific application of quantum theory. We present in this paper a general discussion of elementary particle symmetries or "micro-physics" and the assumptions embodied in this scale of physics. The four force fields are discussed and a uniform formalism is developed for them (Sec. VI). Certain implications result from this systematic formalism.

Man is tied to the concept of constancy and symmetry in his attempt to understand the universe. One expression of the constant aspect in the space-time manifold is the universal constants. Physicists like to create "nice," concise theories, that is theories that are (if possible) simple in form and contain a high degree of symmetry in their mathematical structure and in the case of "equations of motion" a high degree of symmetry in their transformation properties.

Prior to the advent of relativity and the possibility that space is curved and even possibly has "dents" embedded in it, it was thought that space was "nice" and flat (Euclidian), isotropic and symmetrical. This is what is meant by the word "nice" in this context. In fact, it is quite apparent that such simplicity would not explain the great variety that is observed to exist in our universe and one should not be surprised to find partial symmetries in elementary particle physics. With the breakdown of parity in weak interactions comes the concept of "partial conservation," i.e., conservation only
in some interactions and not others. Much study of possible violation of CP (or T) and TCP invariance has been undertaken.\textsuperscript{9-12} We define the concept of partial symmetry or partial conservation laws as the "almost proposition."

We shall consider the role of the "breakdown" of conservation principles in partial symmetries and generalized bootstrap dynamics necessary for a possible derivation of the spectrum of elementary particles from the quanta? units.\textsuperscript{13} The concept of bootstrap is a description of force in particle physics where the structure of elementary particles is described as a coupling of that particle with its self and other particles to produce that particle. This particle concept is defined as a composite particle description. See G. Chew\textsuperscript{14,15} for further details of the theory. The concept of bootstrap dynamics as a description of force may be generalizable to more than just elementary particle physics. It may indeed be true as G. Chew suggest, "that to understand zero-mass phenomena through self-consistency may require bootstrapping space-time itself." In order to derive the spectrum of observed particles (see Table II), one would have to bootstrap from the matter-energy content of the space-time manifold. This could hopefully be done from the quantal unit description of the space-time manifold.\textsuperscript{7}

The bootstrap concept is an aspect of co-identification. Quantities or aspects of nature, such as the universal constants, are co-identified in that they are on an equal footing and not functionally dependent on each other in the sense of a "dependent variable." As in the bootstrap concept, "nuclear democracy" (G. Chew's terminology) among bootstrapped
particles, co-identified quantities are mutually independent of each other and yet primarily dependent on all other aspects of reality.

A generalized bootstrap model must necessarily include all the four force fields functioning in the space-time manifold and include the inner coupling between different force fields. In the geometric interpretation of the four force fields in the manifold, it may be possible to formulate a generalized bootstrap model. Usually each force field is considered separately except weak and strong interactions. Coupling between fields often results in symmetry breaking as the mass splitting of strongly interacting particles due to weak and electromagnetic nonconservation laws. This is an aspect of the "almost proposition" in micro-phenomenon. A "geometric" bootstrap model may also resolve some of the problems with microscopic causality. G. Chew discusses in detail the philosophies of the "bootstrappers" and "fundamentalists" or those looking for basic building blocks of Nature such as the quark models.
III. "FUNDAMENTAL" PARTICLE MODELS AND QUARKS

Some recent work by M. MacGregor on a new quark model is of interest. His work suggests that one may formulate quarks in terms of other particles, more "fundamental" or in terms of other quarks. It may be another "step down" as one goes from atomic to nuclear particles and from nuclear particles to their internal constituents such as the "quarks" that M. Gell-Mann named. The charges and spins of the three quarks proposed by M. Gell-Mann are given in Table III.

Each quark \( \pi, \nu, \) and \( \lambda \) has its antiquark counterpart \( \bar{\pi}, \bar{\nu}, \) and \( \bar{\lambda} \). In each case for the anti-quark particle there is a sign change in front of each factor in Table III.

It is possible, with the properties of the three quarks to "build up" the properties of "quarkable" particles. Hadrons and mesons are quarkable and Leptons and the photon are not in this model. An example is the quark representation of the proton whose quantum numbers are \( Q = 1, \ I_3 = 1/2, \ B = 1, \ S = 0, \ Y = 1 \). The unique solution is \( \pi \nu \nu \). Another quarkable particle is the omega minus, \( \Omega^- \) with quantum numbers, \( Q = -1, \ I_3 = 0, \ B = 1, \ S = -3, \ Y = -2 \). Its quark representation is \( \lambda \lambda \lambda \). For particle quantum numbers, see Table V. Note that a baryon has baryon charge unity and that strangeness, \( S \), and hypercharge, \( Y \), are defined in Sec. IV C. The quark representation of mesons can be done with two quarks, for example, the \( k \) meson or kaon exists in four states, \( k^+ \) and \( k_0 \) and \( \bar{k}_0 \) and since it is not a baryon, it has \( B = 0 \). Then, for example, the positive kaon, \( k^+ \), having the quantum numbers \( Q = 1, \ I_3 = 1/2, \ S = 1, \ Y = 1 \) can be made up of quarks \( \pi \bar{\lambda} \). The other particles can be similarly represented from
Table IV. Experiments are being conducted to detect quarks; they are thought to have mass of from 3 to 10 BeV. If then, three quarks make up a proton, then the energy relation of 30 BeV making up around 1 BeV must hold. It is like the phenomenon of "three ten-ton trucks smashing together to make a Model T Ford roadster," not impossible, but needs some explanation. In spite of difficulties with the model, quarks may be experimentally found but then the question of their being the "most fundamental building block" must still be answered. E. Teller and L. Schiff have suggested a strong correlation between the existence of quarks and magnetic monopoles as mentioned in Ref. 13. The quark and magnetic monopole models may represent a certain division and structure of matter such as the atomic state but a state of matter that can be further subdivided into a co-identified state as the co-identified universal constants, in which all states of matter can be represented.

Let us review the manner in which elementary particle physics is formulated and some of the assumptions the principle that elementary particles obey.
IV. SYMMETRY PRINCIPLES, CONSERVATION LAWS, COUPLING CONSTANTS, AND "ELEMENTARY" PARTICLES

A. Eight Basic Assumptions of "Elementary" Particle Physics

The primary attempt of present day particle physicists is to understand the basic elementary particle properties and the system of interaction of these particle properties. Over a hundred particles and their antiparticles have been discovered and various classification schemes for them based on their "properties" have been developed. The assumptions these classification systems make are, (1) particles can be described as "free entities" apart in space-time from all other particles,\(^2^1\) (2) these particles are completely described by symmetry properties and conservation laws and coupling constants (or dimensionless numbers representing the interaction strength), (3) there are four\(^2^2\) force fields comprising all the possible types (or strengths) of interactions between these particles (or aggregates of these particles), (4) the "classical" conservation laws are assumed to hold, as conservation of energy, linear, and angular momentum. [These conservation laws hold as invariances under rotations translations (in space-time) or the Lorentz invariance assumption under the proper Lorentz group (excluding space or time inversion)], (5) invariance under time reversal\(^1^1,^1^2\) and charge conjugation and CPT, (6) the existence of antiparticle counterparts for each particle [from assumption (4) and (5) here], (7) spin and statistics [of a system involving more than one particle "weakly" interacting (not forming a tightly bound state as a composite particle)] are related, (8) two more specific assumptions directly relate to
present day "Gell-Mann-Ne'eman type"$^{23-25}$ SU$_3$ group symmetry theories are: (a) a general causality$^{26}$ or localness condition which implies analyticity of the S-matrix scattering amplitudes,$^{27}$ (b) a one-to-one correspondence between poles in the S-matrix amplitude, and an elementary (or nuclear) particle. Stable particles lie on the physical sheet, unstable lie on the unphysical sheet.

Let us look at what type of theories develop from these assumptions, look at what kind of form or structure they have, what seems to be inadequate about them, and what can be implied from this work about the fundamental aspects of nature and the constants of nature. We shall then in turn see what these fundamental constants and quantized quantities infer about the properties of elementary particles.$^6,^7,^{13}$

The basic assumption about a set of symmetry properties for elementary particles is very significant in that symmetries usually result in conservation laws. Some symmetry quantities are exact (completely conserved) in all four force fields and some are not.

Let us look at these eight assumptions more closely, and see what can be inferred from them, and also what fundamental significance they may have. We shall also see what happens upon relaxing and modifying these assumptions as to causality, and the structure of space-time, dual aspects in nature, partial and nonexact microscopic symmetries and also the cosmological implications$^1,^2$ of the structure and properties of elementary particles. Our present day comprehension of the physical world relies on the assumption that there are four basic co-existing forces in the universe. They are gravitational, weak (or decay), electromagnetic, strong (or nuclear) forces.
B. The Four Basic Force Fields

A summary of their relative strengths is given in Table V. The strengths of these fields and their interaction time (which is closely related to the field strength) and range is given. The relative strength or interaction time is given only approximately. For comparison, the quantal force is also included in Table V. It is interesting to note, that in a scale where the nuclear force strength is taken to be about unity, the relative quantal force strength is about $10^{40}$ and the gravitational force strength is about $10^{-40}$. The quantal units are manifest in both micro and macrophenomena that is in both nuclear and cosmological features. The interaction strengths of the four force fields (see Table V), because of their range, are thought to apply exclusively to other microphenomena (short range, strong forces) or macrophenomena (long range, weaker forces).

One might state the conventional hypothesis in the following way. The nuclear, electromagnetic and weak forces are manifest in microscopic phenomena. This is the exclusive domain of short range nuclear and weak interaction forces. Both electromagnetic and gravitational forces are aspects of macroscopic phenomena. The weak forces, although a great deal stronger than the gravitational force, takes no part in macroscopic phenomena because it is so short range. Essentially the gravitational forces are too weak to be observed in microphenomena.28

It may be in fact that certain geometrical and symmetry properties of gravitational forces do effect microphenomena and conversely strong interactions may be manifest in macrophenomena. This is, it is believed what is
indicated by the quantal units. Microphenomena does make up or constitute macrophenomena in the sense that atoms are the "building blocks" of matter.

As stated before all information about particles is obtained by studying their interaction (target and "observer" particle). The total set of scattering events is described by the S matrix which is the collection of all scattering amplitudes is called the S matrix.  

Singularities in the S matrix come about through the unitarity condition on the S matrix (preserves the length of state vectors) and is associated with actual physical states. A one-to-one correspondence between poles in the S-matrix amplitude and nuclear particles is made (point 8b this section).  

The S matrix is a Lorentz-invariant analytic function of all momentum variables with only those singularities required by the unitarity condition. The requirement of simultaneous unitarity in all the different channels of the S matrix obtained by switching incoming and outgoing particles (crossing channels) is an extremely highly restrictive condition.

In recent years a great deal of discussion has centered about the so-called "composite" structure of "elementary" particles that is a description of these particles as "bound states" of other particles. In the particular case where the components of particle (such as a nucleon) is itself that particle (or a nucleon), the properties, mass and spin, etc., are determined on a dynamical basis termed bootstrap dynamics by Charles Zemach. Such an example is the nuclear
resonance state, or "Regge recurrence,"\textsuperscript{29} the N
or \( \Delta \) or \( 3,3 \) particle (an unstable particle with \( I = 3/2 \) and
\( J = 3/2 \)). This composite particle can be described by a set of diagrams
for \( N \approx \pi + N \).\textsuperscript{32,33}

One makes the assumption of analytic continuation in angular
momentum \( J \) for \( \text{Re} \ J \) sufficiently large.\textsuperscript{34} This is the Regge idea.
The position of the poles corresponds to the mass of the particles and
the residue of the pole corresponds to the coupling constant. For two-
body strong interaction in the S matrix, implies that all strong poles
are Regge (or composite) poles. A "total" bootstrap calculation is
not without its difficulties as the forces coming from the exchange of
particles with spin greater than \( 1/2 \) are, in general, singular and the
corresponding N/D dispersion equations are singular. Attempts to get
around these difficulties have been made by the introduction of
arbitrary adjustable parameters.

Let us turn our attention to the quantum numbers involved in
these calculations and try to understand the implications of partial
symmetries.

C. Isotopic Spin as a Partially Conserved Quantity and
Other Quantum Numbers

The isotopic spin operator is interpretable as a generator of
the unitary symmetry groups \( SU_2 \) and \( SU_3 \) (and higher symmetry groups).

In high-energy physics calculations, little is known about the
Hamiltonian, \( H \), but one can gain information about the state functions
by use of symmetry groups because the criterion that \( H \) commutes with
the symmetry group operator can be used. (This is because the wave
function of commuting operators can be simultaneous eigenfunctions of the two operators.)

The generators of the groups forming the (infinitesimal) Lie algebra represent conserved quantities. For example, the three components of \( J \) spin (angular momentum spin) form the generators of the \( SU_2 \) (special unitary) group in "real" space. The three components of \( I \) spin (isotopic spin), \( I_+ \), \( I_- \), and \( I_z \) are the generators of \( SU_2 \) also where the symmetry transformations occur in isotopic spin space. The commutation relations obeyed by the \( I \) spin components are of the type \([I_+, I_-] = -iI_z\).

The generators of \( SU_3 \) are the three components of \( I \) spin and \( Y \), hypercharge and four other quantities involving \( Y \) and \( Q \) (electric charge). There are thus eight independent generators for traceless \( 3 \times 3 \) matrices. The \( O_3^+ \) group of rotations is homomorphic to the \( SU_3 \) group.

In studying a great number of reactions in high-energy physics (strong and weak interactions plus electromagnetic), certain quantities appear obviously to be completely conserved such as baryon number, \( B \) or \( N \), lepton number, \( L \) and electric charge, \( Q \) and the classical quantities \( J \) (and spin) energy and \( L \) (see Table VI).

In \( SU_3 \) we have the expression,

\[
Q = I_z + \frac{B + S}{2}
\]

where \( S \) is the strangeness quantum number (and \( I_z = Q - \frac{1}{2} B \) for the \( SU_2 \) representation). Referring back to the four force fields, it has
been experimentally observed that isotopic spin conservation is violated in electromagnetic interactions; this gives rise to the octet mass splitting in the SU\(_3\) formulation. It has also been observed by Lee and Yang\(^8\) that parity (or space inversion) is violated in weak interactions.

Quantities such as P, I and CP (C is the antiparticle charge conjugation operator) are not completely (for all four fields) conserved, but are partially conserved and are an aspect of partial symmetries in the microscopic world. Until recently CPT appears to be completely conserved,\(^1^2\) where T is the time reversal operator but this is now an open question. (See Table VI for the conservation of the quantum numbers and Table IV for the quantum number assignment of "stable" elementary particles.)
V. "STATISTICAL AVERAGES" AND PARTIAL SYMMETRIES

As we see, the "almost proposition" actually puts the Gell-Mann-Ne'eman group theory symmetry on a stronger footing than the assumption that "microscopic physics" obeys a perfect symmetry group that have not yet been discovered.

The lack of perfect symmetry (spatial and rhythmic patterns in time) is readily observed in the macroscopic ("man-sized") world, such as the partial symmetry of the shapes of the petals of a rose. A statistical average over these shapes, summed and averaged over a reasonably large random sampling of roses would yield a perfectly symmetric shaped representation of a rose. Because man is a member of this scale, he need not take this sum to discern the property of the rose.35

When man looks at the microscopic world, he has to take a statistical sum over many events (such as resonance sums over particle scatterings or decay events) in order to ascertain aspects of the properties of the elements of the microscopic scale.

The division between macroscopic and microscopic aspects of nature is not arbitrary in the sense of a scale relationship between aspects in nature as determined by such quantities as the quantal unit, \( \hbar \). Indeed, this is recognized in the division between classical36 (macro) physics where \( \hbar \) is not significant and quantum physics (micro), where it is significant. (See Table I.) The scaling depends on the observer, man, who detects (unaided by instruments) macroscopic events and therefore formulates (for many thousands of years) a philosophy based on these macroscopic (and other scales) worlds, and therefore makes certain assumptions about their properties.
The sum over resonance states, on the looking at many microscopic events in elementary particle physics, is necessary because the "man-size" scale is macro--not microscopic.

When the sum over the "form" or "structure" (the symmetry) of events in the microscopic world, the irregularities still stand out such as the mass splitting in the SU$_3$ octets due to partial or almost conservation of certain quantities for some but not all of the four force fields.

If we were able to view the world of elementary particles as particles ourselves (not the man-size scale) we would undoubtedly see more irregularities when we did not sum over the symmetry events. We can no longer retain the philosophical assumption of microscopic perfection (perfect symmetries) and macroscopic imperfection (partial symmetries). In actuality, though, the quantum description of the microscopic world is based on the laws of probabilities and the observables are represented as

$$\int \psi^* \psi \, dt \quad \text{or} \quad \langle n|n' \rangle$$

which indeed represents an "ignorance" of actual individual events in this scale. We shall see that the preconditioning by our existence to the preassumptions made in the "macroscopic philosophy" make a presupposition about the micro-world perfect symmetry that in fact there is no evidence for. In actuality there is a great deal of evidence to the contrary in the existence of (1) partially conserved quantities in elementary particle physics, and (2) also, the non-Abelian nature (or
noncommutative nature) of the transformation group generators (canonically conjugate variables) that give rise to the quantum mechanics. Such is the case for

\[ [E,t] \geq \hbar \quad (3a) \]

and

\[ [p,x] \geq \hbar, \quad (3b) \]

quantum mechanics would not exist if the elements \( E \) and \( t \) commuted or \( p \) and \( x \) commuted, or any set of canonically conjugate variables commuted: for \( \hbar \) would be zero!

There are three presuppositions that lend to the main content of causality: they are, (1) the preassumption about the perfect symmetry of the microscopic world, (2) that "nature takes the easy way out," or the simplest solution [this concept is embodied in the principle of least action] and has both macroscopic (classical) and microscopic (quantum) manifestations, and (3) Lorentz invariance (and constancy of the velocity of light).

Because of the invalid presupposition about microscopic perfect symmetry, we have been led to invalid ideas about causality and analyticity in elementary particle physics.

We shall see that the existence of the fundamental length \( \ell = 10^{-33} \) cm, and a total set of such quantities lead to the existence of these partial microscopic symmetries and thus a new concept of causality.\(^{39}\)
The assumption that nature takes the simplest or easiest way out tends to lend to this presupposition about the microscopic world being perfectly symmetric, but indeed, if it were, the vast and varied universe that does exist could not, as for one thing quantum mechanics could not exist, and of course by "magnification of microscopic phenomena," macroscopic events are determined. Macroscopic partial symmetries are so obvious to us as a deviation from systems with a great deal of symmetry, but lacking perfect symmetry. It appears, for example, that two or more simple, perfect things (such as two sine waves) can be added together to form a complicated system that lacks perfect symmetry, for example, and oscilloscope pattern. (See Fig. 1.)

To macroscopically resolve a figure like (1c) into its original sine wave components requires electronic components (that experience an $I^2R$ loss) so that in fact to gain a greater symmetry in the macroscopic world requires an increase in entropy, so that macroscopic perfection and symmetry and order make implications about irreversibility. So does microscopy order (for example, the mixing of the atoms of two unlike gases and then to chemically separating them).
VI. FOUR FORCE FIELD COUPLING CONSTANTS

Force field strengths can be "measured" by what is termed a coupling constant. Force strengths appear in Lagrangians and Hamiltons as coefficients. Essentially, a coupling constant represents a coupling of two fields of the same force field. Very little work has been done in the coupling of fields of different force fields. The usual coupling constants are defined between two or more "particle" fields of the following type: (1) gravitation, the coupling between two gravitational fields arising from two or more mass particles (as an emitter and detector), (2) weak interactions, the coupling between two "elementary" particle fields, as between two leptonic fields in muon decay or a leptonic field coupled to a hadronic field as in the beta decay process, (3) electromagnetic coupling constants for the coupling of electromagnetic fields arising from the presents of electric charge or more generally electric and magnetic monopole charge, (4) strong interactions coupling constants between hadronic force fields. The force field description for strong interactions appears to be very complex either formulated in a quark model or bootstrap model. For the relative force strengths in dimensionless form, see Table V.

The essential feature of a coupling constant is its constancy and it is therefore proposed that it represents a constant aspect of nature and therefore can be expressed uniquely in terms of the universal constants (such as \( c, \hbar, G, Q, \) and \( k \)).

A force field can be represented by a current. The interactions of currents describe a dynamic process and the coupling constants represent a constant coefficient of this process or as the coefficient
in the interaction Hamiltonian. Let us briefly discuss the usual form of the four force field coupling constants. See Eq. (21) for the electromagnetic Hamiltonian as an example.

First, consider the gravitational coupling constant in its dimensional form as,

\[ G_{\text{grav}} = \frac{8\pi G}{c^4} = \frac{8\pi}{F} = 2.07 \times 10^{-48} \text{(Gauss-cm)}^{-2} \]  

as in Refs. 1 and 2. As previously noted, since the quantal force \( F = c/G \), the quantity \( 8\pi G/c^4 = 8\pi/F \). In this form, the gravitational coupling constant is expressed in terms of universal constants \( G \) and \( c \).

In Table V, the dimensionless form of the gravitational coupling constant appears; it is of the order of \( 10^{-39} \) to \( 10^{-40} \). To derive this quantity from the dimensional coupling constant appearing in Eq. (4), one multiplies by a factor based on the proton mass, \( m_p = 1.67 \times 10^{-24} \text{gm} \) to form a dimensionless quantity. Since \( 8\pi G/c^4 \) is in units of (Gauss-cm)$^{-2}$ or dyne$^{-2}$ or sec$^2$/gm cm, the dimensional coupling constant is multiplied by \( \lambda_c (\text{cm})m_p(\text{gm})/t_c^2 \) (sec$^2$), where \( \lambda_c \) is the proton Compton wavelength or \( \lambda_c = \frac{\hbar}{m_p c} = 2.10 \times 10^{-14} \text{cm} \) and \( t_c = \lambda_c/c = 7.0 \times 10^{-24} \text{sec} \) as in Refs. (6) and (13). This factor is about \( 5 \times 10^{10} \text{gm/sec}^2 \) and therefore we have

\[ 8\pi G/c^4 = 2.07 \times 10^{-48} \text{sec}^2/\text{gm cm} \times 8.2 \times 10^9 \text{gm cm/sec}^2 = 10^{-39} \text{ to } 10^{-40}. \]

There are other calculations which are based on the electron mass,

\[ m_e = 9.11 \times 10^{-28} \text{gm} \]  

and an electron Compton wavelength of

\[ \lambda_c = 3.86 \times 10^{-11} \text{cm} \]  

and the corresponding time for a light signal to
travel this distance of $t = \lambda_c / c = 6.58 \times 10^{-22}$ sec. The factor

$$\lambda_c (\text{cm}) \frac{m_p (\text{gm})}{t_c^2 (\text{sec}^2)}$$

thus becomes $7.4 \times 10^{\frac{1}{4}} \text{gm-cm/sec}^2$ giving a gravitational coupling constant of about $10^{-43}$. One conventionally bases the conversion from the dimensional gravitational coupling constant to the dimensionless form based on the proton mass. This is an arbitrary choice. Dimensionless coupling constants are formed for the comparison of magnitude of force strengths of the four force fields as the dimensional coupling constants are not in equivalent units.

Second, the theory of weak interactions has been developed in analogy to quantum electrodynamics where $A_\mu$ and $\gamma_\mu$ play similar roles in gauge invariance,

$$\partial_\mu A_\mu = 0 \quad (5)$$

and also for the conserved vector current (c.v.c.),

$$\partial_\mu J^V_\mu = 0 \quad (6)$$

and the nonconserved axial vector current,

$$\partial_\mu J^A_\mu \neq 0. \quad (7)$$

The weak coupling constant is given in the weak interaction Hamiltonian as,

$$H_{\text{weak}} = \frac{W}{\sqrt{2}} J_\mu J^\nu_\nu + \text{h.c.} \quad (8)$$

where h.c. is the Hermitian conjugate and the current $J_\mu$ has the general form $A_\mu.$
in the V-A theory ($\gamma_\mu$ is the vector part and $\gamma_\mu \gamma_5$ is the axial vector part of the current). In effect $W$ is the probability that the two currents will interact.\(^{43}\) The value of $W$ is given as $W = 1.42 \times 10^{-49}$ erg-cm\(^3\), to be more precise, two weak coupling constants should be defined. They are $g_V$ and $g_A$ where $g_V$ is the vector coupling constant and $g_A$ is the axial vector coupling constant.\(^{45,46}\) It is found phenomenologically that within a few percent,\(^{47}\)

\[
\frac{g_A}{g_V} \approx -1.25. \quad (10)
\]

We can represent $\beta$ decay by $J_\mu$ as a "bare" non-leptonic or hadronic current,

\[
J_\mu = \bar{\psi}_p \gamma_\mu (1 - \gamma_5) \psi_n \quad (11)
\]

and for the V-A form of the current,

\[
J_\mu = \bar{\psi}_p \gamma_\mu \left( 1 - \frac{g_A}{g_V} \gamma_5 \right) \psi_n \quad (12)
\]

and $J_\nu$ as a leptonic current,

\[
J_\nu = \bar{\psi}_e \gamma_\mu (1 + \gamma_5) \psi_e \quad (13)
\]

where $\psi_e$ is the electron-neutrino, also we define

\[
\bar{\psi} = \psi^* \gamma_5. \quad (14)
\]
We can write for the $\beta$ decay interaction Hamiltonian,

$$H_{\text{int}} = \frac{W}{\sqrt{2}} \bar{\psi}_p \gamma_\mu (1 - g_A/g_V) \gamma_5 \psi_n \bar{\psi}_e \gamma_\mu (1 + \gamma_5) \psi_e + \text{h.c.} \quad (15)$$

Phenomenologically for $^{0_{14}}$, beta decay,

$$g_V = \frac{W}{\sqrt{2}}. \quad (16)$$

For the muon decay interaction Hamiltonian we have,

$$H_{\text{int}} = \frac{W'}{\sqrt{2}} \bar{\psi}_\nu \gamma_\mu (1 + \gamma_5) \psi_\mu \bar{\psi}_0 \gamma_\mu (1 + \gamma_5) \psi_\nu + \text{h.c.} \quad (17)$$

Note that the subscript $\mu$ on $\psi_\mu$ stands for the muon wave function and the subscript $\mu$ on $\gamma_\mu$ stands for the index to be summed over.

If the universality of the weak interactions holds, then $W = W'$. In the absence of strong interactions, this would be so, that is the beta decay and muon coupling constants would be equal. The vector coupling constant is not changed by the presence of strong interactions, that is, it is not renormalized for a zero changing strangeness current, unless vector part is conserved.

The experimental lifetime of muon decays give $W' \sim 10^{-5} m_p^{-2}$, where $m_p$ is the mass of a proton. By definition $W'$ is called the Fermi coupling constant, and has dimensions of $\ell^{-2}$ or $m^{-2}$ (for $\hbar = c = 1$). The Hamiltonian, $H$, is an energy density in dimensions of $m\ell^{-3}$ or $m^2$ and the current has dimensions of a density or $\ell^{-3}$ or $m^3$. When $W'$ is obtained from the muon lifetime, one does not need to take radiative corrections into account.
For an experimental lifetime of $\tau_\mu \sim 2.2 \times 10^{-6}$ sec, we have,

$$\tau_\mu^{-1} = \frac{W^2 \, m_\mu^5}{24(2\pi)^3}$$

or $W' \sim 10^{-5} \, m_p^{-2}$. The units of $m_p^{-2}$ are chosen somewhat arbitrarily and are found to be useful.

In forming the dimensionless Fermi coupling constant for Table V, one uses a factor of $f^2$ or $m^{-2}$. The conversion from $f^2$ to $m^{-2}$ or to $m^{-1}$ can be seen to relate to G. N. Lewis' units, as $1 \text{ gm} = \hbar/c \text{ cm}^{-1}$ so that $1 \text{ cm}^2 = (\hbar/c)^2 \text{ gm}^{-2}$. Actually, the expression $\lambda_c = \hbar/mc$ is utilized, setting $\hbar = c = 1$ so that $\lambda_c = m^{-1}$ where $\lambda_c$ is length.

To calculate the dimensionless form of the weak interaction coupling constant for Table V, again an arbitrary choice is made based on the proton or electron mass. The dimensional coupling constant is given as, $W = 1.42 \times 10^{-49} \text{ erg cm}^3$. As we mentioned, the actual coupling constant depends on the decay process considered and whether strong interactions are "turned on." The factor to multiply $W$ by to form its dimensionless counterpart is constructed out of the units of $(\text{erg cm}^3)^{-1}$ or $\text{sec}^2/\text{gm cm}^5$. The factor based on the proton mass then becomes $t_c^2/m_p \lambda_{cp}^5 = 7.6 \times 10^{14}$. Thus $W$ in its dimensionless form is $1.08 \times 10^{-4}$ based on the proton mass. Based on the electron mass, the conversion factor to a dimensionless coupling constant is $5.3 \times 10^{36}$ so that the dimensionless weak coupling constant becomes $7.52 \times 10^{-13}$. Values quoted for the dimensionless weak coupling constant lie in the range of about $10^{-5}$ to $10^{-15}$ depending on the conversion factor, based
on the mass of the proton or electron or muon and also depending on the decay time of the decay process considered. Usually in weak interaction theory, the conversion to the dimensionless form is based on the electron or muon mass since it is a leptonic process. The corresponding decay time for $W \approx 10^{-14}$ is $10^{-9}$ sec, see Table V. It is interesting to note that one can obtain a dimensionless form of the weak interaction coupling constant in terms of the quantal energy and the proton Compton wavelength as,

$$\frac{W}{(\lambda_p)^3} \frac{1}{E} \approx 10^{-23} \quad \text{(dimensionless)} \quad (19)$$

where $W$ is the coupling constant and $E$ is the quantal energy. The value is approximately the value of the weak decay constant in Table V. For a further discussion of nuclear properties and quantal units see Ref. 13. More on a new unifying representation of the four force field coupling constants in terms of universal constants will be given in the next section.

Third, the electromagnetic coupling interaction is the best understood quantitative quantum mechanical description of a force field. In electromagnetics the force can be calculated in terms of one or more photons but the primary contribution to the electromagnetic force is from one photon exchange accounting to about 98 percent of the coupling constant. The series in the perturbation expansion of the sum of Feynman diagrams converges rapidly, each extra photon contributing a decreasing factor of $1/137$, which is $\alpha$, the fine structure constant,
for each diagram summed. This is the reason for the great success of quantum electrodynamics:

The electromagnetic coupling constant can be expressed as a coefficient of the electromagnetic current and thus, appears as a coefficient of the electromagnetic Hamiltonian. The electromagnetic current with higher order terms is given by,

\[ J_\mu \propto i e \bar{\psi} \gamma_\mu \psi \]  \hspace{1cm} (20)

and the Hamiltonian is given by,

\[ H^\text{em} = - J_\mu A^\mu = i e (\bar{\psi} \gamma_\mu \psi) A_\mu \]  \hspace{1cm} (21)

(see Ref. 13). The quantity \( e^2 \) is the electromagnetic coupling constant. The coupling constant is actually \( e^2/\hbar c \), but in the usual convention \( \hbar = c = 1 \), has been taken. Sometimes the factor \( e^2/4\pi \) is used where \( 4\pi \) is the factor in \( \hbar \) so that \( \hbar = c = 1 \), has been used. No dimensional conversion factors are necessary to put the electromagnetic coupling constant in dimensionless form for Table V.

The purpose in using the form \( e^2/4\pi \), as done by others, is to be consistent with the use of \( \hbar = c = 1 \), in calculating the other coupling constants.

In Ref. 13 we discussed J. Schwinger's \( \hbar g \) form of the electromagnetic coupling constant that places electric and magnetic "charge" on an equal footing. The relation in Ref. 13, \( e g/\hbar c = n \) expresses the form of the electromagnetic coupling constant both in terms of electric and magnetic monopole charge. In Sec. IX we shall discuss this form of coupling constants more completely.
Fourth, the strong interaction coupling constant is least understood of the four force fields and appears to be the most "complex" of the force fields. Of course, the least understood something is, the more complex it appears. The strong nuclear force is very short range. Instead of obeying an inverse-square law as does the gravitational and electromagnetic forces, the strong force has a very strong repulsive "core," an attractive force of somewhat larger distances, falling asymptotically to zero at about the nuclear radius of $10^{-13}$ cm or 1 fm.

One hypothesis in the description of force fields has been that forces are produced by the exchange of a particle or particles. This concept of forces is quite well accepted and is basic to the bootstrap description of hadrons.

If the exchange particle model is correct, then for gravitational forces the exchange particle would be the gravitation (zero rest mass and spin 2) to fit into the ideas of quantum field theory. In electromagnetic theory, as we mentioned, the photon (rest mass zero and spin 1) is the exchange particle and in weak interactions, the intermediate vector Boson (IVB) is hypothesized as the exchange particle. More discussion on the IVB in Sec. IX. Yukawa hypothesized that nuclear forces were produced by the exchange of an "intermediate mass particle" between the mass of an electron and the mass of a proton. For nuclear decay times of $\tau = 2 \times 10^{-23}$ sec, the mass of the predicted exchange particle would be the mass of the pion (pi meson spin zero and mass $\pi^+$, 276 times the mass of an electron and $\pi^0$, 268 times the mass of an electron). This particle was looked for in cosmic ray studies. The
Muon mass was discovered first, but did not strongly interact with nuclear matter. It is a lepton, not a meson and is misnamed. From the relation \( r = \frac{\lambda_s}{c} \) where \( \lambda_s = \hbar/mc \) for \( \lambda_s \), the range of the nuclear force \( 1.4 \times 10^{-13} \) cm, then \( m \) is about 271 electron masses, a very good prediction. For pion exchange, the coupling constant \( g \), corresponding to \( e \) of the photon coupling in electromagnetism, satisfies,

\[
\frac{g^2}{\hbar c} = G' = 15
\]

(22)

where \( g \) is the baryon "charge" (not to be confused with the magnetic monopole charge in \( e \mathfrak{g}/4\pi c = n \). [See Eq. (29).] A perturbation expansion in terms of multiple exchanges of pions diverges rapidly since \( g^2/\hbar c \gg 1 \) and is therefore not possible to use, as such, in strong interaction theory. There are doubts raised by R. P. Feynman and others as to whether it is possible to develop strong interaction theory in a consistent manner with quantum field theory. The Yukawa potential,

\[
\Phi = g \frac{e^{-r/r_0}}{r}
\]

(23)

contains \( g \) as a "strength" coefficient and \( r_0 \) is the range of the interaction. This simple description of the nuclear force is incomplete and inexact, but highly successful theory in its use to predict a great amount of experimental results!

In analogue to \( e^2/\hbar\pi \) for \( \hbar = c = 1 \) in electromagnetism, the coupling constant for strong interactions is defined as

\[
g_s^2/\hbar\pi = G_s'.
\]

(24)
A typical cross section for a nuclear scattering process will be of the order,

$$\sigma \sim (g_s^2/4\pi)^2 \chi A_N$$  \hspace{1cm} (25)

where $A_N$ is a typical nuclear area. Then,

$$\sigma = (g_s^2/4\pi)^2 (1/m_e)^2 \approx (g_s^2/4\pi)^2 \times 2 \times 10^{-26} \text{ cm}^2. \hspace{1cm} (26)$$

For the photon production process such as $\gamma + P \rightarrow N + \pi$ depending linearly on $e^2/4\pi$, then for the electromagnetic cross section for this process,

$$\sigma \approx (e^2/4\pi)(g_s^2/4\pi) \approx (g_s^2/4\pi) \times 10^{-28} \text{ cm}^2. \hspace{1cm} (27)$$

By comparison of Eq. (26) to Eq. (27), gives

$$g_s^2/4\pi \approx 1 \text{ or } g_s^2 \approx 12. \hspace{1cm} (28)$$

This is the quantity, $g_s$, that appears as coefficients in strong interaction Hamiltonians and Lagrangians. There are many specific forms of the strong interaction coupling constant depending on the particular model used.

Now that we have briefly discussed the four force field coupling constants in their conventional form, let us look at their expression uniquely in terms of universal constants on a unified basis. As stated before, the universal constants represent the fundamental constant aspect of the manifold and therefore all constant aspects of the manifold should be expressible in terms of them. Feynman says "you know the particles and the couplings and so you know everything. Physics
in a nutshell. The problem is that it's hard to know the particle dynamics, bootstraps or quarks, and the coupling constants (particularly the weak and strong, or finite range forces). Process or dynamics as well as constancy must be considered. We will represent the fundamental constant aspect in the coupling constants or the constant aspect of the force field amplitudes at an (interaction) vertex.

Two fundamental aspects result from the unified form of the four force field coupling constants; they are universal correspondence of quantum and classical physics and generalized complimentary or the dual aspect of force fields. In Sec. VII we present the coupling constants expressed in terms of the universal constants. In Sec. VII we shall present a discussion of magnetic monopoles which is relevant to Sec. VIII.
VII. MAGNETIC MONOPOLES AND "CHARGE" SYMMETRY

Recent work in electrodynamics has been towards unifying both electric and magnetic phenomenon and treating some aspects of magnetic phenomenon in analogy with electric phenomena. There is much recent interest in magnetic monopole theories by T. F. Shiff, E. Teller, J. Schwinger, and others. J. Schwinger discusses a "charge" quantization condition, which puts magnetic and electric monopole charge on an equal footing. This condition is expressed as

\[ \frac{e}{\hbar c} = n \]  \hspace{1cm} (29)

where \( n \) is an integer, \( e \) is the electric charge (carried by a spin-1/2 field) and \( \mu \) is the magnetic charge (carried by another spin-1/2 field) in his consideration. Note from before, we defined \( \hbar \) as \( \hbar' \).

We can see the analogy between Eq. (29) and the fine structure constant,

\[ \frac{e^2}{\hbar c} = \alpha. \]  \hspace{1cm} (30)

Of course \( \alpha \) is not an integer, but is given by \( \alpha = 1/137.036 \). A quantized condition must be expressible in terms of an integer, so that its expression as a wave phenomenon is not "wiped out" by its continuation in time. It appears that the expression in Eq. (29) is therefore a quantum condition whereas Eq. (30) is not. It appears that, by considering the existence of magnetic monopoles, a quantum theory of change can be formulated and that Eq. (29) represents a more complete expression of "charge" than Eq. (30), since \( n \) is an integer and \( \alpha \) is not.
E. Teller\textsuperscript{54} points out that experimental detection of magnetic monopoles will require very high energy experiments. Recently L. W. Alvarez, et al.,\textsuperscript{55} have discussed the use of superconducting elements in the detection of magnetic monopoles and D. Sivers\textsuperscript{56} discusses some theoretical aspects that are relevant to experimental detection of magnetic monopoles.
VIII. GENERALIZED COUPLING CONSTANTS

Each of the four force field coupling constants can be expressed in terms of a "pole charge" pair and other universal constants. This is done by forming analogous expressions of Eq. (29). In classical electromagnetic theory, the coupling constant is given by $\frac{e^2}{\hbar c} = \alpha$, the fine structure constant and in Schwinger's quantized in terms of electric and magnetic charge. The electromagnetic coupling constant becomes $e^2/\hbar c = n$ where $n$ is an integer greater than or equal to one. By analogy each coupling constant for each force field in Table VI is expressed in terms of a "classical" analogy to $e^2/\hbar c$ and a quantized analogy to $e^2/\hbar c = n$. For example, for the strong force field, we have for the "classical" form of the coupling constant $g^2/\hbar c = G_s$ where $G_s$ is about 15 as discussed in Sec. VI. In the strong force field, the quantized expression for the coupling constant is given by $g_Q^2/\hbar c = N$ where $g_s = g_Q$ in the form of an "electric strong pole particle." Let us compare Table V and Table VII.

In order to calculate the dimensionless form of the coupling constants in Table V, one can use the form such as $e^2/\hbar c = \alpha$ or $e^2/4\pi = \alpha$ for the assumption that $h = c = 1$ or $e^2 = \alpha$ for the assumption that $\hbar = c = 1$. This can be done for all four force fields. The relative force field strengths are the same for $h = c = 1$ or $\hbar = c = 1$ or for $\hbar$ and $c$ equal to their dimensional value. For the strong field, we have then $g_s^2/4\pi = G_s$ from Sec. VI [Eq. (24)] and $g^2/\hbar c = G_s'$ [Eq. (22)] where $G_s'$ is dimensionless and is given values of 1 to 15. Then $g_s$ is dimensionless and $g_s = (4\pi G_s')^{1/2}$ where
The quantity \( g \) is dimensional and is given by \( g = (G_s t')^{1/2} \) where \( t' = \alpha h \) and \( g_s \) and \( g \) are related by \( g_s^2/4\pi = g^2/hc = G_s \) so that \( g_s = 4\pi g/\alpha' \). If \( \alpha' \) is taken to be in units of \((\text{esu})^2\) from the relation \( \alpha' = e^2/\alpha \) where \( e \) is in esu then by analogy \( g \) can be in esu units. As stated in Sec. VI, the relation \( e^2/4\pi = \alpha \) assumes that \( h = c = 1 \) so that \( e^2/4\pi = e^2/hc = \alpha \) where we denote the dimensionless form of \( e \) as \( e' \) where \( e' = (4\pi \alpha)^{1/2} \). Also \( e \) in esu is given by \( e = (\alpha')^{1/2} \) or the usual esu charge on an electron.

In Sec. VI, the weak interaction constant was given as \( W' \simeq 10^{-5} m_p^{-2} \) for the universal Fermi interaction or as \( g_w^2/4\pi = W' \). For typical nuclear times \( t = 10^{-23} \) sec then \( \tau = (4\pi g_w^2)^{1/2} t \) and exponentially weak decays go on the order of \( \tau \sim 10^{-9} \) sec giving, \( g_w^2/4\pi = W' = 10^{-7} \). The reason that this differs from Table V is that weak interactions are often calculated in terms of \( W' = (g_w^2/4\pi)^2 \simeq 10^{-14} \) which is the interaction matrix element squared instead of \( W' = g_w^2/4\pi = 10^{-7} \). For the expression \( W' = 10^{-5} m_p^{-2} \) the mass \( m_p \) or \( m_B \) can be taken as that of the proton (from \( \beta \) decay) or the intermediate vector boson propagating the interaction currents. As stated before, we take \( W' \) to be the Fermi type coupling or \( W' = g_w^2/4\pi m_p^{-2} \).

For the gravitational coupling constant relative strength, one can calculate its relative strength by the ratio of the gravitational force between a proton and an electron to the electrical force between a proton and an electron since the potential equations of these two force fields is of the same form (see Table V). We have
where the $1/r$ dependence drops out. See Eq. (4), where $G$ is the universal gravitational constant and is analogous to the Fermi constant, $W'$. We can then write the gravitational coupling as

$$
\frac{\mathcal{g}}{e\&m} = \frac{Gm_p m_e}{e^2} \approx 10^{-40}
$$

(31)

where the $1/r$ dependence drops out. See Eq. (4), where $G$ is the universal gravitational constant and is analogous to the Fermi constant, $W'$. We can then write the gravitational coupling as

$$
\mathcal{g}_G^2/4\pi = G m_p^2 = \gamma \approx 10^{-39}
$$

for the dimensionless form of the coupling constant $\mathcal{g}_G^2$. For the dimensional form, we have $m^2/\hbar c = \gamma$.

In each of the four force fields we then have the analogous "classical" coupling constant in Table VII as,

\begin{align*}
\frac{\mathcal{g}}{e^{\prime}} &= G^{\prime}, \\
\frac{e^2}{e^{\prime}} &= \alpha, \\
\frac{w^2 m_p^2}{e^{\prime}} &= W^{\prime}, \\
\frac{m^2}{e^{\prime}} &= \gamma,
\end{align*}

(32a) (32b) (32c) (32d)

for $e^{\prime} = e\hbar$. These are basically of the form $P^2/e^{\prime} = C$ where $P$ is a force field "charge strength" and $C$ is the dimensionless coupling constants given in Table V.

In analogy to Schwinger's quantum form of the electromagnetic coupling constants, $eg/\hbar c = n$, we can form

$$
\frac{\mathcal{g}_Q \mathcal{g}_M}{e^{\prime}} = N,
$$

(33a)
\[ \frac{e}{\mathcal{Q}} = n, \quad (33b) \]

\[ \frac{W_Q W_M m_p^2}{\mathcal{Q}} = \eta, \quad (33c) \]

\[ \frac{m_Q m_M}{\mathcal{Q}} = N, \quad (33d) \]

for \( \mathcal{Q}' \equiv \mathcal{Q}h \). These are of the form \( \mathcal{Q}M/\mathcal{Q}' = n \) where \( n \) is an integer. It is postulated that each pole pair, \( \mathcal{Q}M \) in analogy to the \( e \) product of electric and magnetic charge or monopoles are universal constants.

In the form of Table VII all coupling constants are then expressed uniquely in terms of universal constants. In "Nuclear Democracy," the mass of the proton and electron are not universal constants but they do carry the universal constant of charge. It should be noted that "conventional" forms of coupling constants use these masses to "convert" to dimensionless forms but this is a method, not a fundamental theory.

Each force field in the quantum formulation is characterized by three quantities. The pair of field "pole charges" and \( \mathcal{Q}' \), where \( \mathcal{Q}' \equiv \mathcal{Q}h \). All coupling constants are expressible uniquely in terms of universal constants. The quantity, \( \mathcal{Q}' \) is a universal constant, made up of \( c \) and \( h \) and, as for example in electromagnetics \( e \) and \( \mathcal{Q} \) are universal constants; so are \( g_Q \) and \( g_M \), \( W_Q \) and \( W_M \), and \( m_Q \) and \( m_M \). Each force field contains two "pole" terms which are universal constants. The "nonuniqueness" or difficulty in a universal expression for the strong and weak coupling constants is due to these force fields greater dependence on the bootstrapping in terms of the other force.
fields because of the finite range of these forces. This is an aspect
of the "almost proposition" and is seen as symmetry breaking principle
in elementary particle physics. The quantities $\epsilon'$ and $\epsilon''$ and the
quantal units occupy equal footing with and can be considered as universal
constants because of the space-time bootstrap concept! The quantal units
and universal constants are co-identifiable and neither one is more
fundamental than the other.

Co-identification implies a lack of dependence in the sense of
functional dependence to an independent entity and yet the bootstrap
concept implies a set of objects that are of equal status by being
mutually dependent on the rest of the set. Such is the case for the
universal constants. (Co-identification is discussed in Sec. II.)

Universal correspondence is exemplified by the uniformity of this
form of the four force field (FFF) coupling constants. The correspondence
between the "classical" form of the coupling constant, such as,
$e^2/4\pi c = \alpha$ and the "quantum" form as $e\phi/\pi c = n$ demonstrates for each
force field, the principle of universal correspondence. Generalized
complementarity appears in the coupling constants as the dual nature
of the "poles" as the pole pair $e$ and $\phi$ in electromagnetic theory.
In the quantum form, we have an integer whereas in the classical form
$\alpha$, the fine structure constant is not an integer. See Ref. 49.

The uniform formalism of the four force field coupling constants
leads to the prediction of and is consistent with: 1) the intermediate
vector Boson in weak interaction theory, 2) the magnetic monopole of
force strength about 100 times the electric charge (for lowest order
quantal charge), 3) a dual or pair of strong interaction charges 58 or
exchange particles, 4) two forms of gravitational matter (one perhaps being anti-matter). The uniformity of the pair or dual form of the force field as coexisting or co-identified "poles" is consistent with a generalized bootstrap principle as is the principle of partial symmetries.

The convention of choosing "a set of units" where $\hbar = c = m_e = 1$ has been mentioned in this and earlier sections. It is stated that no inconsistency results from this "change of units." We term the units of a universal constant its dimensionality and when the convention

$$\sigma_0 = c = m_e = 1$$

is used, we are essentially setting something (the dimensionality of a universal constant) equal to nothing since unity has no units or dimensionality. At best, there is a loss of information by this convention and at worst, it is incorrect in the strict sense of a mathematical equation.

We have spoken of the co-identification of universal constants (and quantal units) that exist on an equal footing and mutually dependent on all other universal constants, i.e., to their magnitude and dimensionality. We have also presented the concept that the essential feature of a universal constant is its constancy. This aspect of constancy is absolute, not relative, and is not contingent. Absolute constancy is independent of the manner in which it is formulated. The manner in which this constancy is formulated in a particular set of units is significant in expressing the manner in which one universal constant is related to another, i.e., how it is co-identified with other universal constants. In this sense then, using the convention $\sigma_0 = c = m_e = 1$ is denying the basic co-identified nature of the universal constants. This is the
reason for our use of $\gamma = \frac{\hbar c}{\mu}$ in Table VII and elsewhere in this paper where the universal constants are used with their relative numerical values and dimensionality. Of course, whether one uses cgs, MKS, or other units is arbitrary.
IX. GENERALIZED BOOTSTRAP AND PARTIAL SYMMETRIES

There has been the search for a set of "fundamental elementary particles" to act as building blocks for the hadron spectra and perhaps all of reality. Empedocles' (500-430 B.C.) four elements of Fire, Water, Air, and Earth were further broken down into chemical compounds, molecules to atom rearrangement by the alchemists and then atoms to nuclei and electrons and nuclei into protons and neutrons by the radiation physicists, and nucleons postulated to be composed of quarks. In each respective case, these entities can be decomposed into their "building blocks" using energies which are relatively small compared to the rest mass of the "parent" entity. In the case of hadrons, the energies needed to form quarks are larger than the rest mass of a hadron in order to decompose it into a specific nondetermined number of other hadrons each of which has approximately the same mass and is not more or less "elementary" than any other one. This is the essence of the bootstrap concept! The bootstrap concept avoids the conceptual difficulty of building blocks out of building blocks out of--and so on.

The quarks, the lowest dimensional, lowest mass representation of the symmetry group SU$_3$, can also be expressed in terms of other hadrons and therefore can be considered as part of the hadron family and are consistent with the bootstrap concept. Actually, one has to bootstrap the space-time manifold itself. In this description the geometry of the space-time manifold itself has co-identified elements, the quantal units. Or, in another manner of description, a co-identified set of universal constants. Each constant is not functionally dependent on any other constant, but mutually dependent on all others. Their mutual
existence is what determines "the shape of the universe." We have demonstrated an aspect of this in Refs. 1 and 2, in our closed universe model. As stated before, co-identification is a generalization of the bootstrap concept to essentially bootstrapping the space-time manifold; its matter-energy and other physical variable content in the form of quantal units. In Ref. 15, G. Chew suggested bootstrapping space-time itself as mentioned before. The hadrons spectra should be derivable from this bootstrap process, and the mass spectra should result from the bootstrapping of the quantal mass, \( m \gtrapprox 10^{-5} \) gm.

The coupling constants of the four force fields are also co-identified. That is, no single force field can be completely expressed in a close theoretical framework without considering the others. All force fields mutually depend on the geometric structure of the space-time manifold which give rise to them. In summary, each co-identified coupling constant can be represented by a pair of pole charges and other universal constants which are co-identified. This co-identification is a manifestation of the more fundamental one in the space-time manifold. This pair relation of poles is another manifestation of the basic dual aspect of the space-time manifold which is also manifest in the generalized Heisenberg relations.\(^6\),\(^7\)
X. CONCLUSION

In part, the notion of the various branches of physics comes about through the action of various of the four force fields, gravitation, weak, electromagnetic, and strong interactions. Recently Taylor, Parker, and Langenberg proposed that the universal constants may act in a theoretical manner to unify the various branches of physics. An example of this concept is presented in the present paper. A uniform formalism of the four force fields, in terms of universal constants, is presented. Several implications result from this formalism, for example; (1) a magnetic monopole in the electromagnetic interaction, of force strength $10^2$ times that of the electric charge for lowest order monopole, (2) an intermediate vector Boson (IVB) in weak interactions, (3) two forms of strongly interacting "charges" or exchange "particles," (4) two forms of gravitational matter which are perhaps redundant or one form may be "antimatter."

A generalized concept of the bootstrap process is presented and defined in terms of physical variables, expressed uniquely in terms of universal constants, termed quantal units. It is proposed that the quantal units, which are manifest in classical, quantum, and relativistic physics, represent the geometrical structure of the space-time manifold. Therefore this generalized bootstrap process, does not bootstrap particles, per se', but bootstrap space-time itself. G. Chew recently made this conjecture.
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FOOTNOTE AND REFERENCES

* Work performed under the auspices of the U.S. Atomic Energy Commission.


2. E. A. Rauscher, Closed Cosmological Solutions to Einstein's Field Equations, Lawrence Radiation Laboratory Report UCRL-19767, June 1, 1970.


4. M. Planck, Theory of Heat Radiations (Dover, New York, 1959) p. 175. The author is grateful to P. Lieber for this reference.


20. Quote from Margaret L. Silbar (private communication).

21. The only system that attempts to overcome the free particle assumption is the bootstrap dynamics which considers composite systems of particles.

22. In some theories a fifth (medium strong) force is introduced to explain "discrepancies" in other theories. Sometimes also a sixth medium weak force is introduced to "prevent" or explain the mass splitting effect in terms of a perfect set of symmetries.
27. Particle properties are studied through interactions during scattering events.
28. It is thought by some that neutron stars represent the case where both nuclear and gravitational forces are effective. It appears that certain geometrical and symmetry properties of gravitational forces do affect microphenomena and conversely strong interactions are manifest in macrophenomena. Indeed microphenomena does affect macrophenomena.
29. G. Chew "... to know the S matrix is to know all that can possibly be known about the subatomic world." Nucleon Structure (Hofstadter and Schiff), 1967. In fact one must consider only interaction and not free particles, which do not exist.
30. One gives up the space-time continuum (or causality) in its usual Kantian sense (Immanuel Kant).
31. The Mandelstam representation is a means of representing a relation between direct and crossed channels.
35. J. L. Oldham (private communication), University of California, Livermore Lawrence Radiation Laboratory, 1968.
36. In Ref. 13, we have discussed the manner in which other quantities other than \( \alpha \) determine scale. These quantities are other universal constants or combinations of universal constants as in quantal unit form.

37. The microscopic world "makes up" or constitutes the macroscopic world.

38. The use of the word "ignorance" in this context indicates the lack of ability to obtain complete information about asymptotic wave functions \( \psi \) at all "points" in space-time and therefore this ignorance is a unreasonable one.


40. R. P. Feynman, talk at University of California-Berkeley, April 23, 1968.

41. William Rarita (private communication) 1967, Department of Physics, University of California-Berkeley.

42. Hadrons are a strongly interacting particle, leptons (muons, electrons, and neutrinos) do not couple to strong currents.


44. Y. Ne'eman, Particle Symmetries and Space Time Curvature, Annals of Physics 31, 391 (1965).

45. N. Cabibbo (advanced studies at CERN), Lecture Series, University of California, Lawrence Radiation Laboratory, May, 1968.

For a review of the Dirac theory for electromagnetic and weak interactions see G. Kallen (Ref. 43) and Brandeis Summer Institute, 1962, *Elementary Particle Physics and Field Theory* (W. A. Benjamin, Inc., New York, 1963).


When force fields currents of different forces are coupled, then symmetry splitting results as the mass splitting in SU$_3$ of strongly interacting particles due to weak interactions. This is an aspect of the "almost proposition."

The principle of universal correspondence was introduced by P. Lieber and is presented in Ref. 13. It is a generalization of Bohr's principle of correspondence which specifically applies to $\chi$; the principle of universal correspondence applies to a generalized set of canonically conjugate relations in terms of universal constants and quantal units as in Refs. 6, 7, and 13. The term generalized complimentarity also refers to the generalized set of canonically conjugate variables as the set of generalized Heisenberg relations in Refs. 6 and 7.


E. Teller "Quarks and Magnetic Monopoles" talk at Lawrence Radiation Laboratory, Livermore, January 10, 1966.

56. D. Sivers, Possible Binding of a Magnetic Monopole to a Particle with Electric Charge and a Magnetic Dipole Moment, Lawrence Radiation Laboratory Report UCRL-19794, April 28, 1970.

57. In each interaction the matrix element for the interaction is given as $M \sim G_s'$ for strong forces, $M \sim \alpha$ for electromagnetic forces, $M \sim W'$ for weak forces, and $M \sim G'_{\text{grav}}$ for gravitational forces. The square of the interaction matrix element gives the characteristic cross section for the process $\sim |m|^2$.

58. It may be that in Table VII $g_Q = g_m = g_s$ in an analogous manner to $\pi^0$ being its own antiparticle in Table II.
Table I. Universal Quantal Units

<table>
<thead>
<tr>
<th>Quantal unit in terms of force, $\ell$, $\tau$, and $\tau'$</th>
<th>Numerical value of quantal unit $^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ell = \left(\frac{\hbar}{c^3}\right)^{\frac{1}{2}}$</td>
<td>$1.60 \times 10^{-33}$ cm</td>
</tr>
<tr>
<td>$t = \left(\frac{\hbar}{c^3}\right)^{\frac{1}{2}}$</td>
<td>$5.36 \times 10^{-44}$ sec</td>
</tr>
<tr>
<td>$m = \left(\frac{\hbar}{Gc^3}\right)^{\frac{1}{2}}$</td>
<td>$2.82 \times 10^{-5}$ gm</td>
</tr>
<tr>
<td>$E = \left(\frac{\hbar^2}{Gc}\right)^{\frac{1}{2}}$</td>
<td>$1.25 \times 10^{16}$ ergs</td>
</tr>
<tr>
<td>$p = \left(\frac{\hbar^2}{Gc}\right)^{\frac{1}{2}}$</td>
<td>$4.16 \times 10^{10}$ gm-cm/sec</td>
</tr>
<tr>
<td>$L = \hbar$</td>
<td>$1.06 \times 10^{-27}$ erg-sec</td>
</tr>
<tr>
<td>$F = \frac{\hbar^4}{Gc}$</td>
<td>$1.22 \times 10^{49}$ dynes</td>
</tr>
<tr>
<td>$c = c$</td>
<td>$3.00 \times 10^{10}$ cm/sec</td>
</tr>
<tr>
<td>$p = \frac{\hbar^2}{Gc^2}$</td>
<td>$3.66 \times 10^{59}$ dyne cm/sec</td>
</tr>
<tr>
<td>$\rho = \frac{\hbar^2}{G^2c^2}$</td>
<td>$6.50 \times 10^{93}$ gm/cm</td>
</tr>
</tbody>
</table>

$^a$ The quantal units are expressed in terms of the universal quantal force, $F = \frac{\hbar^4}{Gc}$, $\tau$, $\tau'$, and $c$. The quantities, $\tau$ and $\tau'$, are defined as $\tau = \hbar/c$ and $\tau' = c\hbar$.

$^b$ In the evaluation of the quantal units, the values of $\tau = 3.50 \times 10^{-38}$ gm-cm and $\tau' = 3.15 \times 10^{-17}$ erg-cm have been used.
Table II. This is the list of some of the most basic "elementary" particles.

This data is taken from the review of particle properties, UCRL-8030 by N. Barash-Schmidt, et al., revised January, 1970. Stability is defined with respect to strong interactions.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Spin</th>
<th>Mass (MeV)</th>
<th>Decay Products</th>
<th>Mean Life (sec)</th>
<th>Antiparticle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega^-$</td>
<td>3/2</td>
<td>1672</td>
<td>$\Xi^0 + \pi^- + \Lambda k^-$</td>
<td>$10^{-10}$</td>
<td>$\Omega^+$</td>
</tr>
<tr>
<td>$\Xi^-$</td>
<td>1/2</td>
<td>1318</td>
<td>$\Lambda + \pi$</td>
<td>$10^{-10}$</td>
<td>$\Xi^0$</td>
</tr>
<tr>
<td>$\Xi^0$</td>
<td>1/2</td>
<td>1311</td>
<td>$N + \pi$</td>
<td>$10^{-10}$</td>
<td>$\Xi^+$</td>
</tr>
<tr>
<td>$\Sigma^+$</td>
<td>1/2</td>
<td>1189</td>
<td>$N + \pi$</td>
<td>$10^{-10}$</td>
<td>$\Sigma^0$</td>
</tr>
<tr>
<td>$\Sigma^-$</td>
<td>1/2</td>
<td>1139</td>
<td>$N + \pi$</td>
<td>$10^{-10}$</td>
<td>$\Sigma^+$</td>
</tr>
<tr>
<td>$\Lambda^0$</td>
<td>1/2</td>
<td>1157</td>
<td>$p + e^- + \nu_e$</td>
<td>$10^3$</td>
<td>$\Lambda^0$</td>
</tr>
<tr>
<td>$\bar{\Omega}$</td>
<td>1/2</td>
<td>1115</td>
<td>$N + \pi$</td>
<td>$10^{-10}$</td>
<td>$\Omega^-$</td>
</tr>
</tbody>
</table>

Nucleons

<table>
<thead>
<tr>
<th>Particle</th>
<th>Spin</th>
<th>Mass (MeV)</th>
<th>Decay Products</th>
<th>Mean Life (sec)</th>
<th>Antiparticle</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>1/2</td>
<td>939</td>
<td>$p + e^- + \nu_e$</td>
<td>$10^3$</td>
<td>$\bar{n}$</td>
</tr>
<tr>
<td>p</td>
<td>1/2</td>
<td>938</td>
<td>stable</td>
<td></td>
<td>$\bar{p}$</td>
</tr>
</tbody>
</table>

Mesons

<table>
<thead>
<tr>
<th>Particle</th>
<th>Spin</th>
<th>Mass (MeV)</th>
<th>Decay Products</th>
<th>Mean Life (sec)</th>
<th>Antiparticle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta^0$</td>
<td>0</td>
<td>548</td>
<td>$\gamma + \gamma$</td>
<td>$10^{-20}\tau$</td>
<td>$\eta^0$</td>
</tr>
<tr>
<td>$k^+$</td>
<td>0</td>
<td>494</td>
<td>$\mu^+ + \nu_\mu$</td>
<td>$10^{-10}$</td>
<td>$k^-$</td>
</tr>
<tr>
<td>$k_1^0$</td>
<td>0</td>
<td>498</td>
<td>$k_1^0 \rightarrow 2\pi$</td>
<td>$10^{-8}$</td>
<td>$k_0^0$</td>
</tr>
<tr>
<td>$k_2^0$</td>
<td>0</td>
<td>498</td>
<td>$k_2^0 \rightarrow 3\pi$</td>
<td>$10^{-16}$</td>
<td>$k_1^0$</td>
</tr>
<tr>
<td>$\pi^0$</td>
<td>0</td>
<td>140</td>
<td>$\mu + \nu_\mu$</td>
<td>$10^{-8}$</td>
<td>$\pi^-$</td>
</tr>
<tr>
<td>$\pi^+$</td>
<td>0</td>
<td>135</td>
<td>$\gamma$</td>
<td>$10^{-16}$</td>
<td>$\pi^0$</td>
</tr>
</tbody>
</table>

Table II continued on next page
Table II continued

<table>
<thead>
<tr>
<th>Particle</th>
<th>Spin</th>
<th>Mass (MeV)</th>
<th>Decay Products</th>
<th>Mean Life (sec)</th>
<th>Antiparticle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leptons</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu^-$</td>
<td>1/2</td>
<td>105</td>
<td>$e^- + \bar{\nu}<em>e + \nu</em>\mu$</td>
<td>$2 \times 10^{-6}$</td>
<td>$\mu^+$</td>
</tr>
<tr>
<td>$e^-$</td>
<td>1/2</td>
<td>1/2</td>
<td>stable</td>
<td>$e^+$</td>
<td></td>
</tr>
<tr>
<td>$\nu_e$</td>
<td>1/2</td>
<td>0</td>
<td>stable</td>
<td>$\bar{\nu}_e$</td>
<td></td>
</tr>
<tr>
<td>$\nu_\mu$</td>
<td>1/2</td>
<td>0</td>
<td>stable</td>
<td>$\bar{\nu}_\mu$</td>
<td></td>
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<tr>
<td>Photon</td>
<td>$\gamma$</td>
<td>1</td>
<td>stable</td>
<td>$\gamma$</td>
<td></td>
</tr>
<tr>
<td>Graviton</td>
<td>G</td>
<td>2</td>
<td>stable</td>
<td>$G$</td>
<td></td>
</tr>
</tbody>
</table>
Table III. Quark Quantum Numbers

<table>
<thead>
<tr>
<th>Quark, Q</th>
<th>Charge, Q</th>
<th>Isotopic Spin, $I_3$</th>
<th>Boyon Number, $B$</th>
<th>Strongness, $S$</th>
<th>Hypercharge, $Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>$\frac{2}{3}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{3}$</td>
<td>0</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>$-\frac{1}{3}$</td>
<td>$-\frac{1}{2}$</td>
<td>$\frac{1}{3}$</td>
<td>0</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$-\frac{1}{3}$</td>
<td>0</td>
<td>$\frac{1}{3}$</td>
<td>1</td>
<td>$-\frac{2}{3}$</td>
</tr>
</tbody>
</table>
# Table IV: The Hypercharge, Strangeness and Isotopic Spin Assignments of the "Stable" Particles

<table>
<thead>
<tr>
<th>Particle</th>
<th>Q</th>
<th>B</th>
<th>Y</th>
<th>S = Y - B</th>
<th>$I_3 = Q - \frac{Y}{2}$</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega^{-}$</td>
<td>-1</td>
<td>1</td>
<td>-2</td>
<td>-3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\Xi^{0}$</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>-2</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\Xi^{-}$</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>-2</td>
<td>$-\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\Sigma^{+}$</td>
<td>+1</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>+1</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\Sigma^{0}$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$\Sigma^{-}$</td>
<td>-1</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>$-\frac{1}{2}$</td>
</tr>
<tr>
<td>$\Lambda^{0}$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>p</td>
<td>+1</td>
<td>1</td>
<td>+1</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>n</td>
<td>0</td>
<td>1</td>
<td>+1</td>
<td>0</td>
<td>$-\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$k^{+}$</td>
<td>+1</td>
<td>0</td>
<td>+1</td>
<td>+1</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$k^{0}$</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>+1</td>
<td>$-\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\overline{k^{0}}$</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$k^{-}$</td>
<td>-1</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\pi^{+}$</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\pi^{0}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$\pi^{-}$</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>$-\frac{1}{2}$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Table V. Relative Force Strengths

<table>
<thead>
<tr>
<th>Force</th>
<th>Coupling Constant or Relative Strength</th>
<th>Range (fm)</th>
<th>Interaction Time (sec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantal Force</td>
<td>$10^{40}$</td>
<td>$10^{-20}$</td>
<td>$10^{-44}$</td>
</tr>
<tr>
<td>Strong (nuclear)</td>
<td>1 to 10</td>
<td>1 fm</td>
<td>$10^{-23}$</td>
</tr>
<tr>
<td>Electromagnetic</td>
<td>$10^{-2}$ (or 1/137)</td>
<td>$\infty$</td>
<td>$10^{-21}$</td>
</tr>
<tr>
<td>Weak (decay)</td>
<td>$10^{-11}$ to $10^{-14}$</td>
<td>short range</td>
<td>$10^{-9}$</td>
</tr>
<tr>
<td>Gravitational</td>
<td>$10^{-39}$</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
</tbody>
</table>
Table VI. Conservation of Quantum Numbers

<table>
<thead>
<tr>
<th>Interaction</th>
<th>P</th>
<th>I</th>
<th>M</th>
<th>S</th>
<th>or</th>
<th>Y</th>
<th>N</th>
<th>CPT</th>
<th>C</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>strong</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td></td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>e and m</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td></td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
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<td>weak</td>
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<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
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<tr>
<td>gravitation</td>
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<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

+ Means conserved.
- Means not conserved.
<table>
<thead>
<tr>
<th>Force</th>
<th>Potential Equation</th>
<th>&quot;Classical&quot; Coupling Constant</th>
<th>Quantum Coupling Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td>( \phi_s = g \frac{2}{2} \frac{e^{-\mu r}}{r} )</td>
<td>( \frac{g^2}{\hbar c} = G_s' )</td>
<td>( \frac{g_Q g_M}{\hbar c} = N )</td>
</tr>
<tr>
<td>Electromagnetic</td>
<td>( \phi_{\text{el}} = \frac{1}{4\pi \varepsilon_0} \frac{e^2}{r} )</td>
<td>( \frac{e^2}{\hbar c} = \alpha )</td>
<td>( \frac{eQ}{\hbar c} = n )</td>
</tr>
<tr>
<td>Weak</td>
<td>( \frac{W^2 m^2}{\hbar c^2} = W' )</td>
<td>( \frac{W Q M m^2}{\hbar c^2} = \eta )</td>
<td></td>
</tr>
<tr>
<td>Gravitational</td>
<td>( \phi_g = \frac{m^2}{r} )</td>
<td>( \frac{m^2}{\hbar c} = \gamma )</td>
<td>( \frac{m Q M}{\hbar c} = N )</td>
</tr>
</tbody>
</table>
Fig. 1. (A) Perfect sine waves. (B) Two sine waves added together (with two different amplitudes). (C) Many sine waves added together.
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