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Publication Date
1992-05-01
Submitted to Water Resources Research


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May 1992
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This work was supported in part by the Assistant Secretary for Conservation and Renewable Energy, Office of Renewable Energy Technologies, Geothermal Technology Division, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

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ABSTRACT

Two-phase (gas-liquid) flow experiments were conducted in horizontal artificial fractures. The fractures were between glass plates, either smooth or artificially roughened by gluing a layer of glass beads. One smooth fracture was studied, with an aperture of 1 mm, and three rough fractures: one with the two surfaces in contact and two without contact.
Videotape observations revealed flow structures similar to those observed in two-phase flow in a pipe, with structures depending on the gas and liquid flow rates.

Measurement of pressure gradient, flow rates, and gas saturation were correlated according to two approaches:
- Porous media approach (relative permeability concept)
- Two-phase flow in a pipe (Lockhart-Martinelli model and homogeneous flow model)

Relative permeability curves were found to be similar to conventional curves in porous media, but were not solely a function of saturation.

The Lockhart and Martinelli equation might be useful for rough calculations of pressure drop.

By treating the two phases as one homogeneous phase and by defining a two-phase Reynolds number, a correlation of the friction factor with Reynolds number was found in a smooth fracture.

**INTRODUCTION**

Two-phase flow in fractured rocks occurs in recovery of petroleum of natural gas and of coalbed methane, in exploitation of geothermal energy and in isolation of radioactive waste. Models to predict two-phase flow in fractures are therefore of practical interest, but little is known of the laws governing such flow. In the subsurface environment, flow is generally through a network of intersecting fractures and the study of two-phase flow in a single fracture is basic to understanding flow in fracture networks. Conceptually, fracture flow can be considered either as a limiting case of flow in a porous medium or as a limiting case of pipe flow. Historically, the porous-medium approach has been used for
situations involving subsurface flow. This approach emphasizes the importance of capillary and viscous forces, with negligible inertial forces. Under conditions of high velocity and relatively open fractures (in which inertial forces are strong and capillary forces weak), such as those intersecting a production well at shallow depths, two-phase fracture flow may approach the limiting case of two-phase pipe flow.

The approach most commonly taken to model two-phase flow in a single fracture is to treat the fracture as a two-dimensional porous medium, and write Darcy's law for each phase. For horizontal flow:

\[ V_{ls} = -\frac{k_0 K_{rl}}{\mu_L} \nabla P_L \]  

\[ V_{gs} = -\frac{k_0 K_{rg}}{\mu_G} \nabla P_G \]

where \( V \) is velocity, \( \mu \) is viscosity, \( P \) is pressure, \( k_0 \) is the intrinsic permeability and \( K_r \) the relative permeability. Subscripts \( L \) and \( G \) represent liquid and gas respectively, and subscript \( S \) represents superficial velocity (also called Darcy velocity). Although all equations are written for liquid and gas, any two or more immiscible phases could be represented.

The relative permeability factors account for the fact that each phase interferes with the flow of the other, and (at least in porous media) the \( K_{rl} \) and \( K_{rg} \) functions are highly dependent upon phase saturation. Due to lack of data, it is generally assumed for modeling purposes that in fractures the relative permeability to each phase is equal to its saturation; i.e. that neither phase interferes with the flow of the other and \( K_{rl} + K_{rg} = 1 \). This assumption is based on the experimental work by Romm [1966], in which oil and water were confined in different regions of a smooth fracture by controlling the
wettability of the fracture surfaces and also on analysis of field data from geothermal reservoirs [Pruess et al., 1983, 1984]. But theoretical analysis and numerical simulations by Pruess and Tsang [1990] showed that significant phase interference would occur in a rough fracture. This was confirmed by the experimental work of Persoff et al. [1991] and Fourar et al. [1991, 1992].

Fracture-wall roughness is important in single-phase flow because it increases friction and causes streamlines to be crooked even in laminar flow. In two- (or multi-) phase flow, wall roughness causes the aperture to vary from point to point in the fracture. Regions of smaller aperture (like smaller pores in a porous medium) are more attractive to the wetting phase, and generally constitute the flow path for that phase. In our research we conducted two-phase flow experiments in smooth and rough fractures with openings on the order of 1 mm. Models for two-phase flow in both porous media and pipes were examined for fit of the data. The results are qualitatively the same for all fractures. The results obtained with the smooth fracture (h = 1.05 mm) and one rough fracture (h1 = 0.94 mm) are mainly presented in this paper.

**APPARATUS**

The apparatus is shown in Figure 1. The fracture is consisted of two horizontal glass plates, 1 m long and 0.5 m wide. The plates were either smooth or artificially roughened. One set of experiments (S) was done with the smooth plates. Three sets of experiments were done with the rough plates: one with the rough surfaces in contact (R1) and two with the surfaces spaced apart (R2 and R3). The smooth fracture was assembled by placing 1 mm strips of stainless steel along the no-flow boundaries. Rough surfaces were made by applying a 0.3 mm layer of transparent epoxy cement to the surfaces and gluing a single layer of 1 mm glass beads to each plate. Figure 2 shows a sample of the
roughness pattern. For experiment R1, the two surfaces were placed in contact and silicone caulk was used to seal the no-flow boundaries. Two additional experiments, R2 and R3, were done by disassembling R1 and reassembling it with 3 mm stainless steel strips and caulk along the no-flow boundaries. Therefore R2 had approximately 1 mm clearance between the rough surfaces. R3 was prepared by disassembling R2 and reassembling it with more silicone caulk along the no-flow boundaries, slightly increasing the clearance between the rough surfaces slightly (by about 0.1 mm). Steel bars were tightened in place to prevent the glass from bulging at high flow rates for all fractures.

The injector consisted of 500 stainless steel tubes of 1 mm OD. and 0.66 mm ID. Air and water were injected through alternating tubes to achieve uniform distribution of flow at the inlet. Air was injected at constant pressure and its volume flow rate, corrected to standard pressure, was measured by an inline rotameter. Water was injected by a calibrated pump. At the outlet of the fracture, gas escaped to the atmosphere and water was collected in a decanter and recycled.

Nine liquid-filled pressure taps were cemented into holes drilled along the center line of the lower plate. Any pair of taps could be connected by valves to a differential transducer. This arrangement was designed to non uniform pressure gradients, but in the experiments the pressure gradient was always found to be uniform along the length of the fracture. Since the measured pressure gradient varied rapidly as the two taps were contacted by air or water, only the time-averaged values were recorded.

The fracture was initially saturated with water, and water was injected at a constant rate through the fracture for each experiment. Air injection was started and increased stepwise through a range of flow rates. When steady state was reached for each flow rate, the
pressure gradient and liquid saturation were measured. Then the fracture was resaturated with water and the experiment was repeated several times at different liquid flow rates.

Liquid saturation was measured by a volume-balance method. The water volume in the decanter was measured at the start of the experiment with the fracture completely saturated with flowing water, and again when steady state had been reached at each air flow rate. Changes in the water volume in the decanter were then used to compute a water balance, from which the liquid saturation in the fracture was determined. The liquid saturation values thus obtained in the smooth fracture were checked by comparing them with photographs of the experiments. Good agreement was found and the volume-balance method was used to measure liquid saturation in the rough fractures, which could not be estimated from photographs. Videotape and still photography were used to record the distribution and motion of the phases through the fracture.

Pressure drop in single-phase gas flow was too small to be measured reliably. Pressure drop in single-phase liquid flow for the four fractures is shown in Figure 3.

In two-phase flow, essentially the same range of flow structures was observed in both smooth and rough fractures with and without contact, as shown in Figure 4. These flow structures were constantly in motion, never stopping, even momentarily. The flow structures varied over the range of liquid and gas flow rates studied as shown in Figures 5 a and b. In these figures, the volume flow rates of gas and liquid have been converted to superficial velocities by dividing by the width (0.5 m) and by the hydraulic aperture of the fracture, calculated as described in the next section. The observed pressure gradients and liquid saturations are plotted as smooth contours in Figures 5 a and b. Note that the contours of pressure gradient and saturation contours do not show sharp breaks as they
pass from one flow-structure region to another. This suggests that a single model may be adequate to describe flow in all regions from bubbles to film.

**DISCUSSION**

Calculation of hydraulic aperture

The hydraulic aperture of each fracture was computed from the single-phase liquid-flow data shown in Figure 3. Laminar flow through a fracture with smooth parallel sides obeys Darcy's law with intrinsic permeability [Witherspoon et al. 1980]:

\[
k_0 = \frac{h^2}{12}
\]

where \( h \) is the uniform aperture of the fracture. In Figure 3, the smooth-fracture data plot as a straight line, while the rough-fracture data plot as parabolas. Deviation from linearity for rough fractures indicates deviation from Darcy's law but does not necessarily indicate turbulent flow. Such deviation has been observed in porous media [see reviews by Houpeurt, 1974, and Temeng and Horne, 1988] and in rough fractures [Schrauf and Evans, 1986]. The deviation from Darcy's law is attributed to inertial forces, which are negligible in comparison to viscosity forces at small Reynolds number. The inertial forces are proportional to the square of the velocity and are independent of the viscosity. The relationship between pressure gradient and flow rate is then written:

\[
-\nabla P = \frac{12\mu}{h^3} Q + B \frac{\rho}{h^3} Q^2
\]
where \( Q \) is the volume flow rate per unit width of the fracture and \( B \) is a dimensionless number measuring the roughness. For a rough fracture, \( h \) is the hydraulic aperture. Values of \( h \) and \( B \) determined from the parabolas are shown in Figure 3. From the slope of the line for the smooth fracture, \( h \) equals 1.05 mm, which agrees well with the known value of 1 mm. These values for \( h \) were used for subsequent data analysis.

The hydraulic aperture is in a sense the average aperture available for flow in the rough fracture. We would expect \( h \) to be greater for R3 than for R2, and for both R2 and R3 to be greater than 1 mm, because there is 1 mm clearance between the rough surfaces. There are at least three points of contact between the rough surfaces for R1, and the actual aperture must vary between zero and almost 2 mm. The calculated value for R1 is 0.94 mm.

Hydraulic apertures found in the field are generally smaller than those in our experimental fractures. Romm [1966] states that most fractures are in the range of 0.015 to 0.04 mm. But Raven et al. [1988] inferred fracture hydraulic apertures from pumping tests and found many to be in the 0.1 to 0.2 mm-range.

**Two-phase flow structures**

The flow structures observed show more similarity to the structures observed in pipe flow than to the structures expected for a porous medium, because they were constantly in motion. It is generally assumed that the wetting phase occupies the smallest pores and the non-wetting phase occupies the largest pores in a porous medium. Accordingly, for any fixed value of wetting phase saturation, each phase occupies its own network of pores through which only it flows, and each network is continuous from inlet to outlet. The phase occupancy of the pore space, which determines the relative permeability, is
solely a function of the saturation. Fluids move through these fixed pore networks but the pore networks themselves do not move unless the flow rates change. In our experiments, however, we see quite the opposite. Only one phase is continuous, with the other phase flowing as discrete drops or bubbles, except perhaps in the complex region. In general, no spot in the fracture is ever occupied continuously by either phase. The flow structures are essentially similar whether the glass plates are smooth, rough with contact or rough without contact.

The observed flow structures may result from the nature of the artificial roughness. The aperture in the rough fracture varies from point to point, making some location with smaller apertures more attractive to the wetting phase and others more attractive to the non-wetting phase. But all parts of the fracture are equally attractive to either phase viewed on scale of 1 cm. The wetting fluid would have to be dispersed into tiny disconnected droplets in order to have it occupy small pores. Surface tension acts to prevent this by minimizing interfacial area and continuous injection of wetting phase fluid forces it into larger pores. The aperture also varies from point to point in a natural rock fracture, but it does not vary from zero to a maximum in so short a distance (the correlation length of the aperture is longer). This allows formation of stable connected pore networks belonging to each phase. Stable pore occupancy instead of moving flow structures might have occurred in our artificially rough fractures at very low flow rates (inertial forces negligible).

**INTERPRETATION**

Correlation of flow rates, pressure drop, and saturation is a major goal of our research work. We have examined porous-medium and pipe-flow models, and an equivalent homogeneous flow to see how well they fit the data.
The generalized Darcy model

Since this model is based on viscous pressure drop, we expect only poor agreement. However, we would like to test the model which is used in petroleum industry.

For the porous-media approach, we suppose that the two-phase flow in a fracture is governed by the generalized Darcy law (1) and (2).

In these equations, relative permeability expresses the degree to which each phase impedes the flow of the other. The capillary pressure is negligible in our experiment. Then \( \nabla P = \nabla P_L = \nabla P_G \), where \( \nabla P \) is the observed pressure gradient under two-phase flow conditions.

Substituting (3) into (1) and (2),

\[
K_{rL} = -\frac{12\mu_L V_{LS}}{h^2 \nabla P} \quad (5)
\]

\[
K_{rG} = -\frac{12\mu_G V_{GS}}{h^2 \nabla P} \quad (6)
\]

The calculated \( K_{rL} \) and \( K_{rG} \) are plotted against the measured saturation in Figures 6 a and b. The various symbols represent individual experiments in which \( V_{LS} \) is held constant and \( V_{GS} \) increases stepwise in these and following figures. These curves are qualitatively similar to conventional curves obtained in porous media. However, a family of curves is found (instead of one single curve as in porous media) depending on \( V_{LS} \). Relative permeabilities are therefore not solely functions of saturation under these conditions. In Figures 7 a and b, the data of Figures 6 a and b are plotted as \( K_{rL} \) vs. \( K_{rG} \).
These figures show that the sum of $K_{rL}$ and $K_{rG}$ is less than one at all saturations. Thus significant phase interference occurs even in the smooth fracture and relative permeabilities are not linearly dependent on saturation as commonly assumed for reservoir simulations.

**Lockhart and Martinelli’s Model**

Because of the similarity of flow structures to those observed in two-phase pipe flow, we examined the data to see if they could be fit by the *Lockhart and Martinelli model* [1949, see also *Perry* and *Chilton* 1973, and *Wallis* 1969]. The advantage of this model is that it accounts for inertial forces.

In two-phase flow, the pressure gradient is greater than pressure gradient would be for either phase flowing alone at the same flow rate. The Lockhart-Martinelli model expresses this inequality by factors $\Phi$ which in turn are correlated against a parameter $\chi$:

$$\Phi_L = \frac{\nabla P}{\nabla P^*_L}$$  \hspace{1cm} (7)

$$\Phi_G = \frac{\nabla P}{\nabla P^*_G}$$  \hspace{1cm} (8)

$$\chi = \frac{\nabla P^*_L}{\nabla P^*_G}$$  \hspace{1cm} (9)

where $\Phi_G$ and $\Phi_L$ are respectively the gas and liquid multipliers, $\chi$ is the Martinelli parameter, $\nabla P$ is the observed pressure gradient under two-phase flow conditions and $\nabla P^*_L$ and $\nabla P^*_G$ are the pressure gradients that would exist for gas or liquid flowing at the same flow rate with no flow and zero saturation of the other phase. These quantities
are computed from known flow rates using equation (4), which takes the inertial forces and the values of h and B shown in Figure 3 into account. For the smooth fracture, 
\[ \chi = \frac{\mu_L V_L}{\mu_G V_G} \]. Note that for simplicity \( \Phi_G, \Phi_L, \) and \( \chi \) are defined differently here than in the cited publications.

When (7) and (8) are compared with (5) and (6) for the smooth fracture, \( B = 0 \), it is apparent that \( \frac{1}{\Phi_L} \) and \( \frac{1}{\Phi_G} \) are analogous to \( K_{rL} \) and \( K_{rG} \), respectively. The Lockhart-Martinelli model is a generalization of the Darcy relative-permeability model for non-Darcy flows.

The plot of \( \frac{1}{\Phi_L} \) and \( \frac{1}{\Phi_G} \) against \( \chi \), which increases monotonically from 0 at single-phase gas flow to 1 at single-phase liquid flow, is shown in Figures 8 a and b. These curves are similar to the ones in Figures 6 a and b. It appears that there is no unique relationship between \( \frac{1}{\Phi} \) and \( \chi \), and also the sum of \( \frac{1}{\Phi_L} \) and \( \frac{1}{\Phi_G} \) is less than one. Similar to Figures 6 a and b, the data for each value of \( V_{LS} \) fall on a different curve, although plotting according to the Lockhart-Martinelli model makes the data more nearly collapse onto a single curve, especially for the rough fractures.

Furthermore the data appear to fit the Lockhart-Martinelli model even better when plotted as \( \Phi_G \) versus \( \chi \) (Figures 9 a and b). Curves of the empirical Lockhart and Martinelli relationship are (Delaye et al. 1981) also plotted in Figures 8 a and b and Figures 9 a and b:

\[ \Phi_L = 1 + \frac{C}{\sqrt{\chi}} + \frac{1}{\chi} \]  
(10)

\[ \Phi_G = 1 + C\sqrt{\chi} + \chi \]  
(11)

where the value of C depends on whether each phase is laminar or turbulent:
Good agreement is obtained with smooth fracture experimental data and with the $\Phi_G$ relationship for laminar-laminar flow ($C = 5$) (Figure 9 a). This suggests that the Lockhart and Martinelli model might be useful for rough calculations of pressure drop, although not for liquid saturation. In discussing Lockhart and Martinelli's original model, Gaxley and Bergelin (1949) presented two-phase pipe flow data that behave similarly to our data in Figures 8 and 9. This observation suggests that if the Lockhart-Martinelli model can be modified to better fit our fracture flow data, the same modification might also improve the model's value in predicting head losses in pipe flow.

Homogeneous flow model

Another empirical approach based on pipe flow models is to treat the two phases like a single homogeneous phase, define average values for the fluid properties and express the pressure gradient in terms of a friction factor which can be empirically correlated with a two-phase Reynolds number. For two-phase flow in a horizontal pipe [Delhaye et al, 1981]:

$$-\nabla P = \frac{\Pi}{A} \tau_w + \nabla \left[ S_L \rho_L V_L^2 + S_G \rho_G V_G^2 \right]$$

(10)

where $A$ and $\Pi$ represent the pipe area and perimeter, $\tau_w$ is the average wall shear stress, and $V_L$ and $V_G$ are the local fluid velocities related to superficial
velocities by \( V_L = \frac{V_{LS}}{S_L} \) and \( V_G = \frac{V_{GS}}{S_G} \). The two terms on the right-hand side of equation (10) can be regarded as frictional and accelerational components of the pressure gradient.

The accelerational component of the pressure gradient cannot be calculated, because only average, not local, values are known for \( S_L \), and hence for \( V_L \) and \( V_G \). However the accelerational component can be estimated for any data point, assuming either of two limiting cases: (i) \( S_L \) (and therefore \( V_L \)) remains constant through the fracture and \( V_G \) increases as gas bubbles expand due to reduced pressure or (ii) \( V_G \) remains constant through the fracture so that \( S_L \) decreases as gas bubbles expand. The term \( (S_L \rho_L V_L^2 + S_G \rho_G V_G^2) \) is evaluated at inlet and outlet and the difference is compared to the observed pressure drop. This quantity is small under either assumption for all the data points in the smooth fracture. We therefore disregard the accelerational component and attribute the pressure gradient to friction.

From Figure 3, we see that the accelerational contribution to pressure gradient was negligible in the smooth fracture for single-phase flow, but not in the rough fractures. The homogeneous flow model is therefore expected to fit the data better for smooth fractures than for rough ones.

We express the wall shear forces in terms of a friction factor, \( C_f \), and a mean hydraulic diameter according to the conventional practice for single-phase pipe flow. The average wall shear stress is:

\[
\tau_w = \frac{1}{2} C_f \rho_m V^2 \tag{11}
\]

where \( V \) is the superficial velocity of the mixture.
and $\rho_m$ is the mean density:

$$\rho_m = \frac{Q_L \rho_L + Q_G \rho_G}{Q_L + Q_G}$$

Substituting (11) into (10),

$$-\nabla P = 2C_f \rho_m \frac{V^2}{D}$$

where $D$ is the hydraulic mean diameter (defined as four times the hydraulic radius; in a fracture $D = 2h$).

We now define $Re_2$, the two-phase Reynolds number, using the hydraulic diameter of the fracture, and the flow-rate weighted average fluid properties.

$$Re_2 = \frac{2hV \rho_m}{\mu_m}$$

where $\mu_m$ is the mean viscosity of the mixture. We use the definition adopted by Dukler [1964], which is consistent with our definition of average density:

$$\mu_m = \frac{Q_L \mu_L + Q_G \mu_G}{Q_L + Q_G}$$

With the data from our two-phase flow experiments, we plot $C_f$ against $Re_2$ for the smooth fracture in Figure 10a, and one rough fracture in Figure 10b. The data for
single phase liquid flow and the smooth-fracture single-phase experimental data reviewed by Romm [1966] are also plotted for the smooth fracture. (Note that our values of $Re$ are greater than Romm's by a factor of four. This results from using $D = 2h$ as the characteristic length for defining $Re$ rather than $r = h/2$).

Several useful features can be noted for these plots. The correlation is indeed better for the smooth fracture than the rough ones. First, we note that the data follow a line with slope $= -1$ at small $Re$, the same as for single-phase flow.

The line summarizing Romm's data shows a break in slope to $-0.25$ above $Re = 2400$; this marks the boundary between laminar and turbulent flow. Our data also show a slope break at $Re_2 = 1000$. Flow regimes for each point can be determined by comparing the points in Figure 10a with those in Figure 5a. Examination shows that bubbles flow appears on either side of the break in slope. Therefore we caution that equality of $Re_2$ does not imply dynamic similarity of the flow, as $Re$ would for single-phase flow.

$C_f$ is found to be greater for two-phase flow than for single phase flow by a factor of 1.5 below $Re_2 = 1000$.

**CONCLUSIONS**

Two-phase (air-water) flow experiments were conducted in smooth and artificially roughened fractures with a hydraulic aperture of approximately 1 mm. Gas and liquid superficial velocities ranged from 1.3 to 500 cm/sec and from 0.4 to 40 cm/sec respectively. Under these conditions, the results of the experiments support the following conclusions:
1. No static flow paths are formed for each phase, but rather moving flow structures in which generally only one phase is continuous. These structures vary with gas and liquid flow rates.

2. The data show that the relative permeabilities are not linearly dependent on saturation, contrary to what is commonly assumed. The data can not be fit using either the two-phase Darcy model with relative permeability or the Lockhart-Martinelli model. The data fall on curves showing the same general behavior (phase interference) in both cases, but different curves result from different liquid velocities. However, the Lockhart-Martinelli relationship might be useful for rough calculations of pressure drop.

3. The data for flow in a smooth fracture can be fit by correlating the friction factor $C_f$ with the Reynolds number for two-phase flow $Re_2$, with similar results as for single-phase flow.

ACKNOWLEDGEMENTS

The authors appreciate their helpful discussions with Karsten Pruess. Mr. Persoff's work and part of Mr. Fourar's, was sponsored by the Assistant Secretary for Conservation and Renewable Energy, Office of Renewable Energy Technologies, Geothermal Technology Division of the U.S. Dept. of Energy, under contract no. DE-AC03-76SF00098.
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Gaxley and Bergelin (1949) in Lockhart and Martinelli (1949).


Fractures, presented at Sixteenth Stanford Geothermal Workshop, Stanford University, Jan 25-27.


Figure 1. Experimental apparatus.
Figure 2. Photograph of rough surface used in the experiments. Bead diameter = 1 mm.
Figure 3. Pressure drop in single-phase liquid flow plotted against flow rate. Parabolic curves for the rough-walled fractures indicate deviation from Darcy's law. Values of $h$ and $B$ fit the data to equation (4).
Figure 4. These flow structures were observed in both smooth and rough fractures. In these photographs, flow is from left to right. In the smooth fracture, liquid contained dye and appears dark; in the rough fractures, liquid contained no dye and appears light.
Figure 5a. Flow structure map with contours of pressure gradient and liquid saturation for smooth fracture.
Figure 5b. Flow structure map with contours of pressure gradient and liquid saturation for rough fracture R1 with contact (results for rough fractures R2 and R3 are similar).
Figure 6a. The generalized Darcy model: Relative permeability plotted against saturation.
Figure 6b. The generalized Darcy model: Relative permeability plotted against saturation.
Figure 7a. Relationship between gas and liquid relative permeabilities according to generalized Darcy model.
Figure 7b. Relationship between gas and liquid relative permeabilities according to generalized Darcy model.
Figure 8a. Lockhart-Martinelli model: Relationship between $1/\Phi$ and $\chi/(1+\chi)$. 

**Smooth fracture (h = 1.05 mm)**

- Liquid-Gas
- Laminar-Laminar
- Turbulent-Laminar
- Laminar-Turbulent
- Turbulent-Turbulent

<table>
<thead>
<tr>
<th>Liquid velocity, VLS (m/s)</th>
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<tr>
<td>□ 6.89E-02</td>
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<tr>
<td>■ 9.19E-02</td>
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<tr>
<td>□ 1.37E-01</td>
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Figure 8b. Lockhart-Martinelli model: Relationship between $1/\Phi$ and $\chi/(1 + \chi)$.
Smooth fracture (h = 1.05 mm)

Liquid-Gas
- laminar-laminar
- turbulent-laminar
- laminar-turbulent
- turbulent-turbulent

Liquid velocity, VLS(m/s)
- 6.89E-02
- 9.19E-02
- 1.37E-01
- 1.82E-01
- 2.26E-01
- 2.72E-01
- 3.17E-01
- 3.64E-01
- 4.11E-01

Martinelli's parameter $\chi$

Figure 9a. Data to fit Lockhart-Martinelli model.
Figure 9b. Data to fit Lockhart-Martinelli model.
Smooth fracture (h = 1.05 mm)  

Single-phase flow

- Experimental data
- Laminar flow
- Turbulent flow

\[ C_f = \frac{24}{Re} \]

\[ C_f = \frac{0.079}{Re^{0.25}} \]

Liquid velocity, VLS (m/s)

- 6.89E-02
- 9.19E-02
- 1.37E-01
- 1.82E-01
- 2.26E-01
- 2.72E-01
- 3.17E-01
- 3.64E-01
- 4.11E-01

Figure 10a. Plot of friction factor \( C_f \) against two-phase Reynolds number \( Re_2 \).
Rough fracture (h1 = 0.94 mm)

Liquid velocity, VLS (m/s)

- 2.24E-02
- 4.49E-02
- 8.93E-02
- 1.33E-01
- 1.77E-01
- 2.20E-01

Figure 10b. Plot of friction factor $C_f$ against two-phase Reynolds number $Re_2$. 

Single-phase flow
- Laminar flow
- Turbulent flow

$C_f = \frac{24}{Re}$

$C_f = \frac{0.079}{Re^{0.25}}$