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February 2, 1965
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There is currently great interest in the phase of forward elastic scattering amplitudes at high energy. In a simple diffraction picture, one expects a purely imaginary amplitude. However, recent Coulomb interference measurements\(^1\)-\(^6\) in \(pp\) and \(\pi p\) scattering have shown that the real part is substantial, of order 20 to 30\%, up to 26 GeV/c. Various dispersion relation calculations\(^7\)-\(^9\) have confirmed that something of this order may be expected, but they do not explain what mechanism produces it.

The present note points out that simple Regge pole models predict a substantial real part at the energies in question, in surprisingly good agreement with experiment. We employ the latest \(\pi N\), \(NN\), and \(NN\) total cross section data\(^5\),\(^10\) from 6 to 26 GeV/c, to fix the parameters of the forward scattering amplitudes, taking three Regge poles for \(\pi N\) and four for \(NN\). The ratio of the real to the imaginary part of the forward scattering amplitude is illustrated and compared with experiment in Figs. 1 and 2.

Let us define and normalize the spin-averaged forward elastic amplitude \(A(0)\) for each process, such that the optical theorem reads

\[
\sigma_T = \text{Im} A(0) \quad , \quad (1)
\]
where $\sigma_T$ is the corresponding total cross section. Then in a high-energy approximation each Regge pole contributes to $A(0)$ a term of the form

$$A_i(0) = B_1 \left[ 1 + \exp(-i\pi a_i) \right] / \sin \pi a_i (E/E_0)^{a_i-1}. \quad (2)$$

Here $i$ labels the Regge pole, $a_i$ is its trajectory at squared momentum transfer $t = 0$, $B_i$ is a real coefficient measured in millibarns, $E$ is the total laboratory-system energy of the bombarding particle, and $E_0$ is an arbitrary scale parameter which we choose to be 1 GeV. Note that the phase of $A_i(0)$ is determined by $a_i$, through the "signature factor" (in braces), and is therefore directly related to the energy dependence.

For $\pi N$ scattering at least three Regge poles are needed. The Pomeranchuk pole $P$ gives the asymptotic limit; a second vacuum pole $P'$ and the $\rho$ pole give the differences of the $\pi^+ p$ amplitudes from the asymptotic limit and from each other. The signature $\pm$ in Eqs. (2) is $+$ for $P$ and $P'$, $-$ for $\rho$. Let us take the coefficients $B_i$ above to refer to $\pi^- p$ elastic scattering (for $\pi^+ p$, $B_\rho$ changes sign). Since the fit to data is not very sensitive to the precise values of the $a_i$, we fix them at suitable values $a_p = 0.5$, $a_{P'} = 0.6$. Then by a least-squares fit to the total cross sections the $B_i$ are determined: $B_p = 19.9 \text{ mb}$, $B_{P'} = 18.1 \text{ mb}$, and $B_\rho = 2.4 \text{ mb}$.

The predicted ratios $\text{Re} A(0)/\text{Im} A(0)$ for $\pi^+ p$ scattering are shown in Fig. 1. They agree with the experimental determination in sign, in magnitude, and in giving a larger value for $\pi^+ p$ than for $\pi^- p$; however, the experimental uncertainties are rather large. The dispersion calculation
of reference 9 agrees closely with our $\pi^-p$ curve but gives a $\pi^+p$ curve displaced upwards.

For $NN$ and $\bar{NN}$ scattering, at least two more poles are usually invoked. One is the negative-signature $\omega$ pole (which is supposed to include any contribution from the $\phi$ pole, lying near, with the same quantum numbers). The other is the $R$ pole, proposed by Pignotti. However, the experimental uncertainties in the data we use are such that the $R$ contribution that is determined is not significantly different from zero, and we ignore this term. Let us take the coefficients $B_i$ in Eq. (2) to refer to $\bar{p}p$ elastic scattering; for $\bar{n}p$ the $\rho$ term changes sign; for $pp$ the $\omega$ and $\rho$ terms change sign; for $np$ the $\omega$ term changes sign. We fix $\alpha_\omega = 0.5$, with $\alpha_\rho$, $\alpha_{\bar{p}}$, and $\alpha_{\bar{n}}$ as before; then by a least-squares fit to the total cross sections we determine

$$B_p = 36.2 \text{ mb}, \quad B_{\bar{p}} = 33.8 \text{ mb}, \quad B_\rho = 1.0 \text{ mb}, \quad \text{and} \quad B_\omega = 21.0 \text{ mb}.$$

The predicted ratios $Re A(0)/Im A(0)$ for $pp$ and $\bar{p}p$ scattering are shown in Fig. 2. The experimental points refer only to $pp$. Below 10 GeV/c there is a marked divergence between prediction and experiment; the former becomes steadily more negative while the latter (in a region not illustrated) finally becomes positive below 1.5 GeV/c. Above 10 GeV/c, however, the agreement with experiment is surprisingly good. There have been several dispersion calculations which agree roughly with one another and with experiment. Söding's calculation, for example, gives roughly 70% of our $pp$ values (above 10 GeV/c); it also agrees rather closely with our $\bar{p}p$ values.

Regge pole models are designed for high energies. As the energy is lowered, the various correction terms play more and more important roles.
We may expect the real part of the forward scattering amplitude to be especially sensitive to these corrections, since it is in a sense a correction term itself—coming wholly from the secondary trajectories. Thus a divergence of the prediction from experiment is to be expected at lower energies.

The parameters of our Regge poles have been fixed by total cross sections only. A complete Regge pole model should also fit elastic angular distributions; this seems to present no serious difficulty, but the best fit to this wider range of data generally gives slightly different parameters. However, in the cases we have studied the change is small; the curves in Figs. 1 and 2 should be little altered by a more complete fit to data.

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FOOTNOTES AND REFERENCES

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FIGURE CAPTIONS

Fig. 1. The ratio of the real to the imaginary part of the forward amplitude for $\pi^+p$ and $\pi^-p$ scattering. The curves are Regge pole predictions. The data are from reference 1: the inner error flags are statistical; the outer ones are estimated limits of systematic error.

Fig. 2. The ratio of the real to the imaginary part of the forward spin-averaged amplitude, for $pp$ and $p\bar{p}$ scattering. The curves are Regge pole predictions. The data refer only to $pp$ scattering. Where double error flags are shown, the inner ones are statistical and the outer ones are estimated limits of systematic error.
Fig. 2
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