Lawrence Berkeley National Laboratory
Recent Work

Title
A FRESH LOOK AT BOSE-EINSTEIN CORRELATIONS

Permalink
https://escholarship.org/uc/item/4kz9v7s9

Author
Hofmann, W.

Publication Date
1987-03-01
A FRESH LOOK AT BOSE-EINSTEIN CORRELATIONS

W. Hofmann

March 1987
DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
A Fresh Look at Bose-Einstein Correlations

Werner Hofmann

Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

March 1987

Talk presented at the
Workshop on Electronuclear Physics with Internal Targets
Stanford Linear Accelerator Center
January 5-8, 1987
Recent experimental data on Bose-Einstein (BE) correlations between identical bosons are reviewed, and new results concerning the interpretation of the BE enhancement are discussed. In particular, it is emphasized that the classical interpretation of the correlation function in terms of the space-time distribution of particle production points cannot be directly applied to particle production in high energy reactions.

Bose-Einstein (BE) correlations between like-sign pions, also known as the GGLP effect, have first been observed over 25 years ago and have been of continued interest since. In this paper, I will summarize recent progress in our understanding of the BE effect. First the "classical" BE effect and its interpretation is summarized. Next, I will show that the classical description is not appropriate for high-energy reactions, point out where modifications are required, and discuss the extent to which experimental results support these ideas. I will briefly mention the experimental problems which complicate the study of BE correlations, and end with some concluding remarks. For a more complete review of recent experimental results, the reader is referred to Ref. 1.

The classical "setup" to study BE correlations is indicated in Fig. 1: given a (large) number of fixed, identical, incoherent ("chaotic") pion emitters with lifetime \( \tau \) and a spatial distribution \( \rho(r) \) (with a characteristic width \( R \)), plus two distant detectors looking for the simultaneous emission of two identical pions with four-momenta \( p_1 = (E_1, p_1) \) and \( p_2 = (E_2, p_2) \).

Fig. 1. Amplitudes interfering in the creation of the Bose-Einstein enhancement for identical bosons
For any pair of emitters, there are two ways for the particles to propagate to the detectors, and those two amplitudes interfere. Summing over all pairs of emitters, it is easy to show that the resulting two-particle correlation function \( C \) is essentially the square of the four-dimensional Fourier transform of the (normalized) distribution \( \rho(r) = \rho(r,t) \) of emission points

\[
C = \frac{\sigma^{(2)}(p_1,p_2)}{\sigma_0^{(2)}(p_1,p_2)} = 1 + \left( \int d^4r \rho(r)e^{i qr} \right)^2
\]

(1)

with

\[
q = p_1-p_2 = (q_0,q)
\]

Here \( \sigma^{(2)}(p_1,p_2) \) denotes the measured two-particle cross section, and \( \sigma_0^{(2)}(p_1,p_2) \) stands for the two-particle cross section in the absence of BE symmetrization. Since all emitters are assumed to have identical lifetimes, the Fourier transform factors into a term depending only on \( q_0 = E_1-E_2 \) and a term depending on three-momentum difference \( q = p_1-p_2 \) :

\[
C = 1 + |f(q)g(q_0)|^2.
\]

For large \( q \) or \( q_0 \) the integral vanishes and we obtain \( C = 1 \); for small momentum differences \( C \) rises and reaches \( C = 2 \) for \( q = q_0 = 0 \). In other words, BE statistics predict that identical bosons will be preferentially emitted in the same quantum state, i.e. \( |q| R < 1 \) and \( q_0 \tau < 1 \) (we use \( \hbar = c = 1 \) everywhere). Since the correlation function \( C(q) \) is rather insensitive to details of the distribution \( \rho(r) \) — it is e.g. virtually impossible to distinguish a gaussian distribution in space from a group of emitters arranged on the surface of a sphere — experiments are typically limited to the determination of the effective source radius \( R \) and the lifetime \( \tau \). In case the events exhibit a preferred axis, such as in \( e^+e^- \) annihilation into jets of hadrons, one can make further statements concerning the shape of the distribution of emitters ("spherical" or "cigar-like" or "pancake-like") by studying the effective source size as a function of the angle between \( q \) and the event axis. To clarify the nomenclature, let me point out that in the following I will use the term 'particle source' for the set of individual particle emitters; the measured 'source size' corresponds to (at least in the classical interpretation) the rms width of the distribution of emitters.

At a first glance, the interpretation given by Eqn. (1) works extremely well: considering e.g. two rather different pion sources, namely heavy ion collisions at 1.8 GeV/nucleon and \( e^+e^- \) annihilations at 29 GeV cms energy, we find in both cases a two-pion correlation function which is constant for large momentum transfers, and rises for small momentum differences (Fig. 2). For the heavy-ion
system, the correlation length of about 70 MeV/c translates into a characteristic source size of ~3 fm — just about the size of the composite nuclear system — whereas for e+e- annihilation the enhancement extends over a larger range in q, resulting in an effective source size of about 0.7 fm, consistent with the expected range of the confinement forces responsible for particle production.

Fig. 2. (a) Two-pion correlation function measured in Ar + KCl collisions at 1.8 GeV/Nucl, as a function of the momentum difference |q| for small qo. (b) Two-pion correlation function obtained in e+e- annihilation at 29 GeV cms energy for small qo, as a function of qT. qT is the component of q perpendicular to the total momentum p1+p2 of the pion pair.

However, several authors have recently pointed out that Eqn. (1) is not appropriate to describe BE correlations among particles produced in high energy reactions. As we shall see, several of the basic assumptions are violated: 1) particle emitters are typically not at rest, but move with high velocity with respect to each other; 2) because of this motion, the spectra of different emitters (as observed in a common frame, such as the lab frame) will not be identical; 3) for Eqn. (1) to hold, the spectra should be approximately constant over a range |q| = 1/R; however momentum spectra in e+e- reactions, e.g., show strong variation over a range of a few
100 MeV/c. Finally one may question if the different emitters are actually incoherent.

In order motivate these statements and to show how the interpretation of BE correlations has to be modified to suit high-energy reactions, I need to discuss the present model of the space-time evolution of particle production in high-energy reactions, as it has evolved over the last decade or so. I will use $e^+e^-$ annihilation as the simplest example. At $t=0$, a quark and an antiquark are created from a virtual photon (Fig. 3).

"Source size":
\[ \approx 30 \text{ fm at PEP} \]

Fig. 3. Space-time evolution of particle production in $e^+e^-$ annihilation into hadrons

They recede from each other at close to the speed of light, feeding energy into the color force field which builds up between them. At early times, corresponding to short gluon wavelengths, perturbative QCD can be used to describe the structure of this color field; at later times, large coupling constants cause any perturbative treatment to break down, and we have to resort to the phenomenological picture of a color flux tube ("string") spanned from quark to antiquark. Such a string provides a linear confinement potential, in agreement with measurements and consistent with results obtained using QCD on discrete space-time lattices. The energy
stored in this color field is ultimately released through the production of new quark-antiquark pairs, which screen the color field and which recombine to form colorless hadrons. Since the decay of the color field will occur on a typical time scale $\tau_0$ in the rest frame of the corresponding string segment, particle production points will scatter about the hyperbola $t^2 - z^2 = \tau^2_0$. On average, the primary quarks will propagate over a distance $\gamma \tau_0 = (\sqrt{s}/2m)\tau_0$ before they are confined to a hadron. We expect $\tau_0$ to be of the order of typical hadron sizes; $m$ is a typical hadronic mass scale, $O(m_p)$. At PEP energies — $\sqrt{s} = 29$ GeV — this picture implies a longitudinal extent of the distribution of particle production points of about 30 fm, as compared to a transverse extent of order 1 fm (the diameter of a flux tube).

Since this general model relies mainly on invariance arguments, and since all models with specific dynamics constructed so far agree with it, there is considerable confidence in this picture. Why, then, is this large source size not observed experimentally? The key to the answer lies in the observation that for such a space-time evolution the production point and momentum of an emitted particle are highly correlated. An emitter moving along the $z$-axis with a velocity $\beta$ will typically decay at a distance $z_{lab} = \beta \gamma \tau_0$ from the origin, and the average $z$-component of momentum of one of its daughters will be $<p_{z,lab}> = \beta \gamma E_0$, where $E_0$ is its average energy in the rest frame of the emitter; hence $<p_{z,lab}> \propto z_{lab}$. This correlation implies that particles created at opposite "ends" of the event are never nearby in phase space. As a consequence, BE correlations will show no evidence of a large source size. This is most easily demonstrated in the example of two decaying "fireballs" of radius $R$ and lifetime $\tau$ moving rapidly in opposite directions (Fig. 4). BE statistics enhances two-particle production near the diagonal $p_{z1} = p_{z2}$ (neglecting transverse momenta, for simplicity). We note that regions where the enhancement occurs are populated by particle pairs originating from the same fireball, never from opposite fireballs. The BE correlation length is therefore determined by the fireball size $R/\gamma$ (as seen in the lab), and not by the two-fireball separation $D \approx \gamma \tau$!

For the more general case of $e^+e^-$ jets, it is easy to show that each of the emitters indicated in Fig. 3 will spread particles over approximately $\pm 0.7$ units in rapidity $y = (1/2) \log (1+\beta_2/1-\beta_2)$, centered at the rapidity of the emitter (assuming isotropic
emission in its rest frame). Particle distributions from different emitters will overlap in momentum space provided that the rapidity difference $\Delta y$ of the emitters is of the order of one unit or less. In a comoving frame, this in turn implies a maximum separation of the emitters $\Delta z = \tau_0 \sinh(\Delta y) = \tau_0$. In such frame, the BE correlation length both in longitudinal momentum difference and in energy difference is therefore of order $1/\tau_0$. The equality of space and time scales is a natural consequence of the covariant description.

![Diagram](image)

Fig. 4. Simple model to illustrate BE correlations for moving emitters with $\beta = 1$. Lorentz boosts result in $p_z > 0$ for most particles emitted from 'b', and in $p_z < 0$ for most particles from 'a'. The lower plot indicates the resulting two-particle density. In the region of the BE enhancement, $p_{z1} \approx p_{z2}$ (indicated by the black band), both particles tend to stem from the same fireball.

The correlation length in transverse direction is determined by the flux tube diameter, which is of the same order as $\tau_0$. Since the BE correlation length is similar for $q$-vectors parallel and perpendicular to the jet (= z) axis, we would expect the distribution of particle emitters to appear roughly spherical, and not cigarlike with a large ratio of major to minor axes, as one might naively expect based on Fig. 3.

More detailed studies\textsuperscript{2,6} confirm these features: one finds that

- the correlation function $C$ depends mainly on the (invariant) square of the four-momentum transfer $Q^2 = -q^2 = (p_1 - p_2)^2$, and hence in general cannot be represented in the form $C = 1 + |f(q)g(q_0)|^2$
- the apparent source size, determined from the correlation length in $Q^2$, is of order $\tau_0$
• the source appears essentially spherical
• the measured source size is almost independent of the cms energy and the momentum of the pion pair

Let me briefly discuss one explicit implementation of BE effects - a modification of the Lund hadronization model proposed first by Andersson and myself, and later studied in detail by Artru and Bowler. The basic idea is simple: consider a typical space-time diagram for particle production via string decay into quark-antiquark pairs (top diagram in Fig. 5). In this scheme, break-up points of the string uniquely determine particle momenta; the energy of a particle is proportional to the distance between the production points of its quarks, and its momentum is proportional to the difference in quark production times.

Fig. 5. Space-time structure of quark fragmentation in e+e- annihilation, as predicted in the Lund string model. The space-time area swept by the color field is denoted by A and gives rise to the production amplitude \( M = e^{i\xi A} \). An exchange of the two central particles results in a change of that area by \( \Delta A \), with a corresponding change in amplitude and phase.

It is plausible that the matrix element M describing the decay of the color string is given by \( M = e^{i\xi A} \), where \( \xi = \kappa + iP/2 \). The (invariant) space-time area spanned
by the string is denoted by $A$. The real part of $\xi A$, $\kappa A$, is essentially the classical string action ($\kappa$ denotes the energy per unit length, $\kappa = 1$ GeV/fm). The imaginary part, $PA/2$, describes the breaking of the string by quark-antiquark production at a constant rate $P$ per unit length. In order to properly symmetrize production amplitudes for final states containing several identical bosons, we need to sum over all diagrams corresponding to permutations of those particles. In the context of BE correlations between two given pions, let us consider the effect of exchanging those two pions. Swapping two particles will change the space-time area swept by the string, and hence both the amplitude and phase of $e^{i\xi A}$ (bottom diagram in Fig. 5). Given the known magnitudes of $\kappa$ and $P^9$, it is easy to see that the interference pattern between the amplitudes corresponding to Fig. 5 is dominated by the phase change of order $\Delta \phi = \kappa A = Q^2/2\kappa$. As a result, amplitudes interfere constructively for $Q^2 < \kappa = (0.4 \text{ GeV})^2$ and cause a BE enhancement at low $Q^2$, compared to an effectively incoherent superposition for larger $Q^2$.

As in the classical case, $C(q)$ reaches a limiting value $C = 2$ for $q = q_0 = 0$, indicative of complete chaoticity of the source. However, whereas in the classical case the chaoticity is built in via the assumption that emission phases vary randomly from emitter to emitter and from event to event, here the strong momentum dependence of the amplitude $e^{i\xi A}$ guarantees virtually random phases between amplitudes corresponding to different permutations of particles, unless the final state contains two pions with almost identical momenta.

I should point out here that much of our revived interest in BE correlations results from this point of view — BE correlations as a measure of multiparticle production amplitudes and their phases — as opposed to the classical geometrical interpretation, which suffers from conceptual difficulties for systems with dimensions of the order of the wavelength of the emitted particles.

In the remainder of this paper, I will summarize relevant experimental data (with strong emphasis on results from $e^+e^-$ colliders) and discuss potential drawbacks in the experimental procedures. To begin, let us see if there is indeed evidence that BE correlations depend only on $Q^2$, and not on $q$ and $q_0$ in a factorizable fashion.
Fig. 6 demonstrates the that BE enhancement is certainly seen in the variable $Q = \sqrt{Q^2}$. A clean distinction between the "classical" form based on Eqn. (1)

$$C(q,q_0) = 1 + \alpha e^{-R_2^2 q^2} e^{-\tau^2 q_0^2}$$

(where we have for simplicity used a gaussian space-time distribution of emission points; the "fudge" factor $\alpha$ will be discussed later) and the relativistically invariant form (note the different sign of the $q_0$ term)

$$C(q,q_0) = 1 + \alpha e^{-R_2^2 q^2} = C = 1 + \alpha e^{-R_2^2 q^2} e^{+R_2^2 q_0^2}$$

however turns out to be rather difficult, since $q$ and $q_0$ are of course highly correlated.

Fig. 6. Correlation coefficient $C$ as a function of $Q = \sqrt{Q^2}$, measured in $e^+e^-$ annihilation at 29 GeV cms energy$^5$. Full line: fit to the data based on Eqn. (3). Dashed line: prediction of the model of ref. 6. Possible dilution of the BE correlation due to long-lived resonances is not included in the model curve. Predictions of the model of Ref. 2 exhibit a very similar shape.

The correlation function $C(q,q_0)$ as given by Eqn. (2) and (3) is displayed in Fig. 7 (a) and (b), respectively. The region $q_0^2 \geq q^2$ is kinematically forbidden. Therefore, the $q_0$ dependence cannot be derived simply by looking at $C(q,q_0)$ for small $q$. To circumvent this problem, experiments have traditionally reported $C$ in terms of the Kopylov-Podgoretski variables $q_T$ and $q_0$ instead of $q$ and $q_0$ (see also Fig. 2). Here $q_T$ denotes the component of $q$ perpendicular to $p_1 + p_2$. However, the $q_0$ dependence of $C(q_T,q_0)$ is quite different from that of $C(q,q_0)$. Using $q^2 = q_{||}^2 + q_T^2$, with $q_{||}^2 = q_0^2 \gamma^2/(\gamma^2 - 1)$ and $\gamma = (E_1 + E_2)/m_{\pi\pi}$ we can rewrite Eqn. (2) as

$$C(q_T,q_0) = 1 + \alpha e^{-R_3^2 q_T^2} e^{-R_3^2 q_0^2 \gamma^2/(\gamma^2 - 1)} e^{-\tau^2 q_0^2}$$

(2').
The four-momentum transfer $q^2$ in Eqn. (3) may be expressed in terms of $q_T$ and $q_0$ as $q^2 = q_T^2 + q_0^2/(\gamma^2 - 1)$. Eqn. (3) can then be written as

$$C(q_T, q_0) = 1 + \alpha e^{-R^2 q_T^2} e^{-R^2 q_0^2/(\gamma^2 - 1)} \quad (3').$$

We note that the $q_T^2$ dependence of $C(q_T, q_0)$ in Eqs. (2') and (3') is identical to the $q^2$ dependence of $C(q^2, q_0)$ in Eqn. (2), and the $q^2$ dependence of $C(q^2)$ in Eqn. (3), respectively, and is independent of the energy of the pion pair. The $q_0^2$ dependence, however, is strikingly different for Eqn. (2') as compared to (3'). While both forms show $C(q_T, q_0)$ decreasing with increasing $q_0$, the range in $q_0$ over which the production of pion pairs is enhanced is always of order $1/R_3 \approx 1/\tau$ in (2') (for relativistic systems), whereas the correlation length in $q_0$ for (3') increases proportional to $\gamma$ for high-energy pion pairs. Since the absolute normalization of $C$ is somewhat arbitrary — many experiments normalize $C$ to unity for "large" $q_0" the weak $q_0$ dependence of Eqn. (3') for high-energy pairs means that essentially no correlation is detected. The range in correlation patterns is demonstrated in Fig. 8(a), where we have plotted $C(q_T \to 0, q_0)$ for different cuts on the pion momentum. As expected from Eqn. (3'), the $q_T^2$ dependence is almost independent of cuts and closely reflects the original $q^2$ dependence (Fig. 8(b)). The present experimental situation concerning the $q_0$ dependence of $C(q_T, q_0)$ for small $q_T$ is somewhat unclear\(^1\). Whereas in hadron-hadron reactions a clear $q_0$ dependence of the type
\[ C(q_T \rightarrow 0, q_0) = 1 + \alpha e^{-\beta q_0^2} \]  

(4)

with \( \beta = R_3^2 \) is observed, some \( e^+e^- \) experiments report a similar \( q_0 \) dependence, but others find little variation of \( C(q_T, q_0) \) with \( q_0 \). The inconsistency between experiments may partly reflect the considerable systematical problems (see later). However, it is also obvious that in contrast to the stable \( q_T \) dependence the observed \( q_0 \) dependence of \( C(q_T \rightarrow 0, q_0) \) depends strongly on the explicit and implicit cuts applied to the pion sample, and on the momentum distribution, i.e. the center-of-mass energy. Except for most recent works\(^{13,14}\), this aspect has received (too) little attention.

(a) (b)

Fig. 8 (a) Resulting correlation function \( C(q_T, q_0) \) with \( q_T < 0.1 \) GeV/c for different cuts in the pion momentum, assuming that the BE correlation is described by Eqn. (3) with \( R = 0.7 \) fm. Curves correspond to different experimental cuts: 1) \( p_\pi < 0.5 \) GeV/c, 2) no momentum cut, but minimum opening angle \( \theta_{\pi\pi} > 10^\circ \), 3) no cuts, 4) \( p_\pi > 0.5 \) GeV/c and 5) \( p_\pi > 1 \) GeV/c (top to bottom). In analogy to typical experimental procedures, \( C \) is normalized to unity in the region \( 1.0 \) GeV < \( q_0 < 2.0 \) GeV. (b) \( C(q_T, q_0) \) with \( q_0 < 0.1 \) GeV. The curve is virtually independent of the pion momentum range.
The observation of a weak $q_0$ dependence of BE correlations between high-momentum pions is used by the CLEO group\textsuperscript{14} as an argument in favor of the description according to Eqn. (3) as compared to Eqn. (2). Unfortunately, the main evidence — absence of a $q_0$ dependence, as displayed in Fig. 3 of their paper — depends strongly on the maximum $q_T$ allowed; Fig. 6 of the same paper indicates a significant $q_0$ dependence, once $q_T$ is chosen above the region potentially influenced by detector problems.

A possibly more straightforward way to distinguish between the descriptions of Eqs. (2) and (3) is to investigate whether there is a positive correlation for large and approximately equal $|q|$ and $q_0$ (see Fig. 7). Using a global fit of the measured $C(q,q_0)$ the TASSO group\textsuperscript{13} reports a preference for Eqn.(3) over Eqn.(2). However, the statistical errors on the large $|q|$, large $q_0$ data are such that the evidence, though statistically significant, is by no means striking.

In conclusion the kinematical dependence according to Eqn. (3) appears indeed favored over Eqn. (2), but higher statistics and higher quality data would certainly be welcomed!

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig9.png}
\caption{Size parameter $R$ of the pion source, determined according to Eqn. (3) in various reactions, as a function of the cms energy\textsuperscript{1,5,13,14,15}.}
\end{figure}
Another essential prediction of the new class of models is that the BE correlation length in $Q - \sqrt{-q^2}$ is virtually independent of the reaction energy, the dipion momentum, and the angle between $q$ and the event axis. Fig. 9 shows a summary of effective radii $R$ determined using Eqn. (3) for different reaction types over a wide range of cms energies; given the systematic problems to be discussed later, the data are consistent with each other and point to an effective radius of about 0.7 - 1 fm. The source shape is consistent with approximate spherical symmetry\textsuperscript{5,13,14} (Fig. 10) and independent of the $\gamma$-factor of the pion pair (Fig. 11).

Both in Figs. 2 and 6 we note that $C$ does not seem to reach the predicted value $C = 2$ for vanishing momentum difference $q$ of the two pions. Parametrization of the BE enhancement in terms of a gaussian (Eqn.(3)) typically yields $\alpha \approx 0.5 - 0.6$ instead of $\alpha = 1$ (after correction for particle misidentification, detection efficiency etc.); see Fig. 12. The two exceptions are BE correlations in $J/\Psi$ decays and in two-photon collisions, for which $\alpha$ near 1 is measured. Several explanations have been put forward for the deviation of $\alpha$ from 1: BE correlations are e.g. absent for coherent particle emitters\textsuperscript{2,3}, hence $\alpha < 1$ could be evidence for a partial coherence of the source.

Fig. 10. Apparent size of the pion source in $e^+e^-$ annihilation at 29 GeV, determined using Eqn. 3, as a function of the viewing angle with respect to the jet axis\textsuperscript{5}. Curves are based on the assumption the the pion emitting region is a three-dimensional ellipsoid, with a transverse size $R_0$ and a longitudinal extent $cR_0$, for $c=1$ (dashed), $c=2$ (solid) and $c=3$ (dotted).
A much simpler explanation is that the measured value of $\alpha$ is usually obtained from an extrapolation of data at finite $Q^2$ to $Q^2 = 0$ and is therefore sensitive to the assumptions concerning the $Q^2$-dependence of the BE enhancement. The usual gaussian shape is used mainly for convenience and has no strong theoretical motivation. In fact, the recent models discussed above predict shapes which are much more peaked for $Q \rightarrow 0$. As shown in Fig. 6, the models are in reasonable agreement with data in the range typically covered by experiments, $Q > 50$ MeV, and nevertheless extrapolate to $C = 2$ for $Q \rightarrow 0$.

Fig. 11. Size parameter $R$ of the pion source in high-energy $e^+e^-$ annihilation, as a function of the boost $\gamma = E_{\pi\pi}/m_{pp}$ of the pion pair$^{13}$.

Fig. 12. Parameter $\alpha$ determined from fits of $C(Q^2)$ according to Eqn. (3), for different reaction types as a function of cms energy$^{1,5,13,14,15}$. Data points are corrected for particle misidentification (except for the ISR data), but are not corrected for the reduction in $\alpha$ due to pions from decays of long-lived particles.
Another reason for a non-gaussian shape is pion production by long-lived resonances such as $\omega$, $\eta$, and $\eta'$. For pions created in such decays, the effective source size is of the order $1/\Gamma_{\text{resonance}} > 20 \text{ fm}$. Correspondingly, such pions contribute$^{16}$ to the BE enhancement only for small $Q < 10 \text{ MeV}/c$ - a region not covered by experimental data, resulting in an underestimate of $\alpha$ (Fig. 13). The absence of detectable BE correlations for pion daughters from long-lived particles has been demonstrated experimentally using pions from $K_0^*$ decays$^5$.

This suppression of the BE enhancement due to long-lived resonances may also explain the striking difference between the large $\alpha$-values measured on the $J/\Psi$ and in $\gamma\gamma$ interactions, and the much smaller $\alpha$ observed at higher energies in the $e^+e^-$ continuum. Fragmentation models$^9$ indicate that at high energy only 25-30% of all pion pairs can contribute to a BE enhancement for $Q > 25 \text{ MeV}$, compared to 40% or more for $J/\Psi$ decays. The difference is caused by the absence of heavy-quark production at the lower energies (and in $\gamma\gamma$, assuming vector-meson dominance), and, rather indirectly, by the softer spectrum of secondaries. These predictions must be taken with a grain of salt, however, since the decrease in the effective $\alpha$ is very sensitive to the rates of $\eta$ and $\eta'$ production$^{17}$, which are not well measured and probably overestimated in current fragmentation models$^{17}$. The CLEO group has nevertheless attempted to correct their data for resonance effects and obtain $\alpha$ consistent with unity after correction$^{14}$ (Fig. 14).

Fig. 13. Expected $Q$-dependence of the two-pion correlation function $C$, assuming that 50% of all pairs contain at least one decay product of a long-lived resonance of decay width $\Gamma$. The dotted region indicates the region typically covered by data points.
Given the limited precision of the data and the uncertainties in the modeling, it is very difficult to draw any clear-cut conclusion at this point. Obviously, there are several mechanisms which explain $\alpha_{\text{measured}} < 1$ in a rather natural fashion; its seems premature to invoke partially coherent emitters. Clearly, more detailed data would help!

However, major technical problems stand in the way of more precise measurements. Let us first consider the $\sigma^{(2)}(p_1, p_2)$ term in the definition of $C$ (Eqn. (1)): particle pairs in the interesting region $p_1 = p_2$ tend to overlap in the detector and create pattern recognition problems. Furthermore, since the BE effects occurs only for identical particles, some particle identification is required, otherwise the data has to be corrected for a (typically 30%) contamination from other species. Finally, one needs to remove (or correct for) pions from very long-lived particles such as $K_0^S$ or $\Lambda$, and ideally one would want to reject pions from particles with $C$ or $B$ quarks. These corrections introduce additional uncertainties. Finally, the rate of pairs at low $Q$ decreases rapidly with $Q$, since the available phase space goes like $Q^2$.

Fig. 14. Two-pion correlation function $C$ as a function of $q_T$ (see Fig. 2), for $q_0 < 0.1 \text{ GeV}$. (a) uncorrected data, (b) corrected for the fraction of non-interfering pion pairs from decays of long-lived particles. From CLEO$^{14}$
Even worse, however, are the problems caused by the $\sigma_0^{(2)}(p_1,p_2)$ term in Eqn. (1). Obviously, BE effects cannot simply be "switched off" in the experiment in order to determine $\sigma_0^{(2)}$. One technique is to approximate $\sigma_0^{(2)}(p_1,p_2)$ by the product of single particle densities $\sigma^{(1)}(p_1)\sigma^{(1)}(p_2)$. This procedure removes the BE enhancement, but it also removes correlations caused e.g. by phase space constraints, superposition of different event types etc., and can result in a serious overestimate of $C(q=0)$. Another solution is to use unlike particles, i.e. unlike-sign pion pairs, to derive $\sigma_0^{(2)}$. The problem here is that while natural correlations due to phase space etc. are taken into account, the unlike-sign pion sample shows many additional correlations due to resonance decays and local charge conservation. Furthermore, acceptance corrections will usually not cancel when comparing like-sign to unlike-sign pion pairs. Even if great care is taken in handling all these problems, one is typically left with a $O(10\%)$ systematic uncertainty on the parameter $R$ for "easy" data samples - such as global BE correlations in $e^+e^-$ annihilation. For more difficult samples such as pions produced in $\nu N$ reactions\(^1\) (where event characteristics such as the hadronic mass $W$ vary from event to event) or for specific phase space region in $e^+e^-$ events, systematic errors due to the $\sigma_0^{(2)}$ determination can easily reach 50%; the systematic problems in the determination of $\alpha$ are even worse.

Let me summarize: I feel that BE correlations provide a rather interesting way to study multiparticle production dynamics; however, given our limited understanding of even the simplest cases ($e^+e^-$) and the experimental problems discussed above, I don't view BE correlations at this moment as a powerful diagnostic tool for more complicated processes such as electron scattering off large nuclei. Topics I would like to see studied (most likely in $e^+e^-$) include: the precise shape at low $Q^2$, the detailed dependence on $q$ and $q_0$ (or similar variables, see Ref. 13), and the effect (and rates) of resonances. As to serious applications in nuclear physics, I feel that one first needs to understand results from simple ($e^+e^-$) systems in a quantitative way.

This work was supported by the US Department of Energy under contract number DE-AC03-76SF00098, and by the Alfred P. Sloan Foundation. The author wants to thank M.G. Bowler and X. Artru for valuable correspondence, and acknowledges interesting and helpful discussions with G. Goldhaber, I. Juricic and M. Suzuki.
References

1. For recent reviews, see: G. Goldhaber, LESIP I Workshop, Bad Honnef, Germany (1984), and LBL-19417 (1985); G. Goldhaber and I. Juricic, LESIP II Workshop, Santa Fe, NM, 1986, and LBL-21531 (1986); and references given there.

2. M.G. Bowler, Z. Phys. C29, 617 (1985). This reference contains a very pedagogical introduction to theory of BE correlations; a more rigorous treatment is given in Ref. 3.


This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Department of Energy to the exclusion of others that may be suitable.