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Harry G. Heard
September 24, 1957

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ABSTRACT

This report concerns itself with the radial magnetic gradient of the Bevatron, a weak-focusing proton synchrotron. The effects of pole-tip configuration, eddy currents, and residual field upon the radial extent of useable magnetic field are discussed. Analytical treatments are given of the perturbing field required to change the radial gradient at any point, the field distribution resulting from an excited pair of windings, and the current distribution required to yield a given field distribution. Three methods of exciting pole-face winding are discussed comparatively.
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I. INTRODUCTION

The ring-shaped magnet, which provides the guide field for protons being accelerated in the Bevatron, is composed of four quarter-circle magnets of 50-ft radius which are separated by 20-ft straight sections. Each quadrant is subdivided into thirty-six 2-1/2 deg. wedge-shaped sectors. In addition to the outer yoke structure, which provides the return path for the magnetic flux threading the accelerator aperture, each sector contains an upper and a lower pole base and an upper and a lower pole tip (see Fig. 1). The main field for this H-shaped magnet is produced by coils located outside of the vacuum liner and disposed in pairs above and below the geometric median plane. Local control of the average magnetic field in each sector is obtained by the excitation of windings which surround the pole base slabs. Some control is exercised over the average magnetic-field gradient in each quadrant by pole-face windings that are distributed over the pole tips.

If particles are to execute stable oscillations about orbits within the aperture of a weak-focusing synchrotron, the magnetic field must decrease with radius within narrow and predictable tolerance limits. It is convenient to express this variation as

\[ H = H_0 \left( \frac{r}{r_0} \right)^{-n} \quad (0 < n < 1) \quad (1) \]

where \( H_0 \) is the instantaneous mean field at \( r = r_0 \) and \( n \) is the field exponent. Considerations of injection efficiency, gas scattering, frequency tolerance, damping of synchrotron oscillations, and various particle resonances indicate that \( n = 0.64 \pm 0.14 \) will permit stable operation with minimum losses.

As the flux density increases, saturation of the pole-tip iron reduces the radial width of the usable field region. Pole-face windings were included in the original design so as to counteract this effect. Soon after the Bevatron was operated as an accelerator, it was determined that the radial width of field was adequate to contain the particle oscillations as well as provide adequate aperture for frequency-tracking errors. The provision of high-current corrections thus proved unnecessary. The major function of the pole-face windings at present is to correct the \( n \)-value of the magnetic field at the start of the magnet cycle. These windings are also used to reduce the ripple field in the aperture and thereby decrease the ripple structure of secondary beams.\(^1\)

\(^1\)Harry G. Heard, A New Method for Controlling the Magnetic Field in the Aperture of Synchrotrons, UCRL-3427, May 1956.
Fig. 1. A Sector of the Bevatron magnet.
II. MAGNETIC-FIELD CONFIGURATION

The measured value of the uncorrected magnetic-field index is shown as a function of radial position in Fig. 2 for several values of magnetic field. It will be noted that the average n-value at 287 gauss (corresponding to injection) approaches 1/2. This low n-value permits coupling between horizontal and vertical betatron oscillations and results in substantial beam loss.  

After the Bevatron had been in operation for some time, it was discovered that a considerable reduction in beam loss could be effected by raising the average value of n by approximately 0.1 with pole-face windings. These windings are self-excited throughout the magnet cycle. Their effect decreases nearly hyperbolically with magnetic field so that corrections are negligible at 5000 gauss.

III. FACTORS AFFECTING THE MAGNETIC FIELD GRADIENT

The deviation of the observed n-value from that calculated from the gap geometry is due to the residual field distribution, and the steady-state eddy-current field. Although these perturbations cause a net reduction in the n-value during the entire acceleration cycle, their effect is most significant during the first 200 millisec.

The residual-field distribution depends upon the geometry of the magnet, the intensity of magnetization of the iron, and the mode of decay of the magnetic field. Measurements have been made of the residual-field distribution for a magnet pulse rate of 10 pulses per min. The magnetic field increased to 16 kgauss in 1.75 sec and decreased to the residual level in 2.4 sec. These data appear in Fig. 3. In general the residual field is seen to decrease with radius; this effect is large enough to cause a net increase in n-value.

The eddy-current effects may be divided into two categories. There is a time delay of the order of millisec. between the magnet flux and the excitation current. This time lag must be considered in the design of frequency tracking equipment if the oscillator frequency is controlled by a sample of magnet current. In addition there is a quasi steady-state magnetic-field perturbation due to the average rate of change of magnetic field. The eddy currents, which circulate in the 1/2-in. iron laminations in the return

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2Glen R. Lambertson, Testing the Magnetic Field of the Bevatron, UCRL-2818, November 1954.


Fig. 2. Radial variation of magnetic-field index at low fields pole-face windings not excited.
Fig. 3. Radial variation of the average residual magnetic field.
yoke and in the 0.050-in. stainless steel wall of the vacuum liner, have the net effect of reducing the value of $n$ over the aperture. While these effects persist over the entire acceleration cycle, they are most serious at injection wherein the $n$-value is reduced by approximately 11%. The change in $n$-value due to the steady-state eddy-current field is shown in Fig. 4. It will be noted that the reduction in $n$ on the outer radius is approximately twice as great as on the inside radius. These data were obtained by a comparison of the uncorrected magnetic-field gradient at 287 and 1255 gauss.

To obtain the magnetic-field distribution due to the quasi steady-state eddy-current field, one must measure the field with the magnet excited from a dc source. Because this measurement has not been made, one can only calculate the expected effect of the average eddy currents. The main effect of the average eddy-current field is to cause the $n$-value to decrease on the outside radius and increase on the inside radius.

IV. POLE-TIP DESIGN

Three factors which affect $n$ were considered in the design of the Bevatron pole tips. The average gradient of the field was obtained by introducing a linear taper in the pole-tip faces. The value of the gap taper may be obtained from the following considerations. If the iron is assumed to have infinite permeability, then the product of the gap, $g$, and field, $B$, is a constant $Bg$ so that

$$B_g = \text{const.} \quad (2)$$

$$\frac{\delta B}{B} = -\frac{\delta g}{g}. \quad (3)$$

Now from the definition of $n$ we have

$$\frac{\delta B}{B} = -n \frac{\delta r}{r}. \quad (4)$$

Substituting and integrating, we obtain

$$g = g_o \left(\frac{r}{r_o}\right)^n \quad (5)$$

Thus $n < 1$ corresponds to a gap that increases with radius. The constants for the Bevatron gap are $g_o = 13$ in. for $r_o = 599.3/8$ in. and $\delta g/\delta g = 13.68 \times 10^{-3}$ in/in for $n = 0.63$. The value of $n$ was increased from $n = 0.60$ to $n = 0.63$ in the last phase of the model-magnet testing program when it became evident that eddy-current effects would cause the effective $n$-value to decrease at low fields.

Because of the finite radial extent of the pole tips, fringing fields exist at the edges. A slight increase in the radial width of useable field at low flux densities can be obtained if the pole tips are contoured so as to decrease
Fig. 4. Change in the Bevatron magnetic-field index at injection (~287 gauss) due to eddy currents.
the gap near these boundaries. The configuration of the pole tips shown in Fig. 5 was determined by potential plots and magnetic measurements on the 1/7-scale model. The net effect of this change is to increase the effective width of the pole tip by approximately 5%.

As the magnetic field increases above 10,000 gauss, saturation of the pole tips causes the useful radial width of the magnetic field to decrease. The large aperture required for efficient injection is not needed at higher energies, as the amplitude of a particle's radial oscillation decreases. Considering this and the desirability of obtaining a high flux density per unit peak current, triangular cutouts were made in the pole tip laminations at the outer edges. The loss of iron has very little effect at low magnetic fields. As the iron saturates, however, these holes have a net effect of increasing the peak flux density by 7% (see Fig. 6). The increase in peak flux density is obtained at the expense of usable radial width of field. The radial extent of the usable field has been found to be adequate. Although the peak flux density does increase, the peak energy of the accelerator does not increase by the same amount. This is because the region of usable field increases in radius at high flux densities, a direct result of pole-tip saturation produced by the cutouts. There is a net gain in energy, however, as the fractional increase in flux is greater than the fractional increase in radius.

V. POLE-FACE WINDING DESIGN

Four quantities must be considered in the design of pole-face windings. These include the amount of correction desired, its time dependence, the portion of the aperture occupied by the windings and their support structure, and the voltage induced in the winding by the main-field pulse.

The pole-face windings were originally designed to correct the field exponent of the magnet at 12 to 15,500 gauss. Numerical analysis indicated that peak currents of the order of 2000 amp would be required if a maximum field width of 17 in. was to be obtained. This high peak current dictated the installation of windings fabricated from 3/8-in. diameter water-cooled copper pipes with adequate mechanical structure to resist forces of 200 lbs/ft. The support structure for these windings projects 1/2 in. into the useful aperture from each pole tip. A total of 21 pairs of windings were distributed circumferentially over the pole tips. These windings are spaced in 3-in. radial increments over the pole tips. All windings were insulated from one another and from the pole tips for 1000 v. Vacuum feed-throughs and electrical interconnections were provided at each end of each of the windings in the straight sections.

The magnitude and sign of the voltage induced in a pole-face winding depends upon the location of the winding within the aperture. The point of division of magnet flux between the inner and outer leg yokes of the magnet is centered at 582 in. at low magnetic fields. As the magnetic field increases, this point shifts to larger radii. Thus the induced-voltage distribution is asymmetrical and varies with time. At 300 gauss the induced voltage is zero at 582 in. and increases linearly with radius with a slope of approximately 0.6 volts/inch/quadrant.
Fig. 5. Bevatron pole-tip configuration (not to scale). Ref. UCRL print 3Q2766.
Fig. 6. Effect of notched pole tips on magnetization curve of 1/16-scale bevatron model.
VI. DETERMINATION OF THE CORRECTION FIELD

The distribution of the correction field required to achieve a given n-value may be approximately determined from the following analysis. Consider the instantaneous value of the total magnetic field in the geometric median plane of the aperture. For convenience let the reference point \( r_0 \) be located on the centerline of a pair of excited pole-face windings. Decompose the total magnetic field \( H(r) \) into the main field \( h(r) \) and the perturbation field \( g(r) \) due to the excitation of that pair (upper and lower) of pole-face windings. Then for \( r_2 > r_1 > r_0 \) we have

\[
H(r_1) = h(r_1) + g(r_1 - r_0)
\]

and

\[
H(r_2) = h(r_2) + g(r_2 - r_0) .
\]

Now use the notation

\[
x = \frac{x_1 + x_2}{2} \quad \text{and} \quad \Delta x = x_2 - x_1 ,
\]

where \( x = g, r, h, \) and \( H \) as required. From the definition of \( n \) it follows that the desired value of \( n \) is

\[
- n = \frac{\Delta r}{\Delta r} \left( \frac{\Delta H}{H} \right) = \frac{\Delta r}{\Delta r} \left[ \frac{\Delta h + \Delta g}{g - h} \right] ,
\]

but for \( h \gg g \), which is always true, the value is

\[
- n \approx \frac{\Delta r}{\Delta r} \left( \frac{\Delta h}{h} \right) \left[ 1 + \frac{\Delta g}{\Delta h} \right] .
\]

For convenience, define \( n_0 \) as the uncorrected value of \( n \):

\[
- n_0 = \frac{\Delta r}{\Delta r} \left( \frac{\Delta h}{h} \right) \quad \text{and} \quad \Delta n = n_0 \left( \frac{\Delta g}{\Delta h} \right) .
\]

Then we have

\[
n = n_0 + \Delta n
\]

and

\[
\Delta g = h \left( \frac{\Delta h}{h} \right) \left( \frac{\Delta n}{n_0} \right) .
\]

From (11) and (12), the desired value of \( n \), the uncorrected n-value and the magnetic data for the Bevatron magnet, one can determine the field distribution required. Note that these relations do not specify the current distribution required to achieve this correction field.
VII. THEORETICAL BEHAVIOR OF A PAIR OF POLE-FACE WINDINGS

Consider the change in the main field distribution caused by the flow of a unit current in a pair of pole-face windings. For simplicity assume that the pole tips are semi-infinite in extent, have very high permeability, and are separated by a constant gap, \( l \). Then the change in flux density in the median plane because of a current \( I \) in each winding of a pair of pole-face windings, at a distance \( r \) from the center of the winding, measured in the median plane may be expressed in terms of images as

\[
B_z = \sum_{k=0}^{\infty} B_k \left( \frac{r}{x_k} \right),
\]

where we have

\[
B_k = \frac{\mu_0}{2\pi} \left( \frac{4I}{x_k^2} \right),
\]

and

\[
x_k^2 = r^2 + (k + 1/2)^2 l^2
\]

is the distance from the point of interest to the image of order \( k \). Thus we obtain

\[
B_z = \frac{8\mu_0}{\pi l^2} \sum_{k=0}^{\infty} \frac{r}{\left( \frac{2r}{l} \right)^2 + (2k + 1)^2}. \tag{15}
\]

For the Bevatron this may be written as

\[
\frac{B_z}{I} = 0.03806 \, \xi \, \text{ (gauss/amp) in the median plane} \tag{16}
\]

where we have

\[
\xi = \frac{1}{5.105} \sum_{k=0}^{\infty} \frac{r}{\left( \frac{2}{13} \right)^2 r^2 + (2k+1)^2}. \tag{17}
\]

Values of \( \xi \) are given in Table I as a function of the distance from the center of a pair of pole-face windings.

The magnetic-field distribution per unit current was measured with the 1/7-scale model for a main field of 300 and 15,500 gauss and on the full scale magnet at 15,500 gauss. The agreement of these data with this model was sufficiently good that all further calculations for magnetic-gradient correction were based upon the theoretical model.
Table I

Variation of $\xi$ with distance, $r$, from the center of a pair of pole-face windings

<table>
<thead>
<tr>
<th>$r$ inches</th>
<th>$\xi$</th>
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<tr>
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</tr>
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</tr>
<tr>
<td>15</td>
<td>0.999</td>
</tr>
<tr>
<td>18</td>
<td>1.000</td>
</tr>
</tbody>
</table>

VIII. CALCULATION OF THE CURRENT DISTRIBUTION

Once the required correction-field distribution and pole-face winding field-current characteristics have been determined, the current distribution can, in principle, be computed from the relation

$$B_z(r_i) = \sum_k \sum_i B_{ki} \left( \frac{r_i}{x_{ki}} \right).$$  

(18)

In general it will only be possible to generate an approximation to the desired field distribution. The tolerance on $n$ is large enough, however, that a sufficiently good match may be obtained. Figure 7 shows the uncorrected and corrected values of $n$ for the Bevatron.

IX. EXCITATION OF THE POLE-FACE WINDINGS

Three factors must be considered in the selection of a voltage source to force current through the pole-face windings. These include the desired current-time characteristic, the magnitude of the time-dependent induced voltage and the electrical characteristics of the pole-face windings.

The current-time characteristic for correction of the magnetic field index is, fortunately, of simple form. A current of the form

$$I = I_0 (1 + at) \quad (0 < t < 1.0 \text{ sec.})$$  

(19)
Fig. 7. Effect of pole-face windings on the magnetic field index at injection.
is applied to the pole-face windings. The increase in current with time
\( (a \geq 1.0) \) insures that the correction in \( n \) will not vanish too rapidly as
the main field increases.

The equivalent circuit of a pole-face winding of one-quadrant length is
an inductance of 48 \( \mu \) henry and a resistance of \( 63 \times 10^3 \) ohms in series.
Because this corresponds to a time constant of approximately 750 \( \mu \)sec, the
circuit acts very much like a pure resistance for the rates of change of
current of interest in correcting the magnetic-field gradient. Therefore,
once the voltage-time characteristic of the voltage source the voltage-time
characteristic of the induced voltage of each winding, and the current dis-
tribution in the windings are known, the values of the series resistance for
a shunt-excited circuit are determined.

The pole-face windings may be excited by an external generator \(^7\) or may
be self-powered from auxiliary windings on the magnet, provided the voltage
on the auxiliary windings exceeds the induced voltage on the pole-face
windings. The latter method is very reliable and is used in the Bevatron.
The excitation voltage is developed by a 5-turn loop of 1-0 flexible cable
which couples the inner-leg-yoke flux of the magnet. The correct current
distribution is obtained from a matrix of resistors which permits separate
excitation of the windings.

The pole-face windings may be either connected in series or in parallel.
If they are connected in series, the same current will flow in each winding,
and the general effect of varying the series current will be to vary the
average value of \( n \) over the entire aperture at once. The pole-face wind-
ings were originally connected in this way to permit a survey of the range
of \( n \)-values at injection that corresponded to maximum accelerated beam. It
was found that pole-face-winding correction currents corresponding to
values of \( n=0.65 \pm 0.05 \) gave optimum charge acceptance.

The disadvantages of series operation of pole-face windings are twofold.
First, the plus and minus induced voltages sum to a value of the order of
600 to 700 \( V \) peak. This had led to spark-over in pressure ranges where
Paschen's-Law discharges can occur. Second, it is difficult to make
secondary field corrections between windings to remove local regions of
poor \( n \)-values. The shunt-connected pole-face-winding circuit now in use
in the Bevatron is shown in Fig. 8. Note that there are two internal loops
that carry currents to correct for a low \( n \)-value in the region of 618 in. and
a high \( n \)-value in the region of 577 in. Through the use of these secondary
corrections, it has been possible to increase the useful radial aperture of
the Bevatron to 46 in. at injection.

ACKNOWLEDGMENTS

The author wishes to express his appreciation to William M. Brobeck,
Glen R. Lambertson, and Lloyd Smith for their consultations and encouragement.

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\(^7\) Blewett, Blewett, Moore, and Smith, Rev. Sci. Inst. 24, 773 (1953).
Fig. 8. Simplified network for self-excited, shunt-connected pole-face windings.