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On the Theory of Strengthening by Coherent Ordered Precipitates

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On the Theory of Strengthening by Coherent Ordered Precipitates

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In recent papers both we [Glazer and Morris, 1987a] and Ardell and Huang [1988] have discussed the strengthening contribution of coherent, ordered precipitates whose size is near that at which Orowan looping replaces dislocation shearing as the mechanism of dislocation bypass. This regime is of considerable technological as well as academic interest since it occurs near peak strength in precipitation-hardened alloys. The Al₃Li (δ') and Ni₃Al (γ) precipitates were considered explicitly. The strengthening contribution of these precipitates is largely due to the antiphase boundary that is created when they are sheared, although coherency strains also contribute to the strength of the γ precipitate. While there are many points of agreement between the two sets of authors, who use basically similar models of the critical resolved shear stress (CRSS), the analyses differ in several respects and suggest rather different values for the antiphase boundary energy of the δ' phase, whose value was a major point of the exercise.

Because of the complexity of the dislocation interactions that govern yielding behavior in real crystals all tractable models that treat the problem are based on strong simplifying assumptions. Nonetheless, they serve three important functions. First, they advance the qualitative understanding of plastic deformation by identifying the critical mechanisms and phenomena that govern the process. The value of the model in this respect depends on the physical accuracy of its assumptions. Second, if the assumptions of the model describe the process to first-order, then the model may be used to predict strengthening behavior from a microstructural description. Finally, physical properties that are not used as input to the model may be derived from it (in these studies, the antiphase boundary energy). These latter two items are the quantitative consequences of the model. Their accuracy is affected by both the physical assumptions of the model and the approximations in the mathematics used to implement it.

Given the strength of the basic assumptions that underlie all of these models it is difficult to decide their relative value on the basis of their quantitative agreement with experimental data. The agreement (or lack of it) is as likely to arise from cancelling approximations as from numerical accuracy. However, their qualitative differences are, at least in principle, amenable to experimental investigation, and deserve particular study since they govern our understanding of the hardening process.

The most important qualitative difference between the models advanced by ourselves and by Ardell and Huang concerns the nature of the yielding process near peak
strength in a material that contains a realistic distribution of precipitate sizes. In our work the peak strength is assumed to correspond to a situation in which the largest of the hardening precipitates are looped rather than sheared. This, we claim (Glazer and Morris [1987b, 1988]), is a straightforward consequence of the manner in which the strengths of obstacles of different sizes sum to produce the CRSS. Although actual precipitate size distributions in precipitation-hardened alloys vary considerably [e.g. Mahalingham, Gu, Liedl and Sanders, 1987] the diameter of the largest precipitates is significantly greater than the average value, \(\langle d \rangle\). Precipitate coarsening models predict that the size of the largest precipitates will be 1.5\(\langle d \rangle\) or more [e.g. Lifshitz and Slyosov, 1961; Ardell, 1972; Brailsford and Wynblatt, 1979; Davies, Nash and Stevens, 1980]. Hence in our model of hardening by coherent precipitates [Glazer and Morris, 1987a] the peak strength is assumed to be reached when a precipitate of size 1.5\(\langle d \rangle\) would be looped by the Orowan mechanism.

Ardell and Huang [1988] also associate the peak strength in alloys hardened by coherent ordered precipitates with the transition from shearing to looping, but assume that peak strength occurs when the looping diameter is equal to the average planar diameter of the precipitate, near \(\frac{\pi}{4}\langle d \rangle\). They do not discuss this choice in the context of the theory of obstacle strengthening, but rather suggest that it is in agreement with several experimental observations.

This difference between the two criteria for peak strength has both qualitative and quantitative significance. The quantitative difference is substantial. If \(\frac{\pi}{4}\langle d \rangle\) is selected, the predicted antiphase boundary energy is 70% higher than if 1.5\(\langle d \rangle\) is selected. This difference dwarfs all other factors that differ between the analyses of Ardell and Huang [1988] and Glazer and Morris [1987a], and is responsible for most of the difference between their respective values for the antiphase boundary energy of \(\delta^\prime\).

However, the qualitative difference is, we believe, even more important, since it affects the nature of hardening at peak strength. The criterion proposed by Ardell and Huang requires a substantial re-thinking of the existing theories of the CRSS. The theories known to us that treat the CRSS of a solid that contains a distribution of hardening precipitates can be fit by a summing rule of the form

\[
\tau^q = \sum \tau_i x_i^{q/2} \tau_i^q
\]

where \(\tau\) is the CRSS of the entire array, \(x_i\) and \(\tau_i\) are the volume fraction and CRSS, respectively, due to the \(i\)th obstacle, and the exponent \(q > 1\). As shown in detail elsewhere (Glazer and Morris [1987b, 1988]) a quadratic summing rule (in fact, any summing rule with \(q > 1\)) leads to the prediction that the value of \(\langle d \rangle\) at peak strength is less than the looping radius, and is sensitive to the shape of the precipitate distribution and especially to the size of the largest precipitates. Both Ardell and Morris and their coworkers, as well as many other investigators, have used summing rules with exponents higher than 1.5 in their other work [see Ardell, 1985 for a review]. If Ardell and Huang are correct in their assumption then either summation laws like equation (1) are invalid or something important is missing from the theory of hardening by distributions of precipitates. There are relevant
physical phenomena that are not accounted for in the simple models that yield summing rules like eq. (1), such as the transition from paired to unpaired dislocation motion as looping replaced shearing as the dominant mechanism of obstacle passage. However, no analysis known to us suggests that these invalidate the summing rule prior to peak strength.

The question of the relative timing of the onset of Orowan looping and the occurrence of peak strength should be resolvable by experiment. However, the available experimental data is decidedly ambiguous. The ambiguity is indicated by the fact that we [1987a] and Ardell and Huang [1988] cited a partly overlapping set of references to support very different conclusions. The interpretation of prior experimental work is complicated by a number of factors, including differing definitions of the minimum looped diameter (average diameter when some precipitates are looped vs. smallest precipitates that can be looped whatever the average diameter), the low sensitivity of strength to ⟨d⟩ that is often found at peak strength, the fact that the studies were done on relaxed samples, often after straining by several per cent, and the fact that the investigators were unaware that the issue might be controversial.

On re-reading in light of Ardell and Huang's [1988] discussion, we must concede that that the experimental data we cited [1987a] do not support our interpretation of the looping phenomenon as clearly as we thought at the time. On the other hand, they do not establish the position taken by Ardell and Huang [1988] either. Of the seven separate investigations Ardell and Huang [1988] cite in support of their position, by our reading only one [Chaturvedi, et al., 1976] shows unambiguously that particles of average size are looped at peak strength. However, the system studied in this case, Ni-Ti, has a large misfit energy that may stabilize loops at smaller size and allow continued strengthening beyond that point. Only one piece of prior work [Miura, 1986] specifically addressed this issue for the Al-Li system that is the major subject of controversy. Both the wording and the micrographs in Miura's paper are ambiguous with respect to the degree of looping at peak strength, and the author himself believes that the issue is undecided [Miura, 1989].

It follows that the available experimental data does not resolve the issue of the relative importance of looping and shearing at peak strength in alloys hardened by coherent precipitates. The issue is a critical one since it qualitatively constrains the kinds of models that can be used to treat hardening by ordered precipitates, and qualitatively affects understanding of the manner in which obstacles of different strength interact. Probative experiments are needed. New insight into the coupling of superdislocations in the peak strength regime would be particularly helpful since decoupling is neglected in the current models.

The model proposed Ardell and Huang [1988] differs from ours [Glazer and Morris, 1987a] in several other details. The most important is the manner in which the results of Bacon, Kocks and Scattergood [1973] are used to incorporate the line tension and dislocation segment interaction. The model of Bacon, et al. [1973] is the most thorough study known to us, but it describes the dislocation self-interactions in an array of impenetrable obstacles in an average sense. It considers neither the effect of randomness in the array of obstacles nor the increased importance of self-interactions when the obstacles are penetrable; both are important to the problem at hand. The issues involved in applying
the results of Bacon, et al. are subtle, but the numerical consequence of the different approximations used by Ardell and Huang and by us are relatively small. A definitive theoretical treatment can only be obtained by building a model that includes self-interactions in the initial mathematical formulation.

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