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Abstract We review the "slope anomaly" data for 2-neutron transfer between heavy ions. Where one or both partners are deformed, the 2-neutron transfer always shows about the same dependence on distance-of-closest approach D as does 1-neutron transfer. Where both partners are spherical, the 2-neutron transfer falls off as the square of the 1-n transfer with one known exception 208pb(64Ni,62Ni)210pb. We propose that the enhancement of 2-n transfer comes from the pairing attraction between the neutrons and that the enhancement requires availability of high-l orbitals to form a tight wave packet in angle. A simple model is derived for numerical estimation. Prospects for testing Coriolis antipairing by heavy ion 2-n transfer are re-examined.

There is an unresolved puzzle regarding the dependence of 2-neutron transfer on distance of closest approach of heavy nuclei. We gave a brief review of the situation for deformed nuclei last year. 1

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From early Breit-Ebel theory the general logarithmic dependence of 1-neutron transfer cross sections on distance of closest approach 0 is understood. Indeed the slopes generally agree with the square-barrier WKB tunneling exponent depending on neutron separation energy and neutron mass.

Work of von Oertzen et al. for Sn-on-Sn isotopes and by Sapotta et al. for Ni on $^{208}$Pb show the expected slope behavior for nearly all these systems with spherical partners. Work of various authors working with deformed systems (cf. Ref. 1) also show the expected slope behavior for 1-neutron transfer.

The puzzle arises when examining the slopes of 2-neutron transfer. In all cases studied for one or both partners being deformed the 2-neutron slopes are comparable to the 1-neutron slopes. The simple theoretical prediction, whether sequential or simultaneous 2-neutron transfer dominates, is for the 2-neutron slope to be about twice that of the 1-neutron. That is, the 2-neutron transfer probability should fall off with distance exponentially and as the square of the 1-neutron transfer probability. In almost all cases with spherical partners the expected double slope appears, except that the addition of 2-neutrons to $^{208}$Pb shows the anomalously small slope of the deformed cases, whereas the other transfers removing neutron pairs from $^{208}$Pb all show the expected behavior of double slope.
TABLE I  The parameter $c_x$, which describes the slope of the transfer probability with the distance $D$ (Ref. 4).

<table>
<thead>
<tr>
<th>$x$</th>
<th>-2$n$</th>
<th>-1$n$</th>
<th>+1$n$</th>
<th>+2$n$</th>
<th>+3$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{208}$Pb + $^{58}$Ni</td>
<td>1.04</td>
<td>1.04</td>
<td>1.96</td>
<td>3.2</td>
<td></td>
</tr>
<tr>
<td>$^{208}$Pb + $^{64}$Ni</td>
<td>1.04</td>
<td>1.01</td>
<td>1.21</td>
<td>2.33</td>
<td>3.4</td>
</tr>
</tbody>
</table>

We show in our Table I the slopes determined by Sapotta et al.\(^4\) (their Table V). Only the addition of two neutrons to $^{208}$Pb has the anomalously low slope.

In order to characterize the anomalously small slopes quantitatively, Himmele et al.\(^5\) used the language of an effective excitation energy of the transferred pair. That excitation terminology has been used in later papers of others, though whether the excitation is thought of as real or virtual is not generally defined. Anyway, the work of Ref. 4 and the recent results of Guidry and collaborators with Ni or Sn-on-Dy isotopes seem to show\(^6\) mostly "cold" transfer at and near the yrast line. Thus, there cannot be much real excitation if the excitation picture has any validity.

We shall below argue that the enhanced 2-neutron tunneling, where observed, is mainly a pairing effect. Why should enhanced tunneling (small slopes) be seen in some cases and not in others? Based on the present limited data set we postulate that to realize pairing-force energy lowering during simultaneous 2-neutron transfer at least one partner must have available near the Fermi surface two or more high-j orbitals. With such high-j orbitals available the neutron pair can form a tight correlation in angle, thus exploiting the short-range pairing force.
Examination of a Nilsson level diagram will show that for deformed rare-earth or actinide nuclei the various j orbitals are split widely by the quadrupole field; there are always at least two high-j orbitals available to form a tight angular wave packet. For the transfer on $^{208}\text{Pb}$ the addition of neutrons goes into $g_{9/2}$, $i_{11/2}$, and $j_{15/2}$ region, whereas the removal of neutrons must use the $p_{1/2}$, $f_{5/2}$, and $p_{3/2}$, primarily.

To be somewhat more quantitative in making this argument we note that zero-angle lobe of a Legendre function squared has an angular width of about $1/L$ radians. At moderately large $L$ the Legendre function can be approximated as a cosine wave within a $1/(\sin \theta)^{1/2}$ envelope. Thus, a linear combination of at least two high-$L$ orbitals is needed to cancel the side lobes and realize a wave packet of width $1/L$. If one high-j orbital were sufficient to form a tight wave packet, then some of the Sn-on-Sn two-neutron transfers of Ref. 3 would show slope anomalies by virtue of the $h_{11/2}$ orbitals.

Consider the two-neutron transfer as a two-dimensional barrier penetration problem in the center-to-center direction coordinates. The value of the angular integral will fix the depth of the pairing potential in the radial coordinate. (This will be illustrated shortly in Figs. 1 and 2.)

The width of the 45 degree valley in the $r_1 - r_2$ plane is governed mainly by the n-n force range. Hence, for an angular force integral not too small, the cross width of the wave packet should be independent of separation distance, and the energy tied up in the zero point energy of the cross-mode should be constant.

Thus, the angular integral should directly give the
DEPENDENCE OF HEAVY-ION NEUTRON TRANSFER ... barrier lowering. Let us do the angular integral using the Cartesian approximation. We take the coordinates of the first neutron about the center of the nucleus in which the neutrons originate as \( x_1, y_1, \) and \( z_1 \), where the z-axis is along the line of centers and \( \ell \) is an average angular momentum of the nucleon orbitals involved. Neutron 2 has coordinates \( x_2, y_2, \) and \( z_2 \). Let \( R \) equal the nuclear radius from which shell model orbitals are assigned. Take

\[
\psi(x_1,y_1,x_2,y_2) = N \exp[-(\ell^2/2R^2)(x_1^2 + y_1^2 + x_2^2 + y_2^2)],
\]

which is an approximation to a forward-angle Legendre function, with \( N \) the normalization factor. We ignore spin and antisymmetrization.

\[
V(\vec{r}_1 \vec{r}_2) = -V_1 \exp\left[-\frac{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}{r_0^2}\right] \tag{1}
\]

\[
V(z_1 z_2) = -V_1 \exp[-(z_2-z_1)^2/r_0^2]
\]

\[
\int\int\int\int \psi^* \exp\left[-\frac{((x_2-x_1)^2 + (y_2-y_1)^2)}{r_0^2}\right] \psi \, dx_1 \, dx_2 \, dy_1 \, dy_2.
\tag{2}
\]

where \( V_1 \) is the Gaussian strength parameter for the \( nn \) force and \( r_0 \) is the range.

Changing variables to relative \( \xi \) and center of mass \( \chi \), we have

\[
\xi = 2^{-\frac{1}{2}}(x_2 - x_1) \tag{3}
\]

\[
\chi = 2^{-\frac{1}{2}}(x_2 + x_1) \tag{4}
\]

and similarly for the \( y_i \) coordinates.
where we have chosen a normalization that makes the transformation canonical.

\[ V(z_2 - z_1) = -V_1 \exp[-(z_2 - z_1)^2/r_0^2] \]

\[ \left\{ N^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left[ -\frac{\xi^2}{R^2} \left( X^2 + \xi^2 \right) - \frac{2\xi^2}{r_0^2} \right] dX d\xi \right\}^2. \]  \hspace{1cm} (5)

The integral is squared because it is identical in \( x_1 \) and \( y_1 \) variables.

Normalization \( N \) is

\[ 1 = N^2 \left[ \int_{-\infty}^{\infty} \exp \left[ -\frac{\xi^2}{R^2} X^2 \right] dX \right]^4 \]  \hspace{1cm} (6)

\[ V(z_2 - z_1) = -V_1 \left( 1 + \frac{2R^2}{\xi r_0^2} \right) \exp[-(z_2 - z_1)^2/r_0^2] \]  \hspace{1cm} (7)

The critical \( \ell \) value of orbitals to allow exploitation of the pairing attraction is

\[ \ell_c = \sqrt{2R/r_0} = \text{Nuclear radius/neutron-neutron force range} \]

For \( ^{208}\text{Pb} \), this is about 8, so \( \ell_{11/2} \) and \( j_{15/2} \) are needed.

Finally, we obtain

\[ \Delta B = -V_1 \left( 1 + 2R^2/\ell^2 r_0^2 \right)^{-1} \]

\[ \Delta B = -13.5 \text{ MeV for } \ell = 6 \]

\[ \Delta B = -2.3 \text{ MeV for } \ell = 2 \]

with \( V_1 = 35 \text{ MeV}, r_0 = 1.4 \text{ fm}, R = 7.5 \text{ fm}. \)

Having performed the angular integrals, we are now able to represent the potential surface in the radial
coordinates of the two neutrons. Figures 1 and 2 show contour plots of the potential for a system of two particles along the internuclear (Z) dimension. Figure 1 involves two rounded square wells of the size of tin nuclei at a surface-to-surface distance of 6 fm with no pairing interaction ($V_1$ of Eq. (1) set to zero). Figure 2 is the same except for addition of the pairing interaction with

\[ V(z_1, z_2) = -V_0 \sum_{i=1,2} \left\{ \left[1 + \exp((-15)/0.6)\right]^{-1} - \left[1 + \exp((-3)/0.6)\right]^{-1} \right\} \]

with $V_0 = -40$ MeV.

FIGURE 1. Contour plot of the potential of a system of two neutrons in nearby Woods-Saxon rounded square wells. The abscissa represents the coordinate of neutron 1 along the center-to-center axis, and the ordinate is the corresponding value for neutron 2. Contours are at 10 MeV intervals.

\[ V(z_1, z_2) = -V_0 \sum_{i=1,2} \left\{ \left[1 + \exp((-15)/0.6)\right]^{-1} - \left[1 + \exp((-3)/0.6)\right]^{-1} \right\} \]

with $V_0 = -40$ MeV.
FIGURE 2. Same as Figure 1 but with a short range attractive Gaussian pairing term added (Equation 2) with $V_2$ ($\equiv V_1$ times the angular integral) taken as 12 MeV and the range $r_0$ taken as 1.4 fm.

$V_2 = 12$ MeV and $r_0 = 1.4$ fm. One sees there are four potential wells. A point in the lower left-hand well denotes the location of both neutrons in the left-hand (parent) nucleus. The upper right-hand well denotes the final state location of both neutrons in the right-hand nucleus. The other two wells represent the intermediate states of one neutron having transferred. Thus, sequential transfer is the tunneling from lower left-hand well via one of the intermediate wells to the upper right-hand well. Simultaneous transfer is the direct tunneling along the 45 degree diagonal to the upper right-hand final state. If there is no pairing potential gain along the 45 degree diagonal, then it is easy to show for finite square-well
DEPENDENCE OF HEAVY-ION NEUTRON TRANSFER . . .

potentials that the slope for transfer probability vs. distance of closest approach is the same for sequential or simultaneous transfer. In the latter case the barrier is twice as high, but the path length for tunneling is shorter by the square root of two, thus, giving the same WKB integral. In Figure 2 we see that the pairing interaction term has carved a valley along the 45 degree line, cutting the central plateau and thus facilitating tunneling. Of course, the initial and final states of the two neutrons in the wells are lowered also by the pairing interaction, but the configuration space in the wells is larger than that where both neutrons are in the barrier. Hence, tunneling is enhanced by the interaction.

The angular integral evaluated previously will determine the depth of the pairing-induced valley along the 45 degree line. The width of the valley is governed by the range of the attractive short-range pairing force. The most general solution would involve solving coupled-channel equations through the barrier. It is convenient to convert to relative-center-of-mass coordinates, equivalent to a rotation of axes by 45 degrees. For the purpose of rough estimates here we may use the vibrationally adiabatic approximation to the two-dimensional barrier penetration problem. That is, we may subtract the zero-point energy across the valley (relative motion) from the available pair center-of-mass motional energy for barrier penetration along the valley.

For the Pb transfer case above, we assume the $\ell = 2$ and $\ell = 6$ cases give about the same cross-valley zero-point energy. Hence the barrier for 2-neutron tunneling is reduced by $13.5 \text{ MeV} - 2.3 \text{ MeV} (= -11 \text{ MeV})$ in the example. This lowering accounts for the slope anomaly.
NUCLEAR SQUID EFFECTS

Broglia and collaborators\(^7\) have showed on single j-shell models that there is an oscillation of pair transfer matrix elements as one increases the cranking velocity. Nikam, Ring, and Canto\(^8\) have recently made theoretical calculations on a more realistic system that confirm that pair transfer matrix elements might undergo sharp drops or even sign changes at modest cranking velocities near where alignment of \(i_{13/2}\) orbitals sets in. The numerical example of Ref. 7 is \(^{166}\text{Hf}\), the neutron number of which is the same as \(^{162}\text{Dy}\), a target in the experimental studies on the "spin spectrometer" at Oak Ridge. Thus, it seems appropriate to search for this effect in the existing data. To gain a better understanding of the transfer process and its slope problem for deformed nuclei it is obvious that we need studies of more exclusive nature. That is, we need to be able to examine transfer to low angular momentum and low excitation states separately from other processes.

The nuclear squid prediction does not essentially alter the goals set forth for heavy-ion 2-neutron transfer in the theoretical paper of Guidry, et al.\(^9\) a few years ago. There it was pointed out that Coulomb excitation on the inward path might pump a deformed target to rotational levels high enough that Coriolis anti-pairing (CAP) reductions in the pair transfer could be measured. Later classical orbital calculations\(^10\) reproduced here in Figure 3 held the hope that Pb ions on rare earths might drive the spin at closest approach up to the first \(i_{13/2}\) pair alignment ("band-crossing" or "backbending"). What is new in the SQUID paper is that the 2n-transfer strength might drop more sharply near the alignment than previously...
FIGURE 3. Classical orbital calculations of the distributions of rotational angular momentum at closest approach for several heavy ion collision systems. Note that the high-spin tail of the Sn-on-Gd system extends into the rare-earth backbending (spin alignment) region of interest. 

believed, and the CAP effects might be seen by careful analysis with projectiles as light as Sn, though heavier beam work is still desirable.
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7. R. Broglia, in Proceedings International Summer School on Theory of Nuclear Structure and Reactions, La Rabida, Huelva, Spain, June 23-July 6, 1985, Pairing phase transitions in nuclei (see Figure 7).


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