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Cross-sectional Variation of Measurement Error and Predictability of Earnings and Stock Returns

By

Jung Hoon Kim

A dissertation submitted in partial satisfaction of the requirements for the degree of

Doctor of Philosophy

in

Business Administration

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

Professor Richard G. Sloan, Chair
Professor Patricia M. Dechow
Professor Shai Levi
Professor Adam Szeidl

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Cross-sectional Variation of Measurement Error and Predictability of Earnings and Stock Returns

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by

Jung Hoon Kim
Abstract

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Doctor of Philosophy in Business Administration

University of California, Berkeley

Professor Richard G. Sloan, Chair

In capital markets research, market expectation of future earnings plays a vital role. However, almost all proxies inevitably measure the market expectation of future earnings with error, which results in unsatisfactory empirical outcomes in prior research (e.g., small empirical values of earnings response coefficient and poor quality estimates of expected rates of return). Using analysts’ consensus forecasts, this study investigates how noisy measurement of the market expectation of future earnings affects the predictability of future earnings and stock returns. Based on the errors-in-variables approach, this study first provides a framework to capture cross-sectional variation of the measurement error in analysts’ consensus forecasts. With this framework in place, this study documents that analysts’ consensus forecasts with more measurement error have less ability to predict future earnings and stock returns, and that incorporating information about cross-sectional variation of the measurement error can improve the predictability of future earnings and stock returns. These findings will be useful to accounting research that relies on the market expectation of future earnings and to practitioners seeking to forecast profitability and stock returns.
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Chapter 1

Introduction

A large body of capital markets research relies on market expectation of future earnings. Since the market expectation of future earnings cannot be observed, proxies are often employed. However, they inevitably measure the market expectation of future earnings with error that varies cross-sectionally, which results in unsatisfactory empirical results in prior research although conclusions mostly go in the right direction.\(^1\) Therefore, if cross-sectional variation of the measurement error can be estimated (although the signs and magnitudes of individual measurement errors may be unknown), better specified empirical test models can be generated.

With this notion, using analysts’ consensus forecasts, this study investigates the effect of measuring the market expectation of future earnings with error on the predictability of future earnings and stock returns. For this purpose, analysts’ consensus forecasts provide the appropriate basis because analysts’ consensus forecasts deviate from the market expectation of future earnings due to several error sources such as staleness, analysts’ inability and analysts’ bias. Towards this end, based on the errors-in-variables approach and characteristics of analysts’ consensus forecasts at the time of prediction, this study first provides a framework that captures cross-sectional variation of the measurement error in analysts’ consensus forecasts. With this framework in place, this study reports that analysts’ consensus forecasts that measure the market expectation of future earnings with more error have less ability to predict future earnings and stock returns. This study then analytically documents that incorporating information about cross-sectional variation of the measurement error enhances specification of prediction models and simulation results confirm this. Empirical test results also show that incorporating information about cross-sectional variation of the measurement error improves the predictability of future earnings and stock returns. The results also generally hold in out of sample tests.

This study contributes to the extant literature in several respects. First, and most important, building on prior findings, this study shows that noisy measurement of the market expectation of future earnings causes inference problems in empirical research and the effect continues to exist for quite a long period. To address the potential problem caused by measuring the market expectation of future earnings with error, this study directly uses information about cross-sectional variation of the measurement error while most prior studies use the proxy for the measurement error.\(^2\)

Second, the main focus of this study is to improve the predictability of future earnings

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\(^1\) For example, the general consensus is that empirical values of earnings response coefficient are much smaller than theoretical values. As documented in the review paper by Kothari (2000), with the discount rate of 10%, earnings response coefficient should theoretically be 11 but most empirical estimates range from 1 to 3 only. Since earnings response coefficient is a contemporaneous mapping of change in earnings to change in firm value, smaller than expected earnings response coefficient would be reported if a proxy measured the market expectation of future earnings with error. The review paper by Easton (2007) indicates that measuring the market expectation of future earnings with error could cause poor quality estimates of expected rates of return.

and stock returns, whereas most prior studies focus primarily on earnings response coefficients.\textsuperscript{3} Towards this end, it is analytically and empirically shown that incorporating information about cross-sectional variation of the measurement error can improve the predictability of earnings and stock returns and enhance the specification of prediction models.\textsuperscript{4} Based on the errors-in-variables approach, this paper provides a framework that captures cross-sectional variation of the (ex-ante) measurement error in analysts’ consensus forecasts by distinguishing the effect of the (ex-ante) measurement error from that of (ex-post) earnings news on the coefficient and $R^2$. This framework can be easily extended to other accounting research that relies on the market expectation of future earnings.

Finally, this study reports that the coefficient and $R^2$ from the regression of the stock price on the analysts’ consensus forecast decrease as the measurement error in analysts’ consensus forecasts increases. This is direct evidence that analysts’ consensus forecasts with large measurement error do not well explain the market expectation of future earnings. Not many prior studies directly examine the relation between the stock price and measurement error.

This study is organized as follows. Chapter 2 summarizes related prior research. Chapter 3 develops models and testable hypotheses. Chapter 4 reports the simulation results. Chapter 5 explains the sample selection process. Empirical findings are discussed in Chapter 6 and Chapter 7. Chapter 8 concludes this study.

\textsuperscript{3} Exceptions include Beaver et al. (1980), Das and Lev (1994) and Frankel and Lee (1998). Beaver et al. (1980) provides weak evidence that including stock price to control for measuring permanent earnings changes with error, may improve the predictability of future earnings over a random walk with drift model. Das and Lev (1994) shows that the predictability of stock returns can be improved by using a non-linear relation between earnings and stock returns. However, they do not pinpoint why the predictability can be improved. Frankel and Lee (1998) shows that analysts’ forecast error can be used to predict future stock returns. However, they do not use the errors-in-variables approach.

\textsuperscript{4} To the best of my knowledge, this is the first study to analytically document improvement of the specification of prediction models by incorporating information about cross-sectional variation of the measurement error.
Chapter 2

Relation to Prior Research

This study is closely related to the prior research that investigates the effect of measuring the market expectation of future earnings with error. Trueman (1993) presents a model where researchers erroneously measure the market expectation of future earnings based on analysts’ forecasts, which results in a non-linear relation between earnings surprise and stock returns. Ryan and Zarowin (1995) allows for the measurement error in earnings to explain variation of earnings response coefficients and $R^2$s across different model specifications. The findings of Trueman (1993) and Ryan and Zarowin (1995) imply that correction of noisy measurement of the market expectation of future earnings may enhance results of the empirical research that relies on the market expectation of future earnings. With the similar insight, in examining earnings response coefficients, some prior research tries to mitigate the potential problem caused by measuring the market expectation of future earnings with error. This stream of research includes Beaver et al. (1980) (stock price), Brown et al. (1987) (firm size), Kothari and Sloan (1992) (leading period stock returns), Beneish and Harvey (1998) (non-linear function), Machuga (2000) (statistical method) and Bartov et al. (2001) (dispersion of analysts’ forecasts) among others. This study is different from prior studies in two respects. First, this study directly uses information about cross-sectional variation of the measurement error while other studies use the proxy for the measurement error. Second, this study focuses on improvement in the predictability of future earnings and stock returns by incorporating information about cross-sectional variation of the measurement error, whereas most prior studies focus mainly on earnings response coefficients.5

This study is also related to the prior research that examines the effect of precision of a signal about future earnings on the market reaction.6 The main finding is that when a signal is not precise, the market response (i.e., earnings response coefficient) is decreasing in the magnitude of imprecision (i.e., non-linear relation between earnings surprise and stock returns). This stream of research includes Imhoff and Lobo (1993), Subramanyam (1996), Kinney et al. (2002) and Burgstahler and Chuk (2010) among others.7 This study is different from prior studies in that this study employs the errors-in-variables approach to explain the effect of ex-ante noisy measurement of market expectation on the predictability of future earnings and stock returns, whereas most prior studies rely on the Bayesian model to account for various ranges of earnings response coefficients.

Some prior research including Ali et al. (1992), Elgers and Lo (1994), Frankel and Lee (1998), Guay et al. (2005), Gode and Mohanram (2008) and Hughes et al. (2008), proposes methods to mitigate analysts’ forecast errors. This study is similar to those prior studies in its method for estimating cross-sectional variation of the measurement error in analysts’ consensus forecasts. However, this study uniquely provides a framework that captures cross-sectional variation of the (ex-ante) measurement error by distinguishing the effect of (ex-ante) errors.

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5 Exceptions include Beaver et al. (1980), Das and Lev (1994) and Frankel and Lee (1998) as mentioned earlier.
6 Precision refers to the extent that a signal captures the market expectation of future earnings.
7 Prior studies including Cheng et al. (1992), Freeman and Tse (1992) and Das and Lev (1994) also document a non-linear relation between earnings surprise and stock returns. They attribute their results primarily to the earnings persistence.
measurement error from that of (ex-post) earnings news on the coefficient and $R^2$ based on the errors-in-variables approach.

This study uses the determinants of the analysts’ forecast error documented in prior studies. Abarbanell et al. (1995) shows that variance of the measurement error in analysts’ consensus forecasts increases in the dispersion of analysts’ forecasts.8 Prior research including Abarbanell and Bernard (1991), Mendanhall (1991) and Clement and Tse (2003), reports that analysts’ forecast errors are serially correlated. This study additionally identifies two more determinants based on the insight that transitory earnings can cause noisy measurement of the market expectation of future earnings.9

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8 Based on the finding of Abarbanell et al. (1995), Bartov et al. (2001) uses the dispersion of analysts’ forecasts as a proxy for noisy measurement of the market expectation. Prior research including Imhoff and Lobo (1992), Kinney et al. (2002) and Burgsthieler and Chuk (2010) also documents that the dispersion of analysts’ forecasts is positively related to the analysts’ forecast error.

9 The determinants of the analysts’ consensus forecast error employed in the study are based on the (ex-ante) characteristics of analysts’ consensus forecasts while prior studies that try to mitigate the analysts' forecast error identify the determinants of the analysts’ forecast error based on the firm characteristics.
Chapter 3

Model and hypotheses development

Using analysts’ consensus forecasts, this study examines the effect of noisy measurement of market expectation of future earnings on the predictability of (long-term) future earnings and stock returns. For this purpose, analysts’ consensus forecasts provide the appropriate basis because analysts’ consensus forecasts deviate from the market expectation of future earnings due to several error sources such as staleness, analysts’ inability and analysts’ bias. Figure 1 contrasts the case where all analysts’ forecasts correctly (and rationally) represent the market expectation of future earnings (Case 1) against the case where each analyst’s forecast measures the market expectation of future earnings with error (Case 2). Figure 1 indicates that the measurement error in analysts’ consensus forecasts can occur due to analysts’ characteristics even when the underlying market expectation of future earnings is certain. Figure 1 also suggests that the measurement error in analysts’ consensus forecasts may vary cross-sectionally according to the extent that each error source contributes to the aggregate measurement error. This cross-sectional variation of the measurement error should have implications for the predictability of future earnings and stock returns when predictions are made based on analysts’ consensus forecasts.

Based on the errors-in-variables approach, this study provides a framework to estimate cross-sectional variation of the measurement error in analysts’ consensus forecasts although the measurement error itself may be unknown. With this framework in place, this study then explores the method to improve the predictability of future earnings and stock returns.

3.1 Effect of measurement error in analysts’ consensus forecasts on predictability of future earnings

Figure 2 depicts the timeline of predicting actual earnings at time t+1. From Figure 2, the prediction model that forecasts actual earnings at time t+1 with the analysts’ consensus forecast at time t can be established as in (1):

\[
A_{i,t+1} = \beta_0 + \beta_{M} M_{i,t} + n_{i,t+1} + e_i, \text{ with } R^2_{M}\n
A_{i,t} = \beta_0 + \beta_{F} F_{i,t} + e_i, \text{ with } R^2_{F}
\]

where

- \(A_{i,t+1}\): Actual earnings at time t+1
- \(M_{i,t}\): Market expectation at time t of actual earnings at time t+1
- \(F_{i,t}\): Analysts’ consensus forecast at time t for actual earnings at time t+1
- \(n_{i,t+1}\): Earnings news during time t+1
- \(e_i\): Measurement error in analysts’ consensus forecast at time t.

According to the errors-in-variables approach, (1) implies that if the actual earnings at time t+1

\[\text{10}\] This study implicitly assumes that the market expectation is exogenously given.
\[\text{11}\] It is possible that analysts’ consensus forecasts happen to coincide with the market expectation of future earnings although each analyst’s forecast measures the market expectation of future earnings with error. However, this is not likely the case for most of the firms covered by many analysts.
are predicted with the analysts’ consensus forecast at time $t$ as a proxy for the market expectation, the coefficient ($\beta_{EIV}$) and $R^2$ ($R^2_{EIV}$) are attenuated as in (2) because analysts’ consensus forecasts measure the market expectation with error ($e_t$)\(^{12}\):

$$
p \lim_{n \to \infty} \beta_{EIV} = \beta_{EIV} \frac{\text{Var}(M_{t+1}^{\text{true}})}{\text{Var}(M_{t+1}^{\text{true}}) + \text{Var}(e_t)}
$$

$$
p \lim_{n \to \infty} R^2_{EIV} = R^2_{EIV} \frac{\text{Var}(M_{t+1}^{\text{true}})}{\text{Var}(M_{t+1}^{\text{true}}) + \text{Var}(e_t)}\frac{\text{Var}(M_{t+1}^{\text{true}})}{\text{Var}(M_{t+1}^{\text{true}}) + \text{Var}(n_{t+1})}
$$

The formal derivation of (2) is provided in Appendix A. (2) also indicates that if the measurement error in analysts’ consensus forecasts varies cross-sectionally, the analysts’ consensus forecasts that measure the market expectation with more error translate less into future earnings and, in turn, have less ability to predict future earnings. Similar intuition applies to the prediction of long-term future earnings (i.e., five year ahead mean actual earnings).\(^{13}\)

**Hypothesis 1:** Coefficients and $R^2$ decrease as measurement error in analysts’ consensus forecasts increases when analysts’ consensus forecasts are used to predict future earnings.

From (2), it should be noted that earnings news during time $t+1$ ($n_{t+1}$) is the measurement error to the dependent variable that is not correlated with the (ex-ante) measurement error in analysts’ consensus forecasts since it was not anticipated at the time of prediction.\(^{14}\) Hence, attenuation in the coefficient is purely attributable to the measurement error in analysts’ consensus forecasts, but $R^2$ is decreasing in both variance of the measurement error in analysts’ consensus forecasts and variance of earnings news during time $t+1$. These properties of the coefficient and $R^2$ reported in (2) are used to construct the proxy for the (ex-ante) measurement error in analysts’ consensus forecasts (“meproxy”), which is used to test the hypotheses.

### 3.2 Improvement of earnings prediction and model specification from consideration of cross-sectional variation of measurement error

In the previous sub-section, it was shown that the predictive ability of analysts’ consensus forecasts varies cross-sectionally according to the extent of the measurement error in them. This sub-section investigates the effect of ignoring cross-sectional variation of the measurement error on the predictability of future earnings and provides the method to improve the predictability of future earnings. Towards this end, the coefficient and $R^2$ are derived for the case where two groups of analysts’ consensus forecasts are aggregated to predict future earnings when each group is subject to the different degree of measurement error. For this analysis, it is assumed that both $A_{t+1}$ and $F_{t+1}^{\text{true}}$ are vector with $n+k$ observations respectively. The first $n$ observations of

\(^{12}\) If analysts’ consensus forecasts were rational, they would be the same as the market expectation of future earnings. However, since the observed analysts’ consensus forecasts contain measurement error, they become noisy measurement of the market expectation of future earnings.

\(^{13}\) For the prediction of long-term earnings, analysts’ consensus forecasts are used as a proxy for the market expectation of long-term earnings.

\(^{14}\) If there were no earnings news during time $t+1$, the dependent variable should be equal to the market expectation at time $t$. When earnings news comes out, actual earnings at time $t+1$ becomes different from the market expectation at time $t$. Therefore, earnings news adds error to the dependent variable.
$F_{it}^{t+1}$ measure the market expectation with a certain degree of error ($e_1$) and the remaining $k$ observations measure the market expectation with a different degree of error ($e_2$) as in (3)\textsuperscript{15}:

$$A_{i,t} = \beta_e + \beta_{true}M_{it}^{t+1} + n_{i,t} + e_i \quad \text{with } R_{true}^{2}$$

$$A_{i,t} = \beta_e + \beta_{true}(F_{it}^{t+1} - e_i) + n_{i,t} + e_i = \beta_e + \beta_{POOLED}F_{it}^{t+1} + z_i \quad \text{with } R_{POOLED}^{2}$$

\text{where}

$A_{i,t}$: Actual earnings at time $t+1$

$M_{it}^{t+1}$: Market expectation at time $t$ of actual earnings at $t+1$

$F_{it}^{t+1}$: Analysts’ consensus forecast made at time $t$ for actual earnings at $t+1$ ($F_{it}^{t+1} = M_{it}^{t+1} + e_1$ for first $n$ observations and $F_{it}^{t+1} = M_{it}^{t+1} + e_2$, for remaining $k$ observations)

$e_i$: Measurement error in analysts’ consensus forecast at time $t$ ($e_i = e_1$ for first n observations and $e_i = e_2$, for remaining $k$ observations)

$n_{i,t}$: Earnings news during time $t+1$.

With the standard assumptions, the probability limit of the coefficient ($\hat{\beta}_{POOLED}$) and $R^2$ ($R_{POOLED}^2$) under this setting can be derived as in (4):

$$\lim p\hat{\beta}_{POOLED} = \frac{\text{Var}(M_{it}^{t+1})}{\text{Var}(M_{it}^{t+1}) + \frac{n}{n+k}\text{Var}(e_1) + \frac{k}{n+k}\text{Var}(e_2)}$$

$$\lim pR_{POOLED}^2 = \frac{\text{Var}(M_{it}^{t+1})}{\text{Var}(M_{it}^{t+1}) + \frac{n}{n+k}\text{Var}(e_1) + \frac{k}{n+k}\text{Var}(e_2)} \frac{\text{Var}(M_{it}^{t+1}) + \text{Var}(n_{i,t})}{\text{Var}(M_{it}^{t+1}) + \text{Var}(n_{i,t})}$$

The formal derivation is provided in Appendix A.\textsuperscript{16} (4) shows that aggregation without considering cross-sectional variation of the measurement error obscures the predictive ability of each group since the coefficient and $R^2$ are decreasing in the weighted average of measurement error variance of each group with the weight determined based on the number of observations of each group. It should also be noted that the coefficient and $R^2$ are more biased toward those of the group with large measurement error.\textsuperscript{17} Therefore, the predictive ability of the group with small measurement error would be especially impaired if observations were aggregated without considering cross-sectional variation of the measurement error.

The results of (4) imply that better earnings predictions and model specifications can be generated if information about cross-sectional variation of the measurement error can be incorporated. Applying this notion, this study investigates how the coefficient and $R^2$ change if information about cross-sectional variation of the measurement error is incorporated as in (5) where the coefficients are allowed to vary using an indicator according to the cross-sectional variation of the measurement error:

$$A_{i,t} = \beta_e + \beta_{true}M_{it}^{t+1} + n_{i,t} + e_i \quad \text{with } R_{true}^{2}$$

$$A_{i,t} = \beta_e + \beta_{true}(F_{it}^{t+1} - e_i) + n_{i,t} + e_i = \gamma_1 + \gamma_2F_{it}^{t+1} + \gamma_3D + \gamma_4F_{it}^{t+1} \cdot D + x \quad \text{with } R_{ext}^{2}$$

\text{15} To understand the effect of cross-sectional variation of the measurement error in analysts’ consensus forecasts, it is assumed that earnings news during time $t+1$ has the same distribution for both groups.

\text{16} These results can be readily extended to any number of groups.

\text{17} Das and Lev (1994) and Burgstahler and Chuk (2010) note the similar intuition.
where

\( A_t \): Actual earnings at time \( t+1 \)

\( M^{t+1}_t \): Market expectation at time \( t \) of actual earnings at time \( t+1 \)

\( F^{t+1}_t \): Analysts’ consensus forecast made at time \( t \) for actual earnings at time \( t+1 \) (\( F^{t+1}_t = M^{t+1}_t + e_t \) for first \( n \) observations and \( F^{t+1}_t = M^{t+1}_t + e_2 \) for remaining \( k \) observations)

\( e_t \): Measurement error in analysts’ consensus forecast at time \( t \) (\( e_t = e_1 \) for first \( n \) observations and \( e_t = e_2 \) for remaining \( k \) observations)

\( D_t \): 1 when \( F^{t+1}_t = M^{t+1}_t + e_1 \) and 0 otherwise

\( n_{t,i} \): Earnings news during time \( t+1 \).

Then, the coefficient and \( R^2 \) for (5) can be expressed as in (6):

\[
\begin{align*}
\beta_1^\prime & = \beta_{t2} \frac{\text{Var}(M_t^{t+1})}{\text{Var}(M_t^{t+1}) + \text{Var}(e_t)} \\
\beta_1 & = \beta_{t2} \left( \frac{\text{Var}(M_t^{t+1})}{\text{Var}(M_t^{t+1}) + \text{Var}(e_2)} - \frac{\text{Var}(M_t^{t+1})}{\text{Var}(M_t^{t+1}) + \text{Var}(e_1)} \right) \\
\beta_2 & = \beta_{t2} \left( \frac{n}{n+k} \frac{\text{Var}(M_t^{t+1})}{\text{Var}(M_t^{t+1}) + \text{Var}(e_1)} + \frac{k}{n+k} \frac{\text{Var}(M_t^{t+1})}{\text{Var}(M_t^{t+1}) + \text{Var}(e_2)} \right) \frac{\text{Var}(M_t^{t+1})}{\text{Var}(M_t^{t+1}) + \text{Var}(n_{t,i})}
\end{align*}
\]

The formal derivation is provided in Appendix A.\(^{18}\) Comparing (4) and (6) reveals that \( R^2 \) increases when information about cross-sectional variation of the measurement error is incorporated.\(^{19}\) This is formally shown in (7) and unreported simulation results confirm this finding:

\[
\begin{align*}
p\lim R^2_{\text{MEV}} - p\lim R^2_{\text{Pooled}} & = \frac{(n+k)^2 \text{Var}(M_t^{t+1})^2 (\text{Var}(M_t^{t+1}) - 1) + nk \text{Var}(M_t^{t+1})(\text{Var}(e_1) - \text{Var}(e_2)) + \text{Var}(e_2)}{(n+k)(\text{Var}(M_t^{t+1}) + \text{Var}(e_1))(\text{Var}(M_t^{t+1}) + \text{Var}(e_2))((n+k) \text{Var}(M_t^{t+1}) + n \text{Var}(e_1)) \text{Var}(M_t^{t+1}) + k \text{Var}(e_2)) \text{Var}(M_t^{t+1}) + \text{Var}(n_{t,i})} R^2_{\text{MEV}}
\end{align*}
\]

**Hypothesis 2:** Predictability of future earnings and \( R^2 \) of earnings prediction model are improved if information about cross-sectional variation of measurement error is incorporated.

3.3 Estimation of cross-sectional variation of measurement error in analysts’ consensus forecasts

To empirically test Hypothesis 1 and Hypothesis 2, cross-sectional variation of the measurement error needs to be estimated since the market expectation of future earnings and measurement error in analysts’ consensus forecasts cannot be observed. Towards this end, the analysts’ consensus forecast error is employed, which is defined as the absolute difference between actual earnings at time \( t+1 \) and the first available analysts’ mean consensus forecast after the actual earnings at time \( t \) is announced, scaled by the stock price at the end of the third

---

\(^{18}\) These results can also be easily extended to any number of groups.

\(^{19}\) This argument is consistent with Sheffrin (1996). As Sheffrin (1996) puts it, if any information that is correlated with the forecast errors that is available at the time of forecasting, it would be possible to improve the forecast by incorporating this correlation into forecasting.
month after fiscal year end of time \( t \) as in (8). The second equality in (8) is derived from the relation among the actual earnings at time \( t+1 \), analysts’ consensus forecast at time \( t \) and market expectation at time \( t \) as depicted in Figure 2:

\[
\text{Analysts' consensus forecast error}_{t+1} = \frac{|A_{t+1} - F_{t+1}|}{P_t} = \frac{|M_{t+1}^{r} + n_{r,t} - (M_{t+1}^{r} + e_t)|}{P_t} = \frac{|n_{r,t} - e_t|}{P_t} \tag{8}
\]

where
- \( A_{t+1} \): Actual earnings at time \( t+1 \)
- \( M_{t+1}^{r} \): Market expectation at time \( t \) of actual earnings at time \( t+1 \)
- \( F_{t+1}^{r} \): Analysts’ consensus forecast at time \( t \) for actual earnings at time \( t+1 \)
- \( P_t \): Stock price at time \( t \)
- \( n_{r,t} \): Earnings news during time \( t+1 \)
- \( e_t \): Measurement error in analysts’ consensus forecast at time \( t \).

The reason for choosing (8) as a starting point is that the cross-sectional variation of the measurement error can be reasonably estimated if individual measurement errors can be identified as shown in (9):

\[
\text{Var}(e_t) = E(e_t^2) - [E(e_t)]^2 = \frac{1}{n} \sum_{i=1}^{n} e_{i,t}^2 = \frac{1}{n} \sum_{i=1}^{n} |e_{i,t}| \tag{9}
\]

where
- \( e_t \): Measurement error in analysts’ consensus forecast at time \( t \).

Since the future earnings cannot be observed at the time of prediction, (8) cannot be directly used. Instead, a proxy is constructed based on the characteristics of analysts’ consensus forecasts at the time of prediction (i.e., ex-ante determinants of the analysts’ consensus forecast error). The following variables are used as the determinants.

1. **Standard deviation of analysts’ consensus forecast at time \( t \)** \( \left( \frac{\text{std}(F_{t}^{r})}{P_{t}} \right) \): In prior research including Abarbanell et al. (1995) and Bartov et al. (2001), the standard deviation of the analysts’ forecasts (i.e., dispersion) is used as a proxy for the measurement error in analysts’ forecasts.\(^{21}\)
2. **Analysts’ consensus forecast error at time \( t-1 \)** \( \left( \frac{\text{abs}(A_t - F_{t-1})}{P_{t-1}} \right) \): Prior research including Abarbanell and Bernard (1991), Mendanahall (1991) and Clement and Tse (2003) reports that analysts’ consensus forecast errors are serially correlated.
3. **Absolute difference between actual earnings at time \( t \) and analysts’ consensus forecast at time \( t \) for actual earnings at time \( t+1 \)** \( \left( \frac{\text{abs}(A_t - F_{t+1}^{r})}{P_t} \right) \): This determinant indicates either that analysts do not reflect new information embedded in actual earnings at time \( t \) or that a large

\(^{20}\) This is, in spirit, similar to the measure used in Elgers and Lo (1994). Elger and Lo (1994) uses the signed analysts’ forecast error as a proxy for the market’s earnings expectation error.

\(^{21}\) Imhoff and Lobo (1992), Kinney et al. (2002) and Burgstahler and Chuk (2010) also document that the dispersion of analysts’ forecasts is positively related to the analysts’ forecast error.
proportion of earnings is inherently transitory.

(4) Absolute difference between analysts’ consensus forecast at time t for actual earnings at
time t+1 and at time t+2 (\(\frac{\text{abs}(F_{t+1} - F_{t+2})}{P_t}\)): This determinant mainly captures the measurement error from transitory earnings. Both forecasts are required to be made in the same month.

The measurement error proxy (“meproxy”) at time t is then constructed by multiplying each determinant at time t by the respective coefficient from the regression (10) based on the data between the sample starting year (i.e., 1990) and time t-1:

\[
\frac{\text{abs}(A_t-\bar{F}_{t+1})}{P_t} = \alpha_1 + \alpha_2 \frac{\text{std}(F_{t+1})}{P_t} + \alpha_3 \frac{\text{abs}(A_t-\bar{F}_{t+1})}{P_t} + \alpha_4 \frac{\text{abs}(A_{t-1}-\bar{F}_{t+1})}{P_t} + \alpha_5 \frac{\text{abs}(F_{t+1}-F_{t+2})}{P_t} + \epsilon_t \tag{10}
\]

where
- \(A_t\): Actual earnings at time t+1
- \(A_t\): Actual earnings at time t
- \(\bar{F}_{t+1}\): Analysts’ consensus forecast made at time t for actual earnings at time t+1
- \(\bar{F}_{t+1}\): Analysts’ consensus forecast made at time t-1 for actual earnings at time t
- \(\bar{F}_{t+2}\): Analysts’ consensus forecast made at time t for actual earnings at time t+2
- \(P_t\): Stock price at time t
- \(P_t\): Stock price at time t-1.

From (8), it is noted that analysts’ consensus forecast errors capture both the measurement error in analysts’ consensus forecasts at time t and earnings news during time t+1. 23 24 Therefore, whether the measurement error proxy (“meproxy”) primarily captures the (ex-ante) measurement error in analysts’ consensus forecasts is an empirical question. From (2), it is noted that if the measurement error proxy (“meproxy”) successfully captures the measurement error in analysts’ consensus forecasts at time t, the coefficients and R²s from the prediction model where actual earnings at time t+1 are predicted with the analysts’ consensus forecast at time t, are expected to decrease as the “meproxy” increases. 25 To empirically operationalize this intuition and test the hypotheses, quintile ranks are formed on the “meproxy” for each year. Then, for each rank, future earnings are predicted with the analysts’ consensus forecast at time t to investigate changes in the coefficients and R²s across the quintile ranks. 26

22 In spirit, this approach is similar to Ali et al. (1992), Guay et al. (2005), Gode and Mohanram (2008) and Hughes et al. (2008). This study uses it to proxy cross-sectional variation of the measurement error with the assumption that the measurement error itself is not observed, whereas prior studies use it to correct measurement error with the assumption that it represents (at least a portion of) the measurement error.

23 It should be noted that if there were no earnings news during time t+1, (3) would be exactly equal to the measurement error in analysts’ consensus forecasts at time t.

24 Prior studies including Barron (1998), Barron and Stuerke (1998), Barron et al. (2009) and Yeung (2009) show that the dispersion of analysts’ forecasts also captures the (ex-ante) uncertainty about the firm’s future earnings (i.e., firm value). However, this study does not explicitly model the (ex-ante) uncertainty with the assumption that the uncertainty is mostly resolved by earnings news during time t+1 and mostly affects R²s as shown in (2).

25 If the measurement error proxy (“meproxy”) only captured earnings news during time t+1, no noticeable changes in the coefficients would be expected as the “meproxy” changes. Instead, R²s would decrease as the “meproxy” increases.

26 As mentioned, the “meproxy” also captures a portion of earnings news during time t+1. Therefore, R²s are expected to decrease at a faster rate than the coefficients. Similar intuition applies to the prediction of long-term earnings (i.e., five year ahead mean actual earnings).
Hypothesis 1': If measurement error proxy ("meproxy") captures (ex-ante) measurement error in analysts’ consensus forecasts, coefficients and $R^2$ decrease as "meproxy" increases when analysts’ consensus forecasts are used to predict future earnings.

3.4 Effect of cross-sectional variation of measurement error on analysts’ consensus forecast’s ability to explain stock price

Since the test of Hypothesis 1 can be rather mechanical\(^\text{27}\), to directly test whether the measurement error proxy ("meproxy") successfully captures the measurement error in analysts’ consensus forecasts, it is important to examine how the relation between the analysts’ consensus forecast and stock price changes as the measurement error in analysts’ consensus forecasts cross-sectionally varies.\(^\text{28}\) Towards this end, a simple earnings capitalization-based valuation model (11) is employed\(^\text{29}\):

$$\frac{P_r^t}{r} = \frac{1}{r} \left( F_{r+1}^t - e_t \right) \quad (11)$$

where:
- $P_r^t$: Stock price at time $t$
- $M_r^t$: Market expectation at time $t$ of actual earnings at time $t+1$
- $F_{r+1}^t$: Analysts’ consensus forecast at time $t$ for actual earnings at time $t+1$
- $e_t$: Measurement error in analysts’ consensus forecast at time $t$
- $r$: Market implied expected rate of return at time $t$.

(11) implies that the measurement error in analysts’ consensus forecasts causes the coefficient to be smaller than $1/r$ and the coefficient and $R^2$ decrease as the measurement error increases when the stock price at time $t$ is regressed on the analysts’ consensus forecast at time $t$. 

Hypothesis 3: Coefficient and $R^2$ decrease as measurement error proxy ("meproxy") increases when current stock price is regressed on analysts’ consensus forecast.

Hypothesis 3 implies that analysts’ consensus forecasts that contain larger measurement error explain less of the current market value of the firm and the market expectation of future earnings.

3.5 Effect of measurement error in analysts’ consensus forecasts on predictability of future stock returns

Hypothesis 1 and Hypothesis 3 also suggest that the measurement error in analysts’ consensus forecasts may well affect the earnings-based predictability of future stock returns. To examine this intuition, the effect of measuring the market expectation of future earnings with error on the predictability of future stock returns is examined. The prediction of future stock

\(^{27}\) The test of Hypothesis1 based on the “meproxy” can be mechanical because the “meproxy” is constructed based on the difference between the analysts’ consensus forecast and future earnings.

\(^{28}\) This argument is based on the insight that stock price is formed based on the market expectation of future earnings.

\(^{29}\) In this valuation model, it is assumed that earnings in all future periods will be paid out or, alternatively, reinvested earnings will earn $r$. Refer to Kothari and Zimmerman (1995).
returns is based on earnings yield at time t defined as the analysts’ consensus forecast at time t scaled by the stock price at time t.\textsuperscript{30} Figure 3 depicts the timeline of predicting one year future stock returns. The relation (12) follows from (11) and the notion that one year future stock returns is the sum of current expected returns and unexpected returns from the changes in the market expectation of future earnings due to the earnings news during t+1:\textsuperscript{31}

$$
\text{Ret}_t = \frac{1}{P_t} + \frac{M_{t+1}}{P_t} + \frac{F_{t+1} - e_t}{P_t} + n_{t+1}/P_t
$$

(12)

where

$\text{Ret}_t$: One year future stock returns

$P_t$: Stock price at time t

$M_{t+1}^t$: Market expectation at time t of actual earnings at time t+1

$F_{t+1}^t$: Analysts’ consensus forecast at time t for actual earnings at time t+1

$e_t$: Measurement error in analysts’ consensus forecast at time t

$n_{t+1}$: Earnings news during time t+1

$r$: Market implied expected rate of return at time t.

(12) indicates that when future stock returns are predicted with earnings yield based on the analysts’ consensus forecast, the measurement error in analysts’ consensus forecasts impairs the predictability of future stock returns. This is intuitively appealing since analysts’ consensus forecasts that measure the market expectation with more error should be less associated with the market reaction. Therefore, improvement in the model specification and predictability of future stock returns is expected if information about cross-sectional variation of the measurement error is incorporated.\textsuperscript{32} Similar intuition applies to the prediction of long-term future stock returns (i.e., five year future stock returns).

**Hypothesis 4:** Coefficient and $R^2$ decrease as measurement error proxy (“meproxy”) increases when earnings yield (i.e., analysts’ consensus forecast scaled by current stock price) is used to predict future stock returns. Predictability of future stock returns and $R^2$ of stock returns prediction model are improved if information about cross-sectional variation of measurement error is incorporated.

\textsuperscript{30} As Fama (1991) indicates, the predictability of earnings yield (or dividend yield) may be attributable to mispricing or to price being high relative to earnings (or dividends) due to low expected returns.

\textsuperscript{31} Returns news may be an additional possible source of measurement error in $\text{Ret}_t$, since the current model only captures the news related to earnings.

\textsuperscript{32} The predictability of stock returns in this study does not mean to beat the market. The assumption is that the equilibrium future stock returns are given based on the market expectation of future earnings and we can approach more closely the equilibrium future stock returns by incorporating information about cross-sectional variation of measurement error.
Chapter 4

Simulation tests

In this chapter, before empirical tests with actual financial data are conducted, simulation tests are implemented to investigate whether the analytical insights developed in Chapter 3 are valid and to examine effects of the measurement error present in a noisy observed predictor on the degree of the bias in the coefficient estimate and $R^2$, and how to improve the prediction model specifications and forecasting based on a noisy observed predictor.

Simulation tests provide an appropriate test bed for validating the analytical insights since they are based on randomly generated ideal numbers. For the simulation tests, the total of 40,000 observations is used, which consists of 4 groups of 10,000 randomly generated observations. First, a true predictor ($x^*$) is randomly generated from the normal distribution with mean of 2 and standard deviation of 1. Then, a dependent variable ($y$) is defined as the sum of 10 times true predictor ($x^*$) and the disturbance ($\varepsilon$) that has a normal distribution with zero mean and standard deviation of 6.\textsuperscript{33} Next, a noisy observed predictor ($x$) is assumed to measure the true predictor ($x^*$) with error. It is defined as the sum of the true predictor ($x^*$) and the measurement error ($e$ or $u$) that has a normal distribution with zero mean and a certain variance. The details about each variable are described in Appendix B.

4.1 Coefficient estimate and $R^2$ when all observations contain measurement error

Table 1 reports the results of the coefficient estimates and $R^2$s when all observations contain the measurement error based on the following simple prediction model in (13).

$$y = \beta_0 + \beta_{x\varepsilon} x + w \quad (13)$$

where

$y$: Independent variable
$x$: Noisy observed predictor

Panel 1A reports the coefficient estimates and $R^2$s based on the simulation tests. As predicted in Hypothesis 1, the coefficient estimates and $R^2$s decrease as the measurement error variance increases. The coefficient estimate and $R^2$ decrease to 0.998 and 7.23% when the measurement error variance is 9 from 5.031 and 36.92% when the measurement error variance is 1.

To examine whether the decrease in the coefficient estimate and $R^2$ based on the simulation is consistent with what the analytical insights developed in Chapter 3, the coefficient estimate and $R^2$ are computed based on (2) for each measurement error variance. Almost identical to the results from the simulation tests, the results from the theoretical predictions (i.e., computation based on (2)) show that the coefficient estimate and $R^2$ decreases to 1.000 and 7.35% when the measurement error variance is 9 from 5.000 and 36.77% when the measurement error variance is 1.

Figure 4 depicts the pattern of the coefficient estimates and $R^2$s as the relative measurement error variance to the variance of the true predictor increases. Both the coefficient estimates (Figure 4A) and $R^2$s (Figure 4B) show the decreasing trend as the relative measurement error variance.

\textsuperscript{33} Results remain the same regardless of the distribution of the dependent variable ($y$).
measurement error variance to the variance of the true predictor increases. It should be noted that there is a convex relation between the degree of the bias and the relative measurement error variance to the variance of the true predictor. This implies that even small amount of measurement error can much bias the coefficient estimates and $R^2$s but the marginal effect of increasing measurement error variance on the bias is decreasing.

4.2 Coefficient estimate and $R^2$ when half of observations contain measurement error

Table 2 reports the results of the coefficient estimates and $R^2$s when the half of the observations contains the measurement error and the other half does not contain the measurement error. Panel 2A reports the coefficient estimates and $R^2$s based on the simulation tests. Consistent with the results reported in Table 1, the coefficient estimates and $R^2$s decrease as the measurement error variance increases. It should be noted that the bias in the coefficient estimates and $R^2$s is smaller than when all observations of the noisy observed predictor are measured with error as reported in Table 1.

To examine whether the bias in the coefficient estimate and $R^2$ based on the simulation is consistent with what the theory predicts, the theoretical bias in the coefficient estimate and $R^2$ is computed based on (4) with zero variance for the half of the observations and respective variance for the other half of the observations. As reported in Panel 2B, for all cases, the coefficient estimate and $R^2$ from the simulation test are very close to what the theory predicts (i.e., computation based on (4)).

Next, it is examined how the simple identification of the observations that contain the measurement error improves the prediction model specifications although the perfect measurement error distribution is still unknown. The indicator variable is used to identify the observations with the measurement error in the prediction model as in (14).

\[
y = \gamma_0 + \gamma_1x + \gamma_2D + \gamma_3xD + \nu \tag{14}
\]

where

- $y$: Independent variable
- $x$: Noisy observed predictor
- $D$: 1 when $x$ contains measurement error and 0 otherwise

As shown in (7), $R^2$s should increase when the indicator variable is used since both the theoretical predictions and simulation tests assume that the variance of the true predictor is 1. As reported in Panel 2A and 2B (i.e. second column for each case), both simulation test results and theoretical predictions are very close and show that $R^2$ increases for all cases. The difference in $R^2$ between when the indicator variable is used and when the indicator variable is not used is consistent with what the theory predicts based on (7).

Figure 5 also shows that the improvement in the prediction model specifications (Figure 5B) and predictive ability of the noisy observed predictor (Figure 5A). The improvement becomes more prominent as the measurement error variance becomes larger.

In conclusion, the results reported in Table 2 and Figure 5 imply that simple identification of the measurement error improves the specification of the prediction models.

4.3 Coefficient estimate and $R^2$ when sample consists of multiple groups and each group has different measurement error distribution
This subsection examines the bias in the coefficient estimate and $R^2$ when the sample consists of four groups and each group has a different measurement error distribution. The results are documented in Table 3. It is noted that the results of the simulation tests reported in Panel 3A and the theoretical predictions (i.e., computation based on (4)) reported in Panel 3B are very close. Hence the focus of the discussion lies in how the prediction model specification is improved as the degree of the identification of the observations with the measurement error increases. In Table 3, the second columns are the coefficient estimates and $R^2$'s when all observations are pooled regressed. The third columns report the coefficient estimates and $R^2$'s when the observations with large measurement error (i.e., measurement error with variance of 4 or 9) are identified with the indicator variable as in (15) with the assumption that the observations with small measurement error can be roughly distinguished from those with large measurement error.

$$y = \tau_0 + \tau_x x + \tau_{D_1}x + \tau_{D_2}x + \chi$$  \hspace{1cm} (15)

where

$y$: Independent variable

$x$: Noisy observed predictor

$D_1$: 1 when observations contain measurement error with variance of 4 and 9 and 0 otherwise

The fourth columns report the coefficient estimates and $R^2$'s when the observations with the measurement error are identified with the indicator variable as in (16) with the assumption that each measurement error distribution cannot be separately identified although all observations with the measurement error can be identified.

$$y = \gamma_0 + \gamma_x x + \gamma_{D_1}x + \gamma_{D_2}x + \gamma_{D_3}x + \nu$$  \hspace{1cm} (16)

where

$y$: Independent variable

$x$: Noisy observed predictor

$D$: 1 when $x$ contains measurement error and 0 otherwise

The last columns document the coefficient estimates and $R^2$'s when the observations with each different measurement error distribution are separately identified with the respective indicator variable as in (17) with the assumption that the exact distribution of each measurement error is still unknown.

$$y = \gamma_0 + \gamma_x x + \gamma_{D_1}x + \gamma_{D_2}x + \gamma_{D_3}x + \gamma_{D_4}x + \gamma_{D_5}x + \nu$$  \hspace{1cm} (17)

where

$y$: Independent variable

$x$: Noisy observed predictor

$D_1$: 1 when $x$ contains measurement error with variance of 1 and 0 otherwise

$D_2$: 1 when $x$ contains measurement error with variance of 2 and 0 otherwise

$D_3$: 1 when $x$ contains measurement error with variance of 9 and 0 otherwise

The third columns document the increase in $R^2$ when the observations with large measurement error are separately identified. $R^2$'s increase from 16% to 29%. As reported in the fourth columns, the simple identification of the observations with measurement error also helps to improve the model specifications. $R^2$'s increases to around 28% from 16% in the second columns. The separate identification of the different measurement error distribution further
improves the model specifications. $R^2$s in the last columns are around 33%, which is around 5% increase from the third columns.

The same conclusion can be drawn from Figure 6. It should be noted that for the analysis in Figure 6, the sample consists of four groups with equal number of observations. The first group of observations contains no measurement error. The second group of observations contains the measurement error with zero mean and variance of $Var(e)$. The third group of observations contains the measurement error with zero mean and variance of $4Var(e)$. The last group of observations contains the measurement error with zero mean and variance of $9Var(e)$. Figure 6 presents the pattern of the coefficient estimates and $R^2$s as the relative measurement error variance of the second group ($Var(e)$) to the variance of the true predictor varies from 0 to 9. In all cases, the separate identification of the observations with measurement error improves the specifications of prediction models. The improvement is more prominent for the case of separate identification of all observations with measurement error and separate identification of each measurement error distribution. When only the observations with large measurement error are separately identified, to the certain point (i.e. $Var(e)$ is around 1), the improvement in $R^2$ is increasing and even larger than $R^2$ from the prediction model with the indicator variable that separately identifies all the observations with the measurement error. Then, the improvement in $R^2$ starts to decay and become smaller than $R^2$ from the prediction model that separately identifies all the observations with the measurement error. This is because there is an effect of separating the observation with the small measurement error from the observations with the large measurement error becomes ambiguous as the smallest measurement error variance ($Var(e)$) becomes larger. However, $R^2$ is still greater than the prediction model without the indicator variable.

In conclusion, the results reported in Table 3 and Figure 6 suggest that if the observations with measurement error can be separately identified (even if the exact measurement error distribution is still unknown) in any way, better specified prediction models can be generated. Moreover, the predictive ability of the noisy observed predictor is also improved since the observations with the different degree of the measurement error are allowed to have different predictive ability.

### 4.4 Summary from simulation tests

This study examines the effect of the measurement error embedded in the noisy observed predictor on coefficient estimates and $R^2$s based on the simulation tests. The bias in the coefficient estimates and $R^2$s depends on the weighted average of the measurement error variance of each group with the weight determined based on the number of observations of each group if the sample consists of the several groups. These results imply that pooling the observations with the different degree of the measurement error obscures the interpretation of the coefficient estimate and $R^2$. Therefore, separate identification of the different measurement error distribution substantially improves the prediction model specifications with respect to $R^2$s and the predictive ability of the noisy observed predictor. Even approximate identification of the observations with measurement error induces the improvement.

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34 The convex relation between the degree of the bias and the relative measurement error variance to the variance of the true predictor also contributes to the results in Figure 6B. After a certain point, the marginal effect of increasing the measurement error variance is not increasing as much.
Chapter 5

Sample selection procedure for empirical tests

For the empirical tests, the I/B/E/S Summary History and CRSP databases are mainly used. Between 1990 and 2007, all firm years whose analysts’ mean consensus forecast is available in the I/B/E/S database are first identified. For ease of portfolio construction, only December ending firm years are selected. Since the prediction of long-term future earnings and stock returns is the main focus of this paper, only the first analysts’ consensus forecasts made after the prior year earnings are announced are selected. In terms of other data availability, it is required that actual earnings, immediate prior year’s actual earnings, analysts’ consensus forecast for the immediate prior year and next year, stock price at the end of the third month after prior fiscal year end, standard deviation of analysts’ consensus forecasts be available in the I/B/E/S and total assets at prior fiscal year end be available from Compustat. To compute the standard deviation of analysts’ consensus forecasts (i.e., dispersion), the number of forecast estimates is required to be greater than 1 (i.e., 2 or greater). “One year future stock returns” is computed with monthly raw stock returns and accumulated for 12 months beginning in the fourth month after the prior fiscal year end. “Five year future stock returns” is computed in the same manner for 60 months. For the firms that are delisted during the accumulation period, -30% delisting returns are assigned to NYSE and AMEX firms and -55% delisting returns are assigned to NASDAQ firms according to Shumway (1997) and Shumway and Warther (1999).

Since the empirical tests are conducted based on per share financial information, all relevant variables are adjusted to per share basis after controlling for stock splits and stock dividends. To account for outliers, all variables except for stock returns are winsorized at 1% level for both extremes, and observations whose stock price is less than $5 are eliminated. The sample selection procedure provides 21,740 firm year observations between 1990 and 2007 (16,684 firm year observations between 1990 and 2004 for the test of predicting five year mean actual earnings and five year future stock returns). The variable definitions are described in Appendix C and D.

35 Results do not change even when fiscal ending month is not restricted.
36 Tests are also conducted with the samples whose number of forecast estimates is greater than 3 and 5 respectively and the results are qualitatively similar.
37 Given that the small stocks (i.e., stock price less than $5) tend to contain more measurement error, the results become even stronger when the small stocks are included.
Chapter 6

Results of empirical tests

6.1 Analysts’ consensus forecast error regressions

Table 4, Panel A reports the results of annual Fama-MacBeth regressions of the analysts’ consensus forecast error on the ex-ante determinants (i.e., characteristics of analysts’ consensus forecasts) between 1990 and 2006. The results show that all determinants are significantly positively related to the analysts’ consensus forecast error. As shown in (2), analysts’ consensus forecast errors capture both (ex-ante) measurement error in analysts’ consensus forecasts and (ex-post) earnings news. To control for the effect of (ex-post) earnings news, the annual Fama-MacBeth regressions reported in Table 4, Panel A are repeated with the contemporaneous stock returns included. The unreported results still show that all determinants are significantly positively related to the analysts’ consensus forecast error even after controlling for the (ex-post) earnings news and the coefficients are almost identical to those reported in Table 4, Panel A. This indicates that the proxy for the measurement error in analysts’ consensus forecasts can be successfully constructed with the identified determinants.

The measurement error proxy (“meproxy”) is constructed by multiplying each determinant at time t by the weight for the respective determinant (i.e. respective coefficient) derived from the analysts’ consensus forecast error regression reported in Table 4, Panel A based on the data between 1990 and time t-1. Panel B reports the Pearson correlation between the measurement error proxy (“meproxy”) and analysts’ consensus forecast error. The high correlation (0.623) implies that the measurement error proxy (“meproxy”) is successfully constructed with reasonable precision.

6.2 Descriptive statistics

Descriptive statistics for the variables used in predicting future earnings and stock returns are reported in Table 5. The measurement error proxy (“meproxy”) has mean of 0.025 and median of 0.014, suggesting that it is slightly right skewed. Consistent with the optimism in analysts’ forecasts documented in prior research, the mean and median of analysts’ consensus forecasts are greater than those of both one year ahead actual earnings and five year ahead mean actual earnings.

6.3 Effect of cross-sectional variation of measurement error in analysts’ consensus forecasts on predictability of future earnings and evaluation of measurement error proxy (“meproxy”)

38 It should be noted that the measurement error proxy (“meproxy”) still captures the information related to the earnings news due to the (ex-ante) uncertainty about firm’s future earnings (i.e., firm value) that is resolved by earnings news.
39 For example, to construct the “meproxy” for year 2000, each determinant of year 2000 is multiplied by the respective coefficient from the analysts’ consensus forecast error regression using the data between year 1990 and year 1999.
Table 6, Panel A presents the results of the regressions for predicting one year ahead actual earnings (for 1991 ~ 2007) and five year ahead mean actual earnings (for 1991 ~ 2004) with analysts’ consensus forecasts. Consistent with the implications of the errors-in-variables approach, both coefficients and $R^2$s are smaller than one for all prediction models, which implies that analysts’ consensus forecasts measure the market expectation of future earnings with error. It should be noted that the coefficients of the regression models for predicting five year ahead mean actual earnings are generally smaller than those of the regression models for predicting one year ahead actual earnings. This indicates that analysts’ consensus forecasts measure the market expectation of long-term earnings with more error.

To investigate whether the measurement error proxy (“meproxy”) actually captures the (ex-ante) measurement error in analysts’ consensus forecasts, quintile ranks are formed on the measurement error proxy (“meproxy”) for each calendar year where rank 1 is the portfolio with the smallest “meproxy” and rank 5 is the portfolio with the largest “meproxy”. Then, for each rank, actual earnings at time $t+1$ is predicted with the analysts’ consensus forecast at time $t$ to investigate changes in the coefficients and $R^2$s across the quintile ranks. Consistent with Hypothesis 1’, the coefficients and $R^2$s reported in Table 6, Panel A show a decreasing pattern as the “meproxy” becomes larger. This result confirms that the measurement error proxy (“meproxy”) successfully captures the measurement error in analysts’ consensus forecasts at time $t$. Consistent with Hypothesis 1, this also implies that the measurement error varies cross-sectionally and analysts’ consensus forecasts that measure the market expectation with more error have less ability to predict future earnings. It should be noted that $R^2$s are generally decreasing at a faster rate than the coefficients with some exceptions. This suggests that the measurement error proxy (“meproxy”) also captures the effect of (ex-post) earnings news.

To examine whether the specification of earnings prediction models is improved when information about cross-sectional variation of the measurement error is incorporated, the interaction term between the analysts’ consensus forecast at time $t$ and an indicator for each rank is included in model (2) and (4) of Table 6, Panel B. $R^2$s in model (2) and (4) are compared with $R^2$s in model (1) and (3) respectively. Consistent with Hypothesis 2, when information about cross-sectional variation of the measurement error is incorporated as in model (2) and (4), the adjusted $R^2$s are greater. This implies that the sum of all squared prediction errors (i.e., the squared difference between future earnings and the predicted value from each model) is smaller for the prediction model with information about cross-sectional variation of the measurement error. The significantly negative coefficients on the interaction terms confirm the results reported in Table 6, Panel A.

## 6.4 Effect of cross-sectional variation of measurement error on analysts’ consensus forecast’s ability to explain stock price

Table 7 presents the results of the regression of the stock price at time $t$ on the analysts’ consensus forecast at time $t$. Consistent with Hypothesis 3, the results in Table 7, Panel A show

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40 For example, for the one year ahead actual earnings prediction model, the coefficient of rank 3 is 96% ($= 0.860/0.896$) of the coefficient of rank 2 whereas $R^2$ of rank 3 is 88% ($= 0.7158/0.8165$) of $R^2$ of rank 2.

41 A denominator is identical for both models.

42 The coefficients on the interaction terms should be interpreted with much care. Since quintile ranks are scaled to lie between 0 and 1 for the results reported in Table 3, Panel B, the coefficients on the interaction terms represent the difference in the coefficient between rank 1 and rank 5.
that the coefficients and $R^2$s are generally decreasing as the measurement error proxy ("meproxy") increases. This implies that analysts’ consensus forecasts that contain larger measurement error explain less of the market expectation of future earnings. Table 7, Panel B provides the evidence that the adjusted $R^2$s increase when information about cross-sectional variation of the measurement error is incorporated. This implies that noisy measurement of the market expectation of future earnings can affect the results of value relevance studies. Therefore, implications of the measurement error in an independent variable should also be considered in the value relevance studies.

6.5 Effect of cross-sectional variation of measurement error in analysts’ consensus forecasts on predictability of future stock returns

Table 8 presents the results of the regressions for predicting one year and five year future stock returns with earnings yield. The results of Table 8, Panel A show that the coefficients and $R^2$s generally decrease as the measurement error proxy ("meproxy") increases. It should be noted that the coefficients of the prediction model based on all observations (i.e., 0.491 for one year future stock returns prediction and 2.559 for five year future stock returns prediction) are only greater than those of the prediction model for rank 5. Therefore, predicting future stock returns based on the coefficient from the prediction model that ignores cross-sectional variation of the measurement error would be inefficient and misleading.

To examine whether the specification of stock returns prediction models is improved when information about cross-sectional variation of the measurement error is incorporated, the regressions are conducted with the interaction term included between earnings yield and an indicator for each rank as in model (2) and (4) of Table 8, Panel B. Consistent with Hypothesis 4, when information about cross-sectional variation of the measurement error is incorporated as in model (2) and (4), the adjusted $R^2$s are larger and improvement is even greater than the improvement of the earnings prediction models. This implies that the sum of all squared prediction errors (i.e., the squared difference between future stock returns and the predicted value from each model) is smaller for the prediction model with information about cross-sectional variation of the measurement error. The significantly negative coefficients on the interaction terms confirm the results reported in Table 8, Panel A. The predictability of five year future stock returns provides the evidence that the impact of a given year’s noisy measurement of the market expectation of future earnings continues to exist for quite a long period. To examine whether growth in earnings is the main driver of the results, actual earnings growth for the next five years is included in each of the future stock returns prediction models and results do not change (not reported).

Consistent with the findings in Trueman (1993), the results in Table 8 imply that when researchers measure the market expectation of future earnings with error and they do not incorporate information about cross-sectional variation of the measurement error, spurious results can arise in the tests of a relation between earnings and stock returns, and the framework offered in this study helps to mitigate related issues.

6.6 Out of sample tests for improvement in predictability of future earnings and stock returns

43 In their review paper on the value relevance study, Holthousen and Watts (2000) quotes: “If the amount is fraught with too much measurement error, the researcher would not detect a significant relation.”
To confirm the results reported in Table 6, the out of sample tests for predicting future earnings are conducted according to the following procedure:

(i) Measurement error proxy is constructed in the same way as described in 6.1.
(ii) For each prediction year, two types of prediction regressions are conducted using the data of data years as in Table 6, Panel B.
(iii) For each prediction year, predicted values are computed by multiplying each independent variable by the respective coefficient from step (iii) for each of two prediction models.
(iv) Mean absolute difference (MAD) between the respective predicted values and future earnings is computed for each model. MAD1 denotes MAD based on the prediction model without information about cross-sectional variation of the measurement error and MAD2 denotes MAD based on the prediction model with information about cross-sectional variation of the measurement error.

The results reported in Table 9, Panel A show that MAD1 is greater than MAD2 for all years and differences are significant at 1% level (difference for the prediction of one year ahead actual earnings for 2004 is significant at 10% level).

To confirm the results reported in Table 8, the out of sample tests for predicting future stock returns are conducted based on the same procedure as those for predicting future earnings. Table 9, Panel B reports that MAD2 is generally smaller than MAD1 with some exceptions. The results are weaker than the prediction of future earnings because the prediction of future stock returns would be affected by more factors than the prediction of future earnings.

In conclusion, the results in Table 9 confirm that incorporating information about cross-sectional variation of the measurement error improves predictability of future earnings and stock returns.
Chapter 7

Robustness checks

7.1 Effect of earnings persistence

Prior research including Freeman and Tse (1992) and Das and Lev (1993) documents the S-shaped relation (i.e. non linear relation) between unexpected earnings and unexpected stock returns, arguing that this finding is due to earnings persistence and large unexpected earnings contains more transitory components that are more heavily discounted by the market. Since the measurement error proxy (“meproxy”) used in this study is constructed based on the unexpected earnings that prior research uses as a proxy for earnings persistence, one might think results in this study are driven by earnings persistence (i.e. transitory earnings) rather than the measurement error in a noisy predictor. To examine whether this is the case, two tests are conducted.

Ali and Zarowin (1992) reports that unexpected earnings converge to earnings level rather than earnings changes if earnings follow IMA(1,1) process and tend to be transitory. Their finding implies that if the results are entirely driven by earnings persistence (i.e. if quintile ranks in this study represent the degree of earnings persistence), including earnings level should induce a similar level of earnings response coefficients across quintile ranks. Although it is not plausible to include future earnings level in forecasting models, this study artificially includes the future earnings level to investigate whether the results are driven by earnings persistence. 1 year actual earnings deflated by stock price at the end of third month from prior fiscal year end is used as a proxy for the future earnings level. As shown in Table 10, Panel A, earnings response coefficients are still decreasing as the measurement error proxy (“meproxy”) increases. Sum of two earnings response coefficients are also decreasing. Ali and Zarowin (1992) also confirms this result. This suggests that measurement error effects seem dominating earnings persistence effects.

Stock returns represent the change in market’s expectation on future earnings (i.e. dividend or cash flows). Therefore, if earnings are less persistent, stock returns tend to be more volatile. To control for earnings persistence, stock returns volatility defined as the standard deviation of daily raw stock returns for the prediction period is included in all prediction models. Table 10, Panel B reports that the forecasting coefficients and R²’s are still decreasing as the measurement error proxy (“meproxy”) increases. This again confirms that the measurement error is the main driver of the results.

Tests are also conducted with earnings volatility and earnings growth for next five years and the results are qualitatively similar (not reported). Results reported in Table 10 are consistent with the finding of Das and Lev (1994). They find that a non linear relation between unexpected earnings and unexpected stock returns still exists even after special items are removed.

7.2 Effect of negative analysts’ consensus forecasts and size

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44 For 5 year stock returns forecasting, standard deviation of daily raw stock returns over the next 5 years are included. For the rest of the tests, standard deviations of daily raw stock returns over the next 1 year are included.
Results reported in Table 6, 7 and 8 reveal that as the measurement error proxy (“meproxy”) becomes larger, analysts’ consensus forecasts tend to be more negative. Therefore, to investigate whether the results reported in Table 6, 7 and 8 are driven by the negative analysts’ consensus forecasts, the same analyses are conducted only with the positive analysts’ consensus forecasts. Results reported in Table 11 suggest that the negative analysts’ consensus forecasts are not a main driver of the results presented in Table 6, 7 and 8.

Results reported in Table 6, 7 and 8 also reveal that as the measurement error proxy (“meproxy”) becomes larger, a firm size measured as the prior fiscal year end total assets tends to be smaller. Moreover, even after the negative analysts’ consensus forecasts are removed, the pattern of a firm size does not disappear (not reported). To see whether the firm size accounts for results, the whole tests are again conducted with the information on firm size incorporated. For this end, quintile ranks are formed based on the total assets as of prior fiscal year end and quintile ranks are scaled to be lie between 0 and 1. Then, the information on the firm size variability is incorporated as an interaction term with respective predictors. To remove the effect of negative analysts’ consensus forecasts, only positive analysts’ consensus forecasts are used. The results presented in Table 11 still show the significantly negative interaction terms between respective predictor and quintile ranks based on the measurement error proxy (“meproxy”) in all forecasting models although results become weaker. This implies that firm size is not a main driver of the results.

7.3 Tests with undeflated measurement error

Cheong and Thomas (2010) reports that analysts’ forecast error and dispersion do not vary with the scale. To reflect this finding, all tests are repeated with the measurement error proxy (“meproxy”) based on undeflated measurement error regressions and the results are similar except for the 1 year stock returns forecasting. For 1 year stock returns forecasting, forecasting coefficient of rank 1 is smaller than that of rank 2 and 3 (not reported).

7.4 Tests with deflated earnings

Main purpose of tests reported in Table 6 is to document the evidence that measurement error in analysts’ consensus forecast has implications in forecasting undeflated future periodic or permanent earnings. Many prior studies examine forecastability of future earnings based on the earnings deflated by stock price. To be consistent with prior studies, the same tests are conducted with analysts’ consensus forecasts, 1 year actual earnings and 5 year mean actual earnings scaled by stock price at the end of third month after fiscal year end. Results are qualitatively similar to the tests with undeflated earnings although results are weaker (not reported).

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45 Test with market value at prior year fiscal year end produces similar results.
46 Median total assets as of prior fiscal year end for rank 1 is about 1800m and for rank 5 is about 600m.
Chapter 8

Concluding remarks

Using analysts’ consensus forecasts, this study investigates implications of measuring the market expectation of future earnings with error for predicting future earnings and stock returns. With the framework that estimates cross-sectional variation of the measurement error in analysts’ consensus forecasts based on the errors-in-variables approach, this study reports that the measurement error in analysts’ consensus forecasts impairs the predictability of future earnings and stock returns and causes inference problems. This study also documents that incorporating information about cross-sectional variation of the measurement error can generate better specified prediction models and improved predictions of future earnings and stock returns. The findings of this study can be readily extended to other accounting research that relies on the market expectation of future earnings.

A fruitful future research area extending the results of this study includes finding a method to control for (ex-post) earnings news in capturing the (ex-ante) measurement error in analysts’ consensus forecasts to generate better specified prediction models. Another suggested research area is the test of the market level predictability of future stock returns based on earnings yield. Some recent research including Ang and Bekaert (2007) argues that earnings yield has little or no predictive ability of future stock returns.\(^{47}\)\(^{48}\) The findings of this study suggest that the results of prior studies may be due to the observations that measure the market expectation of future earnings with much error and incorporating information about cross-sectional variation of measurement error (i.e., less weight is given to the observations with large measurement error) would bring different results.


\(^{48}\) It is fully recognized that the results of this study are based on the firm specific predictions and the debate on the predictability of future stock returns using earnings yield is mainly based on the market data. However, if market index were re-constructed by incorporating information about cross-sectional variation of the measurement error, more significant and meaningful results should be noted.
References


Figure 1. Illustration of sources of error in measuring market expectation of future earnings

<table>
<thead>
<tr>
<th></th>
<th>Case 1 (Correct and (rational) forecast by each analyst)</th>
<th>Case 2 (Noisy forecast by each analyst)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market expectation of</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>future earnings</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Analyst 1</td>
<td>10</td>
<td>11 (Optimistic bias)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9 (Staleness)</td>
</tr>
<tr>
<td>Analyst 2</td>
<td>10</td>
<td>8 (Inability)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8 (Herding)</td>
</tr>
<tr>
<td>Analyst 3</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Analyst 4</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Analysts’ consensus forecast</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>Dispersion</td>
<td>0</td>
<td>1.4</td>
</tr>
<tr>
<td>Earnings news</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Actual earnings</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

This figure illustrates a situation where the analysts’ consensus forecast measures the market expectation of future earnings with error. This figure also shows how the future actual earnings become different from the market expectation of future earnings.

Note: It is possible that analysts’ consensus forecasts happen to coincide with the market expectation of future earnings although each analyst’s forecast measures the market expectation of future earnings with error. However, this is not likely the case for most of the firms covered by many analysts.
Figure 2. Timeline of prediction of one year ahead actual earnings

\[ n_{t+1} \]

\( t \) (Prediction) \hspace{1cm} \( t+1 \) (Realization)

\[ F_{t}^{t+1} = M_{t}^{t+1} + e_{t} \]

\[ A_{t+1} = M_{t}^{t+1} + n_{t+1} = F_{t}^{t+1} - e_{t} + n_{t+1} \]

\[ e_{t} \]
- Decreases coefficient
- Increases residual variance (i.e. Reduces R^2)

\[ n_{t+1} \]
- Increases residual variance (i.e. Reduces R^2)

where

\( A_{t+1} \): Actual earnings at time \( t+1 \)

\( M_{t}^{t+1} \): Market expectation at time \( t \) of actual earnings at time \( t+1 \)

\( F_{t}^{t+1} \): Analysts’ consensus forecast at time \( t \) for actual earnings at time \( t+1 \)

\( e_{t} \): Measurement error in analysts’ consensus forecast at time \( t \).

\( n_{t+1} \): Earnings news during time \( t+1 \)
Figure 3. Timeline of prediction of one year future stock returns

\[ \frac{F_{t+1}^{t+1}}{P_t} = \left( M_{t+1}^{t+1} + e_t \right) / P_t \]

\[ \text{Ret}_t = \left( M_{t+1}^{t+1} + n_{t+1} \right) / P_t \]

\[ = \left( F_{t+1}^{t+1} - e_t + n_{t+1} \right) / P_t \]

where

\( \text{Ret}_t \): One year future stock returns

\( P_t \): Stock price at time \( t \)

\( M_{t+1}^{t+1} \): Market expectation at time \( t \) of actual earnings at time \( t+1 \)

\( F_{t+1}^{t+1} \): Analysts’ consensus forecast at time \( t \) for actual earnings at time \( t+1 \)

\( e_t \): Measurement error in analysts’ consensus forecast at time \( t \)

\( n_{t+1} \): Earnings news during time \( t+1 \).
Figure 4. Pattern of coefficient estimates and $R^2$s in $\text{Var}(e) / \text{Var}(x^*)$ when all observations contain measurement error

Figure 4A. Pattern of Coefficient Estimates in $\text{Var}(e) / \text{Var}(x^*)$

Figure 4B. Pattern of $R^2$ in $\text{Var}(e) / \text{Var}(x^*)$

Figure 4 shows the pattern of the coefficient estimates and $R^2$s from the prediction model below as the relative measurement error variance ($\text{Var}(e)$) to the variance of the true predictor increases.

$$y = \beta_0 + \beta_{EIV} x + w$$
Figure 5. Pattern of coefficient estimates and $R^2$'s in $\text{Var}(e)/\text{Var}(x^*)$ when half of observations are measured with error and observations with measurement error are identified with indicator variable.

**Figure 5A. Pattern of Coefficient Estimates in $\text{Var}(e)/\text{Var}(x^*)$**

![Coefficient Estimates Graph](image)

**Figure 5B. Pattern of $R^2$ in $\text{Var}(e)/\text{Var}(x^*)$**

![R2 Graph](image)

Figure 5 shows the pattern of the coefficient estimates and $R^2$'s from the prediction models below as the relative measurement error variance ($\text{Var}(e)$) to the variance of the true predictor increases.

1. $y = \beta_0 + \beta_{EIV} x + w$
2. $y = \gamma_0 + \gamma_1 x + \gamma_2 D + \gamma_3 xD + \nu$ where $D = 1$ when observations contain measurement error.

For this analysis, the half of the observations is assumed to be measured with error. The first prediction model does not use any measurement error indicator. The first prediction model assumes the situation where the observations with the measurement error cannot be identified. The second prediction model uses the indicator for the observations that contain the measurement error. The second prediction model assumes the situation where the observations with the measurement error can be distinguished from those without measurement error.
Figure 6. Pattern of coefficient estimates and $R^2$'s in $Var(e) / Var(x^*)$ when sample consists of four groups with distinctive measurement error distribution and observations with each distinctive measurement error distribution are identified with respective indicator variable.

Figure 6A. Pattern of Coefficient Estimates in $Var(e) / Var(x^*)$

Figure 6B. Pattern of $R^2$ in $Var(e) / Var(x^*)$

Figure 6 shows the pattern of the coefficient estimates and $R^2$'s from the prediction models below as the relative measurement error variance of the second group ($Var(e)$) to the variance of the true predictor increases. For this analysis, the sample consists of four groups with equal number of observations. The first group of observations contains no measurement error. The second group of observations contains the measurement error with zero mean and variance of $Var(e)$. The third group of observations contains the measurement error with zero mean and variance of $4Var(e)$. The last group of observations contains the measurement error with zero mean and variance of $9Var(e)$. 

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1. \( y = \beta_0 + \beta_{\text{IV}} x + w \)

2. \( y = \tau_0 + \tau_1 x + \tau_2 D_1 + \tau_3 x D_3 + \chi \) where \( D_1 = 1 \) when observations contain measurement error with variance of \( 4\text{Var}(e) \) and \( 9\text{Var}(e) \) and \( 0 \) otherwise

3. \( y = \gamma_0 + \gamma_1 x + \gamma_2 D + \gamma_3 x D + \nu \) where \( D = 1 \) when observations contain measurement error

4. \( y = \kappa_0 + \kappa_1 D_1 + \kappa_2 D_2 + \kappa_3 D_3 + \kappa_4 x D_4 + \kappa_5 x D_5 + \kappa_6 x D_6 + \kappa_7 x D_7 + \kappa_8 x D_8 + \kappa_9 x D_9 + \nu \) where \( D_1 = 1 \) when \( x \) contains measurement error with variance of \( \text{Var}(e) \) and \( 0 \) otherwise, \( D_2 = 1 \) when \( x \) contains measurement error with variance of \( 4\text{Var}(e) \) and \( 9\text{Var}(e) \) and \( 0 \) otherwise and \( D_3 = 1 \) when \( x \) contains measurement error with variance of \( 9\text{Var}(e) \) and \( 0 \) otherwise

The first prediction model does not use any measurement error indicator. The first prediction model assumes the situation where the observations with the measurement error cannot be identified. The second prediction model uses the indicator for the observations that contain the measurement error with variance of \( 4\text{Var}(e) \) and \( 9\text{Var}(e) \). The second prediction model assumes the situation where the observations with small measurement error can be distinguished from those with large measurement error although not all observations with the measurement error can be separately identified. The third prediction model uses the indicator for the observations with any measurement error distribution. The third prediction model assumes the situation where the observations with measurement error can be identified although each different measurement error distribution cannot be separately identified. The last prediction model uses the respective indicator for each measurement error distribution. The last prediction model assumes the situation where the observations with the different measurement error distribution can be separately identified although the exact distribution for each measurement error is not known.
Table 1. Coefficient estimates and $R^2$s when all observations contain measurement error

Panel A. Coefficient Estimates and $R^2$s Based On Simulation

<table>
<thead>
<tr>
<th>Error Characteristics</th>
<th>True Predictor</th>
<th>Measurement Error with $N \sim (0, 1)$</th>
<th>Measurement Error with $N \sim (0, 4)$</th>
<th>Measurement Error with $N \sim (0, 9)$</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Observations with Measurement Error</td>
<td>0</td>
<td>40000</td>
<td>40000</td>
<td>40000</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.038 (0.58)</td>
<td>10.103*** (125.51)</td>
<td>16.179*** (223.68)</td>
<td>18.199*** (274.17)</td>
</tr>
<tr>
<td>Noisy Observed Predictor</td>
<td>10.009*** (336.43)</td>
<td>5.031*** (152.99)</td>
<td>2.007*** (82.80)</td>
<td>0.998*** (55.83)</td>
</tr>
<tr>
<td>$N$</td>
<td>40000</td>
<td>40000</td>
<td>40000</td>
<td>40000</td>
</tr>
<tr>
<td>$R^2$</td>
<td>73.89%</td>
<td>36.92%</td>
<td>14.63%</td>
<td>7.23%</td>
</tr>
</tbody>
</table>

***, **, *: Significant at 1%, 5% and 10% level respectively
Numbers in parenthesis are t-statistics (two-tailed)

Panel B. Coefficient Estimates and $R^2$s Based On Theoretical Prediction

<table>
<thead>
<tr>
<th>Error Characteristics</th>
<th>True Predictor</th>
<th>Measurement Error with $N \sim (0, 1)$</th>
<th>Measurement Error with $N \sim (0, 4)$</th>
<th>Measurement Error with $N \sim (0, 9)$</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Observations with Measurement Error</td>
<td>0</td>
<td>40000</td>
<td>40000</td>
<td>40000</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.000</td>
<td>10.000</td>
<td>16.000</td>
<td>18.000</td>
</tr>
<tr>
<td>Noisy Observed Predictor</td>
<td>10.000</td>
<td>5.000</td>
<td>2.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$N$</td>
<td>40000</td>
<td>40000</td>
<td>40000</td>
<td>40000</td>
</tr>
<tr>
<td>$R^2$</td>
<td>73.53%</td>
<td>36.77%</td>
<td>14.71%</td>
<td>7.35%</td>
</tr>
</tbody>
</table>

This table presents the coefficient estimates and $R^2$s from the prediction model below.

$$y = \beta_0 + \beta_{EIV} x + w$$

For this analysis, all observations are assumed to be measured with error.
Table 2. Coefficient estimates and \( R^2 \)'s when half of observations contain measurement error and observations with measurement error are identified with indicator variable

Panel A. Coefficient Estimates and \( R^2 \)'s Based On Simulation

<table>
<thead>
<tr>
<th>Error Characteristics</th>
<th>True Predictor</th>
<th>Measurement Error with N ~ (0, 1)</th>
<th>Measurement Error with N ~ (0, 4)</th>
<th>Measurement Error with N ~ (0, 9)</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Observations with Measurement Error</td>
<td>0</td>
<td>20000</td>
<td>20000</td>
<td>20000</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.038 (0.58)</td>
<td>6.722*** (84.23)</td>
<td>0.038 (0.31)</td>
<td>13.456*** (173.47)</td>
</tr>
<tr>
<td>Noisy Observed Predictor</td>
<td>10.009*** (336.43)</td>
<td>6.703*** (197.35)</td>
<td>10.009*** (182.02)</td>
<td>3.357*** (114.33)</td>
</tr>
<tr>
<td>Measurement Error Indicator</td>
<td>10.064*** (64.39)</td>
<td>16.140*** (100.26)</td>
<td>18.161*** (112.63)</td>
<td>10.009*** (157.67)</td>
</tr>
<tr>
<td>Noisy Observed Predictor * Measurement Error Indicator</td>
<td>-4.977*** (-73.77)</td>
<td>-8.002*** (-118.68)</td>
<td>-9.011*** (-135.25)</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>40000</td>
<td>40000</td>
<td>40000</td>
<td>40000</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>73.89%</td>
<td>49.33%</td>
<td>55.40%</td>
<td>24.63%</td>
</tr>
</tbody>
</table>

***, **, *: Significant at 1%, 5% and 10% level respectively. Numbers in parenthesis are t-statistics (two-tailed). Measurement Error Indicator is 1 if Noisy observed predictor contains measurement error and 0 otherwise

Panel B. Coefficient Estimates and \( R^2 \)'s Based On Theoretical Prediction

<table>
<thead>
<tr>
<th>Error Characteristics</th>
<th>True Predictor</th>
<th>Measurement Error with N ~ (0, 1)</th>
<th>Measurement Error with N ~ (0, 4)</th>
<th>Measurement Error with N ~ (0, 9)</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Observations with Measurement Error</td>
<td>0</td>
<td>20000</td>
<td>20000</td>
<td>20000</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.000</td>
<td>6.667</td>
<td>0.000</td>
<td>13.334</td>
</tr>
<tr>
<td>Noisy Observed Predictor</td>
<td>10.000</td>
<td>6.667</td>
<td>10.000</td>
<td>3.333</td>
</tr>
<tr>
<td>Measurement Error Indicator</td>
<td>10.000</td>
<td>16.000</td>
<td>18.000</td>
<td></td>
</tr>
<tr>
<td>Noisy Observed Predictor * Measurement Error Indicator</td>
<td>-5.000</td>
<td>-8.000</td>
<td>-9.000</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>40000</td>
<td>40000</td>
<td>40000</td>
<td>40000</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>73.53%</td>
<td>49.02%</td>
<td>55.15%</td>
<td>24.51%</td>
</tr>
</tbody>
</table>

Measurement Error Indicator is 1 if Noisy observed predictor contains measurement error and 0 otherwise.
This table presents the coefficient estimates and $R^2$s from the prediction models below.

1. \( y = \beta_0 + \beta_{EIV} x + w \)

2. \( y = \gamma_0 + \gamma_1 x + \gamma_2 D + \gamma_3 xD + \nu \) where \( D = 1 \) when observations contain measurement error.

For this analysis, the half of the observations is assumed to be measured with error. The first prediction model does not use any measurement error indicator. The first prediction model assumes the situation where the observations with the measurement error cannot be identified. The second prediction model uses the indicator for the observations that contain the measurement error. The second prediction model assumes the situation where the observations with the measurement error can be distinguished from those without measurement error.
Table 3. Coefficient estimates and R²’s when sample consists of multiple groups and each group has distinctive measurement error distribution and observations with each distinctive measurement error distribution are identified with respective indicator variable

Panel A. Coefficient Estimates and R²’s Based On Simulation

<table>
<thead>
<tr>
<th></th>
<th>True Predictor</th>
<th>Without Measurement Error Indicator</th>
<th>With Small Measurement Error Indicator</th>
<th>With Measurement Error Indicator</th>
<th>With Indicator for Each Measurement Error Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.038 (0.58)</td>
<td>15.714*** (214.03)</td>
<td>6.722*** (50.50)</td>
<td>0.038 (0.17)</td>
<td>0.038 (0.18)</td>
</tr>
<tr>
<td>Noisy Observed Predictor</td>
<td>10.009*** (336.43)</td>
<td>2.235*** (88.40)</td>
<td>6.703*** (118.31)</td>
<td>10.009*** (101.41)</td>
<td>10.009*** (105.14)</td>
</tr>
<tr>
<td>Small Measurement Error Indicator</td>
<td></td>
<td></td>
<td>10.803*** (68.24)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Noisy Observed Predictor * Small Measurement Error Indicator</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measurement Error Indicator</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Noisy Observed Predictor * Measurement Error Indicator</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Indicator for Error Variance of 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Noisy Observed Predictor * Indicator for Error Variance of 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Indicator for Error Variance of 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Noisy Observed Predictor * Indicator for Error Variance of 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Indicator for Error Variance of 9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Noisy Observed Predictor * Indicator for Error Variance of 9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>40000</td>
<td>40000</td>
<td>40000</td>
<td>40000</td>
<td>40000</td>
</tr>
<tr>
<td>R²</td>
<td>73.89%</td>
<td>16.34%</td>
<td>29.52%</td>
<td>28.15%</td>
<td>33.16%</td>
</tr>
</tbody>
</table>

***, **, *: Significant at 1%, 5% and 10% level respectively
Numbers in parenthesis are t-statistics (two-tailed)
Small Measurement Error Indicator is 1 if Predictor contains measurement error with variance of 4 and 9 and 0 otherwise
Measurement Error Indicator is 1 if Predictor contains measurement error and 0 otherwise
Indicator for Error Variance of 1 is 1 if Predictor contains measurement error with variance of 1 and 0 otherwise
Indicator for Error Variance of 4 is 1 if Predictor contains measurement error with variance of 4 and 0 otherwise
Indicator for Error Variance of 9 is 1 if Predictor contains measurement error with variance of 9 and 0 otherwise
### Panel B. Coefficient Estimates and R²’s Based On Theoretical Prediction

<table>
<thead>
<tr>
<th></th>
<th>True Predictor</th>
<th>Without Measurement Error Indicator</th>
<th>With Small Measurement Error Indicator</th>
<th>With Measurement Error Indicator</th>
<th>With Indicator for Each Measurement Error Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.000</td>
<td>15.556</td>
<td>6.667</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Noisy Observed Predictor</td>
<td>10.000</td>
<td>2.222</td>
<td>6.667</td>
<td>10.000</td>
<td>10.000</td>
</tr>
<tr>
<td>Small Measurement Error Indicator</td>
<td></td>
<td>10.667</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Noisy Observed Predictor * Small Measurement Error Indicator</td>
<td></td>
<td>-5.333</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measurement Error Indicator</td>
<td></td>
<td></td>
<td>16.471</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Noisy Observed Predictor * Measurement Error Indicator</td>
<td></td>
<td></td>
<td>-8.235</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Indicator for Error Variance of 1</td>
<td></td>
<td></td>
<td></td>
<td>10.000</td>
<td></td>
</tr>
<tr>
<td>Noisy Observed Predictor * Indicator for Error Variance of 1</td>
<td></td>
<td></td>
<td></td>
<td>-5.000</td>
<td></td>
</tr>
<tr>
<td>Indicator for Error Variance of 4</td>
<td></td>
<td></td>
<td></td>
<td>16.000</td>
<td></td>
</tr>
<tr>
<td>Noisy Observed Predictor * Indicator for Error Variance of 4</td>
<td></td>
<td></td>
<td></td>
<td>-8.000</td>
<td></td>
</tr>
<tr>
<td>Indicator for Error Variance of 9</td>
<td></td>
<td></td>
<td></td>
<td>18.000</td>
<td></td>
</tr>
<tr>
<td>Noisy Observed Predictor * Indicator for Error Variance of 9</td>
<td></td>
<td></td>
<td></td>
<td>-9.000</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>40000</td>
<td>40000</td>
<td>40000</td>
<td>40000</td>
<td>40000</td>
</tr>
<tr>
<td>R²</td>
<td>73.53%</td>
<td>16.34%</td>
<td>29.41%</td>
<td>28.11%</td>
<td>33.09%</td>
</tr>
</tbody>
</table>

Small Measurement Error Indicator is 1 if Predictor contains measurement error with variance of 4 and 9 and 0 otherwise
Indicator for Error Variance of 1 is 1 if Predictor contains measurement error and 0 otherwise
Indicator for Error Variance of 4 is 1 if Predictor contains measurement error with variance of 4 and 0 otherwise
Indicator for Error Variance of 9 is 1 if Predictor contains measurement error with variance of 9 and 0 otherwise

This table presents the coefficient estimates and R²’s from the prediction models below. For this analysis, the sample consists of four groups with equal number of observations. The first group of observations contains no measurement error. The second group of observations contains the measurement error with zero mean and variance of 1. The third group of observations contains the measurement error with zero mean and variance of 4. The last group of observations contains the measurement error with zero mean and variance of 9.

1. \( y = \beta_0 + \beta_{iv} x + w \)
2. \( y = \tau_0 + \tau_1 x + \tau_2 D_s + \tau_3 x D_s + \chi \) where \( D_s = 1 \) when observations contain measurement error with variance of 4 and 9 and 0 otherwise
3. \( y = \gamma_0 + \gamma_1 x + \gamma_2 D + \gamma_3 x D + \nu \) where \( D = 1 \) when observations contain measurement error
4. \[ y = \kappa_0 + \kappa_1 x + \kappa_2 D_1 + \kappa_3 x D_1 + \kappa_4 x D_2 + \kappa_5 x D_3 + \kappa_6 x D_4 + \nu \] where \( D_1 = 1 \) when \( x \) contains measurement error with variance of 1 and 0 otherwise, \( D_2 = 1 \) when \( x \) contains measurement error with variance of 4 and 0 otherwise and \( D_3 = 1 \) when \( x \) contains measurement error with variance of 9 and 0 otherwise.

The first prediction model does not use any measurement error indicator. The first prediction model assumes the situation where the observations with the measurement error cannot be identified. The second prediction model uses the indicator for the observations that contain the measurement error with variance of 4 and 9. The second prediction model assumes the situation where the observations with small measurement error can be distinguished from those with large measurement error although not all observations with the measurement error can be separately identified. The third prediction model uses the indicator for the observations with any measurement error distribution. The third prediction model assumes the situation where the observations with measurement error can be identified although each different measurement error distribution cannot be separately identified. The last prediction model uses the respective indicator for each measurement error distribution. The last prediction model assumes the situation where the observations with the different measurement error distribution can be separately identified although the exact distribution for each measurement error is not known.
Table 4. Analysts’ consensus forecast error regressions and construction of measurement error proxy (“meproxy”)

Panel A. Fama-MacBeth regression of analysts’ consensus forecast error on determinants

\[
\frac{abs(A_{t,i} - F_t^{t+1})}{P_t} = \alpha_0 + \alpha_1 \frac{std(F_t^{t+1})}{P_t} + \alpha_2 \frac{abs(A_{t-1} - F_t^{t+1})}{P_{t-1}} + \alpha_3 \frac{abs(A_{t-2} - F_t^{t+1})}{P_{t-2}} + \epsilon_t
\]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
<tr>
<td>(a_0)</td>
<td>0.006***</td>
<td>0.014***</td>
<td>0.013***</td>
<td>0.009***</td>
<td>0.003***</td>
</tr>
<tr>
<td></td>
<td>(8.41)</td>
<td>(18.08)</td>
<td>(10.49)</td>
<td>(12.05)</td>
<td>(9.13)</td>
</tr>
<tr>
<td>(a_1)</td>
<td>2.728***</td>
<td></td>
<td></td>
<td>1.871***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(9.77)</td>
<td></td>
</tr>
<tr>
<td>(a_2)</td>
<td></td>
<td>0.604***</td>
<td></td>
<td>0.034***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(48.51)</td>
<td></td>
<td>(3.48)</td>
<td></td>
</tr>
<tr>
<td>(a_3)</td>
<td></td>
<td></td>
<td>0.715***</td>
<td></td>
<td>0.257***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(59.41)</td>
<td></td>
<td>(20.84)</td>
</tr>
<tr>
<td>(a_4)</td>
<td></td>
<td></td>
<td></td>
<td>1.014***</td>
<td>0.166***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(22.33)</td>
<td>(4.42)</td>
</tr>
<tr>
<td># of years</td>
<td>17</td>
<td>17</td>
<td>17</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>Mean Adjusted R(^2)</td>
<td>36.09%</td>
<td>22.86%</td>
<td>27.72%</td>
<td>22.53%</td>
<td>40.40%</td>
</tr>
</tbody>
</table>

***, **, *: Significant at 1%, 5% and 10% level respectively
Numbers in parenthesis are t-statistics based on Fama-MacBeth regression

Panel B. Pearson correlations among measurement error proxy and analysts’ consensus forecast error

\[
\frac{abs(A_{t,i} - F_t^{t+1})}{P_t}
\]

\[\text{meproxy} = 0.623***\]

***, **, *: Significant at 1%, 5% and 10% level respectively

Panel A presents the results of the annual Fama-MacBeth regressions of the analysts’ consensus forecast error on the determinants. For Panel A, total observations are 19,970 firm years between 1990 and 2006. Panel B presents the Pearson correlation between the analysts’ consensus forecast error and measurement error proxy (“meproxy”). For Panel B, total observations are 21,158 firm years between 1991 and 2007. All variables are per share basis. Variable definitions are provided in Appendix B. All variables are winsorized at 1% level for both extremes. Description of each notation is provided below:

\(A_{t,i}\): Actual earnings at time \(t+1\)
\(A_t\): Actual earnings at time \(t\)
\(F_t^{t+1}\): Analysts’ consensus forecast made at time \(t\) for actual earnings at time \(t+1\)
\(F_{t-1}^{t+1}\): Analysts’ consensus forecast made at time \(t-1\) for actual earnings at time \(t+1\)
\(F_{t-2}^{t+1}\): Analysts’ consensus forecast made at time \(t-2\) for actual earnings at time \(t+1\)
\(P_t\): Stock price at time \(t\)
\(P_{t-1}\): Stock price at time \(t-1\)
\(R_{t-1}\): Contemporaneous stock returns.

Note: Measurement error proxy (“meproxy”) is constructed by multiplying each determinant by the respective coefficient from the analysts’ consensus forecast error regression based on the data between 1990 and the immediate prior year.
### Table 5. Descriptive statistics of variables used for predicting future earnings and stock returns

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>75%</th>
<th>Median</th>
<th>25%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Measurement Error Proxy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>meproxy</td>
<td>0.025</td>
<td>0.038</td>
<td>0.026</td>
<td>0.014</td>
<td>0.010</td>
</tr>
<tr>
<td><strong>Independent Variables</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_{c, t+1}$</td>
<td>1.494</td>
<td>1.957</td>
<td>2.030</td>
<td>1.220</td>
<td>0.620</td>
</tr>
<tr>
<td>$F_{c, t+5}/P_t$</td>
<td>0.056</td>
<td>0.066</td>
<td>0.082</td>
<td>0.060</td>
<td>0.039</td>
</tr>
<tr>
<td><strong>Dependent Variables</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_{t+1}$</td>
<td>1.226</td>
<td>2.105</td>
<td>1.910</td>
<td>1.080</td>
<td>0.460</td>
</tr>
<tr>
<td>$A_{t-1,t+5}$</td>
<td>1.156</td>
<td>1.956</td>
<td>1.920</td>
<td>1.128</td>
<td>0.472</td>
</tr>
<tr>
<td>Ret$_t$</td>
<td>0.117</td>
<td>0.562</td>
<td>0.318</td>
<td>0.067</td>
<td>-0.176</td>
</tr>
<tr>
<td>Ret$_5$</td>
<td>0.584</td>
<td>1.286</td>
<td>1.072</td>
<td>0.369</td>
<td>-0.255</td>
</tr>
<tr>
<td>$P_t$</td>
<td>25.597</td>
<td>21.798</td>
<td>31.750</td>
<td>20.250</td>
<td>5.160</td>
</tr>
</tbody>
</table>

This table presents descriptive statistics on the measurement error proxy ("meproxy") and variables used to predict future earnings and stock returns. All variables are per share basis. Variable definitions are provided in Appendix B and C. All variables are winsorized at 1% level for both extremes. Total observations are 21,158 firm years between 1991 and 2007 (16,102 firm years between 1991 and 2004 for five year ahead mean actual earnings and five year future stock returns).
Table 6. Effect of measurement error on predictability of future earnings and evaluation of measurement error proxy (“meproxy”)

Panel A. Effect of cross-sectional variation of measurement error on predictability of future earnings and evaluation of measurement error proxy (“meproxy”)

\[ A_{t,i} = \beta_0 + \beta_iF_{t-1} + \epsilon_i \]

<table>
<thead>
<tr>
<th>Rank of “meproxy”</th>
<th>One Year Ahead Actual Earnings</th>
<th>Five Year Ahead Mean Actual Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>( \beta_\epsilon )</td>
</tr>
<tr>
<td>All</td>
<td>21158</td>
<td>-0.016</td>
</tr>
<tr>
<td>1 (Smallest)</td>
<td>4227</td>
<td>0.105</td>
</tr>
<tr>
<td>2</td>
<td>4234</td>
<td>0.090</td>
</tr>
<tr>
<td>3</td>
<td>4235</td>
<td>0.080</td>
</tr>
<tr>
<td>4</td>
<td>4234</td>
<td>-0.016</td>
</tr>
<tr>
<td>5 (Largest)</td>
<td>4228</td>
<td>-0.532</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rank of “meproxy”</th>
<th>N</th>
<th>( \beta_\epsilon )</th>
<th>( \beta_i )</th>
<th>R(^2)</th>
<th>Pearson correlation</th>
<th>Neg %</th>
<th>Total Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>16102</td>
<td>0.334</td>
<td>0.594</td>
<td>0.3277</td>
<td>0.572</td>
<td>7.66</td>
<td>1302</td>
</tr>
<tr>
<td>1 (Smallest)</td>
<td>3216</td>
<td>0.398</td>
<td>0.765</td>
<td>0.5282</td>
<td>0.727</td>
<td>0.90</td>
<td>1968</td>
</tr>
<tr>
<td>2</td>
<td>3223</td>
<td>0.516</td>
<td>0.665</td>
<td>0.4493</td>
<td>0.670</td>
<td>0.56</td>
<td>1646</td>
</tr>
<tr>
<td>3</td>
<td>3223</td>
<td>0.448</td>
<td>0.620</td>
<td>0.4537</td>
<td>0.674</td>
<td>2.20</td>
<td>1321</td>
</tr>
<tr>
<td>4</td>
<td>3223</td>
<td>0.343</td>
<td>0.552</td>
<td>0.3229</td>
<td>0.568</td>
<td>7.54</td>
<td>1060</td>
</tr>
<tr>
<td>5 (Largest)</td>
<td>3217</td>
<td>-0.317</td>
<td>0.512</td>
<td>0.2436</td>
<td>0.494</td>
<td>27.14</td>
<td>802</td>
</tr>
</tbody>
</table>

Panel B. Comparison of specification of future earnings prediction models

(1) \[ A_{t,i} = \alpha_0 + \alpha_iF_{t-1} + \epsilon_i \]

(2) \[ A_{t,i} = \beta_0 + \beta_iF_{t-1} + \beta_i\text{Rank} + \beta_iF_{t-1}\cdot\text{Rank} + \epsilon_i \]

(3) \[ A_{t,i} = \beta_0 + \beta_iF_{t-1} + \epsilon_i \]

(4) \[ A_{t,i} = \beta_0 + \beta_iF_{t-1} + \beta_i\text{Rank} + \beta_iF_{t-1}\cdot\text{Rank} + \epsilon_i \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>One Year Ahead Actual Earnings</th>
<th>Five Year Ahead Mean Actual Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.016</td>
<td>0.225***</td>
</tr>
<tr>
<td></td>
<td>(-0.67)</td>
<td>(6.48)</td>
</tr>
<tr>
<td>( F_{t-1} )</td>
<td>0.832***</td>
<td>0.918***</td>
</tr>
<tr>
<td></td>
<td>(46.02)</td>
<td>(38.54)</td>
</tr>
<tr>
<td>Rank</td>
<td>-0.556***</td>
<td>-0.134***</td>
</tr>
<tr>
<td></td>
<td>(-10.26)</td>
<td>(-3.11)</td>
</tr>
<tr>
<td>N</td>
<td>21158</td>
<td>21158</td>
</tr>
<tr>
<td>Adjusted R(^2)</td>
<td>59.79%</td>
<td>61.58%</td>
</tr>
</tbody>
</table>

***, **, *: Significant at 1%, 5% and 10% level respectively

Numbers in parenthesis are t-statistics based on heteroscedasticity consistent standard errors

Rank: Represents quintile rank of “meproxy” (Scaled to lie between 0 and 1)

Panel A reports the results of the regression for predicting future actual earnings with analysts’ consensus forecast for each of quintile portfolio formed on the measurement error proxy (“meproxy”). Panel B reports results of the regression for predicting future earnings with analysts’ consensus forecast with an indicator for each rank. Rank is scaled to lie between 0 and 1. Pearson correlation is the Pearson correlation coefficient between respective independent variable and dependent variable. Neg % is the proportion of the negative analysts’ consensus forecasts. Total assets are the median total assets of each rank at time t. All variables are per share basis. Variable definitions are provided in Appendix C. All variables are winsorized at 1% level for both extremes. Total observations are 21,158 firm years between 1991 and 2007 (16,102 firm years between 1991 and 2004 for predicting five year ahead mean actual earnings).
Table 7. Effect of measurement error on explaining stock price with analysts’ consensus forecast

Panel A. Effect of cross-sectional variation of measurement error on explaining stock price with analysts’ consensus forecast

\[ P_t = \beta_0 + \beta_1 F_{i,t-1}^{meproxy} + e_t \]

<table>
<thead>
<tr>
<th>Rank of “meproxy”</th>
<th>Stock Price</th>
<th>N</th>
<th>( \beta_0 )</th>
<th>( \beta_1 )</th>
<th>( R^2 )</th>
<th>Pearson correlation</th>
<th>Neg %</th>
<th>Total Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td></td>
<td>21158</td>
<td>16.689</td>
<td>5.962</td>
<td>0.2866</td>
<td>0.535</td>
<td>7.60</td>
<td>1377</td>
</tr>
<tr>
<td>1 (Smallest)</td>
<td></td>
<td>4227</td>
<td>14.800</td>
<td>11.003</td>
<td>0.3198</td>
<td>0.566</td>
<td>0.73</td>
<td>2139</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>4234</td>
<td>9.182</td>
<td>10.808</td>
<td>0.5681</td>
<td>0.754</td>
<td>0.47</td>
<td>1718</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>4235</td>
<td>10.078</td>
<td>9.222</td>
<td>0.5478</td>
<td>0.740</td>
<td>1.98</td>
<td>1357</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>4234</td>
<td>12.606</td>
<td>7.014</td>
<td>0.3992</td>
<td>0.632</td>
<td>7.16</td>
<td>1101</td>
</tr>
<tr>
<td>5 (Largest)</td>
<td></td>
<td>4228</td>
<td>16.931</td>
<td>2.830</td>
<td>0.1435</td>
<td>0.379</td>
<td>27.65</td>
<td>836</td>
</tr>
</tbody>
</table>

Panel B. Comparison of model specification

(1) \[ P_t = \alpha_0 + \alpha_1 F_{i,t-1}^{meproxy} + e_t \]

(2) \[ P_t = \beta_0 + \beta_1 F_{i,t-1}^{meproxy} + \beta_2 \text{Rank} + \beta_3 F_{i,t-1}^{meproxy} \cdot \text{Rank} + e_t \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Stock Price</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td></td>
<td>16.689***</td>
<td>9.918***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(52.67)</td>
<td>(13.24)</td>
</tr>
<tr>
<td>( F_{i,t-1}^{meproxy} )</td>
<td></td>
<td>5.962***</td>
<td>12.935***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(29.65)</td>
<td>(29.37)</td>
</tr>
<tr>
<td>Rank</td>
<td></td>
<td></td>
<td>5.761***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(6.26)</td>
</tr>
<tr>
<td>Rank ( \cdot F_{i,t-1}^{meproxy} )</td>
<td></td>
<td>-9.628***</td>
<td>-9.628***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(-17.26)</td>
</tr>
<tr>
<td>N</td>
<td></td>
<td>21158</td>
<td>21158</td>
</tr>
<tr>
<td>Adjusted ( R^2 )</td>
<td></td>
<td>28.65%</td>
<td>38.72%</td>
</tr>
</tbody>
</table>

***, **, *: Significant at 1%, 5% and 10% level respectively

Numbers in parenthesis are t-statistics based on heteroscedasticity consistent standard errors

Rank: Represents quintile rank of “meproxy” (Scaled to lie between 0 and 1)

Panel A reports results of the regression of stock price on analysts’ consensus forecast for each of quintile portfolio formed on the measurement error proxy (“meproxy”). Panel B reports results of the regression of stock price on analysts’ consensus forecast with an indicator for each rank. Rank is scaled to lie between 0 and 1. Pearson correlation is the Pearson correlation coefficient between respective independent variable and dependent variable. Neg % is the proportion of the negative analysts’ consensus forecasts. Total assets are the median total assets of each rank at time t. All variables are per share basis. Variable definitions are provided in Appendix C. All variables are winsorized at 1% level for both extremes. Total observations are 21,158 firm years between 1991 and 2007.
Table 8. Effect of measurement error on predictability of future stock returns

Panel A. Effect of cross-sectional variation of measurement error on predictability of future stock returns

\[ \text{Ret}_t = \beta_0 + \beta_1 \frac{F_t^{t+1}}{P_t} + e_t \]

<table>
<thead>
<tr>
<th>Rank of “meproxy”</th>
<th>One Year Future Stock Returns</th>
<th>Five Year Future Stock Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>( \beta_0 )</td>
</tr>
<tr>
<td>All</td>
<td>21158</td>
<td>0.089</td>
</tr>
<tr>
<td>1 (Smallest)</td>
<td>4227</td>
<td>-0.042</td>
</tr>
<tr>
<td>2</td>
<td>4234</td>
<td>-0.021</td>
</tr>
<tr>
<td>3</td>
<td>4235</td>
<td>-0.002</td>
</tr>
<tr>
<td>4</td>
<td>4234</td>
<td>0.037</td>
</tr>
<tr>
<td>5 (Largest)</td>
<td>4228</td>
<td>0.122</td>
</tr>
</tbody>
</table>

\[ \text{Ret}_t = \beta_0 + \beta_1 \frac{F_t^{t+1}}{P_t} + e_t \]

<table>
<thead>
<tr>
<th>Rank of “meproxy”</th>
<th>N</th>
<th>( \beta_0 )</th>
<th>( \beta_1 )</th>
<th>( \beta_{Rank} )</th>
<th>( \beta_{Rank} \cdot \frac{F_t^{t+1}}{P_t} )</th>
<th>Pearson correlation</th>
<th>Neg %</th>
<th>Total Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>16102</td>
<td>0.0436</td>
<td>2.559</td>
<td>0.0189</td>
<td>0.138</td>
<td>7.66</td>
<td>1302</td>
<td></td>
</tr>
<tr>
<td>1 (Smallest)</td>
<td>3216</td>
<td>-0.211</td>
<td>13.133</td>
<td>0.0199</td>
<td>0.315</td>
<td>0.90</td>
<td>1968</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3223</td>
<td>-0.088</td>
<td>10.962</td>
<td>0.0649</td>
<td>0.255</td>
<td>0.56</td>
<td>1646</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3223</td>
<td>0.055</td>
<td>7.886</td>
<td>0.0521</td>
<td>0.228</td>
<td>2.20</td>
<td>1321</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3223</td>
<td>0.327</td>
<td>4.558</td>
<td>0.0296</td>
<td>0.172</td>
<td>7.54</td>
<td>1060</td>
<td></td>
</tr>
<tr>
<td>5 (Largest)</td>
<td>3217</td>
<td>0.528</td>
<td>1.300</td>
<td>0.0111</td>
<td>0.105</td>
<td>27.14</td>
<td>802</td>
<td></td>
</tr>
</tbody>
</table>

Panel B. Comparison of specification of future stock returns prediction models

(1) \[ \text{Ret}_t = \beta_0 + \beta_1 \frac{F_t^{t+1}}{P_t} + e_t \]

(2) \[ \text{Ret}_t = \beta_0 + \beta_1 \frac{F_t^{t+1}}{P_t} + \beta_{Rank} + \beta_{Rank} \cdot \frac{F_t^{t+1}}{P_t} + e_t \]

(3) \[ \text{Ret}_t = \alpha_0 + \alpha_1 \frac{F_t^{t+1}}{P_t} + e_t \]

(4) \[ \text{Ret}_t = \beta_0 + \beta_1 \frac{F_t^{t+1}}{P_t} + \beta_{Rank} + \beta_{Rank} \cdot \frac{F_t^{t+1}}{P_t} + e_t \]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>One Year Future Stock Returns</th>
<th>Five Year Future Stock Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.089***</td>
<td>-0.063***</td>
</tr>
<tr>
<td></td>
<td>(11.41)</td>
<td>(-3.49)</td>
</tr>
<tr>
<td>( F_t^{t+1} )</td>
<td>0.491***</td>
<td>2.970***</td>
</tr>
<tr>
<td></td>
<td>(4.77)</td>
<td>(10.57)</td>
</tr>
<tr>
<td>Rank</td>
<td>0.180***</td>
<td>0.771***</td>
</tr>
<tr>
<td></td>
<td>(7.40)</td>
<td>(16.79)</td>
</tr>
<tr>
<td>Rank ( \cdot \frac{F_t^{t+1}}{P_t} )</td>
<td>-2.748***</td>
<td>-12.291***</td>
</tr>
<tr>
<td></td>
<td>(-8.25)</td>
<td>(-17.37)</td>
</tr>
<tr>
<td>N</td>
<td>21158</td>
<td>16102</td>
</tr>
<tr>
<td>Adjusted ( R^2 )</td>
<td>0.33%</td>
<td>1.88%</td>
</tr>
</tbody>
</table>

***, **, *: Significant at 1%, 5% and 10% level respectively

Numbers in parenthesis are t-statistics based on heteroscedasticity consistent standard errors

Rank: Represents quintile rank of “meproxy” (Scaled to lie between 0 and 1)

Panel A reports results of the regression for predicting future stock returns with earnings yield for each of quintile portfolio formed on the measurement error proxy (“meproxy”). Panel B reports results of the regression for predicting future stock returns with earnings yield with an indicator for each rank. Rank is scaled to lie between 0 and 1. Pearson correlation is the Pearson correlation coefficient between respective independent variable and dependent variable. Neg % is the proportion of the negative analysts’ consensus forecasts. Total assets are the median total assets of each rank at time t. All variables are per share basis. Variable definitions are provided in Appendix C. All variables are winsorized.
at 1% level for both extremes except for future stock returns. Total observations are 21,158 firm years between 1991 and 2007 (16,102 firm years between 1991 and 2004 for predicting five year future stock returns).
Table 9. Out of sample tests for improvement in predictability of future earnings and stock returns when information about cross-sectional variation of measurement error is incorporated

Panel A. Improvement in future earnings prediction when information about cross-sectional variation of measurement error is incorporated

(1) $A_{i,t} = \alpha + \alpha_{t}F_{i} + e_{i}$
(1') $A_{i,t} = \beta + \beta_{t}F_{i} + \beta_{Rank} + \beta_{Rank}F_{i} + e_{i}$
MAD1 = abs($A_{i,t} - P1$) where $P1$ is a predicted value from (1)
MAD1' = abs($A_{i,t} - P1'$) where $P1'$ is a predicted value from (1')
Mean Difference 1 = MAD1 – MAD1’

(5) $A_{i,t+1-5} = \alpha + \alpha_{t}F_{i} + e_{i}$
(5’) $A_{i,t+1-5} = \beta + \beta_{t}F_{i} + \beta_{Rank} + \beta_{Rank}F_{i} + e_{i}$
MAD5 = abs($A_{i,t+1-5} - P5$) where $P5$ is a predicted value from (5)
MAD5’ = abs($A_{i,t+1-5} - P5'$) where $P5'$ is a predicted value from (5’)
Mean Difference 5 = MAD5 – MAD5’

<table>
<thead>
<tr>
<th>Data Years</th>
<th>Prediction Year</th>
<th>N</th>
<th>One Year Ahead Actual Earnings Mean Difference 1</th>
<th>Five Year Ahead Mean Actual Earnings Mean Difference 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990 ~ 1994</td>
<td>1995</td>
<td>1037</td>
<td>0.025***</td>
<td>0.041***</td>
</tr>
<tr>
<td>1990 ~ 1995</td>
<td>1996</td>
<td>1130</td>
<td>0.052***</td>
<td>0.038***</td>
</tr>
<tr>
<td>1990 ~ 1996</td>
<td>1997</td>
<td>1313</td>
<td>0.039***</td>
<td>0.056***</td>
</tr>
<tr>
<td>1990 ~ 1997</td>
<td>1998</td>
<td>1486</td>
<td>0.061***</td>
<td>0.068***</td>
</tr>
<tr>
<td>1990 ~ 1998</td>
<td>1999</td>
<td>1445</td>
<td>0.049***</td>
<td>0.061***</td>
</tr>
<tr>
<td>1990 ~ 1999</td>
<td>2000</td>
<td>1247</td>
<td>0.034***</td>
<td>0.034***</td>
</tr>
<tr>
<td>1990 ~ 2000</td>
<td>2001</td>
<td>1134</td>
<td>0.051***</td>
<td>0.073***</td>
</tr>
<tr>
<td>1990 ~ 2001</td>
<td>2002</td>
<td>1272</td>
<td>0.045***</td>
<td>0.055***</td>
</tr>
<tr>
<td>1990 ~ 2002</td>
<td>2003</td>
<td>1253</td>
<td>0.031***</td>
<td>0.035***</td>
</tr>
<tr>
<td>1990 ~ 2003</td>
<td>2004</td>
<td>1568</td>
<td>0.012*</td>
<td>0.045***</td>
</tr>
<tr>
<td>1990 ~ 2004</td>
<td>2005</td>
<td>1586</td>
<td>0.045***</td>
<td></td>
</tr>
<tr>
<td>1990 ~ 2005</td>
<td>2006</td>
<td>1700</td>
<td>0.026***</td>
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</tr>
<tr>
<td>1990 ~ 2006</td>
<td>2007</td>
<td>1770</td>
<td>0.035***</td>
<td></td>
</tr>
</tbody>
</table>

***, **, *: Significant at 1%, 5% and 10% level respectively
Rank: Represents quintile rank of “meproxy” (Scaled to lie between 0 and 1)
Panel B. Improvement in future stock returns prediction when information about cross-sectional variation of measurement error is incorporated

\( (1) \quad \text{Ret}_t = \beta_0 + \beta_1 \frac{E_{i,1}}{P_t} + \epsilon_t \)

\( (1') \quad \text{Ret}_t = \beta_0 + \beta_1 \frac{E_{i,1}}{P_t} + \beta_2 \text{Rank} + \beta_3 \text{Rank} \cdot \frac{E_{i,1}}{P_t} + \epsilon_t \)

MAD1 = abs(\( \text{Ret}_t - P1 \)) where \( P1 \) is a predicted value from \( 1) \)

MAD1' = abs(\( \text{Ret}_t - P1' \)) where \( P1' \) is a predicted value from \( 1') \)

Mean Difference 1 = MAD1 – MAD1'

\( (5) \quad \text{Ret}_t = \alpha_0 + \alpha_1 \frac{E_{i,5}}{P_t} + \epsilon_t \)

\( (5') \quad \text{Ret}_t = \beta_0 + \beta_1 \frac{E_{i,5}}{P_t} + \beta_2 \text{Rank} + \beta_3 \text{Rank} \cdot \frac{E_{i,5}}{P_t} + \epsilon_t \)

MAD5 = abs(\( \text{Ret}_t - P5 \)) where \( P5 \) is a predicted value from \( 5) \)

MAD5' = abs(\( \text{Ret}_t - P5' \)) where \( P5' \) is a predicted value from \( 5') \)

Mean Difference 5 = MAD5 – MAD5'

<table>
<thead>
<tr>
<th>Data Years</th>
<th>Prediction Year</th>
<th>N</th>
<th>One Year Future Stock Returns</th>
<th>Five Year Future Stock Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mean Difference 1</td>
<td>Mean Difference 5</td>
</tr>
<tr>
<td>1990 ~ 1994</td>
<td>1995</td>
<td>1037</td>
<td>0.002***</td>
<td>0.007</td>
</tr>
<tr>
<td>1990 ~ 1995</td>
<td>1996</td>
<td>1130</td>
<td>0.003***</td>
<td>0.017***</td>
</tr>
<tr>
<td>1990 ~ 1996</td>
<td>1997</td>
<td>1313</td>
<td>0.001*</td>
<td>0.011***</td>
</tr>
<tr>
<td>1990 ~ 1997</td>
<td>1998</td>
<td>1486</td>
<td>0.007***</td>
<td>0.049***</td>
</tr>
<tr>
<td>1990 ~ 1998</td>
<td>1999</td>
<td>1445</td>
<td>-0.006***</td>
<td>0.010*</td>
</tr>
<tr>
<td>1990 ~ 1999</td>
<td>2000</td>
<td>1247</td>
<td>0.001**</td>
<td>0.073***</td>
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<tr>
<td>1990 ~ 2000</td>
<td>2001</td>
<td>1134</td>
<td>0.000</td>
<td>0.039***</td>
</tr>
<tr>
<td>1990 ~ 2001</td>
<td>2002</td>
<td>1272</td>
<td>0.004***</td>
<td>0.008*</td>
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<tr>
<td>1990 ~ 2002</td>
<td>2003</td>
<td>1253</td>
<td>0.009***</td>
<td>-0.011*</td>
</tr>
<tr>
<td>1990 ~ 2003</td>
<td>2004</td>
<td>1568</td>
<td>0.000</td>
<td>0.015***</td>
</tr>
<tr>
<td>1990 ~ 2004</td>
<td>2005</td>
<td>1586</td>
<td>0.003***</td>
<td></td>
</tr>
<tr>
<td>1990 ~ 2005</td>
<td>2006</td>
<td>1700</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>1990 ~ 2006</td>
<td>2007</td>
<td>1770</td>
<td>0.001</td>
<td></td>
</tr>
</tbody>
</table>

***, **, *: Significant at 1%, 5% and 10% level respectively

Rank: Represents quintile rank of “meproxy” (Scaled to lie between 0 and 1)

Panel A presents the results of out of sample tests and compares the mean absolute difference from the future earnings between the predicted value based on the prediction model without the information about cross-sectional variation of measurement error (MAD1) and the predicted value based on the prediction model with the information about cross-sectional variation of measurement error variability (MAD2). Panel B presents results of out of sample tests and compares the mean absolute deviation from the future stock returns between the predicted value based on the prediction model without the information about cross-sectional variation of measurement error (MAD1) and the predicted value based on the prediction model with the information about cross-sectional variation of measurement error variability (MAD2). Mean difference is the mean difference between MAD1 and MAD2. T-test is conducted to examine whether the mean difference is significantly different from zero. For each prediction year, the measurement error proxy (“meproxy”) is constructed by multiplying each determinant by a respective coefficient from the analysts’ consensus forecast error regression based on the data in data years. For each prediction year, analysts’ consensus forecasts are grouped into the quintile portfolio based on “meproxy”. For each prediction year, predicted values are computed by multiplying respective independent variable by the respective coefficient estimated from the respective prediction models based on the data in data years. All variables are per share basis. Variable definitions are provided in Appendix C. All variables are winsorized at 1% level for both extremes.
Table 10. Effect of earnings persistence

Panel A. Effect of earnings levels on earnings response coefficient (ERC)

<table>
<thead>
<tr>
<th>Rank of “meproxy”</th>
<th>Intercept</th>
<th>Coefficient (Earnings surprise)</th>
<th>Coefficient (Earnings level)</th>
<th>Sum of two coefficients</th>
<th>Adjusted R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.001</td>
<td>9.683***</td>
<td>0.144</td>
<td>9.827</td>
<td>5.63%</td>
</tr>
<tr>
<td></td>
<td>(-0.05)</td>
<td>(5.82)</td>
<td>(0.39)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-0.027</td>
<td>8.044***</td>
<td>1.067***</td>
<td>9.111</td>
<td>10.01%</td>
</tr>
<tr>
<td></td>
<td>(-1.07)</td>
<td>(9.10)</td>
<td>(2.69)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-0.011</td>
<td>5.985***</td>
<td>1.287***</td>
<td>7.272</td>
<td>9.11%</td>
</tr>
<tr>
<td></td>
<td>(-0.52)</td>
<td>(9.05)</td>
<td>(4.58)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.029*</td>
<td>3.094***</td>
<td>1.314***</td>
<td>4.408</td>
<td>9.88%</td>
</tr>
<tr>
<td></td>
<td>(1.78)</td>
<td>(9.43)</td>
<td>(5.74)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.145***</td>
<td>1.558***</td>
<td>-0.302**</td>
<td>1.256</td>
<td>3.68%</td>
</tr>
<tr>
<td></td>
<td>(9.68)</td>
<td>(8.74)</td>
<td>(-2.03)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>0.065***</td>
<td>1.919***</td>
<td>-0.213*</td>
<td>1.706</td>
<td>3.10%</td>
</tr>
<tr>
<td></td>
<td>(8.42)</td>
<td>(12.04)</td>
<td>(-1.72)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

***, **, *: Significant at 1%, 5% and 10% level respectively
Numbers in parenthesis are t-statistics based on heteroscedasticity-consistent standard error (two-tailed)
Rank: Represents quintile rank of respective estimated measurement error (Scaled to lie between 0 and 1)

Panel B. Rank analysis with future stock returns volatility

<table>
<thead>
<tr>
<th>One Year Ahead Actual Earnings</th>
<th>One Year Future Stock Returns</th>
<th>Stock Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inter Coeff Vol R²</td>
<td>Inter Coeff Vol R²</td>
<td>Inter Coeff Vol R²</td>
</tr>
<tr>
<td>1 0.25 0.89 -5.11 0.88</td>
<td>0.21 0.75 -2.57 0.02</td>
<td>-2.48 13.73 573.73 0.57</td>
</tr>
<tr>
<td>2 0.33 0.86 -9.29 0.80</td>
<td>0.18 0.54 -3.43 0.01</td>
<td>0.60 13.16 222.63 0.70</td>
</tr>
<tr>
<td>3 0.41 0.83 -12.85 0.74</td>
<td>0.25 0.64 -5.28 0.01</td>
<td>3.15 10.15 178.38 0.53</td>
</tr>
<tr>
<td>4 0.53 0.79 -18.75 0.64</td>
<td>0.21 0.03 -4.58 0.01</td>
<td>9.78 5.68 65.85 0.23</td>
</tr>
<tr>
<td>5 0.81 0.60 -30.76 0.48</td>
<td>0.35 -0.84 -2.96 0.02</td>
<td>13.18 2.56 -30.81 0.13</td>
</tr>
<tr>
<td>All 0.72 0.75 -22.97 0.65</td>
<td>0.27 -0.68 -2.66 0.01</td>
<td>12.15 7.10 38.54 0.33</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Five Year Ahead Mean Actual Earnings</th>
<th>Five Year Future Stock Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inter Coeff Vol R²</td>
<td>Inter Coeff Vol R²</td>
</tr>
<tr>
<td>1 0.96 0.68 -19.62 0.61</td>
<td>0.78 4.98 -17.85 0.07</td>
</tr>
<tr>
<td>2 0.62 0.82 -14.85 0.68</td>
<td>1.17 2.19 -21.31 0.03</td>
</tr>
<tr>
<td>3 1.02 0.65 -25.04 0.52</td>
<td>1.43 1.90 -22.43 0.02</td>
</tr>
<tr>
<td>4 1.15 0.58 -30.20 0.45</td>
<td>1.78 1.25 -26.89 0.02</td>
</tr>
<tr>
<td>5 1.29 0.39 -38.53 0.32</td>
<td>2.42 -0.66 -30.78 0.03</td>
</tr>
<tr>
<td>All 1.29 0.57 -33.31 0.49</td>
<td>1.47 -0.34 -18.88 0.02</td>
</tr>
</tbody>
</table>

***, **, *: Significant at 1%, 5% and 10% level respectively
Rank: Represents quintile rank of respective estimated measurement error (Scaled to lie between 0 and 1)
For Panel A, forecasting coefficients, intercepts and R²s are estimated from the following forecasting model for each quintile.

\[
\text{price stock} = c_0 + c_1 \text{Earnings surprise} + \frac{c_2}{\text{stock price}} + \tau
\]

For Panel B, the regression analysis is conducted with the following respective forecasting models for each quintile. For forecasting of 1 year actual earnings and 1 year stock returns and explaining stock price and earnings response coefficients, stock returns volatility is defined as standard deviation of daily raw stock returns for 365 days beginning 1st day of fourth month after prior fiscal year end. For forecasting of 5 year mean actual earnings and 5 year stock returns, stock returns volatility is defined as standard deviation of daily raw stock returns for 1825 days beginning 1st day of fourth month after prior fiscal year end.

\[
\text{1 year actual earnings} = c_0 + c_1 \text{Analyst's consensus forecast} + c_2 \text{stock returns volatility} + \tau
\]

\[
\text{1 year stock returns} = c_0 + c_1 \text{Earnings yield} + c_2 \text{stock returns volatility} + \tau
\]

\[
\text{Stock price} = i_0 + i_1 \text{Analyst's consensus forecast} + c_2 \text{stock returns volatility} + \rho
\]

\[
\text{1 year size - adjusted stock returns} = i_0 + i_1 \text{Earnings surprise} + c_2 \text{stock returns volatility} + \rho
\]

For each year, observations are grouped into each quintile rank based on the respective estimated measurement error. Rank is scaled to lie between 0 and 1. All variables are per share basis. Variable definitions are provided in Appendix B. All variables are winsorized at 1% level. Total observations are 21,158 firm years between 1991 and 2007 (16,102 firm years between 1991 and 2004 for predicting five year mean earnings and future stock returns tests).
### Table 11. Effect of negative analysts’ consensus forecasts and size

<table>
<thead>
<tr>
<th></th>
<th>One Year Ahead Actual Earnings</th>
<th>One Year Future Stock Returns</th>
<th>Stock Price</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Without</td>
<td>With</td>
<td>With &amp; Size</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.121***</td>
<td>0.101***</td>
<td>0.224***</td>
</tr>
<tr>
<td></td>
<td>(4.48)</td>
<td>(2.35)</td>
<td>(4.53)</td>
</tr>
<tr>
<td>Predictor</td>
<td>0.767***</td>
<td>0.967***</td>
<td>0.711***</td>
</tr>
<tr>
<td></td>
<td>(34.78)</td>
<td>(30.72)</td>
<td>(15.00)</td>
</tr>
<tr>
<td>Rank</td>
<td>-0.073</td>
<td>-0.138**</td>
<td>-0.215**</td>
</tr>
<tr>
<td></td>
<td>(-1.03)</td>
<td>(-2.21)</td>
<td>(-2.21)</td>
</tr>
<tr>
<td>Predictor*Rank</td>
<td>-0.333***</td>
<td>-0.260***</td>
<td>-1.465***</td>
</tr>
<tr>
<td></td>
<td>(-5.69)</td>
<td>(-5.07)</td>
<td>(-3.76)</td>
</tr>
<tr>
<td>Rank(ta)</td>
<td>-0.097*</td>
<td>-0.097*</td>
<td>-0.097*</td>
</tr>
<tr>
<td>Predictor*Rank(ta)</td>
<td>0.314***</td>
<td>0.314***</td>
<td>0.314***</td>
</tr>
<tr>
<td></td>
<td>(5.84)</td>
<td>(5.84)</td>
<td>(5.84)</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>59.78%</td>
<td>63.74%</td>
<td>65.51%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Five Year Ahead Mean Actual Earnings</th>
<th>Five Year Future Stock Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Without</td>
<td>With</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.392***</td>
<td>0.298***</td>
</tr>
<tr>
<td></td>
<td>(14.64)</td>
<td>(6.25)</td>
</tr>
<tr>
<td>Predictor</td>
<td>0.609***</td>
<td>0.893***</td>
</tr>
<tr>
<td></td>
<td>(27.58)</td>
<td>(25.48)</td>
</tr>
<tr>
<td>Rank</td>
<td>0.021</td>
<td>-0.067</td>
</tr>
<tr>
<td></td>
<td>(0.30)</td>
<td>(-1.23)</td>
</tr>
<tr>
<td>Predictor*Rank</td>
<td>-0.472***</td>
<td>-0.337***</td>
</tr>
<tr>
<td></td>
<td>(-8.41)</td>
<td>(-7.62)</td>
</tr>
<tr>
<td>Rank(ta)</td>
<td>0.091*</td>
<td>0.137</td>
</tr>
<tr>
<td>Predictor*Rank(ta)</td>
<td>0.478***</td>
<td>0.478***</td>
</tr>
<tr>
<td></td>
<td>(9.89)</td>
<td>(9.89)</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>40.15%</td>
<td>43.04%</td>
</tr>
</tbody>
</table>

***, **, *: Significant at 1%, 5% and 10% level respectively
Numbers in parenthesis are t-statistics based on heteroscedasticity-consistent standard error (two-tailed)
Rank: Represents quintile rank of respective estimated measurement error (Scaled to lie between 0 and 1)
Rank(ta): Represents quintile rank of prior fiscal year end total assets (Scaled to lie between 0 and 1)

This table repeats the analyses reported in Panel B of Table 5, Table 6 and Table 8 only based on positive analysts’ consensus forecast. A predictor for 1 year actual earnings, 5 year mean actual earnings and stock price is analysts’ consensus forecast. A predictor for 1 year stock returns and 5 year stock returns is earnings yield defined as analysts’ consensus forecast scaled by stock price at the end of third month after prior fiscal year end. A predictor for 1 year size-adjusted stock returns is earnings surprise defined as difference between 1 year actual earnings and analysts’ consensus forecast for a given year, deflated by stock price at the end of third month after prior fiscal year end. For each year, observations are grouped into each quintile rank based on the respective estimated measurement error.
error. For 1 year actual earnings test, the estimated measurement error is constructed as a fitted value from the measurement error regression reported in Table 3, Panel A only based on positive analysts’ consensus forecast. For 5 year mean actual earnings and stock returns test, the estimated measurement error is constructed as a fitted value from the measurement error regression reported in Table 3, Panel B only based on positive analysts’ consensus forecast. Rank is scaled to lie between 0 and 1. Rank(ta) represents quintile rank of prior fiscal year end total assets. Rank(ta) is scaled to lie between 0 and 1. All variables are per share basis. Variable definitions are provided in Appendix B. All variables are winsorized at 1% level for both extremes. Total observations are 21,158 firm years between 1991 and 2007 (16,102 firm years between 1991 and 2004 for predicting five year mean earnings and future stock returns).
Appendix A: Derivation of coefficient and $R^2$ when analysts’ consensus forecast measures market expectation of future earnings with error

1. Derivation of \((2)\)

\[
A_{t+1} = \beta_0 + \beta_{\text{typical}} M_{t+1}^{\text{typical}} + \epsilon_t \quad \text{with} \quad R^2_{\text{typical}}
\]

\[
A_{t+1} = \beta_0 + \beta_{\text{typical}} (F_{t+1}^{\text{typical}} - e_t) + \epsilon_t = \beta_0 + \beta_{\text{typical}} F_{t+1}^{\text{typical}} + \epsilon_t \quad \text{with} \quad R^2_{\text{typical}}
\]

where:

- \(A_{t+1}\): Actual earnings at time \(t+1\)
- \(M_{t+1}^{\text{typical}}\): Rational market expectation at time \(t\) of actual earnings at time \(t+1\)
- \(F_{t+1}^{\text{typical}}\): Analysts’ consensus forecast at time \(t\) for actual earnings at time \(t+1\)
- \(n_{t+1}\): Earnings news during time \(t+1\) with distribution \(N(0, \text{Var}(n_{t+1}))\)
- \(e_t\): Measurement error in analysts’ consensus forecast at time \(t\) with distribution \(N(0, \text{Var}(e_t))\)
- \(\epsilon_t\): Disturbance with distribution \(N(0, \text{Var}(\epsilon_t))\).

Following standard assumptions are assumed to hold:

\[
\text{COV}(M_{t+1}^{\text{typical}}, e_t) = \text{COV}(M_{t+1}^{\text{typical}}, n_{t+1}) = \text{COV}(M_{t+1}^{\text{typical}}, \epsilon_t) = 0
\]

\[
\text{COV}(e_t, n_{t+1}) = \text{COV}(e_t, \epsilon_t) = \text{COV}(n_{t+1}, \epsilon_t) = 0
\]

Then,

\[
p \lim \hat{\beta}_{\text{typical}} = \frac{p \lim \frac{1}{n} \sum_{i=1}^{n} E_{i,i} A_{t+1,i}}{p \lim \frac{1}{n} \sum_{i=1}^{n} (E_{i,i}^2) + \sum_{i=1}^{n} e_{t,i}^2} = \frac{p \lim \frac{1}{n} \sum_{i=1}^{n} \beta_{\text{typical}} (M_{t+1}^{\text{typical}}) \beta_{\text{typical}}^2}{p \lim \frac{1}{n} \sum_{i=1}^{n} \text{Var}(M_{t+1}^{\text{typical}}) + \text{Var}(e_t)}
\]

\[
p \lim R^2_{\text{typical}} = \frac{p \lim \frac{1}{n} \sum_{i=1}^{n} (M_{t+1}^{\text{typical}} + e_t, i) (M_{t+1}^{\text{typical}} + n_{t+1}, i) - \frac{1}{n} \sum_{i=1}^{n} (M_{t+1}^{\text{typical}} + e_t, i) (M_{t+1}^{\text{typical}} + n_{t+1}, i)}{\left[ p \lim \frac{1}{n} \sum_{i=1}^{n} (M_{t+1}^{\text{typical}} + e_t, i)^2 - \frac{1}{n} \sum_{i=1}^{n} (M_{t+1}^{\text{typical}} + n_{t+1}, i)^2 \right]} = \frac{\text{COV}(M_{t+1}^{\text{typical}}, M_{t+1}^{\text{typical}})}{\left( \text{Var}(M_{t+1}^{\text{typical}}) + \text{Var}(e_t) \right) \left( \text{Var}(M_{t+1}^{\text{typical}}) + \text{Var}(n_{t+1}) \right)}
\]

\[
= \frac{\beta_{\text{typical}}}{\text{Var}(M_{t+1}^{\text{typical}})} \frac{\text{Var}(M_{t+1}^{\text{typical}})}{\text{Var}(M_{t+1}^{\text{typical}}) + \text{Var}(n_{t+1})} \frac{\text{COV}(M_{t+1}^{\text{typical}}, M_{t+1}^{\text{typical}})}{\text{Var}(M_{t+1}^{\text{typical}})}
\]

\[
= R^2_{\text{typical}}
\]
2. Derivation of (4)

\[ A_{t;i} = \beta_e + \beta_{\text{rce}} M^{i+1}_{t;i} + n_{t;i} + \epsilon_i \quad \text{with} \quad R^2_{\text{rce}} \]

\[ A_{t;i} = \beta_e + \beta_{\text{rce}} (F^{i+1}_{t;i} - e_i) + n_{t;i} + \epsilon_i = \beta_e + \beta_{\text{pooled}} F^{i+1}_{t;i} + z_i \quad \text{with} \quad R^2_{\text{pooled}} \]

where

\( A_{t;i} \): Actual earnings at time t+1

\( M^{i+1}_{t;i} \): Rational market expectation at time t of actual earnings at time t+1

\( F^{i+1}_{t;i} \): Analysts’ consensus forecast made at time t for actual earnings at time t+1 (\( F^{i+1}_{t;i} = M^{i+1}_{t;i} + e_1 \) for first n observations and \( F^{i+1}_{t;i} = M^{i+1}_{t;i} + e_2 \) for remaining k observations)

\( e_i \): Measurement error in analysts’ consensus forecast at time t (\( e_i = e_1 \) with distribution \( N(0, \text{Var}(e_1)) \) for first n observations and \( e_i = e_2 \) with distribution \( N(0, \text{Var}(e_2)) \) for remaining k observations)

\( n_{t;i} \): Earnings news during time t+1.

\( \epsilon_i \): Disturbance with distribution \( N(0, \text{Var}(\epsilon_i)) \).

Following standard assumptions are assumed to hold:

\[ \text{COV}(M^{i+1}_{t;i}, e_i) = \text{COV}(M^{i+1}_{t;i}, n_{t;i}) = \text{COV}(M^{i+1}_{t;i}, e_1) = \text{COV}(M^{i+1}_{t;i}, e_2) = 0 \]

\[ \text{COV}(e_1, n_{t;i}) = \text{COV}(e_2, n_{t;i}) = \text{COV}(e_1, e_i) = \text{COV}(e_2, e_i) = \text{COV}(n_{t;i}, e_i) = 0 \]

Then,

\[
\lim \frac{1}{n+k} \sum_{i=1}^{n+k} (M^{i+1}_{t;i} + e_1) (\beta_e + \beta_{\text{rce}} M^{i+1}_{t;i} + e_1 + n_{t;i}) + \sum_{i=1}^{n+k} (M^{i+1}_{t;i} + e_2) (\beta_e + \beta_{\text{rce}} M^{i+1}_{t;i} + e_1 + n_{t;i}) \]

\[
= \lim \frac{1}{n+k} \sum_{i=1}^{n+k} (M^{i+1}_{t;i} + e_1) \beta_{\text{rce}} \text{Var}(M^{i+1}_{t;i}) + \frac{n}{n+k} \text{Var}(e_1) + \frac{k}{n+k} \text{Var}(e_2)
\]

\[
\lim R^2_{\text{pooled}} = \lim \frac{1}{n+k} \sum_{i=1}^{n+k} (M^{i+1}_{t;i} + e_1) (M^{i+1}_{t;i} + n_{t;i}) - \frac{1}{n+k} \sum_{i=1}^{n+k} (M^{i+1}_{t;i} + e_1) (M^{i+1}_{t;i} + n_{t;i}) \]

\[
= \left( \lim \frac{1}{n+k} \sum_{i=1}^{n+k} (M^{i+1}_{t;i} + e_1) (M^{i+1}_{t;i} + e_1) + \sum_{i=1}^{n+k} (M^{i+1}_{t;i} + e_1) (\beta_e + \beta_{\text{rce}} M^{i+1}_{t;i} + e_1) \right)
\]

\[
= \left( \frac{\text{Var}(M^{i+1}_{t;i}) + \frac{n}{n+k} \text{Var}(e_1) + \frac{k}{n+k} \text{Var}(e_2)}{\text{Var}(M^{i+1}_{t;i}) + \frac{n}{n+k} \text{Var}(e_1) + \frac{k}{n+k} \text{Var}(e_2)} \right) \frac{\text{Var}(M^{i+1}_{t;i})}{\text{Var}(M^{i+1}_{t;i}) + \frac{n}{n+k} \text{Var}(e_1) + \frac{k}{n+k} \text{Var}(e_2)}
\]

\[
= \frac{R^2_{\text{rce}}}{R^2_{\text{pooled}}}
\]
3. Derivation of (6)
\[ A_{i,t} = \beta_0 + \beta_{n+1} M_{1t} + n_{1t} + e_i \] with \( R^2_{n+1} \)
\[ A_{i,t} = \beta_0 + \beta_{n+1} (F_{1t} - e_i) + n_{1t} + e_i = \gamma_0 + \gamma_1 F_{1t} + \gamma_2 D + \gamma_3 F_{2t} + D + x_i \] with \( R^2_{n+2} \),

where
- \( A_{i,t} \): Actual earnings at time \( t+1 \)
- \( M_{1t}^{n+1} \): Rational market expectation at time \( t \) of actual earnings at time \( t+1 \)
- \( F_{1t}^{n+1} \): Analysts' consensus forecast made at time \( t \) for actual earnings at time \( t+1 \) (for first \( n \) observations and \( F_{1t}^{n+1} = M_{1t}^{n+1} + e_1 \) for remaining \( k \) observations)
- \( e_i \): Measurement error in analysts' consensus forecast at time \( t \) (\( e_i = e_1 \) with distribution \( N(0, \text{Var}(e_1)) \) for first \( n \) observations and \( e_i = e_2 \) with distribution \( N(0, \text{Var}(e_2)) \) for remaining \( k \) observations)
- \( D \): 1 when \( F_{1t}^{n+1} = M_{1t}^{n+1} + e_2 \) and 0 otherwise
- \( n_{1t} \): Earnings news during time \( t+1 \).
- \( e_i \): Disturbance with distribution \( N(0, \text{Var}(e_i)) \).

Following standard assumptions are assumed to hold:
\[
\begin{align*}
\text{COV}(M_{1t}^{n+1}, e_i) &= \text{COV}(M_{1t}^{n+1}, n_{1t}) = \text{COV}(M_{1t}^{n+1}, e_1) = \text{COV}(M_{1t}^{n+1}, e_2) = 0 \\
\text{COV}(e_1, n_{1t}) &= \text{COV}(e_2, n_{1t}) = \text{COV}(e_1, e_i) = \text{COV}(e_2, e_i) = \text{COV}(n_{1t}, e_i) = 0
\end{align*}
\]

For notational convenience, the first \( n \) observations are denoted with sub-vector \( M_{1t}^{n+1}, A_{1t}^{n+1} \) and \( n_{1t} \), and the remaining \( k \) observations are denoted with sub-vector \( M_{2t}^{n+1}, A_{2t}^{n+1} \) and \( n_{2t} \). It should be noted that sub-vectors have the same distribution as the total observations. An upper bar indicates a mean of respective variables:
\[
\bar{M}_{1t}^{n+1} = \bar{M}_{2t}^{n+1} = \bar{M}_{1t}^{n+1} \text{ and } \text{Var}(M_{1t}^{n+1}) = \text{Var}(M_{2t}^{n+1}) = \text{Var}(M_{t}^{n+1})
\]
\[
\bar{A}_{1t}^{n+1} = \bar{A}_{2t}^{n+1} = \bar{A}_{1t}^{n+1} \text{ and } \text{Var}(A_{1t}^{n+1}) = \text{Var}(A_{2t}^{n+1}) = \text{Var}(A_{t}^{n+1})
\]
\[
\bar{F}_{1t}^{n+1} = \bar{F}_{2t}^{n+1} = \bar{F}_{1t}^{n+1} \text{ and } \text{Var}(F_{1t}^{n+1}) = \text{Var}(F_{2t}^{n+1}) = \text{Var}(F_{t}^{n+1})
\]
\[
\bar{n}_{1t} = \bar{n}_{2t} = \bar{n}_{t} \text{ and } \text{Var}(n_{1t}) = \text{Var}(n_{2t}) = \text{Var}(n_{t})
\]

Then, the prediction model for the first \( n \) observations can be expressed as below. It should be noted that the coefficient and intercept are denoted with the same notation as the prediction model with the total observations because the mean of sub-vectors are assumed to be the same as the mean of the total observations. \( R^2 \) for the first \( n \) observations is denoted with \( R^2_{n+1} \).
\[
A_{1t} = \beta_0 + \beta_{n+1} M_{1t} + n_{1t} + I_t = \beta_0 + \beta_{n+1} (F_{1t} - e_1) + n_{1t} + I_t = \phi_0 + \phi_1 F_{1t}^{n+1} + \omega_t
\]
\[
\begin{align*}
\text{plim} \beta_0 &= \beta_{n+1} = \frac{\text{Var}(M_{1t}^{n+1})}{\text{Var}(M_{1t}^{n+1}) + \text{Var}(e_1)} \\
\text{plim} R^2_{n+1} &= R^2_{n+1} = \frac{\text{Var}(M_{1t}^{n+1})}{\text{Var}(M_{1t}^{n+1}) + \text{Var}(e_1)} \\
R^2_{n+1} &= \frac{\text{TSS}_{n+1} - \text{ESS}_{n+1}}{\text{TSS}_{n+1}} = \sum_i \left( \phi_i F_{1t}^{n+1} - \phi_0 \bar{F}_{1t}^{n+1} \right)^2
\end{align*}
\]

The prediction model for the remaining \( k \) observations can be expressed as shown below. \( R^2 \) for the last \( k \) observations is denoted with \( R^2_{n+2} \).
\[
A_{2t} = \beta_0 + \beta_{n+1} M_{2t} + n_{2t} + I_2 = \beta_0 + \beta_{n+1} (F_{2t} - e_2) + n_{2t} + I_2 = \eta_0 + \eta_1 F_{2t}^{n+1} + \omega_t
\]
\[
\begin{align*}
\text{plim} \eta_0 &= \beta_{n+1} = \frac{\text{Var}(M_{2t}^{n+1})}{\text{Var}(M_{2t}^{n+1}) + \text{Var}(e_2)} \\
\text{plim} R^2_{n+2} &= R^2_{n+2} = \frac{\text{Var}(M_{2t}^{n+1})}{\text{Var}(M_{2t}^{n+1}) + \text{Var}(e_2)} \\
R^2_{n+2} &= \frac{\text{TSS}_{n+2} - \text{ESS}_{n+2}}{\text{TSS}_{n+2}} = \sum_i \left( \eta_i F_{2t}^{n+1} - \eta_0 \bar{F}_{2t}^{n+1} \right)^2
\end{align*}
\]
Let $\gamma$ be the coefficient of the independent variable $x$. Based on the above prediction models, the following relations among the coefficients and intercepts can be derived:

$$R^2 = \frac{\text{TSS} - \text{ESS}}{\text{TSS}} = \frac{\sum_{i=1}^{n+k} (\eta_i \hat{F}_i^{(1)} - \eta_i \tilde{F}_i^{(1)})^2}{\sum_{i=1}^{n+k} (\eta_i F_i^{(1)} - \eta_i \tilde{F}_i^{(1)})^2}$$

The prediction model with the total $n+k$ observations and an indicator variable for the last $k$ observations can be expressed as shown below.

$$A_i = \gamma_0 + \gamma_1 F_i^{(1)} + \gamma_2 D + \gamma_3 F_i^{(1)} \cdot D + u_i$$

with $R_{\text{MEV}}^2$ where $D = 1$ when $F_i^{(1)} = M_i^{(1)} + e_2$, and 0 otherwise

Based on the above prediction models, the following relations among the coefficients and intercepts can be derived:

$$\gamma_0 = \phi_0, \gamma_1 = \phi_1, \gamma_2 = \eta_0 - \phi_0, \gamma_3 = \eta_1 - \phi_1.$$

Then,

$$\lim_{n \to \infty} \beta_{\text{MEV}} = \frac{\text{Var}(M_i^{(1)})}{\text{Var}(M_i^{(1)} + \text{Var}(e_1_i))}$$

$$\lim_{n \to \infty} \beta_{\text{MEV}} = \frac{\text{Var}(M_i^{(1)})}{\text{Var}(M_i^{(1)} + \text{Var}(e_2_i))}$$

Let $R_{\text{MEV}}^2$ denote the $R^2$ from the prediction model with total $n+k$ observations. Then, $R_{\text{MEV}}^2$ can be expressed with respect to $R_i^2$ and $R_{\text{MEV}}^2$ as shown below.

$$\lim_{n \to \infty} R_{\text{MEV}}^2 = \lim_{TSS} \frac{\text{ESS}}{\text{TSS}+\text{TSS}} = \lim_{TSS} \frac{\text{TSS}+\text{TSS}}{\text{TSS}+\text{TSS}} = \lim_{TSS} \frac{\text{TSS}+\text{TSS}}{\text{TSS}+\text{TSS}}$$

$$= \frac{n+k}{n+k} R_i^2 = \frac{n+k}{n+k} R_i^2$$

where

$$\text{TSS} = \sum_{i=1}^{n+k} (M_i^{(1)} - \bar{M}_i^{(1)})^2 = \sum_{i=1}^{n+k} (M_i^{(1)} - \bar{M}_i^{(1)})^2 = \text{TSS} + \text{TSS}$$

$$\text{ESS} = \sum_{i=1}^{n+k} (\gamma_1 F_i^{(1)} + \gamma_2 D + \gamma_3 F_i^{(1)} \cdot D - \gamma_1 \tilde{F}_i^{(1)} - \gamma_2 D - \gamma_3 \tilde{F}_i^{(1)} \cdot D)$$

$$= \sum_{i=1}^{n+k} (\gamma_1 F_i^{(1)} + \gamma_2 D + \gamma_3 F_i^{(1)} \cdot D - \gamma_1 \tilde{F}_i^{(1)} - \gamma_2 D - \gamma_3 \tilde{F}_i^{(1)} \cdot D)$$

$$= \sum_{i=1}^{n+k} (\gamma_1 F_i^{(1)} - \gamma_1 \tilde{F}_i^{(1)} + \gamma_2 D - \gamma_2 D + \gamma_3 F_i^{(1)} \cdot D - \gamma_3 \tilde{F}_i^{(1)} \cdot D)$$

$$= \sum_{i=1}^{n+k} (\gamma_1 F_i^{(1)} - \gamma_1 \tilde{F}_i^{(1)} + \gamma_2 D - \gamma_2 D + \gamma_3 F_i^{(1)} \cdot D - \gamma_3 \tilde{F}_i^{(1)} \cdot D)$$

$$= \sum_{i=1}^{n+k} (\gamma_1 F_i^{(1)} - \gamma_1 \tilde{F}_i^{(1)} + \gamma_2 D - \gamma_2 D + \gamma_3 F_i^{(1)} \cdot D - \gamma_3 \tilde{F}_i^{(1)} \cdot D)$$

$$= \sum_{i=1}^{n+k} (\gamma_1 F_i^{(1)} - \gamma_1 \tilde{F}_i^{(1)} + \gamma_2 D - \gamma_2 D + \gamma_3 F_i^{(1)} \cdot D - \gamma_3 \tilde{F}_i^{(1)} \cdot D)$$

where

$$D = \frac{k}{n+k} (\therefore D_i = 0 \text{ for } 1 \leq i \leq n \text{ and } D_i = 1 \text{ for } n+1 \leq i \leq n+k) \text{ and }$$

$$\hat{D} = \frac{k}{n+k} F_i^{(1)} \tilde{D} \text{ for } 1 \leq i \leq n \text{ and } \hat{F}_i^{(1)} \tilde{D} = F_i^{(1)} \tilde{F}_i^{(1)} \text{ for } n+1 \leq i \leq n+k$$
Appendix B: Definition of variables for simulation tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>True Predictor ((x^*))</td>
<td>Normal distribution with (N \sim (2, 1))</td>
</tr>
<tr>
<td>Independent Variable ((y))</td>
<td>True Predictor (* 10 +) Disturbance and Disturbance has a standard normal distribution with (N \sim (0, 36))</td>
</tr>
<tr>
<td>Noisy Observed Predictor ((x))</td>
<td>True Predictor + Measurement Error</td>
</tr>
<tr>
<td>Measurement Error ((e\ or \ u))</td>
<td>Measurement error has a normal distribution (N \sim (0, V))</td>
</tr>
<tr>
<td>Measurement Error Variance ((V))</td>
<td>Measurement Error Variance is assumed to be either the same for all observations or differ across various groups of observations</td>
</tr>
</tbody>
</table>
Appendix C: Definition of variables used for analysts’ consensus forecast error regression

<table>
<thead>
<tr>
<th>Classification</th>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analysts’ Consensus Forecast Error</td>
<td>$\frac{\text{abs}(A_t - F_{t+1}^*)}{P_t}$</td>
<td>Absolute difference between actual earnings at time $t+1$ and first available analysts’ consensus forecast from I/B/E/S after earnings announcement at time $t$ for actual earnings at time $t+1$, deflated by stock price at end of third month after fiscal year end of time $t$.</td>
</tr>
<tr>
<td>Determinants of Analysts’ Consensus Forecast Error</td>
<td>$\frac{\text{std}(F_{t+1}^*)}{P_t}$</td>
<td>Standard deviation of analysts’ consensus forecast at time $t$ for actual earnings at time $t+1$, deflated by stock price at end of third month after fiscal year end of time $t$.</td>
</tr>
<tr>
<td></td>
<td>$\frac{\text{abs}(A_t - F_{t-1}^*)}{P_{t-1}}$</td>
<td>Absolute difference between actual earnings at time $t$ and first available analysts’ consensus forecast from I/B/E/S after earnings announcement at time $t-1$ for actual earnings at time $t$, deflated by stock price at end of third month after fiscal year end of time $t-1$.</td>
</tr>
<tr>
<td></td>
<td>$\frac{\text{abs}(A_t - F_{t+2}^*)}{P_t}$</td>
<td>Absolute difference between actual earnings at time $t$ and analysts’ consensus forecast at time $t$ for actual earnings at time $t+1$, deflated by stock price at end of third month after fiscal year end of time $t$.</td>
</tr>
<tr>
<td></td>
<td>$\frac{\text{abs}(F_{t+1}^* - F_{t+2}^*)}{P_t}$</td>
<td>Absolute difference between analysts’ consensus forecast at time $t$ for actual earnings at time $t+1$ and analysts’ consensus forecast at time $t$ for actual earnings at time $t+2$, deflated by stock price at end of third month after fiscal year end of time $t$. Both analysts’ consensus forecasts are required to be made in the same month.</td>
</tr>
<tr>
<td>Measurement Error Proxy</td>
<td>meproxy</td>
<td>Computed by multiplying determinants at time $t$ by respective coefficient from analysts’ consensus forecast regression at time $t-1$ reported in Table 1, Panel A.</td>
</tr>
</tbody>
</table>

* All variables are per share basis
<table>
<thead>
<tr>
<th>Classification</th>
<th>Variable</th>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Independent Variable</strong></td>
<td>Analysts’ consensus forecast</td>
<td>$F_{t+1}^{t+1}$</td>
<td>First available analysts’ consensus forecast from I/B/E/S after earnings announcement at time $t$ for actual earnings at time $t+1$. It is also further required that consensus earnings forecast be made before nine months from fiscal year end.</td>
</tr>
<tr>
<td></td>
<td>Earnings yield</td>
<td>$\frac{F_{t+1}^{t+1}}{P_t}$</td>
<td>First available analysts’ consensus forecast from I/B/E/S after earnings announcement at time $t$ for actual earnings at time $t+1$, deflated by stock price at end of third month after fiscal year end of time $t$.</td>
</tr>
<tr>
<td><strong>Dependent Variable</strong></td>
<td>One year ahead actual earnings</td>
<td>$A_{t+1}$</td>
<td>Actual earnings at time $t+1$ from I/B/E/S.</td>
</tr>
<tr>
<td></td>
<td>Five year ahead mean actual earnings</td>
<td>$\overline{A_{t+1-t+5}}$</td>
<td>Mean actual earnings between time $t+1$ and time $t+5$ from I/B/E/S. If any of next five years’ actual earnings is not available, it is computed with only available actual earnings.</td>
</tr>
<tr>
<td></td>
<td>One year future stock returns</td>
<td>$\text{Ret}_1$</td>
<td>Monthly raw stock returns accumulated for 12 months beginning in fourth month after prior fiscal year end. If firms are delisted during accumulation period, -30% delisting returns are assigned to NYSE and AMEX firms and -55% delisting returns are assigned to NASDAQ firms.</td>
</tr>
<tr>
<td></td>
<td>Five year future stock returns</td>
<td>$\text{Ret}_5$</td>
<td>Monthly raw stock returns accumulated for 60 months beginning in fourth month after prior fiscal year end. If firms are delisted during accumulation period, -30% delisting returns are assigned to NYSE and AMEX firms and -55% delisting returns are assigned to NASDAQ firms.</td>
</tr>
<tr>
<td><strong>Control Variable</strong></td>
<td>Stock price</td>
<td>$P_t$</td>
<td>Stock price at end of third month after fiscal year end of time $t$.</td>
</tr>
<tr>
<td></td>
<td>Total assets</td>
<td>$\text{LOG}(TA)$</td>
<td>Logarithm of total assets at prior fiscal year end</td>
</tr>
<tr>
<td></td>
<td>Actual earnings</td>
<td>$A_t$</td>
<td>Actual earnings at time $t$ from I/B/E/S.</td>
</tr>
<tr>
<td></td>
<td>Actual earnings yield</td>
<td>$\frac{A_t}{P_t}$</td>
<td>Actual earnings at time $t$ from I/B/E/S, deflated by stock price at end of third month after fiscal year end of time $t$.</td>
</tr>
</tbody>
</table>

* All variables are per share basis