Essays on International Development

A dissertation submitted in partial satisfaction
of the requirements for the degree
Doctor of Philosophy in Management

by
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ABSTRACT OF THE DISSERTATION

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Professor Sebastian Edwards, Chair

The chapters of this dissertation are devoted to the study of three aspects of international development. In the first chapter, I ask whether it is true that a "competitive real exchange rate" is behind China’s success, something that the media and many policy makers sustain, but that is not supported by the most prominent economic models. I develop a model that uses insights from the "unlimited supply of labor" and "de-industrialization" literatures (W.A. Lewis, W. Max Corden and Peter Neary) to gain a better understanding of these issues. In particular, I propose a 3-sector model with labor market frictions that explains how a policy aimed at increasing domestic savings and depreciating the RER can, at the same time, generate real growth through a reallocation of workers from a low-productivity traditional sector into a high-productivity manufacturing sector. The policy is particularly effective in countries with relative abundance of labor, scarcity of agricultural resources, and high barriers for the entry of workers into the manufacturing sector. Empirically, I verify that higher real undervaluation (measured as deviations from PPP)
is positively associated with GDP and manufacturing growth in countries with lower per capita agricultural land and higher rural population. The relationship vanishes and even becomes negative in the opposite cases. Finally, I propose a simple methodology for the identification of real depreciations exogenously induced (i.e. that are not related to changes in productivities or in terms of trade). I find that, during the last 20 years, such episodes have been mainly observed in East Asian developing countries.

In the second chapter, I present a simple, general theoretical framework that explains how individuals’ decisions on the structure of the household, the allocation of time between market and home work, and the degree of elaboration of the purchased market goods, are affected by three key dimensions of technological progress: the rise in market wages, the decline in the price of domestic appliances, and the increase in the access to more elaborated goods and services. In the model, the production of the final consumption good comprises a continuum of tasks (or stages) that increase the degree of elaboration of the good until it is ready to be consumed. Individuals can produce the final good in different ways, using in each case a market good with a particular degree of elaboration, and adding the amount of homework that completes the process of elaboration. The existence of fixed costs in homework provides the incentives to form a household for the joint production and consumption of the final good. The predictions of the model are supported by both, the changes observed in the American household in the last century, as well as the differences in household structure, labor market conditions, access to basic public goods and services, and durable goods currently observed across countries.

Finally, in the third chapter I explore the effects that exogenous aggregate volatility can have on the composition of investment and the pattern of sectoral production of the economy. The key assumption in the proposed mechanism is that aggregate production can be performed by combining different factors of production (other than labor), whose marginal returns are af-
ected in different ways by aggregate uncertainty. In this context, differences in the volatility of aggregate productivity can induce differences in the relative use of the factors of production and, therefore, in the pattern of production of the economy. The mechanism is analyzed in two different settings. First, in small open economy that produces manufactures and commodities based on natural resources, higher (relative) volatility of aggregate productivity increases the dependence of the economy on natural resources. Interestingly, the effect is stronger in economies with higher endowments of natural resources and lower average productivity. Second, in a closed economy with two sectors that differ in the degree of reversibility of their capital, higher aggregate volatility reduces the share of investment and production in the sector with more irreversible capital. The effect can be particularly strong if the elasticity of substitution between sectoral goods is high. In both models, higher concavity of preferences strengthens the change in the patterns of investment and production induced by volatility. Aggregate volatility can negatively affect growth by inducing an otherwise inefficient allocation of resources across sectors. The negative effect on growth can be larger in the long run if, for example, there is endogenous growth associated with the expansion of a sector that uses a very specific and irreversible type of capital.
The dissertation of Juan M. Wlasiuk is approved.

Hugo A. Hopenhayn
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Romain Wacziarg
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2013
To my wife, Milena,
for her love, patience and support
throughout these years;
and to my precious daughters,
Josefina and Julieta,
for bringing so much joy into my life.
# Table of Contents

List of Figures ix

List of Tables x

## 1 The Mechanics of Real Undervaluation and Growth

1.1 Introduction .................................................. 1
1.2 Empirical Regularities ........................................... 5
  1.2.1 Real Exchange Rate Undervaluation and Growth .............. 5
  1.2.2 Savings .................................................... 8
  1.2.3 Manufacturing Production .................................. 9
  1.2.4 Questions and Some Answers ................................ 12
  1.2.5 Labor Productivity in Manufactures and Agriculture .......... 15
  1.2.6 Natural Resources .......................................... 18
1.3 Model .......................................................... 20
  1.3.1 Labor Market ................................................ 21
  1.3.2 Problem of the Individuals .................................. 22
  1.3.3 Equilibrium ................................................ 24
  1.3.4 Comparative Statics ......................................... 28
  1.3.5 Government Interventions ................................... 35
  1.3.6 Main Predictions of the Model ............................... 41
1.4 Testing the Main Predictions of the Model ........................ 43
  1.4.1 Assessing the Relation between Real Undervaluation, GDP Growth, and Manufacturing Production .............. 44
  1.4.2 Regressions ................................................ 51
  1.4.3 Identification of Real Undervaluations Induced by Government Interventions .......................... 58
1.5 Conclusions .................................................... 64

A1 A Measure of "Oversize" of the Tradable Sector .................. 66

## 2 Technological Change and Household Structure

2.1 Introduction .................................................... 68
2.2 Empirical Evidence .............................................. 72
  2.2.1 Technological Change and Household Structure in the United States: 1900-2010 .......................... 72
  2.2.2 International Evidence ..................................... 80
2.3 Model .......................................................... 87
  2.3.1 Benchmark Model .............................................. 87
  2.3.2 The Production Process ..................................... 91
  2.3.3 Technological Progress: Introducing More Elaborated Goods and Services .......................... 97
  2.3.4 Homework Production ......................................... 98
  2.3.5 Technological Progress: Increase in Market Wages and Fall in Prices of Domestic Appliances .............. 101
  2.3.6 Choosing the Optimal Technology .............................. 104
  2.3.7 Summary of Predictions ..................................... 110
2.4 Conclusions .................................................... 111
### 3 Aggregate Volatility, the Composition of Investment, and the Pattern of Sectoral Production

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Introduction</td>
<td>114</td>
</tr>
<tr>
<td>3.2</td>
<td>Model</td>
<td>118</td>
</tr>
<tr>
<td>3.2.1</td>
<td>Two-Sector, Small, Open Economy</td>
<td>118</td>
</tr>
<tr>
<td>3.2.2</td>
<td>Two-Sector, Closed Economy</td>
<td>128</td>
</tr>
<tr>
<td>3.3</td>
<td>Conclusions</td>
<td>136</td>
</tr>
</tbody>
</table>

### References

References | 138
List of Figures

1.1 Undervaluation and per capita GDP Growth .......................... 6
1.2 Undervaluation and Employment in Manufactures .................... 11
1.3 Relative Labor Productivity: Manufactures/Agriculture ............... 16
1.4 Labor Productivity: Manufactures and Agriculture .................... 17
1.5 Model - Equilibrium (3 Cases) ......................................... 27
1.6 Model - Effects of a Reduction of Entry Costs ......................... 29
1.7 Model - Effects of a Change in the Distribution of Rents ............... 29
1.8 Model - Effects of an Increase in Manufacturing Productivity .......... 31
1.9 Model - Effects of Differences in Endowments of Nat. Res. ............ 32
1.10 Model - Diff. in Endowments of Nat. Res. - Gains & Costs ........... 34
1.11 Real Undervaluation & Relative Size of Tradable Sector ............... 60
1.12 Undervaluation & Oversize of Tradable Sector ........................ 61
1.13 Tradable Production (% GDP) & Pc Income ........................... 66
1.14 Estimated Oversize of Tradable Sector (% GDP) ....................... 67

2.1 Household Structure in the U.S., 1900-2010 ........................... 73
2.2 Selected Indicators - United States, 1900-2010 ......................... 75
2.3 Adults per Household & per capita Income ............................ 81

3.1 The Effects of Aggregate Volatility on Investment in Manufacturing .... 127
3.2 The Effect of Aggregate Volatility on Irreversible Investment ........... 136
List of Tables

1.1 Regional Economic Indicators, 1970-2010 ................................................. 7
1.2 Regional Demographic and Natural Resources Indicators, 1990-2010 ................. 19
1.3 Real Undervaluation and Per Capita Income Growth Rate ................................. 50
1.4 Real Underval. and Per Capita Income Growth Rate - Dev. Countries ................. 53
1.5 Real Undervaluation and Manufacturing Production ........................................ 55
1.6 Real Undervaluation and Manufacturing Production - Dev. Countries* ............. 57

2.1 Household Structure Indicators - Selected Countries* ..................................... 82
2.2 Economic and Environmental Indicators - Selected Countries* ........................ 84
2.3 Selected Indicators by Type of Urbanization - China, 2005 .............................. 86
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Chapter 1

The Mechanics of Real Undervaluation and Growth

1.1 Introduction

*Is it true that a competitive exchange rate is behind China’s success?* Over the past two decades, China has experienced one of the most remarkable economic transformations of modern economic history. Millions of workers have migrated from the traditional economy into rapidly expanding industrial areas, manufacturing production and exports have dramatically increased, and per capita income has grown at almost double-digit rates during the last 20 years. Such an outstanding performance is explained, according to many media analysts and policy makers, by a direct manipulation of the exchange rate by the Chinese government, aimed, they say, at keeping the real exchange rate (RER) undervalued and thus fostering exports and growth\(^1\).

This view, however, is not supported by the most prominent economic models which do not predict a positive relationship between an undervalued RER and growth. According to them, either changes in the nominal exchange rate should be accompanied by changes in prices that would virtually leave the RER unchanged, or, if the pressure for appreciation is somehow "controlled" by reducing the domestic demand, the induced real depreciation would introduce a distortion and lead to a misallocation of resources and a *fall* in real output. In this paper, I combine A. Lewis’ idea of *unlimited supply of labor* (Lewis (1954)) with Corden and Neary’s notion of *Dutch disease* (Corden & Neary (1982)) to explain how a policy aimed at depreciating the RER can generate real growth and facilitate the transition from a traditional to a modern economy in countries with relative abundance of labor and scarcity of natural resources. The model’s predictions are consistent not only with the main features of the Chinese experience but also with those of most

---

\(^1\)See, for example, Wiemer (2009), Davis (2012), Lee (2012).
Asian "miracles" and other developing countries in Latin America and South Saharan Africa. The key elements in the proposed mechanism are frictions in the labor market and reallocation of labor across sectors. In the model, high rates of growth are sustained by reallocating labor from a low-productivity traditional sector (agriculture and nontradables) to a high-productivity manufacturing one. With diminishing returns in agriculture, a policy aimed at lowering wages induces migration to manufacturing by increasing the wage differential across sectors, thereby generating simultaneously a real depreciation and growth, and allowing the economy to move to a new steady state with higher per capita income. Interestingly, the effect of such a policy is potentially stronger in economies with low endowments of natural resources (relative to their population), which is precisely the case of most Asian countries.

This paper is related to at least three bodies of literature. First, it is part of the new literature on the effect of persistent real exchange rate depreciation on growth. Most of this literature is empirical and has been devoted to document the relationship between RER and growth and other macroeconomic variables (see, for example, Razin & Collins (1997), Hausmann et al. (2005), Prasad et al. (2006), Eichengreen (2007), Rodrik (2008), Berg & Miao (2010), Jeong et al. (2010), Aflouk & Mazier (2011), and Béreau et al. (2012) among others). As Montiel & Servén (2008) mention, there are basically two proposed theoretical channels to explain a possible positive relationship: the "TFP" channel which assumes that a higher production of tradables has a positive effect on TFP growth, and the "capital accumulation" channel which proposes a link between depreciation, savings, and capital accumulation (see Levy-Yeyati & Sturzenegger (2007) and Montiel & Servén (2008)). Though the mechanism proposed in this paper shares elements with the "capital accumulation" view, it is however a radically different channel whose key element is the reallocation of labor motivated by the depreciation. This key element of the mechanism makes the paper, at the same time, part of a recent literature focused on micro-level resource
misallocation to explain low levels of aggregate TFP. In one of the most recent and influential papers in this field, Song et al. (2011) explain the high rates of growth and savings of China with a mechanism that emphasizes the role of financial frictions and reallocation of resources between firms in the manufacturing sector. In contrast, this work focuses on labor frictions and intersectoral reallocations which, besides explaining the high rates of growth and savings, help to understand the reasons behind the lower real exchange rate as well as the rapid urbanization and decline of the traditional sector observed in these economies. The paper is related in this field with, for example, the work of Banerjee & Newman (1998) and Banerjee & Duflo (2007), and with the recent empirical evidence on productivity and income gaps presented in Bosworth & Collins (2008), McMillan & Rodrik (2011), Gollin et al. (2011), and de Vries et al. (2012). Finally, this paper is related to the literature on structural transformation. Specifically, the model explains how the speed and depth of the transition from an agricultural/traditional to an industrial economy can be affected by interventions aimed at lowering real wages, particularly in countries poor in natural resources.

In the proposed model, workers face a loss of utility (i.e. an entry cost) if they migrate from the traditional economy to the high-productivity manufacturing sector. For a given wage in the manufacturing sector, the entry cost reduces the minimum wage that workers are willing to accept in the traditional economy before migrating to manufacturing, and can generate a wedge between the wages (and productivities) in both sectors. When that happens, a policy that sufficiently lowers the wage in the traditional sector may trigger migration to manufacturing and generate growth through a better allocation of labor. This prediction is consistent with the experience of most East Asian countries (and with the current experience of China in particular), that have sustained high rates of growth for long periods of time by mobilizing employment from an initially overpopulated traditional economy into manufacturing (and appropriately expanding the stock

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2 In a broader sense, the paper is also part of an extensive literature on the causes and consequences of rural-urban labor migration that follows, among others, the work of Harris & Todaro (1970).
of capital), a fact that was already documented by Alwyn Young in his celebrated paper *The Tyranny of Numbers* (Young (1995)) for Hong Kong, Singapore, South Korea and Taiwan. Once the government intervention finishes, the economy behaves as in a standard neoclassical model where growth depends on TFP growth. The experience of Japan after 1995 offers a good example of this.

A particular feature of the theory proposed in this paper is that its predictions regarding the effect of an induced real undervaluation vary depending on the size and productivity of the agricultural sector. Specifically, a more productive agricultural sector and a higher endowment of agricultural land imply either a higher wage in the sector or higher land’s rents, or both, which tends to increase the difference between the actual and the minimum wage workers in the traditional economy are willing to accept before migrating. In this context, inducing migration to manufacturing requires a larger real depreciation (i.e. a larger fall in wage) and, given the higher initial productivity in agriculture, has a smaller final effect on growth. This prediction of the model is consistent with the fact that, as shown in section 1.2, the use of an undervalued RER seems to have been more extended and effective in Asian countries, where the relative endowments of land suitable for agriculture is lower (see Table 1.2). These empirical regularities predicted by the model are also consistent with the empirical analysis performed in section 1.4, that verifies that the East Asian countries have been more prone to experience "exogenous" undervaluations in the last two decades. The empirical analysis also verifies a positive relationship between undervaluation (measured as deviations from PPP) and growth of per capita GDP and manufacturing production, which is stronger in countries with lower endowments of agricultural resources, higher ethnolinguistic fractionalization, and higher initial rural population.

The paper is organized as follows: Section 1.2 describes the empirical evidence from Asia, Latin America and S.S. Africa that motivates the main assumptions of the model. Section 2.3
introduces the model, characterizes the equilibrium, and analyzes possible government interventions aimed at depreciating the RER and their potential effects on growth. Section 1.4 verifies additional empirical regularities predicted by the model using a panel of countries between 1970 and 2010. Section 1.5 concludes.

1.2 Empirical Regularities

In this section I present some empirical regularities related to the relationship between real undervaluation, growth, manufacturing production and employment, and gross savings across different regions. I then provide the intuition of the mechanism that explains the links between these variables and identify the conditions that make them potentially stronger. I finish the section providing empirical evidence that such conditions are mainly observed in Asia, and in China in particular.

1.2.1 Real Exchange Rate Undervaluation and Growth

In an influential and no less controversial paper, Rodrik (2008) argues that undervaluation of the currency (i.e. a real exchange rate depreciation) stimulates economic growth, and that the effect is particularly strong in developing countries. Using a Balassa-Samuelson adjusted PPP method to measure undervaluation, he presents evidence suggesting that the link between RER undervaluation and growth is through an expansion of the industrial sector. Berg & Miao (2010) find similar results in an extension of Rodrik’s work. Other papers that using different measures of RER misalignment document similar regularities are Aguirre & Calderón (2005), Polterovich & Popov (2005), Sallenave (2010), Aflouk & Mazier (2011), and Béreau et al. (2012)\(^3\). In addition, many of these works and others like Sachs & Williamson (1985), Edwards (1988), Ghura & Grennes (1993), Loayza et al. (2004), and Rajan & Subramanian (2011) find evidence of the negative effects of RER overvaluation \(^3\)See Eichengreen (2007) and Aflouk & Mazier (2011) for an extensive review of the theoretical and empirical literature on real exchange rate missalignment and growth.
The case of China is a good example of positive relationship between undervaluation and growth. Figure 1.1 presents data on per capita GDP growth (PPP) and a measure of undervaluation\(^4\) (the same measure used in Rodrik (2008)) for China, India, and two sets with the main non oil-exporting economies of Latin America and South Saharan Africa. At least three observations can be made regarding the relationship between undervaluation and GDP growth based on the figures. First, the relationship does not seem to be positive in the short run, particularly for Latin America where the frequent recessions are usually accompanied by real depreciations\(^5\). Second, the

\(^4\)All the details regarding the estimation of this measure of undervaluation are discussed in Section 1.4.

\(^5\)In Latin America the hyperinflations of the early 1990’s, the Mexican crises of 1995, the Brazilian devaluation of
relationship seems to hold in the long run, but is not monotonic. The Asian countries were on average both, the most undervalued (China 27% and India 52%) and the ones that experienced the highest growth (8.5% and 3.0% respectively), in comparison to the Latin American and African countries that were relatively overvalued during the period (-5% and -11% respectively) and experienced the lowest growth (1.9% and 0.7% respectively). However, it is clear from the comparison between China and India that even in the long run the relationship between undervaluation and growth is not straightforward: though India systematically maintained the most undervalued currency during the period, it was outperformed by China in terms of GDP growth. And, third, the relationship is not equally strong and in the same direction in all countries: it seems to be positive in India, in the African countries, and in China in particular, while in Latin America the relation is either null or negative.

Similar conclusions can be verified when considering a larger sample of countries and a longer period. Table 1.1 presents several economic indicators for countries with per capita GDP (PPP 2005 USD) lower than 10,000 in 1970 in Asia, Latin America and Caribbean, and South Saharan Africa. Columns (2) and (3) show that, besides China and India, the other Asian economies also maintained a relatively high level of undervaluation (21% in average) and experienced high rates of growth (3.2% per capita in average over the entire period). The most relevant examples in this group are South Korea, Singapore, Hong Kong, Indonesia, Malaysia, Thailand and, more recently, Vietnam and Philippines. Their performance is in sharp contrast with that of the African economies, which maintained a high level of overvaluation (-22%) and experienced very low rates of growth (0.5%). The Latin American countries, on the other hand, experienced very modest growth (1.6% per year) despite the fact that they remained relatively undervalued during most of

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6 Overvaluation is simply understood as negative undervaluation.

7 The only countries in these regions that were excluded are Australia, New Zealand, Japan, Barbados and Trinidad & Tobago.
Table 1.1: Regional Economic Indicators, 1970-2010

<table>
<thead>
<tr>
<th>Region</th>
<th>2000US$</th>
<th>% GDP</th>
<th>Undervaluation</th>
<th>% GDP</th>
<th>% Tradables</th>
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<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
<tr>
<td>China</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>1970-74</td>
<td>385</td>
<td>4.8</td>
<td>-0.88</td>
<td>28.8</td>
<td>30.6</td>
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<tr>
<td>2006-10</td>
<td>6,006</td>
<td>10.0</td>
<td>0.23</td>
<td>51.6</td>
<td>33.0</td>
</tr>
<tr>
<td>1970-2010</td>
<td>2,010</td>
<td>7.6</td>
<td>-0.07</td>
<td>38.6</td>
<td>32.1</td>
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<td>India</td>
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<td></td>
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<td></td>
<td></td>
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<tr>
<td>1970-74</td>
<td>894</td>
<td>-0.2</td>
<td>-0.03</td>
<td>16.2</td>
<td>14.5</td>
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<td>2006-10</td>
<td>3,047</td>
<td>6.7</td>
<td>0.54</td>
<td>31.9</td>
<td>15.4</td>
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<td>1970-2010</td>
<td>1,588</td>
<td>3.3</td>
<td>0.35</td>
<td>22.5</td>
<td>15.9</td>
</tr>
<tr>
<td>Rest of East &amp; South Asia</td>
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<tr>
<td>1970-74</td>
<td>1,392</td>
<td>4.1</td>
<td>0.14</td>
<td>15.2</td>
<td>14.8</td>
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<td>2006-10</td>
<td>5,188</td>
<td>3.8</td>
<td>0.26</td>
<td>22.1</td>
<td>21.9</td>
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<td>1970-2010</td>
<td>3,097</td>
<td>3.2</td>
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<td>20.5</td>
<td>19.0</td>
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<td>Latin America &amp; Caribbean</td>
<td></td>
<td></td>
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<td>1970-74</td>
<td>5,445</td>
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<td>0.23</td>
<td>20.0</td>
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<tr>
<td>2006-10</td>
<td>8,739</td>
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<td>-0.10</td>
<td>20.5</td>
<td>16.6</td>
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<tr>
<td>1970-2010</td>
<td>6,948</td>
<td>1.6</td>
<td>0.11</td>
<td>20.2</td>
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<tr>
<td>South Saharan Africa</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>1970-74</td>
<td>1,435</td>
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<td>-0.26</td>
<td>14.8</td>
<td>11.3</td>
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<tr>
<td>2006-10</td>
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<td>10.5</td>
<td>8.0</td>
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<tr>
<td>1970-2010</td>
<td>1,389</td>
<td>0.5</td>
<td>-0.22</td>
<td>11.0</td>
<td>10.4</td>
</tr>
</tbody>
</table>

Note: Population-weighted means by region-year of countries with pc GDP PPP (2000USD) <10,000 in 1970. Number of countries: Asia 31, LAC 30, SSA 46. Data on savings is missing for some country-years, Nigeria is missing in SSA, and Taiwan in Asia. Manufacturing Production/Tradable production is the ratio of Manufacturing/GDP and Total Tradable production/GDP, which is estimated as Industrial Production/GDP+ Agricultural Production/GDP. Source: pc GDP is from PWT 7.1. Undervaluation is estimated using data from PWT 7.1. Gross Savings are from WDI. Manuf. Prod./GDP, Industrial Prod./GDP and Agr.Prod./GDP are from UNSD complemented with de Vries et al. (2012).

the period (11% in average).

1.2.2 Savings

Column (5) of Table 1.1 presents information about savings in these economies. The most striking case is China that, besides maintaining an extraordinary high average saving rate of 38.6% of GDP for more than 40 years, has systematically increased the rate surpassing 50% of its GDP in the late
2000’s. This increase of the saving rate (and particularly the accumulation of foreign assets during the last decade) combined with an acceleration of GDP growth in China is consistent with the evidence presented in Song et al. (2011). But, beyond the case of China, as column (5) of Table 1.1 shows savings have also increased in India and the rest of East and South Asian countries. In fact, with the exception of Bangladesh, Pakistan and The Philippines, the saving rate in all these economies has increased and remained high (above 30% of GDP) during the last 20 years. As Prasad (2011) documents, a common feature across Asian countries during the last decade is the increase in corporate savings, a phenomenon that seems to accompany the high rates of growth and relative undervaluation of the economies of the region. These facts are consistent with the findings of Montiel & Servén (2008), who document a negative relationship between savings and real exchange rate for the fast-growing East Asian economies. These authors point out that, since both the real exchange rate and savings are endogenous variables, we should not, a priori, expect a systematic correlation between them, except for the countries that adopt the real exchange rate as a tool of development policy, which seems to be the case of many Asian economies.

The case of S.S. Africa is, again, on the other extreme. Gross savings, that were already low in the early 1970s, further declined over in the following decades to reach an average of 10.5% of GDP in the late 2000’s. This coincides with the high degree of undervaluation experienced by the region, and seems to be related, among other things, with the large aid inflows experienced by these countries, as evidenced by Rajan & Subramanian (2011).

The Latin American countries maintained a relatively stable but low saving rate through the period. The region maintained its lowest saving rate (17.9% of GDP in average) between 1991 and 2003, a period of large capital inflows, relatively high overvaluation, and high but volatile

---

9 In average, between 1985 and 2010 the amount of Net Official Development Assistance received by the region accounted for more than 5% of GNI (source: WDI).
growth.

### 1.2.3 Manufacturing Production

Columns (5) and (6) of Table 1.1 provide information on the importance of manufacturing production relative to GDP and to total production of tradable goods\(^\text{11}\). Column (5) shows that value added in manufactures grew faster than GDP in China, India and, in particular, in the rest of Asia. Additionally, when looking at the composition of total tradable production, column (6) shows that, as these countries grew, the relative importance of agricultural and mining production diminished, and the production of tradable goods became more dependent on manufactures. In sharp contrast, manufacturing production in S.S. Africa was already low in 1970/74 (at 11.3% of GDP) and grew even slower than GDP through the period, contributing with only 8% of GDP in 2006/10. The composition of the production of tradable goods became more reliant on natural resources during the same period. Lastly, in Latin America, the production of manufactures that was relatively high in the 1970’s both, relative to GDP and with respect to the total production of tradables, was significantly reduced over the following decades.

The experience of the Asian countries is consistent with the evidence presented by Rodrik (2008), who argues that, in low income countries, the positive effect of undervaluation on growth takes place through an expansion of the industrial sector. The contraction of the manufacturing sector in the African countries, on the other hand, is consistent with the literature that highlights the potential adverse effects of overvaluation on the manufacturing sector. These facts are especially in line with the evidence presented by Rajan & Subramanian (2011), who suggest that the poor performance of the manufacturing sector in those countries is related to the real exchange rate.

\(^{11}\text{Total production of tradable goods is estimated as the sum of industrial production (which includes manufacturing and mining) and agricultural production. Specifically, it corresponds to categories A through E of ISIC Rev.3. This measure is a good approximation of total tradable production, but is not exact since it includes electricity, gas and water supply (category F), most of which is non tradable, and does not consider production of exportable services, whose relative importance remains low but has increased in the last years.}\)
appreciation caused by aid inflows. Finally, the evidence on Latin America seems to show a null or somehow negative relationship between undervaluation and manufacturing production.

Complementing the evidence on the relation between undervaluation and manufacturing production, Figure 1.2 illustrates the relationship between undervaluation and the share of employment in manufactures between 1990 and 2005 (2007 for China)$^{12}$. From simple inspection we can see that, besides the differences in levels across countries and regions, employment in manufacturing seems to follow the general trend of undervaluation.

When comparing the evolution of both series, employment in manufacturing (Figure 1.2)

---

$^{12}$Unfortunately there is no data on sectoral employment before 1990 for China and several other countries. The data used here is mainly from Timmer & de Vries (2009) and McMillan & Rodrik (2011).
and GDP growth (Figure 1.1), with that of undervaluation for the different regions, they appear to follow a closer path in China, particularly after the reforms of 1992 and the devaluation of 1994. In China, we observe the three variables, first, moving down together in the aftermath of the Asian crisis (with a slowdown of GDP growth, a fall in manufacturing employment and an real appreciation of the Renminbi), and then moving up together uninterruptedly from 2001 to 2007. In India and S.S. Africa, undervaluation and employment in manufactures follow relatively similar trends, but, in both cases, changes in manufacturing employment do not translate into higher GDP growth as directly as they do in China. In Latin America, though undervaluation seems to be related to employment in manufactures (at least in cases of overvaluation), changes in the latter do not seem to have an impact on GDP growth.

1.2.4 Questions and Some Answers

At least four important questions arise from the previous analysis. First, what is the mechanism, if any, that links real undervaluation with production and employment in the manufacturing sector? Second, how do changes in manufactures translate into higher rates of GDP growth? Third, why do these links seem to be particularly strong in China and in other Asian countries, but weaker in S.S. Africa and practically inexistent in Latin America? And, finally, what role can savings play in instrumenting a policy that effectively undervalue the real exchange rate, and triggers an expansion of the manufacturing sector and boosts growth? I sketch some answers to the first three questions before jumping into the model where all of them are addressed in depth.

Undervaluation and the Value of Manufacturing Production (% GDP). If the law of one price holds for tradable goods (or if at least it holds relatively more for tradables than for nontradables), then a real depreciation (that is, a lower price index after controlling for the Balassa-Samuelson effect) should be associated with a lower relative price of nontradables. There are
basically two possible scenarios in which a fall in the relative price of nontradables is associated with an increase in the value of manufactures relative to GDP. In all other cases, either a real depreciation is associated with a fall in the value of manufacturing, or a raise in the manufacturing (relative to GDP) is linked to a real appreciation. The first exception is a fall in terms of trade or in the productivity of the exportable sector in a country that is a net importer of manufactures (that is, the country is a net exporter of commodities or services). In this case, the value of production of the exportable good falls and, with it, aggregate demand and the demand for nontradables. Since neither the productivity nor the price of manufactures change, the value of production of manufactures relative to both, GDP and total tradable production, increases. This case, however, does not seem to be the case of the Asian countries, which are net exporters of manufactures an importers of commodities. The second possible scenario is one in which the fall in the price of nontradables is caused by an "exogenous" fall in aggregate demand (that is, one that is not related to a change in terms of trade or a fall of productivity in the exportable sector). Here, again, the contraction of aggregate demand will impact negatively in the price (and, probably, the quantity) of nontradables, but will not affect the value of production of tradables, whose productivities and prices remain unchanged. In this case, again, the value of manufactures relative to GDP increases\textsuperscript{13}.

**Undervaluation and Employment in Manufacturing.** The "exogenous" fall in aggregate demand for nontradables also exerts a downward pressure on the sector’s real wage, thereby increasing the incentives of workers for migrating to the tradable sector. Whether there is migration to manufacturing or not depends on two elements: the presence of frictions or entry costs in manufacturing, and the existence of diminishing returns in agriculture. If there are no frictions in the labor market and agriculture is subject to decreasing returns to scale, then there will be migration

\textsuperscript{13}This outcome, in fact, can be used to identify the “exogenous” undervaluations from the endogenous ones, given that in the lattercase the undervaluation is the response to a fall in the value of tradable production (originated in changes in fundamentals). I discuss more about this when testing the implication of the model.
only from nontradables to manufacturing, with no changes in agricultural employment and wages (this assumes constant returns in manufacturing). If, on the other hand, migrating to manufactures is indeed costly for workers (due to, for example, the presence of search costs, the acquisition of new skills, or simply the adaptation to a new environment), and there are constant returns in agriculture, there will be only migration to agriculture, and no change in manufacturing employment and wages. But if there are both, entry costs in manufacturing (though not in agriculture) and diminishing returns in agriculture, then a contraction in the demand for nontradables will induce migration to agriculture, a fall in wages in nontradables and agriculture, and a gap in wages with manufacturing. Once that wedge is sufficiently large, workers will start to migrate to manufacturing. In this case we observe both, a real undervaluation and an increase in employment in manufacturing.

How large the undervaluation should be, and how much labor will be reallocated to manufacturing depend on the nature of entry costs (whether pecuniary or in terms of utility), the initial conditions (how large was the initial wage gap), and the existence of additional sources of income for workers, such as rents from land. This will be discussed later with the model.

**Employment in Manufacturing and GDP Growth.** With entry costs in manufacturing and diminishing returns in agriculture we have that an "exogenous" undervaluation can mobilize workers to manufacturing. We still have to establish the link between growth in employment in manufacturing and real GDP growth. As mentioned, with entry costs in manufacturing, the initial reaction to the fall in demand for nontradables is to reallocate workers to agriculture. If there are diminishing returns in this sector, increasing employment implies lowering the marginal product of labor, thus inducing a fall in wage and a reduction in aggregate production (as measured at initial prices). With a sufficiently large undervaluation, workers will begin to migrate to manufacturing,
where the productivity the and wage are higher. The final effect on aggregate production depends on how large the initial wage gap is (i.e. how inefficient the initial allocation is) and on how many people can potentially be reallocated to manufacturing. If there is initially a sufficiently high productivity gap between manufacturing and the other sectors, then a reallocation of labor toward manufacturing generates growth. If, additionally, there is a large number of workers available in the "traditional" sector, the potential growth associated with an induced undervaluation is high. This is precisely the environment described by Lewis (1954), in which an unlimited (large) supply of labor is available at zero (low) cost.\(^\text{14}\)

**Why in Asia?** With some light about the elements that link an induced undervaluation with employment in manufacturing, and this with real growth, we can outline an answer to the question about the reasons why this mechanism seems to work in China and other Asian economies, but not so much in South Saharan Africa or Latin America. My hypothesis is that, as shown in the next section, the conditions described above (higher entry costs for workers in manufacturing and therefore high productivity gap between sectors, and large shares of employment in the low-productivity sector) are more likely to hold in Asia and, in particular, in China, than in S.S. Africa or Latin America.

1.2.5 **Labor Productivity in Manufactures and Agriculture**

Figure 1.3 displays the ratio of average labor productivities in manufacturing and agriculture between 1990 and 2005 for China, India, and the main non oil-exporting economies of Latin America and S.S. Africa\(^\text{15}\). The figure confirms that there are opportunities for growth in all these regions

\(^\text{14}\)Notice that without the "income gap" between sectors there is no growth associated with increasing manufacturing production. This is the reason why some authors appeal to the presence of nonpecuniary externalities associated with the production of exportables (such as learning by doing effects external to the firm) in order to justify an intervention of this kind. See Eichengreen (2007).

\(^\text{15}\)Labor productivity is estimated as the ratio between value added and employment in each sector.
by reallocating workers to the manufacturing sector, and that those opportunities are particularly relevant in China. The persistence (and even widening) of the productivity gaps is also evidence of the existence of labor market frictions or entry costs in the manufacturing sector. The productivity gap has dramatically increased in China since the reforms in the early 1990s, and has done so in India since 2001. In S.S. Africa the gap has decreased significantly during the entire period, while in Latin America it has done so since the middle 1990s.

These facts are consistent with the evidence presented in an IMF Report (Jaumotte & Spatafora (2006)) that highlights the relatively low labor productivity and large share of employment in agriculture throughout developing Asia and, in particular, in China. In this line, Bosworth & Collins (2008) estimate a productivity gap between workers in the entire industrial sector and those in agriculture of 7 and 4 for China and India respectively in 2004. McMillan & Rodrik (2011) document that the average manufactures-agriculture productivity gap is 2.3 in S.S. Africa, 2.8 in
Latin America, and 3.9 in Asia, and emphasize the role of labor reallocation as a contributor of growth in Asia. Using a more disaggregated database, de Vries et al. (2012) confirm these results.
In a recent work, Gollin et al. (2011) use micro data from households surveys to document that the gap between agriculture and the rest of the economy persists across regions even after controlling by hours worked and human capital. Using their data, we can verify that the gap in China is higher than in most developing countries, and that in Latin America it is particularly low.

Figure 1.4 presents the average labor productivity in manufacturing and agriculture, both expressed in constant PPP USD of 2000. The graphs show that, while Latin America has had the highest levels of labor productivity in both manufactures and agriculture, China had the lowest levels until 1993. In these regions labor productivity in both sectors grew during most of the period, but the relative growth in the manufacturing sector was much higher in China. Interestingly, despite the extraordinary growth in manufacturing productivity after the reforms in the early 1990s, the average level of productivity in that sector by 2005 was still lower in China than in Latin America (which is, at the same time, low relative to the industrialized countries). So the key behind the extraordinary high productivity gap in China is not a particularly high level of productivity in manufacturing, but instead a strikingly low labor productivity in the agricultural sector. The same applies for India, whose agricultural productivity in 2005 remained close to China’s, well below the level of S.S. Africa and Latin America. Analogously, the reason behind the relatively low productivity gap in Latin America is not that manufacturing productivity is low (which is actually higher than China’s), but is related instead with the relatively high labor productivity in agriculture of the region.

1.2.6 Natural Resources

A natural question that follows the previous analysis is why agricultural productivities are so different in Asia and Latin America. Part of the answer is given in column 3 of Table 1.2, which displays the endowment of agricultural (arable and cultivable) land per inhabitant of the different
regions in 1990. The Asian countries have the lowest endowment of agricultural land relative to the size of their population and, among them, China ranks worst. Latin America and Africa, on the other hand, have the highest ratios, with per capita endowments 3 times larger that of China. The numbers become even more contrasting when we consider the share of rural population in each of the regions (column 5). More than 70% of the population in the Asian and African countries lived in rural areas by 1990, compared with 29% in Latin America. A simple calculation allows us to see that in 1990 the endowment of agricultural land per rural inhabitant was about 7.5 times higher in Latin America than in Asia, a number that is in line with the relative labor productivity in agriculture in both regions in the same year.

The pattern persists when considering mineral resources. Column 3 of Table 1.2 shows per capita oil reserves for the different regions in 2002 (before the recent discoveries of oil in Latin America). Again, the East and South Asian countries rank worst. Per capita oil reserves in the Latin American countries were, in average, 9 times higher than in China in that year\textsuperscript{16}. The comparison of total rents from natural resources, which are displayed in columns 1 and 2 of the table, confirms the pattern. Between 1990 and 2010, per capita rents from natural resources were about 6 times higher in Latin America than in China, and almost 14 times higher than in India.

The evidence presented suggests that, in addition to determining the availability of rents, differences in per capita endowments of natural resources may have an important role in explaining the observed disparity in sectoral productivity (and, potentially, wage) gaps across regions. We have verified that such productivity gaps may persist and even increase over time and that, despite its existence, large fractions of the population in these regions remain in low productivity sectors. The persistence of both high productivity gaps and large shares of rural populations in these economies suggest the existence of frictions in the labor market or, more specifically, of high entry costs in the

\textsuperscript{16}The number is still above 3 if Venezuela is excluded. Recent discoveries of oil in the region, elevated the ratio of per capita oil in Latin America to China to more than 50 in 2011, and more than 8 if excluding Venezuela.
Table 1.2: Regional Demographic and Natural Resources Indicators, 1990-2010

<table>
<thead>
<tr>
<th>Region</th>
<th>Natural Resources Rents USD2005</th>
<th>% GDP</th>
<th>pc Agric. Land ha 1990</th>
<th>pc Oil Reserves bbl 2002</th>
<th>Rural Pop. % 1990</th>
<th>Years of Education 1990</th>
</tr>
</thead>
<tbody>
<tr>
<td>China</td>
<td>53.3</td>
<td>4.1</td>
<td>0.114</td>
<td>20.9</td>
<td>73.6</td>
<td>4.9</td>
</tr>
<tr>
<td>India</td>
<td>22.7</td>
<td>3.4</td>
<td>0.202</td>
<td>4.1</td>
<td>74.5</td>
<td>3.0</td>
</tr>
<tr>
<td>Rest of East &amp; South Asia</td>
<td>64.0</td>
<td>5.8</td>
<td>0.170</td>
<td>14.3</td>
<td>68.1</td>
<td>4.2</td>
</tr>
<tr>
<td>Latin America &amp; C.</td>
<td>309.8</td>
<td>6.2</td>
<td>0.343</td>
<td>182.0</td>
<td>29.4</td>
<td>5.2</td>
</tr>
<tr>
<td>S. S. Africa</td>
<td>109.8</td>
<td>14.3</td>
<td>0.347</td>
<td>58.1</td>
<td>71.7</td>
<td>3.2</td>
</tr>
</tbody>
</table>

Population-weighted means by region-year of countries with pc GDP PPP (2000USD) < 10,000 in 1970. Pc Agr. Land is total cultivable and arable land divided by population. Pc Oil Reserves is total oil reserves in 2002/03 (earliest year available) divided by population. Pc Natural Resources Rents USD2005 is approximated by multiplying Total Nat. Res. Rents/GDP (current prices) by pc GDP at constant prices in 2005 USD. Education is average years of schooling of the population above 25 years. Source: Oil reserves are from CIA Factbook. Education is from Barro-Lee. The rest is from WDI.

modern, high-productivity sector. In section 2.3 I develop a model that incorporates these elements and provides an explanation for the observed empirical regularities.

1.3 Model

Consider a small open economy with three sectors: agriculture (commodities), nontradables, and manufactures. Agricultural goods and manufactures are tradables, and their prices $p_A$ and $p_M$ are given internationally. The economy is populated by a continuous of individuals whose total mass is $L$. Individuals supply inelastically 1 unit of labor and consume a basket $(c = c(c_A, c_N, c_M))$ of agricultural, nontradables, and manufacturing goods whose price is $p = \tilde{p}(p_A, p_N, p_M)$.

Agricultural goods are produced by a competitive firm that combines labor ($L_A$) and land ($F$). The amount of land, $F$, is exogenously given, so that the production function of agricultural goods, $Y_A = A_A G(F, L_A)$, exhibits diminishing returns in labor. The production functions of
nontradables and manufactures exhibit constant returns in labor: \( Y_N = A_N L_N \) and \( Y_M = A_M L_M \) respectively.\(^{17}\)

The assumptions made regarding the allocation of the fixed factor’s property rights are of key importance. I begin assuming that the land equally belongs to the entire population, and that land’s rents are equally distributed in it. In particular, I define per capita rents as \( t = r_L F \), which, of course, depend positively on the rental price of land, \( r_L \) and on the absolute amount of land, \( F \).

### 1.3.1 Labor Market

Free mobility of workers between nontradables and agriculture ensures that the wage in both sectors is equalized. On the other hand, mobility of workers from either agriculture or nontradables to manufactures is costly and affects negatively the utility of the individuals. In particular, I assume that there is an entry cost \( E_M \geq 0 \), which is paid in terms of utility, for an individual that enters to the manufacturing sector. This loss of utility represents a variety of costs that an individual might face when entering the sector, and can be associated, for example, with the cost of search, education (acquisition of new specific skills), moving, cultural differences, and/or the cost of adaptation to a different environment and lifestyle.

I refer to the agricultural and nontradable sector as the traditional economy (or \( T \)-sector), whose wage is \( w_T \) and total employment \( L_T = L_A + L_N \). I use the subindex \( M \) to refer to the manufacturing sector (\( M \)-sector), whose wage is \( w_M \) and total employment \( L_M \).

Notice that a linear technology in manufacturing combined with the fact that \( p_M \) is exogenously given completely determines the wage in the sector at \( w_M = p_M A_M \). Also, from the

\(^{17}\)For simplicity and in order to highlight the key role of the labor market, the model only considers labor as a production factor (other than land). Capital can be included as an additional factor in the three sectors WLOG, particularly if it consists of manufactures and is mobile across sectors (that would be a case in which capital is mainly composed of machinery and equipment). In that case the ratio \( K/L \) in the manufacturing sector will be determined by \( p_M \). If, on the other hand, capital consists of final consumption good (this would be the case in which capital is a mix of equipment and infrastructure), a policy that depresses the nontradable sector and generates an migration toward manufacturing, will probably be accompanied by an increase in investment that will affect the demand for nontradables, partially offsetting the initial effect.
first order conditions of the firms producing nontradables, it follows that the price of nontradables is linear in the wage of the traditional sector: \( p_N = \frac{w_T}{A_N} \). Therefore, given the prices \( p_A \) and \( p_M \) and the technology \( A_N \), the price index \( p \) can be expressed as a function only of \( w_T \):

\[
\begin{align*}
\quad p &= \hat{p}(p_A, p_N, p_M) \\
&= \hat{p} \left( p_A, \frac{w_T}{A_N}, p_M \right) \\
&= p(w_T)
\end{align*}
\]

1.3.2 Problem of the Individuals

The problem of an individual that is initially in sector \( S \in \{T, M\} \) consists in choosing a basket of goods \( \{c_A, c_M, c_N\} \) and the sector \( S' \in \{T, M\} \) as to maximize its utility (net of migration costs) subject to its budget constraint. The problem can be formalized as:

\[
V(S) = \max_{\{c_A, c_M, c_N, S'\}} \{ U(c) - I(S' = M|S = L)E_M \} \\
\text{s.t. } p_A c_A + p_N c_N + p_M c_M \leq w_{S'} + t
\]

where \( c = c(c_A, c_M, c_N) \) is final consumption, \( t \) are land’s rents, and \( I(S' = M|S = L) \) is an indicator that takes value 1 if the individual is initially in sector \( T \) and decides to migrate to sector \( M \), and 0 in all other cases.

The problem can be solved in two stages. First, solve for \( \{c_A, c_M, c_N\} \) that maximize \( c(c_A, c_M, c_N) \) for a given income \( w_S + t \). Given the price index \( p = p(p_A, p_N, p_M) \), the solution to this step can be summarized as \( c_S = \frac{w_S + t}{p}, S = T, M \) (that is, the individuals consume their entire income at the given prices). The second stage consists in choosing the sector \( S' \in \{T, M\} \).
An individual in the T-sector. For an individual that is initially in the T-sector, the decision can be formalized as:

\[ V(T) = \max \{ U(c_T), U(c_M) - E_M \} \]

where \( U(c_T) \) is the value of staying in the T-sector, and \( U(c_T) - E_M \) is the value of migrating to manufacturing.

Two important observations can be made here. First, assuming that \( \lim_{L_A \to 0} G_L(F, L_A) = \infty \) (i.e. the marginal product of labor in agriculture approaches infinity as \( L_A \) approaches zero) and that \( \lim_{c_N \to 0} \frac{\partial U(c)}{\partial c_N} = \infty \) (i.e., the marginal utility of nontradable goods increases to infinity as \( c_N \) approaches zero) ensures that agricultural and nontradable goods will always be produced. Therefore, the value of migrating to manufacturing \( U(c_M) - E_M \) will be, in equilibrium, at most equal to the value of staying in the T-sector, \( U(c_T) \) (otherwise, there is complete migration to manufacturing and no production of agricultural and nontradable goods). The second observation is that, if there is indeed migration to manufactures (in the sense that the equilibrium’s labor share in manufacturing is strictly higher than the initial one), the value of staying should be equal to the value of migrating, which implies that \( U(c_T) = U(c_M) - E_M \iff c_T < c_M \iff w_T < w_M \).

That is, the existence of a positive entry cost \( E_M > 0 \), implies that the wage in the traditional sector must be strictly lower than the wage in manufacturing when there is migration. In fact, the "migration condition" \( U(c_T) = U(c_M) - E_M \) implicitly defines the minimum observable wage in the T-sector, \( \bar{w}_T \) such that \( U \left( \frac{w_T + \bar{t}}{p(\bar{w}_T)} \right) = U \left( \frac{w_M + \bar{t}}{p(\bar{w}_T)} \right) - E_M \).

An individual in the M-sector. For an individual that is initially in the M-sector, the decision regarding the sector can be formalized as:
\[ V(M) = \max \{ U(c_M), U(c_T) \} \]

where \( U(c_M) \) is the value of staying in manufacturing, and \( U(c_T) \) is the value of migrating to the \( T \)-sector. In this case, since there is no cost associated with entering the \( T \)-sector, migration to the traditional sector implies that \( U(c_M) \leq U(c_T) \Leftrightarrow w_M \leq w_T \). Eventually, if the equilibrium is such that \( w_M < w_T \), there is complete migration to the traditional sector and no production of manufacturing.

### 1.3.3 Equilibrium

An important feature of the model is that, for a given set of parameters \( \{A_A, A_N, A_M, p_A, p_M, F, E_M\} \), the equilibrium depends on the initial share of employment in the manufacturing sector. Let \( L_{M0} \geq 0 \) be the initial share of labor in the manufacturing sector. It can be proved that there exist a minimum \( L_M \) and a maximum \( \bar{L}_M \), with \( 0 \leq L_M \leq \bar{L}_M \), such that, in equilibrium, \( L_M \) and \( w_T \) are:

<table>
<thead>
<tr>
<th>Equilibrium’s wage and employment:</th>
<th>Initial Empl. in Manuf.</th>
<th>Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_M = L_M ) and ( w_T = w_T )</td>
<td>( L_{M0} \leq L_M )</td>
<td>[1]</td>
</tr>
<tr>
<td>( L_M = L_{M0} ) and ( w_T &lt; w_T &lt; w_M )</td>
<td>( L_M &lt; L_{M0} &lt; \bar{L}_M )</td>
<td>[2]</td>
</tr>
<tr>
<td>( L_M = \bar{L}_M ) and ( w_M \leq w_T )</td>
<td>( \bar{L}_M \leq L_M )</td>
<td>[3]</td>
</tr>
</tbody>
</table>

(in this last case, \( w_M < w_T \Leftrightarrow \bar{L}_M = 0 \))

In other words, for a given set of parameters, the equilibrium’s level of employment in manufacturing \( L_M \) will be in the range \([L_M, \bar{L}_M] \), but the actual level within that range depends exclusively on the initial level \( L_{M0} \). Analogously, the equilibrium’s wage in the traditional sector \( w_T \) will be in the range \([w_T, w_M] \) (unless \( \bar{L}_M = 0 \), which is an extreme case with no production of manufactures at all), but its actual level within that range depends on the initial conditions.
Interestingly, changes in manufacturing employment (relative to its initial level) only take place if the T-sector’s wage is in one of the boundaries of this range: \( L_M \) increases only if \( w_T = w_T \), and decreases if \( w_T = w_M \), but remains unchanged if \( w_T < w_T < w_M \). The existence of this "inaction zone" is directly related to the existence of an entry cost in manufacturing. I now characterize each of these three possible equilibria.

**Case 1: Expansion of Manufacturing Sector**

In case 1, the initial level of employment in manufacturing, \( L_{M0} \), is low relative to the equilibrium level. Achieving equilibrium implies a reallocation of labor toward the M-sector. The equation that determines the wage in the T-sector is the "migration" condition, which can be expressed as

\[
U\left(\frac{w_M + t}{p(w_T)}\right) - U\left(\frac{w_T + t}{p(w_T)}\right) = E_M
\]

and implies that the difference in the utility of individuals in both sectors is exactly equal to the migration cost. From this condition we can extract some important conclusions. First, the fact that the entry cost in manufacturing is expressed in terms of utility implies that differences in \( E_M \) do not translate into proportional differences in wages across sectors. In particular, the higher the concavity of \( U \), the larger the difference between \( w_T \) and \( w_M \) for a given entry cost \( E_M \). In other words, individuals with higher risk aversion will "tolerate" a lower wage \( w_T \) before migrating to manufacturing.

A second important observation has to do with the potential effect of land’s rents or transfers (\( t \)) on the wage \( w_T \). For any strictly concave utility function \( U \) and a given wage in manufacturing \( w_M \), higher rents are associated with a lower wage in the traditional sector \( w_T \) in an equilibrium with migration to manufacturing. That is, in two otherwise identical economies in which employment in the manufacturing sector is expanding, the wage in the traditional sector in
the economy with higher per capita rents (presumably an economy richer in natural resources or capital) should be lower. This is a somehow counter-intuitive result that is key to understanding the mechanism played by real exchange rate undervaluations. The intuition is simple: the larger the rents or transfers \( t \), the higher the level of consumption, and the lower the marginal utility of consumption. A larger difference in wages is therefore necessary to generate the same gap in utilities \( E_M \). In other words, the richer the individuals in terms of rents or transfers not directly associated with their work, the higher their willingness to tolerate a lower wage before deciding to migrate. Again, this result does not mean that the actual wage in the traditional sector \( (w_T) \) of an economy richer in natural resources will necessarily be lower than in poorer ones. Instead, what the result implies is that the wage that triggers migration to manufacturing \( (w_T) \) is lower in those countries and, therefore, less likely to be observed in equilibrium (i.e. the width of the inaction zone is larger in this case).

Once that \( w_T = w_T \), employment in agriculture \( L_A \) is determined by the first order condition of the firm in that sector. Employment in manufacturing \( L_M \) and nontradables \( L_N \) are simultaneously determined by imposing trade balance (or, equivalently, by equilibrium in the nontradable market).

**Case 2: Inaction**

The second case corresponds to situations in which the equilibrium level of employment in manufacturing coincides with its initial level, \( L_{M0} \). The equilibrium wage \( w_T \) is strictly higher than \( w_T \) but lower than \( w_M \), and therefore remains inside the inaction zone.

In this case \( L_M = L_{M0} \), and total employment in the \( T \)-sector remains unchanged \( L_T = L - L_{M0} \). Employment in agriculture, \( L_A \), and the wage in the sector, \( w_T \), are determined by the first order conditions of the firm in agriculture and the trade balance condition.
Case 3: Contraction of Manufacturing Sector

Finally, it may occur that the initial level of employment in manufacturing is actually higher than the level consistent with the equilibrium. The achievement of equilibrium is accompanied in this case by process of de-industrialization. This scenario can occur, for example, if the productivity of the agricultural sector increases, the productivity in manufacturing falls, the relative price of agricultural goods increases (or that of manufactures falls), or simply due to the fact that productivity growth in nontradables is systematically lower than in tradables (i.e. the Balassa-Samuelson effect).

The wage in the traditional sector consistent with a partial de-industrialization (i.e. $0 < L_M = L_M < L_{M0}$) is $w_T = w_M$ (Case 3a). Given this wage, $L_A$ is determined by the first order condition of the firm in agriculture and $L_M$ by the trade balance condition.

In the extreme case in which there is complete de-industrialization (i.e. $L_M = L_M = 0$), the economy functions only with its traditional sectors (Case 3b), so that total employment is split between agriculture and nontradables. Both the wage $w_T$ (which is strictly higher than that in manufacturing, $w_M$) and employment in agriculture $L_A$ are simultaneously determined by the first order condition of the firm in agriculture and the trade balance condition.

Figure 1.5 summarizes the three (actually four) possible equilibria. The two most important things to remember here are that, first, for a given set of parameters (technologies, international prices, entry cost and land’s endowment), the observed equilibrium will depend on the initial level of employment in manufacturing; and, second, we will only observe migration to manufacturing if the wage in the traditional sector is at the lower bound of the inaction zone, $w_T$. 
1.3.4 Comparative Statics

In this section I perform some comparative statics varying different parameters of the model. I analyze changes in entry costs, distribution of rents, manufacturing productivity and, most importantly, endowments of natural resources.

Reduction in Entry Costs

The effect of a fall in the entry cost $E_M$ is straightforward. It follows from equation (1.1) that, as $E_M$ decreases, $w_T$ (the minimum observable wage in the traditional sector) increases, so that the inaction zone narrows from below. Whether this change affects the observed equilibrium depends on the initial equilibrium. For example, if $w_T$ was relatively close to $w_T$ in the initial equilibrium (before the change in $E_M$), then the change in entry costs will probably make the migration condition (1.1) binding and will induce both an increase in $w_T$ and migration toward manufacturing\(^{18}\) (Case A in figure 1.6). If, on the other hand, the initial $w_T$ was relatively distant from $w_T$, it may

\(^{18}\)The increase in $w_T$ will additionally reduce employment in agriculture, reducing at the same time the fixed factor’s rents and so reinforcing the incentives for migration. This is, however, a second order effect whose magnitude does not affect significatively the results.
occur that the reduction in entry costs does not have any effect on the observed equilibrium (Case B in figure 1.6).

Figure 1.6: Model - Effects of a Reduction of Entry Costs

The initial analysis assumed that the rents from the fixed factor were equally distributed across the population. This section analyses the effects of changing the distribution of such rents. Figure 1.7 presents two alternative cases. The first case assumes that the distribution of rents favors the population in the traditional sector (i.e. a more "progressive" distribution). Consequently, given that individuals in the traditional sector dispose of a higher income, the wage that triggers migration, $w_T$, is lower and the inaction zone is wider. The opposite is true if a more "regressive" distribution of rents is assumed (that is, one that favors relatively more the individuals in the manufacturing sector). In this case the lower bound of the inaction zone increases, shrinking the inaction zone and making it more likely for workers in the traditional economy to migrate to manufacturing. In both cases, the potential effect (negative or positive) on $w_T$ from changes in the distribution of rents will be larger the higher the endowment of the fixed factor in the economy.\footnote{The role of the distribution of rents is key in economies with high endowments of natural resources, as is the case of many Latin American countries. If rents are sufficiently high, a redistribution that favors workers in the manufacturing sector can actually eliminate the inaction zone.}

Change in Distribution of Rents

The initial analysis assumed that the rents from the fixed factor were equally distributed across the population. This section analyses the effects of changing the distribution of such rents. Figure 1.7 presents two alternative cases. The first case assumes that the distribution of rents favors the population in the traditional sector (i.e. a more "progressive" distribution). Consequently, given that individuals in the traditional sector dispose of a higher income, the wage that triggers migration, $w_T$, is lower and the inaction zone is wider. The opposite is true if a more "regressive" distribution of rents is assumed (that is, one that favors relatively more the individuals in the manufacturing sector). In this case the lower bound of the inaction zone increases, shrinking the inaction zone and making it more likely for workers in the traditional economy to migrate to manufacturing. In both cases, the potential effect (negative or positive) on $w_T$ from changes in the distribution of rents will be larger the higher the endowment of the fixed factor in the economy.\footnote{The role of the distribution of rents is key in economies with high endowments of natural resources, as is the case of many Latin American countries. If rents are sufficiently high, a redistribution that favors workers in the manufacturing sector can actually eliminate the inaction zone.}
Increase in Manufacturing Productivity

The effects of an increase in the productivity of the manufacturing sector, $A_M$, are mixed. The first and most direct consequence is an increase in the wage of the manufacturing sector, $w_M$, provided that the international price of manufacturing goods remains unchanged. Following this increment in $w_M$, equation (1.1) implies that the lower bound of the inaction zone, $w_T$, increases as well. The intuition of this change is simple: an improvement in the wage in manufacturing increases the incentives for migration from the traditional sector, raising therefore the wage that leaves individuals indifferent between staying and migrating ($w_T$). Moreover, strict concavity of $U$ implies that the change in $w_T$ is proportionally smaller than the increment in $w_M$, implying that the absolute width of the inaction zone actually increases.

The third consequence of an increase in $A_M$ is related to the income effect that accompanies the increase in $w_M$. Specifically, an increase in $w_M$ will increase aggregate income domestic demand for the three goods in the economy\textsuperscript{20} as long as part of the labor force is devoted to manufacturing production (i.e. $L_{M0} > 0$). The natural response of the nontradable sector to a higher demand

\textsuperscript{20}Assuming that the three goods are "normal" goods.

Figure 1.7: Model - Effects of a Change in the Distribution of Rents
is to expand production by adding more workers which, given the diminishing marginal product of labor in agriculture, will be accompanied by an increase in \( w_T \) and, consequently, in \( p_N \). This implies that, since \( p_M \) remains unchanged, an increase in manufacturing productivity is followed by a real appreciation.

Figure 1.8: Model - Effects of an Increase in Manufacturing Productivity

The final effect on the actual size of the productivity gap and on manufacturing employment depend on the initial conditions. If the initial share of labor in manufacturing \((L_{M0})\) is relatively low and the equilibrium wage, \( w_T \), is low and close to \( w_T \) (Case A in Figure 1.8), the increase in \( A_M \) will make the migration condition (1.1) binding and be therefore associated with an increase of the productivity gap and an expansion of the manufacturing sector. This prediction of the model is consistent with the evidence from China, where the increase in manufacturing productivity that started in the early 1990s has been accompanied by, simultaneously, an increase in the productivity gap and an expansion of employment and production in the manufacturing sector. On the other hand, if the manufacturing sector already had a considerable size and the initial wage in the traditional sector was high (close to \( w_M \)), it might happen that the income effect prevails and the increase in \( A_M \) is actually followed by a contraction of manufacturing employment and an expansion of nontradables (Case C in Figure 1.8). Finally, in an intermediate case (with initial \( L_M \)
and $w_T$ at "intermediate" levels - Case B in Figure 1.8), the increase in manufacturing productivity has no effect on manufacturing employment. The composition of aggregate production is affected, however, with manufacturing and nontradable production increasing (due to higher productivity and employment, respectively), and agricultural production diminishing (due to lower employment in the sector).

**Differences in Endowments of Natural Resources**

The most interesting exercise is the one that analyzes the implications of differences in endowments of natural resources. Consider the case of two otherwise identical economies that differ in their initial endowment of land, $F$. Two main consequences arise from that. First, a higher endowment of land increases the productivity of workers in the agricultural sector, and raises both the equilibrium wage in the traditional sector ($w_T$) and the rents from land ($t$). Higher rents and wages in the traditional sector imply an increase in aggregate income which, as in the previous case, is associated with a higher demand for nontradables and a further increase in the equilibrium wage, $w_T$. Therefore, a country richer in natural resources has, in principle, a relatively more appreciated real exchange rate. The second effect has to do with the size of the inaction zone. As mentioned above, the lower bound of the inaction zone, $w_T$, depends negatively on the level of per capita rents, $t$; which implies that countries with higher rents from natural resources will have a lower $w_T$ and a wider inaction zone. In other words, workers that, in addition to their wage, enjoy of higher rents or transfers, have higher "tolerance" to lower wages before migrating to manufacturing. The key is that, due to the existence of rents, a change in the wage translates into a less-than-proportional change in total income and consumption of the individuals.

Figure 1.9 illustrates the equilibria of two countries with different endowment of natural resources. The country richer in natural resources (green) has a higher wage in the traditional
sector and, at the same time, a wider inaction zone. On the other hand, the country poorer in natural resources (red) has a narrower inaction zone and a lower wage in the traditional sector. I now analyze the implications of such differences on the existing inefficiencies in production, and on their possible corrections.

**Production Inefficiency**

From the point of view of production, the allocation of resources is inefficient if the wage in the traditional sector is lower than the wage in manufacturing. Furthermore, the larger the gap in wages between sectors, the more inefficient the allocation. This implies that, everything else equal, the allocation of labor is relatively more inefficient in economies with lower endowment of natural resources, which display a relatively larger wage differential. A correction of that misallocation has, presumably, a potentially larger impact on aggregate production in such countries. The difference in wages, $w_M - w_T$, is in fact an indicator of both the severity of the misallocation and the potential gains that its correction would generate, since it represents the gain in production associated with the reallocation of one worker from the traditional to the manufacturing sector. Figure 1.10 identifies such potential gains for the two countries of the previous example.

The figure also gives an idea of the potential costs of correcting such misallocations. In
fact, if we ask by how much the wage in the traditional sector should change in order to provide incentives for voluntary migration of workers to manufacturing, the answer is simple: it should fall until it reaches the lower bound of the inaction zone, \( w_T \). That is the only wage that leaves workers indifferent between staying and migrating. Workers with higher wage prefer to stay rather than to migrate. Therefore, we can interpret the difference between the equilibrium wage in the traditional sector, \( w_T \), and the lower bound of the inaction zone \( w_T \) as a measure of the necessary distortion that would incentivize voluntary migration to manufacturing. Introducing such distortion is obviously costly, given that it actually exacerbates the misallocation, as verified by the fact that the wage gap between sectors increases.

A key question is, therefore, what are the conditions that ensure that the potential gains associated with the correction of the misallocation surpass the potential costs that such correction would imply. Figure 1.10 helps us to understand the answer to this question by illustrating the potential gains and costs for economies that differ in their endowments of natural resources. The
potential costs are specially high in economies richer in natural resources, whose workers in the
traditional sector enjoy a higher wage ($w_T$) and, due to higher rents and transfers, are willing
to tolerate a lower wage before deciding to migrate to manufacturing (lower $\bar{w}_T$). At the same
time, given that the misallocation is less severe in these economies, they are precisely the ones that
benefit the least from an induced reallocation. In other words, inducing a reallocation of labor
has a relatively high potential cost and low potential gain in economies richer in natural resources.
In economies with scarce natural resources, on the other hand, not only the potential gains from
reallocation are high, but also the necessary distortion that would triggers migration is, in principle,
less severe. In such countries the distortion is still costly, but the potential benefits are, in principle,
higher.

1.3.5 Government Interventions

Though the setting described so far does not assign any particular role to the government, it has
been seen that, under certain conditions, the presence of resource misallocations might justify gov-
ernment interventions aimed at correcting the inefficiencies. The main reason behind this possible
justification is precisely the fact that the equilibrium depends on the initial conditions, and, in
particular, on the initial level of employment in manufacturing. Even though the static nature of
the model is not consistent, in principle, with a dynamic analysis, the key is to understand that
in this context a temporary government intervention that effectively modifies the initial conditions
has, potentially, important long term effects. We can actually consider an exercise with two stages:
In the first stage, the government intervenes with the objective of changing the level of employment
in manufactures. Once employment in manufacturing attains a level consistent with an efficient
long term equilibrium (i.e. one that would imply equal wages across sectors in the second stage),
the government ceases intervening and, in a second stage, leaves the economy working on its own.
In this section I briefly analyze a set of possible interventions that might achieve the desired reallocation of labor. With differences in their instrumentation, the objective in all cases is the same: to have the wage in the traditional sector at the lower bound of the inaction zone so that workers in that sector have the necessary incentives for migrating. This can be achieved by inducing either a fall of the equilibrium wage, \( w_T \), or an increase of the lower bound of the inaction zone, \( w_{LT} \). In the final part of the section, when analyzing the potential issues of each particular policy, I take a more practical approach and consider some elements that are beyond the theoretical framework of the model.

**Policies Aimed at Reducing Domestic Consumption**

We have seen that in order to induce voluntary migration, either the wage in the traditional sector \( (w_T) \) should fall or the lower bound of the inaction zone \( (w_{LT}) \) should increase, or both. A policy that is usually associated with real undervaluations (that is, a lower price of nontradables) and that, in principle, could generate the desired outcomes is one that reduces domestic consumption as measured *in terms of tradable goods*. This type of policy can be instrumented *in practice* in at least two different ways. The *direct* way is to generate a deliberate contraction of aggregate demand, for which tools such as rising taxes, reducing public expenditure, or reducing domestic credit could help. In this case, the real depreciation does not come through a nominal exchange depreciation, but instead through a deflationary process in the nontradable sector. Another (more *indirect*) way of achieving the same outcome is by combining a nominal depreciation and a contention (as opposed to a contraction) of aggregate expenditure as measured in domestic currency, which could imply the use of tools such as sterilization, issuing bonds in the domestic market, controlling the expansion of domestic credit or limiting the access of the public to financing. In this case, the real depreciation comes through a nominal depreciation that raises (relatively more) the prices of tradable goods.
as measured in domestic currency. In both cases the policies might be complemented with the imposition of some sort of controls on capital inflows.

**An Example in Our Model** At the aggregate level, the two policies explained above have a similar effect, which is reducing aggregate consumption as measured in terms of tradable goods. In order to illustrate the way the mechanism works I consider now a simple example within the theoretical framework of our model. The example is by no means exclusive.

**First Stage: Government Intervention** To simplify the analysis, assume that in the first stage the government imposes an ad-valorem income tax, so that individual’s available income is now a fraction $\gamma \in (0, 1)$ of its original income. Assume additionally that the government does not spend domestically the tax’s revenues, but lends them abroad at the international interest rate.

If the original wage in the traditional sector is inside the inaction zone (that is, $w_T > \overline{w}_T$), the contraction in domestic consumption will induce a fall in the price of nontradables$^{21}$, a fall in the wage of the sector, and a reallocation of labor from nontradables to agriculture. This initial depreciation has no effect on manufacturing employment (and in fact has a negative effect in aggregate production) until $w_T$ hits the lower bound of the inaction zone, $\overline{w}_T$. Only when this happens, migration toward manufacturing begins. For the reasons already explained, this initial depreciation might be particularly large and costly in economies rich in natural resources, but relatively small and less distortive in countries with scarce natural resources, whose initial wage is closer to the lower bound.

Additionally, and depending on the concavity of preferences, the tax can have an effect on the lower bound, $\overline{w}_T$. This can be seen in the migration condition from which $w_T$ is determined, which now takes the form

$^{21}$Again, assuming that their income elasticity is strictly positive.
\[ U \left( \frac{w_M + t}{p(w_T)} \right) - U \left( \frac{w_T + t}{p(w_T)} \right) = E_M \]  

(1.2)

If preferences are logarithmic, then a proportional income tax (like the one analyzed here) does not have any effect on \( w_T \), so that once the wage reaches \( w_T \), it remains there and the remaining adjustment is made through reallocation of labor. With less concave preferences (i.e. less concave than logarithmic ones), the lower bound \( w_T \) moves to the left, so a larger depreciation is necessary before migration begins. Finally, if \( U \) is more concave than the logarithmic function, the application of the tax moves \( w_T \) to the right, reducing the necessary depreciation that induces migration. Notice that in this case, if the wage \( w_T \) was initially sufficiently close to \( w_T \), the application of the income tax can actually induce an overreaction with large migration to manufacturing and an appreciation of the real exchange rate.

In all cases, migration takes place until the new trade balance condition is achieved. The difference now is that due to the amount of resources that the government lends abroad, the new trade balance is actually positive and equal to the amount of resources that the government transfers abroad.

When should the intervention stop? Ideally, as mentioned above, the intervention should cease when the level of employment in manufacturing is consistent with an efficient equilibrium, that is, one in which \( w_T = w_M \). In order to determine that level of manufacturing employment it is necessary to have into account the net payments that the country will receive from its higher stock of international assets.

**Final Equilibrium** In the second stage, once the policy is removed, we should expect a large expansion in aggregate demand. The main reasons for this are three. First, the elimination of the tax leaves the households with higher available income. Second, a fraction of the population
migrated from an initially low-income sector to manufacturing, where the wage is higher. And, third, the net payments from the international assets that the economy accumulated in the first stage are now available to be distributed in population. The three effects point to an increase in aggregate demand and, in particular, in the demand for nontradable goods that will have as a consequence a real appreciation, an expansion of the nontradable sector, and a contraction of agriculture.

**Policies Targeting Specific Sectors**

There are other government interventions that potentially have similar (or even better effects) effects. Here, I briefly discuss three of such policies: a tax on nontradable production, a subsidy to manufacturing production, and a subsidy for migrants. The main difference between these policies and the one previously analyzed is that these are targeted at specific sectors of the economy.

**Tax on Nontradables** Consider an ad-valorem tax, $\tau_N$, on nontradable production. The net price received by the producers in the sector is now $p_N(1 - \tau_N)$, so that the wage that they are willing to pay (from the first order condition) is $w_T = p_N(1 - \tau_N)A_N$.

The effect of the tax on the wage depends on the elasticity of demand. If the demand for nontradables is inelastic, the tax will increase $p_N$ but will not have a significant effect on $w_T$. The potential reallocation in this case is very limited. If, on the other hand, the demand for nontradables is relatively elastic, most of the adjustment will come through a reduction in $w_T$. Such fall of the wage will increase employment and production in agriculture and, if the fall makes the wage to hit the lower bound of the inaction zone, it will effectively incentivize migration to manufacturing. Though the relative prices of nontradables increase, this tax induces a depreciation from the producer’s point of view.

The main problem related to this policy has to do with its enforceability in practice, since
the nontradable sector in most developing countries is composed by a large number of small (even one-person) firms that operate with high levels of informality.

Subsidy to Manufacturing Production  Consider now an ad-valorem subsidy, $s_M$, to manufacturing production. The revenue per unit of the firms in the sector is now $p_M(1+s_M)$, so that the wage that the firms are willing to pay is now $w_M = p_M(1+s_M)A_M$. In this case, since the price of manufactures $p_M$ is given, the introduction of the subsidy has the same effect as an increase in the productivity $A_M$, which directly translates into an increase of the wage in the sector. This change in $w_M$ increases $w_T$ and, if sufficiently large, induces migration to manufacturing.

There are, however, some potential practical issues with this policy. First, on the fiscal side, the policy will require either raising taxes or issuing debt. One (possibly good) alternative is combining the subsidy with the tax on nontradables analyzed before. A second potential problem, in practice, arises when the policy has to be finished, since removing the subsidy implies lowering the wage to the workers in the manufacturing sector. A third potential issue has to do, in practice, with the need for controlling aggregate expenditure and avoiding a real appreciation that offsets the initial effect. If, for some reason, the introduction of the subsidy implies an increase of aggregate expenditure beyond the increase in production (that is, an fall in the current account balance), then the higher demand for nontradables and its positive impact on the wage of the traditional sector can actually end up preventing the desired migration of workers. Finally, a fourth potential problem with this policy is that it can give place to (justified) complains for dumping by other countries.

Interestingly, though from the producer’s perspective the policy generates a real depreciation, from the consumer’s perspective it actually induces an appreciation, raising the relative prices of nontradable goods.
Subsidy for Migrants  A more direct policy can consist in subsidizing migration to manufacturing. The effects are similar to the previous case, in the sense that the policy increases $w_T$.

In this case, since the subsidy would only apply to new workers in manufacturing (as opposed to all workers in the sector as in the previous case), the potential fiscal problem is reduced, and the appreciation that would follow the increase in aggregate income should also be smaller. There might be still problems related to the implementation of the policy, particularly if the number of migrants is large. As in the previous case, the policy can also arise complains for dumping from trading partners.

1.3.6 Main Predictions of the Model

According to the model, the potential positive effects of an induced real undervaluation on both, manufacturing employment and real growth, depend on the joint verification of the following conditions:

1. The existence of frictions in the labor market and, in particular, entry costs for workers that want to migrate into manufacturing. These entry costs give place (under certain conditions) to a productivity and income gap between manufacturing and the traditional economy.

2. A low (high) initial share of employment in manufacturing (traditional economy). An initially low share of employment in the high productivity sector implies that, everything else constant, agricultural productivity is low (given the high mass of workers in the sector) and therefore the productivity and income gap is high. Additionally, since per capita income is low, so is the demand for nontradable goods, further increasing employment in agriculture.

3. Low endowments of natural resources (agriculture in particular). Everything else constant, a lower endowment of agricultural resources implies lower labor productivity in agriculture,
and therefore a larger productivity and income gap. In addition, lower rents from natural resources imply that the lower bound of the inaction zone (i.e. the wage in the traditional sector that triggers migration to manufacturing) is higher.

4. Low initial per capita income. The effect on growth rates from reallocating labor from the low-productivity to the high-productivity sector is higher, for a given productivity gap, the lower the initial per capita income.

It is important to notice that all four conditions must hold, to some extent, for a real undervaluation to have a positive effect on manufacturing employment and GDP growth. The first three conditions guarantee the existence of a productivity gap of considerable size, while the fourth condition (which may or may not be a consequence of the first ones) ensures that the relative impact of a given reallocation of labor is high.

**Additional Predictions**

Other important predictions of the model are summarized below:

**Effects from Different Implementations of the Policy**  If the policy is implemented through a reduction of domestic consumption, then the most probable effect is a *depreciation* (a lower wage in the traditional sector), depending on how progressive the policy is. In this case the trade and current account balance should improve during the first stage. Once the policy ends, the higher stock of international assets will generate an inflow of rents that will exacerbate the appreciation of the currency.

If the policy is implemented by redistributing resources internally (for example by subsidizing migration, taxing the nontradable or the traditional sector, and/or subsidizing manufactures), then the depreciation will be moderated and there can even be an appreciation. In this case no
change in the trade and current account balance is observed.

**Asymmetric Effects of Undervaluation and Overvaluation** I briefly analyze here the potential effects of an exogenous overvaluation, which are not symmetric with respect to those of an undervaluation.

If initially $w_T < w_M$, the first effect of a moderate overvaluation is to stop migration to manufacturing (in case there was any), to reduce production of agricultural goods, and to increase the production and prices of nontradables. The wage gap between sectors shrinks. There is no growth if GDP is measured at constant initial prices, but there is an increase of GDP if it is measured at current prices. Aggregate consumption increases and, everything else equal, the trade and current account balance worsens.

If the overvaluation is more severe and the wage in the traditional sector equalizes the wage in manufacturing, there is migration from manufacturing to nontradables (de-industrialization). This is more likely to occur in countries richer in natural resources, where the wage gap is initially narrower. Eventually, a complete deindustrialization may take place, and a further increase in the wage, price, and production of nontradables.

1.4 Testing the Main Predictions of the Model

In this section I use two methods to test empirically the main predictions of the model. The first method consists in verifying that the relationship between real undervaluation and both, growth and industrial development in the last years has been stronger in countries with the proper initial conditions. I construct a measure of undervaluation and a score that summarizes the extent to which a country had, based on the predictions of the model, the proper conditions for an undervaluation. I then perform cross-section regressions for different periods and samples. The second method
consists in identifying the "induced" real depreciations (that is, those that are not explained by changes in fundamentals) and verifying that they are mostly observed in countries that, according to the model, have the best conditions for that kind of policies.

1.4.1 Assessing the Relation between Real Undervaluation, GDP Growth, and Manufacturing Production

In this section I verify that the relation between real undervaluation and both GDP and manufacturing growth is stronger in countries that, according to the model, have the proper conditions for the use of such policies.

A policy that induces reallocation of labor from the traditional to the manufacturing sector (such as the one analyzed here) has, in principle, a potentially higher effect on growth in economies with a large initial productivity gap between sectors, and a large fraction of the labor force in the traditional (low-productivity) sector. According to the model, these conditions should be verified in low-income countries with relatively low endowments of natural resources (agriculture in particular), high frictions in the labor market (entry costs in manufacturing), and low share of employment in manufacturing. In such countries, the relationship between real undervaluation and GDP growth, and between real undervaluation and reallocation of labor to manufacturing should be stronger. I test these predictions in a cross-section of countries\textsuperscript{22}. I begin by specifying the baseline regressions, and then describe the elaboration of the two main variables of the analysis: a

\textsuperscript{22}It is worth mentioning that the results presented below are robust to different specifications with panel data, including panel with country-specific fixed-effects, as well as Difference and System GMM. While the main problem with panel with country fixed-effects is the endogeneity of the regressors due to the inclusion of lags of the dependent variable in the RHS of the equation (e.g. lags of per capita income), the lack of good instruments is the main problem in the case of Difference or System GMM. I deal with these issues by limiting the analysis to the use of cross-section regressions, in which all the included regressors are either exogenous (such as endowments of natural resources) or predetermined in the initial period (e.g. initial per capita income, size of the manufacturing sector, size of the traditional sector, etc.). The costs of restricting the analysis to this type of regressions are at least two: first, the estimated standard errors might suffer from the fact that I have relatively few countries in the sample, and, second, I do not make use of the variation within country. Yet, I prefer to assume those costs and ensuring the reliability of the results.
measure of real undervaluation and a score that summarizes the extent to which a country satisfies on average the multiple conditions that make it a good candidate for the use of this policy. Finally, I present and comment the regressions.

**Baseline Regressions and Data**

The equation I test has the following form:

\[ y_{it_{fin}} = \alpha + X_{it_{ini}}' \beta + \gamma_1 U_{it_{avg}} + \gamma_2 S(X_{it_{ini}}) \times U_{it_{avg}} + \varepsilon_i \]

where \( y_{it_{fin}} \) is the observed outcome for country \( i \) at time \( t_{fin} \) (the end of the period), \( X_{it_{ini}} \) are exogenous or predetermined regressors at time \( t_{ini} \) (the initial period), \( U_{it_{avg}} \) is a measure of the average level of undervaluation over the period, and \( S(X_{it_{ini}}) \) is a score that summarizes the extent to which the regressors \( X_{it_{ini}} \) are in line with the conditions that, according to the model, make a country a good candidate for an undervaluation. In particular, I define the score such that \( S(X_{it_{ini}}) = 0 \) means that the country \( i \) at time \( t_{ini} \) is an ideal candidate, while \( S(X_{it_{ini}}) = 1 \) means that the country has none of the (theoretical) conditions for an undervaluation. I should verify that \( \gamma_1 > 0 \) and \( \gamma_2 < 0 \), meaning that undervaluation is positively associated with the outcome for countries with low \( S \) (i.e. the good candidates), but such association weakens and may even become negative in countries with higher \( S \) (i.e. the ones that do not have the conditions for a real depreciation).

**Dependent Variables**

The dependent variable in the first set of regressions is the average rate of growth of per capita income over the respective period (PPP in 2005$, from PWT 7.1). In the second set of regressions the explained variable is the share of manufacturing in total tradable production (computed as
A Measure of Real Undervaluation

The key variable in the empirical analysis is the measure of real undervaluation. Following Rodrik (2008), I compute my measure of undervaluation as the residual of regressing the logarithm of the real exchange rate on the logarithm of per capita income PPP, as a way of accounting for changes in RER related to the Balassa-Samuelson effect\(^{23}\). The residuals would capture, in principle, deviations of the RER from the level predicted by the country’s per capita income.\(^{24}\)

Specifically, the regression has the following form:

\[
\ln(RER_{it}) = \alpha + \beta \ln(pcIncome_{it}) + I_t + \varepsilon_{it}
\]

where the subindices \(i\) and \(t\) refer to country and year respectively, \(I_t\) are year fixed-effects, and \(\varepsilon_{it}\) is an error term with standard properties. Data for both the RER\(^{25}\) and per capita income PPP are from Penn World Table Version 7.1. In the original regression I use all the countries in the

---

23 The justification for controlling for per capita GDP is that, being a proxy for the relative level of productivity of tradables to nontradables, it controls for the Balassa-Samuelson effect. The question is how well per capita income proxies the relative productivity of tradables. As a verification, I make use of the measures of sectoral labor productivities available in Timmer & de Vries (2009) and McMillan & Rodrik (2011), and construct the ratio of average productivities in tradables and nontradables sectors for the available countries and years. I then compute the same measure of undervaluation using the actual productivity ratio (instead of per capita income). I find that the correlation between both measures of undervaluation is very high, at 0.89, for all the countries and years for which both measures are available. In fact, the only countries for which the measure based on per capita income underestimates undervaluation are Nigeria and Venezuela, the only oil-exporter countries of the sample.

24 Pagan (1984) analyses various issues related to the use of residual generated regressors (as is the case of this measure of undervaluation). He shows that in the model

\[
y = \delta z^* + \gamma (z - z^*) + \epsilon \tag{1.3}
\]

\[
z = z^* + \eta = W\alpha + \eta \tag{1.4}
\]

where \(z^*\) is the predicted or anticipated part of \(z\) and the term \((z - z^*)\) represents the "unanticipated" part of \(z\). "2SLS estimates provide the correct values for \(\delta\) and OLS estimates the correct ones for \(\gamma\). Furthermore, if only "unanticipated" regressors are included in equation (1.3) (i.e. \(\delta = 0\)), then OLS produces the correct estimates of variance and efficient coefficient estimates. This is precisely the case analyzed here.

25 The RER is actually computed as \(RER = \frac{1}{p}\), where \(p\) is "Price Level of GDP, G-K method (US = 100)" in PWT 7.1 (Heston et al. (2012))
database except for Serbia and Georgia (which are notable outliers), for the period 1970-2010.

Measuring Initial Conditions

For initial income I use the logarithm of per capita income (PPP in 2005 $) for the initial year of the respective period.

As proxy for the size of the manufacturing sector I use the share of value added in the manufacturing sector (% of total value added, at current prices)\textsuperscript{26}. The series correspond to category D of ISIC Rev. 3, and is from the U.N. Statistical Division, which contains annual sectoral value added for most countries from 1970 to 2010\textsuperscript{27}.

For entry costs I use two different proxies: average years of schooling (from Barro & Lee (2010)) in the initial year of the respective period, which ideally captures the costs associated with the acquisition of the appropriate skills and human capital (in principle, individuals with initially lower levels of education should face a higher entry cost in case they want to migrate to manufacturing); and the country’s average index of ethnic, linguistic and religious homogenization (computed as $1 - \text{fractionalization}$, from Alesina et al. (2003)), which is intended to capture the migration costs associated with differences in these variables (we could think a priori that lower ethnic, linguistic or religious homogenization imply a higher cost for individuals that migrate from the traditional to the industrial sector).

For the size of the traditional sector I use the percentage of rural population in the initial year of the respective period (from WDI), which provides an idea of the number of workers that can be reallocated to the industrial sector and, at the same time, is as an indicator of the presence

\textsuperscript{26}Unfortunately the availability of data on manufacturing and industrial employment is limited. Timmer & de Vries (2009) offer data for 28 countries for different periods (only 14 of them are currently developing countries). McMillan & Rodrik (2011) complement the dataset with some African countries and China. Still, the number of observations in my regressions was always less than 30 when using these data.

\textsuperscript{27}Specifically for China, the reported data for manufacturing (category D, ISIC Rev.3) for the period 1970-2005 is indeed total value added of the industrial sector (categories C-E, ISIC Rev.3). I instead use data from Timmer & de Vries (2009) for the years 1990-1994, and for the period 1970-1989 I extrapolate the ratio of manufacturing to total industry value added from the period 1990-2010 (which is indeed very stable around 0.8).
of entry costs.

As a proxy for the relative endowment of agricultural resources I use per capita agricultural-arable and cultivable-land (from WDI, average of the period), and as a proxy of mineral resources I use per capita oil reserves (from CIA factbook 2002/03, average of the period)\textsuperscript{28}.

**Computing the Scores**

In order to construct the scores that summarize the extent to which a country satisfies the multiple conditions that make it a good candidate for an undervaluation, I first create six indicator variables:

1. \(I(\text{High Initial Income})\), which takes value 1 if the per capita income of the country in the first year of the period is higher than the sample median in the same year, and 0 otherwise.

2. \(I(\text{High Schooling})\), which takes value 1 if average years of schooling in the country in the first year of the period is higher than the sample median in the same year, and 0 otherwise.

3. \(I(\text{High Urban Pop.})\), which takes value 1 if the percentage of urban population in the country in the first year of the period is higher than the sample median in the same year, and 0 otherwise.

4. \(I(\text{High Aigr. Land})\), which takes value 1 if average per capita agricultural land of the period in the country is higher than the sample median in the same period, and 0 otherwise.

5. \(I(\text{High Homogenization})\), which takes value 1 if average ethnic, linguistic, and religious homogenization in the country is higher than the sample median, and 0 otherwise.

6. \(I(\text{Low \% of Manufacturing in Tradables})\), which takes value 1 if the share of manufacturing production in total tradable production in the country in the first year of the period is lower than the sample median in the same year, and 0 otherwise.

\textsuperscript{28}Per capita oil reserves in each year is estimated with total oil reserves in 2002/3 and each year’s population.
The score used in the first set of regressions (the growth regressions) is the simple average of the first five indicators (high initial income, high schooling, high urban population, high agricultural land, and high homogenization). The score used in the second set of regressions (the ones of the share of manufacturing in tradables) is the simple average of the last three indicators (high agricultural land, high homogenization, and low initial share of manufacturing in tradables). In both cases, a score equal to 0 implies that the country has, from the model’s perspective, the conditions that maximize the potential positive effects of undervaluation on the outcome (either GDP growth or the share of manufacturing in tradables), while a score equal to 1 implies the opposite.

**Periods and Samples**

Two periods are analyzed: 1970 to 2010 (the period for which there is data on sectoral value added from UNSD), and the sub-period between 1982 and 2003 which, by excluding the oil shocks of the 1970s and the more recent commodities boom, presents more stable commodities prices).

There are 180 countries for which the measure of undervaluation is available for these periods. In order to minimize the potential effect of extreme values of undervaluation, I remove 5% from the extremes of the distribution. That is, all observations below the percentile 2.5 and above the percentile 97.5 are dropped. Of the remaining 171 countries, only 116 have complete data for the first set of regressions for the period 1970-2010, and 118 for the period 1982-2003.

Additionally, I consider a subset of developing countries from East and South Asia, Latin America and Caribbean, and S.S. Africa (whose per capita income at the beginning of the respective period was less than 50% that of the U.S. in the same year). There are 73 and 72 such countries with complete data for the first set of regressions.
1.4.2 Regressions

Table 1.3 presents the results of regressing the average rate of growth of per capita income on a set of controls, the measure of real undervaluation, and the interaction of undervaluation with different controls for both periods. All regressions in this table include the following set of controls: initial per capita income ($\ln$), per capita agricultural land ($\text{avg.}$), per capita oil reserves ($\text{avg.}$), initial years of education ($\ln$), initial urban population ($\%$), the index of homogenization, and the initial share of manufactures in GDP ($\%$). Additionally, regressions (2) through (6) (panels A & B) include the interactions between undervaluation and some of the controls. Finally, regression (7) includes the interaction between undervaluation and the score.

Regressions (A1) and (B1) (Table 1.3) provide a measure of the relationship between undervaluation and growth for all countries on average, which is positive and significant at 5% when considering the entire period, but not significant for the period 1982-2003. This result is in line with the prediction of the model that the association between real undervaluation and growth is not necessarily positive for all countries, but only for those with certain conditions. The addition of the interaction term in regressions (2A) & (2B) allows to have an idea of the differential effect of undervaluation on growth depending on the initial level of per capita income. The coefficients of undervaluation and of its interaction with initial per capita income are in both periods significant at 1% or 5%, and their signs imply, as expected, that the relation between undervaluation and growth is positive for countries with low income, but diminishes and eventually becomes negative as the initial income of the country increases (the thresholds are around $11,500 and $5,200 -in 2005USD- for the periods 1970-2010 and 1982-2003 respectively). These results are in line with the predictions of the model and confirm those presented in Rodrik (2008).

Regressions (3 – 6) (panels A & B) in Table 1.3 include the interaction of undervaluation
### Table 1.3: Real Undervaluation and Per Capita Income Growth Rate

**Cross-Section Regressions**

**A - 1970-2010**

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Undervaluation (Avg.)</td>
<td>0.351**</td>
<td>2.985***</td>
<td>0.680**</td>
<td>0.711**</td>
<td>1.733***</td>
<td>0.915**</td>
<td>1.036***</td>
</tr>
<tr>
<td>(0.172)</td>
<td>(1.136)</td>
<td>(0.278)</td>
<td>(0.305)</td>
<td>(0.594)</td>
<td>(0.367)</td>
<td>(0.333)</td>
<td></td>
</tr>
</tbody>
</table>

*Interactions Underval, x:*

- Initial pc Income (Ln)  
  -0.319**  
  (0.128)

- pc Agric. Land (Avg.)  
  -1.267  
  (0.769)

- Initial Yrs. School. (Ln)  
  -0.364*  
  (0.203)

- Homogenization (Index)  
  -2.581**  
  (1.041)

- Initial Urban Pop. (%)  
  -0.014**  
  (0.007)

- Initial Score  
  -1.346***  
  (0.459)

<table>
<thead>
<tr>
<th>[and additional controls]</th>
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</thead>
</table>


R-squared: 0.400 0.427 0.412 0.419 0.432 0.416 0.437

**B - 1982-2003**

<table>
<thead>
<tr>
<th>pc Inc. Growth (Avg.)</th>
<th>(B1)</th>
<th>(B2)</th>
<th>(B3)</th>
<th>(B4)</th>
<th>(B5)</th>
<th>(B6)</th>
<th>(B7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undervaluation (Avg.)</td>
<td>0.026</td>
<td>2.523**</td>
<td>0.354**</td>
<td>0.638*</td>
<td>1.159**</td>
<td>0.594*</td>
<td>0.602**</td>
</tr>
<tr>
<td>(0.120)</td>
<td>(1.005)</td>
<td>(0.171)</td>
<td>(0.384)</td>
<td>(0.482)</td>
<td>(0.325)</td>
<td>(0.240)</td>
<td></td>
</tr>
</tbody>
</table>

*Interactions Underval, x:*

- Initial pc Income (Ln)  
  -0.295***  
  (0.109)

- pc Agric. Land (Avg.)  
  -1.296**  
  (0.499)

- Initial Yrs. School. (Ln)  
  -0.433**  
  (0.208)

- Homogenization (Index)  
  -2.052***  
  (0.754)

- Initial Urban Pop. (%)  
  -0.012**  
  (0.005)

- Initial Score  
  -1.094***  
  (0.304)

<table>
<thead>
<tr>
<th>[and additional controls]</th>
</tr>
</thead>
</table>

Observations: 118 118 118 118 118 118 118

R-squared: 0.292 0.352 0.320 0.337 0.344 0.322 0.352

Robust standard errors in parentheses.  *** p<0.01, ** p<0.05, * p<0.1.  5% of the observations were excluded (the 2.5% with the highest and the 2.5% with the lowest values of undervaluation) to minimize the effect of extreme values.  **Controls:** All regressions include the following set of controls: per capita income (ln, initial year), per capita agricultural land (period average), per capita oil reserves (period average), years of education (ln, initial year), urban population (% , initial year), index of homogenization, share of manufactures in GDP (% , current prices, initial year).  **Score:** Simple average of 5 indicator variables: I(high income), I(high schooling), I(high Agric. land), I(high Homogenization) and I(high Urban Pop.), which take value 1 if the respective variable is above the sample median in the initial year, and 0 otherwise.
with per capita agricultural land, years of education, and urban population. These variables are, in addition to initial income, part of the "initial conditions" that according to the model should potentiate the effect of undervaluation on growth. It can be seen that, again, the sign and significance of the coefficients are in line with the predictions of the model, suggesting that the association between undervaluation and growth is stronger in economies with lower per capita agricultural land, and initially lower levels of education and percentage of urban population. Interestingly, the coefficient of the interaction with agricultural land in regression (A3) (Table 1.3) is not significant. The reason for this might be related to the fact that the period 1970-2010 includes both, the commodities boom of the 1970s, and the more recent one of the 2000s. The changes in international prices in those periods implied an increase in terms of trade for countries richer in natural resources and fall in terms of trade for those relatively poorer, which may weaken and even reverse the relationship between undervaluation, natural resources, and growth. The intuition is confirmed by the fact that the coefficient of the interaction between undervaluation and per capita agricultural land becomes significant at 5% when those periods of high commodity prices are excluded from the sample (regression (B3)).

An important question is whether the interactions in regressions (3) to (6) (panels A & B) in Table 1.3 provide indeed new information, or their coefficients simply reflect the fact that per capita agricultural land, education, and urban population are correlated with initial income (in fact, the $R^2$ in these regressions in both periods is lower than that in regressions (A2) and (B2), which suggests that the explanatory power of these variables, when considered individually, is lower than that of initial per capita income). The answer to this question is given by regressions (A7) and (B7) (Table 1.3), which include the interaction of undervaluation with the score. Despite the simplicity with which the score is calculated, the coefficient of the interaction term is in both periods significant at 1%. Furthermore, both regressions (A7 and B7) produce the highest $R^2$
of their respective period. This suggests that, as predicted by the model, though the individual conditions may help predict the potential effect of a real undervaluation (being per capita income one of the most important), the effect is potentiated when two or more of such conditions hold simultaneously.

In Table 1.4 I present the results of replicating the previous analysis in a sub-sample of developing countries in East and South Asia, Latin America and Caribbean, and S.S. Africa. For each period (1970-2010 and 1982-2003) I present the results of the baseline regression without interaction term (1 and 4), the regression that include the interaction with the score (2 and 5), and a third regression (3 and 6) that includes the interaction with the individual variable (initial condition) that produces the highest $R^2$ (this should be, in principle, the individual condition with highest "traction" in the score).

---

Table 1.4: Real Underval. and Per Capita Income Growth Rate - Dev. Countries
Cross-Section Regressions - Developing Countries in Four Main Regions*

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Undervaluation (avg.)</td>
<td>0.545**</td>
<td>2.894***</td>
</tr>
<tr>
<td></td>
<td>(0.239)</td>
<td>(0.888)</td>
</tr>
<tr>
<td>Interactions Underval. x:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Index Homogenization</td>
<td>-4.932***</td>
<td>(1.791)</td>
</tr>
<tr>
<td></td>
<td>(0.861)</td>
<td></td>
</tr>
<tr>
<td>Initial Score</td>
<td>-1.911**</td>
<td>(0.861)</td>
</tr>
<tr>
<td>Observation</td>
<td>73</td>
<td>73</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.470</td>
<td>0.529</td>
</tr>
</tbody>
</table>

(*) Countries in East Asia, Latin America & Caribbean, South Asia, and S.S. Africa with per capita income lower than 50% that of the U.S. in the initial year. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. 5% of the observations were excluded (the 2.5% with the highest and the 2.5% with the lowest values of undervaluation) to minimize the effect of extreme values. Controls: All regressions include the following set of controls: per capita income (ln, initial year), per capita agricultural land (period average), per capita oil reserves (period average), years of education (ln, initial year), urban population (% initial year), index of homogenization, share of manufactures in GDP (% current prices, initial year). Score: Simple average of 5 indicator variables: I(high income), I(high schooling), I(high Agric. land), I(high Homogenization) and I(high Urban Pop.), which take value 1 if the respective variable is above the sample median in the initial year, and 0 otherwise.
Despite the small size of the sample (73 and 72 observations respectively), the performance of the score as a predictor of the potential effect of undervaluation remains strong. As expected, the explanatory power of initial per capita income diminishes among a group of initially low-income countries. The coefficient of the interaction of initial per capita income with the measure of undervaluation (not reported in the table) is only significant at 10% for the period 1970-2010, and not significant for 1982-2003. Instead, most of the traction for the score is given in this case by the average index of homogeneity (columns (2) and (5) in Table 1.4), implying that the association between real undervaluation and growth is particularly strong in low-income countries with high degree of ethnic, linguistic and religious fractionalization, which, as mentioned above, could be associated with higher migration or entry costs for workers into manufacturing.

**Undervaluation and the Composition of Tradable Production**

In this section, I test some of the prediction of the model regarding the composition of tradable production. In particular, the model predicts that in countries with appropriate initial conditions, a real undervaluation should lead to an increase in the production of manufactures and a (relative) fall in the production of agricultural and other natural resources-based goods. In order to test this particular prediction I replicate the previous regressions but use instead the share of manufacturing in total tradable production at the end of each period as the dependent variable. There are at least two reasons why this variable is a good indicator of the degree of development of the manufacturing sector. First, there is no "right" or expected value for every country, given that the share can vary between 0 and 1 depending on the particular conditions and productive structure of the economy. For example, the share of manufacturing will probably be lower in countries with high endowment of natural resources -say, oil- whose tradable production will be mostly composed of commodities. The second important reason for using this variable is that it is not directly affected by changes
Table 1.5: Real Undervaluation and Manufacturing Production  
Dep. Variable: Share of Manufacturing in Total Tradable Production

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<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Undervaluation (Avg.)</td>
<td>0.074</td>
<td>0.264</td>
<td>0.076</td>
<td>0.081</td>
<td>0.216</td>
<td>0.159**</td>
</tr>
<tr>
<td></td>
<td>(0.056)</td>
<td>(0.352)</td>
<td>(0.080)</td>
<td>(0.056)</td>
<td>(0.135)</td>
<td>(0.077)</td>
</tr>
<tr>
<td>Interactions Undervaluation x:</td>
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<tr>
<td>Initial pc Income (Ln)</td>
<td>-0.024</td>
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<tr>
<td></td>
<td>(0.045)</td>
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<tr>
<td>pc Agricultural Land (Avg.)</td>
<td>-0.005</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(0.212)</td>
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<tr>
<td>Initial Yrs. Schooling (Ln)</td>
<td>-0.008</td>
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<tr>
<td></td>
<td>(0.056)</td>
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<tr>
<td>Homogenization (Index)</td>
<td></td>
<td></td>
<td>-0.270</td>
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<tr>
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<td></td>
<td>(0.261)</td>
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<tr>
<td>Initial Score</td>
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<td>-0.180</td>
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<td>(0.133)</td>
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<tr>
<td>[and additional controls]</td>
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<tr>
<td>Observations</td>
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<td>115</td>
<td>115</td>
<td>115</td>
<td>115</td>
<td>115</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.683</td>
<td>0.684</td>
<td>0.683</td>
<td>0.683</td>
<td>0.685</td>
<td>0.687</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>B - 1982-2003</th>
<th>(B1)</th>
<th>(B2)</th>
<th>(B3)</th>
<th>(B4)</th>
<th>(B5)</th>
<th>(B6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undervaluation (avg. of the period)</td>
<td>0.025</td>
<td>0.303</td>
<td>0.106</td>
<td>0.129*</td>
<td>0.267**</td>
<td>0.157**</td>
</tr>
<tr>
<td></td>
<td>(0.041)</td>
<td>(0.199)</td>
<td>(0.075)</td>
<td>(0.068)</td>
<td>(0.109)</td>
<td>(0.060)</td>
</tr>
<tr>
<td>Interactions Undervaluation x:</td>
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<td></td>
</tr>
<tr>
<td>Initial pc Income (Ln)</td>
<td>-0.033</td>
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<tr>
<td></td>
<td>(0.024)</td>
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<td></td>
</tr>
<tr>
<td>pc Agricultural Land (Avg.)</td>
<td>-0.315</td>
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<tr>
<td></td>
<td>(0.241)</td>
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<tr>
<td>Initial Yrs. Schooling (Ln)</td>
<td>-0.076</td>
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<td>0.769</td>
<td>0.768</td>
<td>0.771</td>
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Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. 5% of the observations were excluded (the 2.5% with the highest and the 2.5% with the lowest values of undervaluation) to minimize the effect of extreme values. Controls: All regressions include the following set of controls: per capita income (ln, initial year), per capita agricultural land (period average), per capita oil reserves (period average), years of education (ln, initial year), urban population (% initial year), index of homogenization, share of manuf. in tradable prod. (level & square, initial year). Score: Simple average of 5 indicator variables: I(high income), I(high schooling), I(high Agric. land), I(high Homogenization) and I(high Urban Pop.), which take value 1 if the respective variable is above the sample median in the initial year, and 0 otherwise.
in the real exchange rate. That is, since both the numerator (i.e. the value of manufacturing production) and the denominator (i.e. the value of total tradable production) are not directly determined by the price of nontradables, changes in the in the relative price of nontradables should not have a direct effect on the ratio\textsuperscript{29}.

Table 1.5 summarizes the results of the regressions that assess the relation between undervaluation and the share of manufacturing in total tradable production. In addition to the measure of undervaluation, all the regressions in the table include the following set of controls: initial per capita income (ln), per capita agricultural land (avg.), per capita oil reserves (avg.), initial years of education (ln), initial urban population (%), the average index of homogenization, and the level and square of the initial share of manufacturing in total tradable production (%). Regressions (A1) and (B1) present the baseline regression with no interaction term, and show that in both periods the relation between real undervaluation and relative production of manufactures is positive but not significant. The coefficients also have the expected sign in regressions (A2 – A5) and (B2 – B5) (which include the interaction of undervaluation with the initial conditions) but they only become significant in the period 1982-2003 and when interacting with initial years of schooling and with the index of homogenization. This implies that the positive association between undervaluation and the relative expansion of manufactures in total tradable production is particularly strong and higher in countries with initially lower levels of education and higher fractionalization.

Finally, regressions (A6) and (B6) include the interaction with the score which is computed in this case as the simple average of four indicator variables: $I(\text{High Income})$, $I(\text{High Agric. Land})$, $I(\text{High Schooling})$ and $I(\text{High Homogenization})$ in the first period of the respective period. In both periods the coefficients become significant and the $R^2$ reaches its maximum, proving again that (in line with the predictions of the model) the association between undervaluation

\textsuperscript{29}This is different in the case of the ratio of manufacturing production to GDP, given that a real depreciation (i.e. a fall in the price of nontradables) will mechanically deflate the value of GDP or inflate the value of manufactures, and therefore increase the ratio.
Table 1.6: Real Undervaluation and Manufacturing Production – Dev. Countries*

Dep. Variable: Share of Manufacturing in Total Tradable Production

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<tr>
<th></th>
<th>(A1)</th>
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<th>(A3)</th>
<th>(A4)</th>
<th>(A5)</th>
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<td>(0.071)</td>
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B - 1982-2003

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<td>0.644</td>
<td>0.654</td>
<td>0.645</td>
<td>0.658</td>
<td>0.667</td>
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</table>

(*) Countries in East Asia, Latin America & Caribbean, South Asia, and S.S. Africa with per capita income lower than 50% that of the U.S. in the initial year. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. 5% of the observations were excluded (the 2.5% with the highest and the 2.5% with the lowest values of undervaluation) to minimize the effect of extreme values. Controls: All regressions include the following set of controls: per capita income (ln, initial year), per capita agricultural land (period average), per capita oil reserves (period average), years of education (ln, initial year), urban population (% initial year), index of homogenization, share of manuf. in tradable prod. (level & square, initial year). Score: Simple average of 3 indicator variables: I(high schooling), I(high Agric. land), and I(high Homogenization, which take) value 1 if the respective variable is above the sample median in the initial year, and 0 otherwise.
and manufacturing development is better assessed when considering a set of initial conditions simultaneously. The results are confirmed when the analysis is performed only with the sub-sample of developing countries, whose results are reported in Table 1.6.

1.4.3 Identification of Real Undervaluations Induced by Government Interventions

One of the most critical issues in almost every analysis that involves the use of the real exchange rate is the fact that, being an endogenous variable, it is difficult to determine whether the observed variations respond to truly "exogenous" government interventions, or simply represent an "endogenous" response to changes in "fundamentals" (such as changes in sectoral productivities or in the terms of trade). The traditional approach to addressing this issue consists in computing deviations of the RER from what is defined as the “equilibrium real exchange rate”\textsuperscript{30}, which depends critically on the adopted definition of RER equilibrium, and on the availability of the relevant data (comparable across countries)\textsuperscript{31}.

I use a different approach here to determine the possible cause behind the observed changes in the RER, which consists in analyzing how the (relative) size of the tradable sector reacts to such movements in the RER. Specifically, the ratio of tradable production to GDP can be expressed as:

\[
\frac{\text{Trad}_{\text{GDP}}}{p_T T} = \frac{p_T T}{p_T T + p_N N T} = \frac{T}{T + \frac{p_N N T}{p_T}} \tag{1.5}
\]

\textsuperscript{30} As explained in IMF (2006) (the 2006 report on the revised and extended methodologies for exchange rate assessments used by the IMF’s Consultative Group on Exchange Rate Issues), there are at least three complementary but different methodologies for the computation of the "equilibrium" RER: the “macroeconomic balance” approach, a reduced form “equilibrium real exchange rate” approach, and an “external sustainability” approach.

\textsuperscript{31} In addition to the lack of data on variables that could determine the equilibrium RER, there are also problems with the comparability across countries of variables that are expressed as indices normalized to a specific year (the typical examples are RER and Terms of Trade indices).
where $p_T$ and $T$ are the price and production indices of tradable goods, and $p_N$ and $NT$ the price and production of nontradables, respectively. According to the model, an endogenous real depreciation (that is, a fall in $\frac{p_N}{p_T}$ originated in changes in parameters of the model) will take place if:

1. the productivity of the tradable sector decreases, in which case $T$ decreases;
2. the productivity of the nontradable sector increases, in which case $NT$ increases;
3. the terms of trade decrease, so that the value of total tradable production ($p_T T$) falls with respect to the value of nontradables ($p_N NT$).

In all these cases the real depreciation will be accompanied by a fall in the ratio of tradable production to GDP. The opposite is also true: either an increase in the productivity of tradables, a fall in the productivity of nontradables, or an improvement in the terms of trade, will induce an endogenous real appreciation and an increase in the share of tradables in GDP.

On the other hand, from expression (1.5) it follows that a real depreciation (appreciation) that does not respond to a change in sectoral productivities or in the terms of trade will be accompanied by a fall (increase) of the term $\frac{p_N}{p_T} NT$ relative to $T$, and therefore an increase (fall) in the ratio of tradable production to GDP.

The four possible cases are summarized in Figure 1.11. This particular prediction of the model allows to better understand to what extent a change in the RER might be associated with an "exogenous" government intervention. As will be seen below, the exercise can be easily performed using only one extra variable (the share of tradable production on GDP, which is perfectly comparable across countries) and has a very intuitive and informative graphic representation.

In the following section I first construct a measure of relative size of the tradable sector that is comparable across countries, and then analyze its interaction with the measure of real
undervaluation in order to determine to what extent those real depreciations are explained by government interventions.

A Measure of "Oversize" of Tradable Production in GDP

I first compute the ratio of tradable production to GDP at current prices using data on gross value added by economic activity (ISIC Rev. 3) at current prices - US dollars from the UN Statistic Division. In particular, I calculate the ratio as the sum of the shares GDP that correspond to Agriculture, Hunting, Forestry and Fishing (categories A-B), and Industry (categories C-E, which includes Mining, Manufacturing, and Utilities)\(^{32}\). In a second step, and in order to account for differences in the ratio of tradable production to GDP related to differences in income levels, I regress the ratio on the logarithm of per capita income and a set of year fixed effects. The residuals from this regression constitute my measure of "oversize" of the tradable sector, and represent to what extent the relative size of the tradable sector of a country is larger than what its level of per capita income justifies.

\(^{32}\)The measure might not be exact due to the fact that, though utilities (water, power, gas, etc.) are in principle tradables, in most cases they are not in practice. As a verification of the potential impact that the inclusion of utilities might have on my measure of tradable production, I used again the sectoral data from Timmer & de Vries (2009) and McMillan & Rodrik (2011) and verified that in average utilities represent only 2% of GDP, with small variations across countries.
capita GDP would predict. The details about the construction of this variable are presented in Appendix A1.

Identifying the Exogenous Undervaluations

In Figure 1.12 I present scatter plots of the measures of oversize of the tradable sector and

Figure 1.12: Undervaluation & Oversize of Tradable Sector
Four Main Regions - Developing Countries

Note: Only countries with population larger than 1 Million and per capita GDP (PPP) lower than 50% that of the U.S. through each period are considered.
Source: Author’s calculations based on data from PWT 7.1 and UNSD.

all the developing countries with populations larger than 1 million in East Asia, Latin America and Caribbean, South Asia, and S.S. Africa. In each period, countries are represented by points whose coordinates are the average of their measures of undervaluation (horizontal) and of oversize of tradable sector (vertical) in the respective period. The figure, then, allows us to graphically identify the countries that, in each period, correspond to the four cases described in figure 1.11.

A first observation that can be made from the figure is that there is significant dispersion in terms of both, undervaluation and oversize of the tradable sector. The period between 1980 and 1989 exhibits the lowest dispersion in terms of oversize of the tradable sector, and the largest dispersion of undervaluation. During those years, while some countries in S.S. Africa (in particular Tanzania, in the SW) and East Asia (Cambodia, Mongolia) experienced large real appreciations, other countries in South Asia (Afghanistan, in the NE) and Latin America (mainly Central American and Caribbean countries, but also Mexico and, for some years Brazil, Peru and Venezuela) experienced important depreciations. The years between 2000 and 2010, on the other hand, exhibit the lowest dispersion in real undervaluation and, coinciding with the boom in commodities prices, the largest dispersion in terms of oversize of tradable sector (being Angola and Rep. of Congo -in the NW- the countries with the most oversized tradable sectors and large endogenous overvaluations, and Eritrea -in the SE- the one with the most undersized tradable sector and a large endogenous undervaluation).

Figure 1.12 allows also to analyze the patterns the different regions have followed over time. A common feature of most countries in the two Asian regions is that their tradable sectors remain relatively oversized during the four periods (the main exceptions are Hong Kong and Singapore in the 1970s and Mongolia in the 1980s). This pattern was combined during the first 20 years (1970-1989) with high dispersion in terms of undervaluation: while Thailand, Korea, Vietnam, Philippines

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33 A country is defined as developing country in a particular year if its per capita income (PPP USD2005) is lower than 50% that of the U.S. in the same year.
and Indonesia remained relatively undervalued for several years, other countries including China were actually overvalued. Beginning in the 1990s however, most Asian countries moved to positions of relative undervaluation. In fact, except for Korea, Indonesia and Papua New Guinea that for some years in the 1990s experienced slight "endogenous" overvaluations, all the East Asian countries were positioned in the NE quadrant between 1990 and 2010, experiencing exogenous (or induced) real undervaluations. As it was seen in Section 1.2, many of these Asian countries have the conditions that, according to the model, make relatively beneficial the use of policies that induce a real depreciation. Interestingly, except for Afghanistan and India for a few years, the South Asian countries remain in SE quadrant of the figure, which suggests that the real undervaluation experienced by those countries is, at least in part, an endogenous response conditions related to the fundamentals of those economies.

An important observation can be made regarding the African countries. The figure shows that between 1970 and 2000 most of the countries in the S.S. African region were relatively overvalued, and in many cases exogenously (SW quadrant). Only in the last decade we observe many of the African nations shifting to the right, attaining levels of undervaluation closer to zero. Compared to other regions, however, most countries in the regions still remained overvalued. These findings are in line with the evidence on the role of foreign aid on the appreciation of the real exchange rate in Africa presented in, for example, van Wijnbergen (1985) and Rajan & Subramanian (2011).

Finally, the countries in Latin America and Caribbean display large dispersion in terms of undervaluation and do not show a consistent pattern over the four periods. The tradable sector in most of the region remained relatively undersized during the 1990s and part of the 2000s, a period characterized by low commodities prices. Following the increase in commodities prices, the trend

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34 This implies that the reasons behind South Asia’s undervaluations may be related, for example, to low levels of productivity in the production of tradables -as verified in Figure 1.3 in the case of India-, and the fact that the recent commodity boom represents for them a fall in their terms of trade. This would also explain why the relationship between undervaluation and growth in this region seems to be weaker than in East Asia (Figure 1.1).
has reverted, in several countries in recent years (Venezuela, Chile, Argentina and Bolivia among others), moving Venezuela and Chile (two of the most important commodities producers of the region) to a position of endogenous appreciation (NW quadrant).

1.5 Conclusions

In this paper I ask whether it is true that a “competitive real exchange rate” is behind China’s success, something that the media and many policy makers sustain but that is not supported by the most prominent economic models. I develop a model that uses insights from the “unlimited supply of labor” and “de-industrialization” literatures (Lewis (1954), Corden & Neary (1982)) to gain a better understanding of these issues. I perform an extensive empirical analysis with the purpose of understanding the key underlying conditions that make China and other Asian economies suitable for policies aimed at depreciating the real exchange rate. I then compare the case of Asia with those of Latin America and South Saharan Africa. I propose a 3-sector model with labor market frictions that explains how a policy that depreciates the real exchange rate (by, for example, increasing domestic savings) can, at the same time, generate real growth through a reallocation of workers from a low-productivity traditional economy into a high-productivity manufacturing sector. The policy is particularly effective in countries with initially low industrial development, relative abundance of labor and scarcity of agricultural resources, and high barriers for the entry of workers into the manufacturing sector. This is precisely the case of many Asian economies. I verify empirically that the association between real undervaluation and growth, and real undervaluation and development of the manufacturing sector is stronger in countries with these conditions. Finally, I propose an empirical strategy for the identification of real depreciations induced by government interventions, and verify that they have mainly been observed in the last two decades in the East Asian economies.

An extremely simplified model was presented here with the purpose of explaining the
proposed mechanism and highlighting the role played by each of its key elements. Future research should explore the quantitative implications of the model in a dynamic environment, introducing capital and endogenous sources of growth associated with the expansion of the industrial sector. Interesting implications regarding the timing of the transition and the pattern of structural transformation and income distribution can result from variations in the instrumentation of the government interventions or the way frictions are modeled.

Despite its simplicity, the theoretical framework developed here contributes to better understand the mechanics of a real undervaluation, its potential effects on the labor market and, more generally, its link with real growth. The theory helps to understand the role played by the recently documented sectoral productivity gaps (Bosworth & Collins (2008), McMillan & Rodrik (2011), Gollin et al. (2011), and de Vries et al. (2012)), and provides an explanation for its puzzling persistence in some developing countries. In the same line, the model provides an explanation for the different patterns of sectoral reallocation of labor observed in Asia, Africa and Latin America, as recently documented by McMillan & Rodrik (2011) and de de Vries et al. (2012).

Before closing, two important comments on some of the limitations of the empirical analysis are in order. First, as is the case with most studies that use cross sectional data, the analysis does not imply causality but simply verifies the existence of the proposed empirical regularities. And second and most importantly, the endogeneity of the decision regarding the use of the RER as a policy instrument arises a clear identification issue that plagues this entire empirical literature. Addressing this issue is one of the greatest challenges of the literature. In spite of this, the results presented here provide useful information for understanding how the association between undervaluation and both, growth and the composition of tradable production, depends on the initial conditions.
A Measure of "Oversize" of the Tradable Sector

Figure 1.13: Tradable Production (% GDP) & Pc Income 1970-2010

Figure 1.13 presents a scatter plot of the share of tradable production in GDP and per capita income for the periods 1970-2010. As can be seen from the figure, there is a clear negative relationship between both variables. Explaining the reasons behind that negative relation would require digging into the literature on structural transformation, and is not the matter of this work (that might be explained, for example, by non-homotheticity of preferences). What I do indeed is, taking that negative relationship as a fact, I construct a measure that informs about how much countries deviate from that particular trend at each moment in time. That measure of "oversize" of the tradable sector is estimated in the same way as the measure of overvaluation: First, I regress the share of tradables in GDP on the logarithm of per capita income and a set of year fixed effects, and, second, I compute the residuals from that regression, which constitute my measure of "oversize" of the tradable sector. Specifically, the regression has the following form:
$Trad_{-GDP_{it}} = \alpha + \beta \ln(pcIncome_{it}) + I_t + \varepsilon_{it}$

where the subindices $i$ and $t$ refer to country and year respectively, $I_t$ are year fixed-effects, and $\varepsilon_{it}$ is a standard error term. Again, I run the regression for the period 1970-2010 and exclude Serbia and Georgia. The estimated coefficient of (ln) per capita income is $-6.5$, with a $t$-statistic (based on robust standard errors) higher than 49. Since per capita income is in logarithmic scale, one way of interpreting of the result is that, in average, the share of tradable production in total GDP falls by 4.5 percentage points when per capita income doubles. The distribution of the residuals of the regression (i.e. my measure of "oversize" of the tradable sector) is displayed in Figure 1.14. We can see that the distribution is centered at zero and, except for a few cases above 30%, most of the observations lie in the range $[-30\%, 30\%]$.
Chapter 2

Technological Change and Household Structure

2.1 Introduction

Technological change affects our life in many ways. In addition to its general positive effect on labor productivity and real wages, technological progress has also increased the availability in the market of both, more elaborated -ready to consume- goods and services, and domestic appliances -durable household goods- that facilitate home production. How have these three dimensions of technological progress affected the structure of the household and its role as a unit of production? Furthermore, can differences in the access to these technologies explain the differences in household structure currently observed across countries? While some works have analyzed the effects of the increase in real wages (on, for example, the allocation of time between market and household activities or on marriage decisions), no attempts have been made in the literature to incorporate all three dimensions of technological change to the analysis. Existing models that, following Barten (1964), do not consider home production\(^1\) and rely on the simple separation between private and public household goods, generate some theoretical predictions about scale economies that, as first proved by Deaton & Paxson (1998), are questionable in light of the empirical evidence\(^2\). On the other hand, models that introduce home production into the analysis usually focus on the features or determinants of one specific type of living arrangement (e.g. marriage, single-adult households, etc.)\(^3\). Finally, to the best of my knowledge, none of the existing works has incorporated to the analysis the fact that market goods may differ in their degree of elaboration, something that has

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\(^1\)See, for example, Muellbauer (1977), Pollak & Wales (1980), Pollak & Wales (1981), Deaton & Muellbauer (1980), and Nelson (1988).

\(^2\)See Logan (2011) for an explanation of the Deaton & Paxson (1998) puzzle, and for details on the literature that followed.

\(^3\)See, for example, Greenwood & Guner (2009).
important implications, particularly in the context of the household.

This paper presents a general theoretical framework that allows to analyze how individual’s decisions about household structure, the allocation of time between market and home work, the use of domestic appliances for home production, and the use of market goods with different degree of elaboration respond to three key aspects of technological progress, namely, the increase in 1) market real wages, 2) the availability of more productive household capital goods -i.e. domestic appliances, and 3) the availability of more elaborated goods and services. In the model, individuals provide themselves with final consumption goods through a production process that combines a market good (as an input) and homework. The market good used as an input can have different degrees of elaboration, which determines the amount of homework necessary to transform the input into final consumption good (for example, individuals can add a large amount of homework to market goods with little elaboration, or may use highly elaborated market goods that require little additional homework in order to be consumed). While market goods are elaborated by firms, homework is performed by individuals in the context of the household combining labor and domestic appliances. In the analysis, a household is defined as a group of individuals (adults) that share the production and consumption of final goods. Two key assumptions are made related to the formation for households. First, there are economies of scale in homework production; and, second, though individuals may enjoy consuming with other adults in the household, their marginal utility of consumption becomes decreasing in the household size once it surpasses a certain level. Thus, while the first assumption provides the individuals with the incentives to join other adults in a household for the production of final consumption goods (especially when the required amount of homework is large), the second assumption prevents the size of the household to increase indefinitely.

According to the model, the three dimensions of technological progress induce, with different intensity, a reduction in the number of individuals per household. The effects on the degree
of elaboration of purchased market goods and on the share of time devoted to market work can be completely different, though. While the increase in real wages makes individuals more likely to use more elaborated market goods that require less homework to be consumed, the decline in the price of domestic goods increases the likelihood of using less elaborated goods that require more homework. In both cases, however, the fraction of time devoted to market work will initially remain constant or decline. It is just when the household size is close enough to its minimum that the share of time in market work will increase. Finally, the decline in the price of market goods with high degree of elaboration induces, simultaneously, a fall in the household size, an increase in the use of market goods that require less homework to be consumed, and an increase in the fraction of time in market work.

The paper is related to at least two bodies of literature. First, the work is part of the literature that studies the economy of the household. Much of the work in this area has been devoted to explaining and verifying empirically the existence of household economies of scale (Barten (1964), Muellbauer (1977), Pollak & Wales (1980), Pollak & Wales (1981), Deaton & Muellbauer (1980), and Nelson (1988), Deaton & Paxson (1998), Logan (2011)). An important prediction of these models (mostly based on Barten (1964)’s model) is that larger households with similar per capita income should spend a larger fraction of income in private goods, a regularity that, as shown by Deaton & Paxson (1998) and more recently confirmed by Logan (2011), is not fully supported by the empirical evidence. Very few other papers in this field, on the other hand, have dealt with the determinants of the existence and structure of the household. Beyond the production and rearing of children (Becker (1981)), the endogenous formation of households has been justified, more recently, by the existence of household public goods (Salcedo et al. (2012)) and by the presence of economies of scale in household production (Greenwood & Guner (2009)). In this line, the works of Salcedo et al. (2012) and Greenwood & Guner (2009) are the ones closest in spirit
with this paper. While Salcedo et al. (2012) explain changes in living arrangements (mainly in the U.S.) only as a function of changes in per capita income in a model with private and household public goods (which, following Barten (1964), has the same empirical issues mentioned above), Greenwood & Guner (2009) explain changes in marriage and divorce decisions based on changes in real wages and in the productivity/price of household goods (they do not distinguish between household capital goods and other inputs). This paper, on the other hand, provides a more general theoretical framework that, in addition to endogenizing the formation of households, introduces the degree of elaboration of household’s inputs as a new margin of decision which, besides enriching the framework, contributes to explaining the empirical findings of Deaton & Paxson (1998) and Logan (2011).

The paper also contributes to the development literature that studies the role of the extended household and, more generally, of the social network in low-income environments. As Cox & Fafchamps (2007) point out in their review of this literature, most recent works in this field have focused on the role of the social network as a "risk-sharing network" in the absence of a formal insurance (Rosenzweig (1988), Townsend (1994), Ligon et al. (2002), Fafchamps & Lund (2003) among others). This literature has shown the existence and relevance of informal risk-sharing networks among the rural poor but, at the same time, has consistently rejected efficient income risk sharing. Furthermore, in their analysis of interpersonal networks in Philippine rural villages, Fafchamps & Gubert (2007) conclude that interpersonal networks "are largely determined by [social and geographical] proximity, and are only weakly the result of purposeful diversification of income risk"\(^4\). Besides risk sharing, the literature has documented the role of kinship networks in other (informal) activities such as labor pooling, child care and fostering, insurance or protection against external events, protection of productive assets, or the joint provision of "public" goods and

\[^4\text{Fafchamps & Gubert (2007), p.327.}\]
services⁵, which provide additional motives for the formation of the structure. Finally, while most of the literature has focused on the "benefits" associated with the interaction with the extended household, several papers have recently pointed out at some of the costs that individuals face in the network, such as the obligation to make transfers to relatives and neighbors, or the existence of harsh social sanctions for those that do not make sufficient transfers to others (Platteau (2000), Di Falco & Bulte (2011), Jakiela & Ozier (2012)). The existence of such (behavior-distorting) costs highlights the need for a better understanding of the determinants of individuals' exposure to (and reliance on) the extended household. This paper contributes to this field in at least two ways. First, it provides a theoretical framework that identifies the conditions under which individuals and households voluntarily join others for the joint provision of (one or more) goods and services (whose production is, for example, subject to economies of scale). And, second, the model highlights the potential role of the government in the determination of such conditions (particularly in poor, rural areas) through, for example, improving the provision of certain public goods and services, or the access to markets or new technologies.

The paper is organized as follows. Section 2.2 presents empirical evidence that motivates the main assumptions of the model. Section 2.3 presents the model and analyzes the consequences of different types of technological progress. Section 2.4 concludes.

2.2 Empirical Evidence

This section presents empirical evidence about the relationship between household structure, different aspects of technological progress (i.e. the increase in market real wages, the availability and productivity of household capital goods, the access to more elaborated goods and services), and the allocation of time between home and market work. I begin with evidence from the United States.

⁵See Cox & Fafchamps (2007) for a review of this literature.
from the year 1900 to 2010, and then extend the analysis to a cross section of countries for which there is available data.

2.2.1 Technological Change and Household Structure in the United States: 1900-2010

Household Structural Change. Figure 2.1 displays the evolution of the size of the household in the United States between 1900 and 2010. As the figure shows, the average household in the U.S. had in 2000 two members less than it had in 1900. Most of the change took place between 1900 and 1980, when the size of the household fell from 4.6 to 2.75. The rate of change, though still negative, diminished significantly since the 1980s, being almost null during the first decade of the new century.

What are the direct causes behind the reduction in the size of the household observed during much of the twentieth century? The first and most obvious reason is the well documented
decline in fertility that took place during the period, and that is seen in the fall in the number of members age 14 or less per household, from 1.6 in 1900 to 0.6 in the middle 1980s. This reduction in the number of children per household, though impressive (the number declined by more than 60%), only explains 50% of the observed change in the total size of the household.

The second reason behind the decline in the size of the household (which explains the other 50% of its change) is, as shown in the figure, the decline in the number of adults and young-adults (i.e. members age 15+) per household, which fell from 3.01 in 1900 to around 2.1 in the middle 1980s. This fact is in line with the evidence presented by Salcedo et al. (2012), who document a fall in the number of adults (18 years and above) per household for the average American from 3.21 to 2.24 between 1850 and 2000 (with most of the decline -from 3.10 to 2.26- taking place between 1910 and 1970), and is confirmed by the spectacular increase in the share of single-person households, from 5% to 25%, registered in last century (displayed also in Figure 2.1). There is a key difference, however, between these two direct drivers of the change in the household size (namely, the decline in the number of children and the decline in the number of adults) that should not be overlooked: The fact that adults have a much more active role than children in both market and household production implies that, while the decline in fertility (which has been traditionally related to the idea of demographic transition and the trade off between the quantity and quality of children) does not imply a fundamental change in the structure of the household, the fall in the number of adults per household represents a change in its productive structure, which has major economic implications. Interestingly, while a vast body of literature has been devoted to study the decline in fertility, little attention has been given to the household structural change associated with the

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6 *Average American* in opposition to *average household*. The average number of persons per household is computed across individuals instead of households, which gives a relatively higher weight to larger households.

7 Indeed, most studies on household size focus on fertility and take as given the structure of the family. In this line are Galor & Weil (1996), Boldrin & Jones (2002), Galor & Weil (2000), Doepke (2004), Boldrin et al. (2005), Greenwood et al. (2005a) and Jones et al. (2010), among others.
reduction in the number of adults\footnote{In recent years some works have studied specific living arrangements, such as the decision of elderly persons of living alone (Costa (1998), Costa (1999), and Bethencourt & Rios-Rull (2009)), or the determinants of young adults living alone (Ermisch & Di Salvo (1997), Regalia & Rios-Rull (2001), and Cobb-Clark (2008)).}. The focus of this paper is, precisely, on the determinants of adults’ incentives for joining (or remaining with) others in a household for the joint production and consumption of goods and services.

**Figure 2.2: Selected Indicators - United States, 1900-2010**

![Graphs showing trends in real wage, per capita income, housing facilities, and durable goods](chart.png)

**Technological Progress** One of the most clear and well documented manifestations of technological progress is the increase in workers’ real wages, which is displayed in Figure 2.2 (A). As the figure shows, American production workers’ hourly compensation increased about 7 times faster than the Consumers Price Index between 1900 and 2010, implying that the (per unit) purchasing...
power of market work in terms of market goods and services increased, in average, by the same magnitude. Most of this increment took place in the period 1900-1980, with annual growth rates of 2.1% and 2.5% in the years 1900-1930 and 1930-1980 respectively. After 1980, on the other hand, though still positive, the pace of growth was remarkably low, with annual growth rates of less than 0.1% and 0.8% in the years 1980-2000 and 2000-2010 respectively.

Technological progress has also greatly improved labor productivity in the household. By definition, the evolution of the real wage captures, in the long run, the average effect of technological progress on labor productivity in the market with respect to a basket of (market-produced) good and services. There are, however, important differences in the way technological change affects the access of individuals to (and the relative price of) the different types of goods and services that conform such a consumption basket. Of particular relevance in the context analyzed here is the extent to which technology has increased the availability (and lowered the relative price) of those goods and services that directly affect the productivity of individuals in the household, such as certain public services and housing facilities (electricity, running water, garbage disposal, security, etc.), as well as household appliances and durable goods (car, refrigerator, washer, dryer, TV, etc.).

Figure 2.2 (B) allows to see how the availability of some key public services, housing facilities, and domestic appliances changed during the last century in the U.S. For each of the series, three stages can be distinguished in the figure. During the years between 1900 and the 1920s the availability of basic services such as running water and electricity increased from around 0 – 20% to almost 50% of the population. During the 30 years that followed, the availability of those services was extended, at decreasing rates, to practically the rest of the population. With some years of delay,
the adoption of the refrigerator by households in the U.S. followed a similar pattern. Between 1920 and the middle 1930s the use of the refrigerator increased exponentially from around 0% to 50% of the population, expanding at the decreasing rates to the rest of the population in the following 30 years. The introduction and expansion of most basic services and domestic appliances by the American households is well documented by Greenwood et al. (2005a), who shows that most housing basic facilities were available by 1960 for almost 90% of American households. These authors document that between 1900 and 1990 the ratio of investment in appliances to GDP increased from 0.15% to almost 0.5%, which raised the ratio of stock of appliances to GDP to almost 2.5%. Figure 2.2 (B) also shows the increase in the availability of motor vehicles in the U.S., though more extended in time, followed a similar trend: an initial period in which vehicles were available only for a small fraction of the population (until the middle 1910s), followed by a period of exponential growth (between middle 1910’s until 1980), and a final period after 1980 in which the ratio tends to stabilize at about 800-850 vehicles per 1,000 people.

A third dimension of technological progress that affects the way households produce and consume is the introduction of more elaborated market goods and services, as well as the improvement in the access to markets where they can be acquired. Unlike the two aspects of technological change previously analyzed (which improve the labor productivity in the market and in the household, respectively), this type of technological innovation simple reduce the amount of homework that households have to perform in order to produce the goods and services they finally consume. For example, while the fall in the price (and the increase in the productivity) of sewing machines increases households' productivity in the production of clothing at home, the fall in the price of ready-to-use clothing (or an improvement in the access to markets where individuals can purchase them) reduces the amount of homework to be performed by individuals, and may even eliminate the need for a sewing machine. Interestingly (and most probably due to the lack of data), this
dimension of technological progress has received little attention in the literature. An exception, however, is the case of food, which has received special attention in the economics and public health literature in recent years. Two of the most important changes that have reduced the time American households devote to food preparation have to do with the use of more processed, ready-to-consume food at home (such as frozen or canned meals)\(^\text{11}\), and with the increase in consumption of food away from home. As Figure 2.2 (C) shows, the share of food expenditure that households devoted to food consumed away from home increased from 15% in 1930 to almost 40% in 1980 (with of the expansion taking place between 1960 and 1980), and continued growing to almost 50% in 2010. Christian & Rashad (2009) document that, despite the fact that the real price of food consumed away from home presents a non-monotonic trend between 1950 and 2007 (increasing until the end of the 1970s, and decreasing in the following years), the share of daily caloric intake from food away from home increased from 18% to 32% between the late 1970s and mid-1990s\(^\text{12}\), which, according to the authors, would be explained by the steady increase in the availability of restaurants and the reduced time costs associated with eating out\(^\text{13}\).

**Time Allocation**  In line with the changes in the structure of the household, and in the different dimensions of technological progress, the way Americans allocate their time between market and home work has also changed. Figure 2.2 (D) presents the evolution of the labor force participation and of total employment, both as a percentage of the working-age population, for the period under analysis. Both indicators express, with some limitations, the way the adult population allocates its

\(^{11}\)For example, the share total groceries expenditures devoted to processed foods increased from 11.6% to 22.9% between 1982 and 2012 (Bureau of Labor Statistics, in Vo (2008)).

\(^{12}\)Data from Stewart et al. (2006).

\(^{13}\)The number of restaurants per 10,000 population in the U.S. increased from 8.8 in 1972, to 14.3 in 1997 (Rashad et al. (2006)), to about 32 in 2013 (based on data from National Restaurant Association [2013 Restaurant Industry - Pocket Factbook]).

Chou et al. (2004) also show that the combination of both, changes in prices as well as the increase in availability of fast food and full-service restaurants has lowered the cost of food consumed away from home relative to that of food consumed at home. Among other authors that document this trend are Gelbach et al. (2009), Powell (2009), Powell & Bao (2009), and Currie et al. (2010).
time between market and non-market activities. Total employment rate, on the one hand, expresses in a more precise way the actual allocation of time, but is more subject to short-term changes in unemployment related to the business cycle (on top of the fact that there is no data for the period 1900-1940). The labor force participation, on the other hand, is a better indicator of the intended (or desired) allocation of time and, because of that, is affected relatively less by short term economic conditions.

As Figure 2.2 (D) shows, the employment rate presents a clear positive trend in the period 1940 and 2000, with a total increment of 13 percentage points (from 50.6% to 63.3%) between the extremes. This implies that, between 1940 and 2000, the percentage of working-age adults effectively engaged in market activities increased by more than 25%. The number, of course, is affected by the fact that the unemployment rate was both, unusually high in 1940 (at 9.6%, as the economy recovered from the Great Depression), and unusually low in 2000 (at only 4%). As mentioned before, we can minimize the impact of short term economic conditions by analyzing the evolution of the labor force participation rate. Three stages can be identified: First, the period between 1900 and 1960, in which the rate oscillated around a mean of 57.4% (with some ups and downs related to the World Wars and the Great Depression); second, the years between 1960 and 1980 in which the rate experienced an upward shift of about 7.5 percentage points; and a third period after 1980 in which the rate has averaged 64.9% (with the last observation being negatively affected by the effects of the Great Recession).

The increase in time devoted to market work suggested by the macro evidence is consistent with the micro evidence recently presented by other authors. Greenwood et al. (2005b), for example, document a remarkable reduction in total hours per week devoted to homework by the average household from 58 in 1900 to 18 in 1975, which is in line with the findings of Aguiar & Hurst (2007), who document a reduction in the per capita weekly hours devoted to non-market work in
the U.S. from 23.5 to 18 between 1963 and 2003 among all non-retired individuals age 21 to 65. McGrattan & Rogerson (2008) also find that, between 1950 and 2005, per capita hours devoted to market work in the U.S. among all individuals age 24–64 increased from 25 to 31 per week. In the same line, but analyzing non-market activities, Hamermesh (2007) documents that, between 1985 and 2003, total "eating" time spent by adult couples decreased by almost 30% (mainly induced by reductions in time shopping, cooking and cleaning up). The reduction in time devoted to home work and the increase in market employment (particularly before the 1980s) is also consistent with the reduction in the share of expenditure devoted to domestic service that, as shown in Figure 2.2 (C), stabilized in the 1980s and 1990s, after maintaining an accelerating downward trend for 5 decades\(^\text{14}\).

**Summary**  Important changes have taken place in the structure of household, in market conditions, and in the way households allocate their time in the U.S. during the last century. The evidence suggests that, during the first three to four decades of the twentieth century, a moderate decrease in the number of adults per household was accompanied by moderate growth in real wages, an initial expansion in the access to basic public services such as electricity and running water, and the introduction and slow adoption of some household durable goods. The four decades that followed (between 1940 and 1980) were characterized by impressive growth in real wages, in the access to public services, and in the use of household domestic appliances and motor-vehicles. During the first two decades of this period, the number of adults per household registered its largest fall, and the allocation of time between home and market work remained relatively stable. During the last two decades of this period (1960-1980), on the other hand, the number of adults per households decreased at a much lower rate, and the labor force participation grew at its highest rate. The use of

\(^{14}\text{Greenwood et al. (2005b), using data until the early 1970s, find a similar trend for the number of domestic workers per household.}\)
of more elaborated goods and services also increased during those years. By 1980, most households had access to basic facilities and durable goods, and even the growth in the ratio of cars to population had decelerated. In the years that followed, as households continued to incorporate more elaborated goods and services, the number of adults per household stabilized around its minimum and the rate of expansion of the labor force participation slowed down.

2.2.2 International Evidence

The documented relationships between the structure of the household, the allocation of time of household’s members, and different aspects of technological progress only refer to the U.S. in a very particular period of time, as it was the twentieth century. In this section I analyze to what extent these regularities are observed in other countries, first, with evidence from a diverse set of countries for which data are available, and, second, with data from different regions of China.

Figure 2.3: Adults per Household & per capita Income
Current International Evidence & U.S. History

Figure 2.3 presents two overlapped scatter plots: one, with the number of adults (and
young adults) per household and per capita GDP for 82 countries for which current data on households are available (only the most recent observation of each country is used), and a second cross plot of the same variables for the United States between 1900 and 2010. Two important conclusions can be extracted from the Figure. First, that there is a strong negative relationship between the size of the household and the level of per capita income in the cross section of countries, particularly for countries with per capita GDP (PPP $2005) above $5,000; and, second, that the evolution in the structure of the household in the United States during the last century is not exceptional, but is part of a general pattern that links the decline in the number of adults per household of a country with its level of development. Finally, the observed dispersion in household size also shows that besides per capita income there are other determinants of the household structure. For example, while China, most former Soviet countries, and several Nordic countries tend to have lower number of adults per household than what their level of income predicts, other countries like Singapore, Cyprus, Malaysia, and even Mexico have households larger than what their income predicts.

Tables 2.1 and 2.2 present additional cross-sectional data for a set of selected countries for which data are available. The data presented for each country refer to the most recent year for which there is information on household structure.

Table 2.1 confirms that, as in the case of the U.S. during the twentieth century, in the cross section of countries the high per capita income is associated with a lower household size, number of adults per household, and shares of households and population living in non-nuclear households\footnote{Non-nuclear household is a household with two or more persons that is not a nuclear household (e.g. extended, composite, or other types of households). A nuclear household is either a couple with children, a couple without children, or a single-parent with children.}. On one extreme, high income countries have both, the smallest households and the lowest number of adults and young adults per household (2.5 and 2.1, in average -columns (2) and (3)). These countries also have the largest fraction of individuals in single-person household (representing, in average, 11.4% of the population and 28.3% of the households -columns (4) and (5)).
## Table 2.1: Household Structure Indicators - Selected Countries*

<table>
<thead>
<tr>
<th>Year (1)</th>
<th>pc GDP PPP $2005 (2)</th>
<th>Persons per HH Total Age 15+ (3)</th>
<th>Single-Person HHs % of Pop. (4)</th>
<th>Non-Nuclear HHs % of HHs (5)</th>
<th>% of Pop. (6)</th>
<th>% of HHs (7)</th>
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<tbody>
<tr>
<td><strong>High-Income Countries</strong></td>
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<tr>
<td>Norway 2001</td>
<td>44,624</td>
<td>2.3</td>
<td>1.8</td>
<td>16.5</td>
<td>37.7</td>
<td>4.6</td>
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<tr>
<td>United States 2009</td>
<td>41,188</td>
<td>2.5</td>
<td>2.0</td>
<td>10.3</td>
<td>27.0</td>
<td>8.2</td>
</tr>
<tr>
<td>Australia 2006</td>
<td>33,020</td>
<td>2.6</td>
<td>2.1</td>
<td>9.7</td>
<td>24.4</td>
<td>13.8</td>
</tr>
<tr>
<td>Japan 2010</td>
<td>30,965</td>
<td>2.4</td>
<td>2.1</td>
<td>13.4</td>
<td>32.4</td>
<td>18.8</td>
</tr>
<tr>
<td>Spain 2009</td>
<td>27,082</td>
<td>2.9</td>
<td>2.4</td>
<td>7.1</td>
<td>20.3</td>
<td>21.7</td>
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<td><strong>Europe</strong></td>
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<tr>
<td>Poland 2002</td>
<td>12,137</td>
<td>2.8</td>
<td>2.3</td>
<td>8.7</td>
<td>24.8</td>
<td>19.9</td>
</tr>
<tr>
<td>Russia 2002</td>
<td>9,546</td>
<td>2.7</td>
<td>2.2</td>
<td>8.2</td>
<td>25.7</td>
<td>34.2</td>
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<td><strong>Latin America</strong></td>
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<td></td>
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<tr>
<td>Mexico 2010</td>
<td>12,481</td>
<td>3.9</td>
<td>2.8</td>
<td>2.2</td>
<td>8.8</td>
<td>35.5</td>
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<td>Brazil 2000</td>
<td>7,909</td>
<td>3.7</td>
<td>2.6</td>
<td>2.4</td>
<td>8.9</td>
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<td>Colombia 2005</td>
<td>7,305</td>
<td>4.0</td>
<td>2.8</td>
<td>2.7</td>
<td>11.1</td>
<td>-</td>
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<td>Nicaragua 2005</td>
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<td>5.2</td>
<td>3.2</td>
<td>0.9</td>
<td>4.6</td>
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<td>Malaysia 2000</td>
<td>10,622</td>
<td>4.5</td>
<td>3.0</td>
<td>1.6</td>
<td>7.1</td>
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<td>China 2005</td>
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<td>2.5</td>
<td>2.8</td>
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<td>India 2011</td>
<td>3,203</td>
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<td>0.8</td>
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<td>-</td>
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<tr>
<td>South Africa 2001</td>
<td>7,691</td>
<td>4.0</td>
<td>2.7</td>
<td>5.6</td>
<td>22.3</td>
<td>-</td>
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<tr>
<td>Burkina Faso 2006</td>
<td>1,051</td>
<td>5.9</td>
<td>3.2</td>
<td>0.9</td>
<td>5.5</td>
<td>53.1</td>
</tr>
</tbody>
</table>

(*) Countries selected based on data availability. There are very limited data on household structure for non-developed countries. Burkina Faso and South Africa, and Malaysia (and China and India to a lesser extent) are the only developing countries with such data in South Saharan Africa and Asia respectively. Source: UNSD, ECLAC, WDI, and censuses. Notes: A household is a small group of persons who share the same living accommodation, who pool some, or all, of their income and wealth and who consume certain types of goods and services collectively, mainly housing and food (UNSD). A nuclear household is either a couple with children, a couple without children, or a single-parent with children. Non-nuclear household is a household with two or more persons that is not a nuclear household (e.g. extended, composite, or other types of households). Persons per Household: (2) is the ratio of total population and total number of households. Members Age 15+ per Household: (3) is the product of total persons per household and the share of total population age 15 or older (WDI). India: Data from Census 2011. In India, a household may contain one or more married couples. Due to lack of data on nuclear-households or number of adults per household, the % of non-nuclear households in India is proxied with the % of married couples living in households with two or more married couples. China: Data from 1% Population Survey 2005 and from 2005 Statistical Yearbook. Due to lack of data on non-nuclear households, the % of population and of households in non-nuclear households in China is proxied with the % of population and households in multi-generational households (i.e. containing three or more generations). By construction, these measures underestimate the % of population and households in non-nuclear households given that they do not contemplate other forms of composite or extended household. Nicaragua: Data from Pop. Census 2005. South Africa: Complemented with data from Census 2001 and Kerr & Wittenberg (2012) and Adams & Trost (2005). (5)), and the lowest share of population in non-nuclear (i.e. composite or extended) households (representing, in average, 13.4% of the population and 9.7% of the households -columns (6) and (7)). On the other extreme, the three countries in the sample with the lowest per capita income
(Burkina Faso (2006), Nicaragua (2005) and India (2011)) have both the largest households (5.3 persons per household, in average) and the highest number of adults per household (3.3, in average). The share of population in single-person households in these countries is less than 1%, and non-nuclear households represent almost 40% of total households (and even a higher fraction of the population). Between these extremes lies the group of upper middle income countries (i.e. with per capita -PPP $2005- income between $7,000 and $12,500), whose household size and number of adults per household is, in average, 3.7 and 2.6 respectively, and in which the fraction of single-person and non-nuclear households is 15.5% and 23.8% respectively.

Table 2.2 presents several economic and environmental indicators that allow to explore in further detail how the documented differences in income and household structure (Table 2.1), are associated with differences in various aspects of technological progress and in the way households allocate time between home and market activities. The table confirms that per capita income is associated with improvements in the three dimensions of technology: market real wage or labor productivity (proxied here with real GDP per employee), the access to basic public services and housing facilities (electricity and piped water in the house) as well as the use of household durable goods or domestic appliances (proxied with motor vehicles per 1,000 population), and the access to markets with more elaborated goods and services (proxied here with the percentage of urban population). Though with some variations, all of these indicators are positively associated with per capita income, and negatively associated with the number of adults per household. In terms of access to basic housing facilities (electricity and piped water) and motor vehicles, the developed countries rank first, and Burkina Faso and India (the countries with the highest number of adults per household), rank worst. These last two countries have also the lowest rates of urban population (22.4% and 31.3%, respectively).

Measuring the fraction of time devoted to market activities is particularly challenging
Table 2.2: Economic and Environmental Indicators - Selected Countries

<table>
<thead>
<tr>
<th>Year</th>
<th>GDP per Employee $2005</th>
<th>Access to Electricity % of Pop.</th>
<th>Piped Water in House % of Pop.</th>
<th>Urban Pop. % of Pop.</th>
<th>Motor vehicles per 1,000 people</th>
<th>Wage &amp; Salaried Workers % of Pop.</th>
<th>Labor Force Part. % of Pop.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
</tr>
<tr>
<td>High-Income Countries</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>United States 2009</td>
<td>88,288</td>
<td>100</td>
<td>-</td>
<td>81.9</td>
<td>815</td>
<td>57.0</td>
<td>73.5</td>
</tr>
<tr>
<td>Norway 2001</td>
<td>87,579</td>
<td>100</td>
<td>100</td>
<td>76.4</td>
<td>525</td>
<td>58.8</td>
<td>79.7</td>
</tr>
<tr>
<td>Australia 2006</td>
<td>66,416</td>
<td>100</td>
<td>-</td>
<td>88.4</td>
<td>671</td>
<td>54.1</td>
<td>75.9</td>
</tr>
<tr>
<td>Spain 2009</td>
<td>65,703</td>
<td>100</td>
<td>99</td>
<td>77.2</td>
<td>591</td>
<td>40.2</td>
<td>73.2</td>
</tr>
<tr>
<td>Japan 2010</td>
<td>62,372</td>
<td>100</td>
<td>98</td>
<td>90.5</td>
<td>591</td>
<td>49.7</td>
<td>74.1</td>
</tr>
<tr>
<td>Europe</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poland 2002</td>
<td>33,677</td>
<td>100</td>
<td>95</td>
<td>61.6</td>
<td>357</td>
<td>31.7</td>
<td>63.8</td>
</tr>
<tr>
<td>Russia 2002</td>
<td>20,623</td>
<td>93.0</td>
<td>78</td>
<td>73.2</td>
<td>202</td>
<td>51.3</td>
<td>70.7</td>
</tr>
<tr>
<td>Latin America</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mexico 2010</td>
<td>30,142</td>
<td>97</td>
<td>89</td>
<td>77.8</td>
<td>275</td>
<td>38.0</td>
<td>65.0</td>
</tr>
<tr>
<td>Brazil 2000</td>
<td>18,189</td>
<td>98.3</td>
<td>86</td>
<td>81.2</td>
<td>164</td>
<td>38.1</td>
<td>71.8</td>
</tr>
<tr>
<td>Colombia 2005</td>
<td>17,819</td>
<td>93.6</td>
<td>85</td>
<td>73.6</td>
<td>57</td>
<td>29.1</td>
<td>69.6</td>
</tr>
<tr>
<td>Nicaragua 2005</td>
<td>8,450</td>
<td>72.1</td>
<td>60</td>
<td>55.9</td>
<td>46</td>
<td>28.9</td>
<td>63.6</td>
</tr>
<tr>
<td>Asia</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Malaysia 2000</td>
<td>25,902</td>
<td>99.4</td>
<td>89</td>
<td>62.0</td>
<td>272</td>
<td>45.7</td>
<td>65.4</td>
</tr>
<tr>
<td>India 2011</td>
<td>8,617</td>
<td>75</td>
<td>23</td>
<td>31.3</td>
<td>18</td>
<td>9.2</td>
<td>57.7</td>
</tr>
<tr>
<td>China 2005</td>
<td>7,292</td>
<td>99.4</td>
<td>59</td>
<td>42.5</td>
<td>47</td>
<td>-</td>
<td>81.3</td>
</tr>
<tr>
<td>South Saharan Africa</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>South Africa 2001</td>
<td>31,142</td>
<td>75</td>
<td>62</td>
<td>57.4</td>
<td>135</td>
<td>29.9</td>
<td>55.0</td>
</tr>
<tr>
<td>Burkina Faso 2006</td>
<td>2,019</td>
<td>14.6</td>
<td>5</td>
<td>22.4</td>
<td>10</td>
<td>5.8</td>
<td>85.2</td>
</tr>
</tbody>
</table>

(*) Countries selected based on data availability. Sources & Notes: GDP per employee (1) is the ratio between total GDP (PPP $2005) and total employed population age 15+ (from WDI). Access to Electricity (2) is from World Energy Outlook 2011 and the REEEP Policy Database and corresponds to 2009 in most cases. Piped Water Inside the House (3) is based on data from UNICEF. The rest is from WDI. Wage & Salaried Workers are those workers who hold the type of jobs defined as "paid employment jobs," that include a basic remuneration that is not directly dependent upon the revenue of the unit for which they work. The main excluded categories are own-account workers, employers, and family contributing workers. The figure is expressed as a % of the total population age 15 or above.

in developing countries. While the labor force participation (or the ratio of total employment to working-age population) is a good indicator in high and middle income countries where market employment is relatively formal and structured, those indicators may not be very accurate in the countries with predominantly informal labor markets, which is precisely the case of the least developed countries. Column (7) of Table 2.2 shows that the group of high-income countries has, in average, a rate of labor force participation of 73.2%, compared with an average of 65.9% among
the group of middle income countries (Poland, Russia, Mexico, Brazil, Colombia, Malaysia and South Africa). In the same line, the rate of population age 15+ that is (formally) waged or salaried is 52% and 37.2% in these two groups of countries. The difference between both rates (the labor participation rate and the rate of waged and salaried workers) is mainly explained by the rate of unemployment and for the share of informal workers (self employed, or family contributing workers). The difference is, clearly, larger in the group middle income countries. However, while the rate of waged and salaried workers is systematically lower in lower income countries (reflecting higher informality of the labor market), the rate of labor force participation is, particularly high in some of the least developed countries\(^{16}\). In Burkina Faso, for example, 85.2% of the working age population is in the labor force, but less than 6% is formally waged or salaried, implying that more than 93% of the labor force is either unemployed or self-employed, or simply a family contributing worker. In a case like this, the high degree of informality (and the precarious conditions) in the labor market make it very blur the difference between market and home work.

Finally, Table 2.3 presents evidence from different regions in China, in 2005. Despite the fact that the structure of the population has been severely affected by diverse government interventions (such as the *one child policy* and other regulations that limit the mobility of individuals across provinces), the patterns observed in the U.S. and across countries, can also be verified across the different types of urbanization in China. Among other things, the table shows that the share of households that live in a self-built house is much lower in the cities (28.5%) than in the rural areas (94.2%), which can be interpreted as an indicator of the extent to which individuals contribute with the elaboration of the goods and services their consume.

In the following section I develop a model that incorporates and explains the documented empirical regularities.

\(^{16}\)The average labor force participation rate among South Saharan countries, excluding South Africa was 73% in 2011 (WDI).
Table 2.3: Selected Indicators by Type of Urbanization - China, 2005

<table>
<thead>
<tr>
<th>Type of Urbanization</th>
<th>City (1)</th>
<th>Town (2)</th>
<th>Village (3)</th>
<th>China (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population (% total)</td>
<td>27.7</td>
<td>17.1</td>
<td>55.2</td>
<td>100</td>
</tr>
</tbody>
</table>

**Family Household Structure**
- Households with 3 or more generations (% Tot. HHs): 13.8, 16.4, 20.9, 18.0
- Population in 3+ generations Household (% Tot. Pop.): 23.1, 26.5, 31.6, 28.4
- Population in single-person Household (% Tot. Pop.): 3.9, 3.1, 2.2, 2.8
- Household Size (Total): 2.9, 3.09, 3.27, 3.13
- Household Size (15+): 2.51, 2.51, 2.56, 2.53

**Household Income**
- Household Annual Income (Yuan, Avg.): 1,492, 1,115, 822, 1,066
- Households with Annual Income under 1000 Yuan (% Tot. HHs): 45.4, 57.2, 75.4, 63.6

**Employment by Occupation (% Tot. Employed)**
- Administration/Professional/Technical: 17.5, 12.7, 4.4, 9.1
- Sales & Services: 25.3, 21.1, 3.9, 12.1
- Clerical & related: 9.6, 5.5, 0.7, 3.7
- Agriculture & related: 15.9, 36.5, 80.4, 57.1
- Manufacturing & Related: 31.2, 23.7, 10.4, 17.8

**Housing (% Tot. HHs)**
- Households living in a self-built house: 28.5, 58.9, 94.2, 69.1
- House Units with Tap Water: 86.3, 66.2, 28.6, 51.8
- House Units whose main cooking fuel is gas or electricity: 73.8, 41.2, 8.6, 33.1
- Housing Rent (Yuan, Avg. Monthly): 281, 194, 164, 248

*Source: Author's calculations based on China 2005 1% Population Survey*

### 2.3 Model

The general model is based on the idea first introduced by Becker (1965), according to which households are producers as well as consumers, and combine time and market goods to produce final goods that directly enter their utility functions. Here, it is assumed that individuals (as opposed to households) are producers and consumers, that combine homework (as opposed to time) and market goods to produce the final consumption good.

Motivated by the presence of economies of scale in production, individuals can voluntarily
join others in a household in which they share production and consumption of the final good. A household, in this context, consists of a group of identical individuals that share production and consumption. A key assumption is that, within the household, both production and consumption of the final good have to be shared with all household members. Another key assumption is that individuals’ utility also depends on the number of individuals with whom consumption is shared. In particular, when the size of the household surpasses a certain threshold, sharing consumption has a negative effect on individuals’ utility. There is, therefore, a trade off between taking advantage of the economies of scale in production, on the one hand, and minimizing the negative impact that sharing consumption has on individuals’ utility, on the other.

2.3.1 Benchmark Model

Consider an economy populated by identical individuals endowed with 1 unit of time, which is split between market and home work. The market wage (in terms of a numeraire) is $w$.

**Household Production and Consumption.** Individuals can join others in a household to share production and consumption. Be $k \geq 1$ the number of individuals in the household.

**Production** Households produce the final consumption good combining a market good and homework. The household production technology can be represented by a cost function $C(x)$, which expresses the total cost (in terms of the numeraire) of producing $x$ units of the final good. $C(x)$ includes both, the cost of market goods as well as the opportunity cost of the time devoted to homework. In order to highlight the role played by economies of scale in household production, I abstract by now from the details of the production function and simply assume that $C'(x) > 0$ for all $x \geq 0$. I provide more structure to the production process below.

The total cost of production is equally distributed among the individuals in the household.
Thus, for the production of \( x \) units of final good, each individual in the household contributes with 
\[
\frac{C(x)}{k}
\] units of the numeraire.

**Preferences and Household Consumption** Individuals’ utility depends on the level of *effective* consumption, \( \hat{x} \), which is a function of the household’s total production, \( x \), and the number of individuals in the household, \( k \). Specifically, when \( x \) units of final good are shared by the \( k \) members of the household, the effective consumption enjoyed by each individual is 
\[
\hat{x} = \frac{x}{\phi(k)},
\] where \( \phi \) is twice continuously differentiable, \( \phi(1) = 1 \), \( \phi'(k) > 0 \) and \( \phi''(k) > 0 \) for \( k \geq 1 \). Assume additionally that the function \( \frac{k}{\phi(k)} \) has a unique maximum at \( k = k^* \), \( 0 \leq k^* < \infty \), and that \( \phi'(1) \leq 1 \).

Notice that *effective* consumption can be expressed as
\[
\hat{x}(x, k) = \frac{x}{k} \frac{k}{\phi(k)} = \bar{x}\psi(k)
\]

where \( \bar{x} = \frac{x}{k} \) is average per capita consumption, and \( \psi(k) = \frac{k}{\phi(k)} \) acts as a household-size adjustment factor. The assumptions made on \( \phi \) imply that \( \psi(k) \) is: increasing for \( k \in [1, k^*) \), reaches its maximum at \( k = k^* \), and decreasing for \( k > k^* \). This implies that, when per capita consumption is independent of the number of individuals in the house (i.e. when returns to scale in household production are constant, so that the cost per unit of final good is constant), effective consumption (and utility) is maximized at \( k = k^* \), and is strictly increasing (decreasing) in \( k \) for \( k < k^* \) (\( k > k^* \)). Finally, \( \phi(1) = 1 \) implies that \( \psi(1) = 1 \), so there is no gain or loss of effective consumption in a single-member household, and \( \phi'(1) \leq 1 \) ensures that \( k^* \geq 1 \).

**Problem of the Individual** The problem of each individual consists in choosing the household size, \( k \), and the level of household production, \( x \), that maximize his utility, \( U(\hat{x}(x, k)) \), subject to
the household’s resource constraint \( wk \geq C(x) \). Given the static nature of the problem, maximizing \( U(\hat{x}(x, k)) \) is equivalent to maximizing effective consumption \( \hat{x}(x, k) = \frac{x}{\phi(k)} \). The formal problem can be written as

\[
\max_{k \geq 1, x > 0} \frac{x}{\phi(k)} \quad \text{s.t.} \quad wk \geq C(x)
\] (2.1)

The first order conditions of the problem imply that

\[
k \frac{\phi'(k)}{\phi(k)} = \frac{C(x)}{x} = \frac{AC(x)}{MC(x)}
\] (2.2)

where \( AC(x) \) and \( MC(x) \) are the average and marginal cost of \( x \), respectively. The LHS of expression (2.2) is related to the slope of the function \( \frac{k}{\phi(k)} \). Specifically, it can be shown that

\[
k \frac{\phi'(k)}{\phi(k)} \begin{cases} < 1 & \iff k < k^* \\ = 1 & \iff k = k^* \\ > 1 & \iff k > k^* \end{cases}
\] (2.3)

Expressions (2.2) and (2.3) imply that the optimal household size, \( k \), is directly related to the notion of returns to scale in household production. Specifically, \( k < k^* \) when the marginal cost is higher than the average cost (i.e. \( MC(x) > AC(x) \)), so that the average cost is increasing and there are, therefore, decreasing returns to scale in production. On the other hand, \( k > k^* \) when \( MC(x) < AC(x) \), so that the average cost is decreasing and there are increasing returns to scale. Finally, \( k = k^* \) when \( AC(x) = MC(x) \), and there are constant returns to scale in the production of the final good.

Furthermore, the combination of expression (2.2) with the household resource constraint
\( wk = C(x) \) and the definition of \( \hat{x} \), allows to write

\[
\frac{w}{x} = \phi'(k)C'(x)
\]  \hspace{1cm} (2.4)

The LHS of equation (2.4) is the final average cost per *effective* unit of consumption for each individual in the household, which is equal (on the RHS) to the product of the marginal cost of production and the marginal increment in the discount function.

Expression (2.4) allows to compare the impact that the use of different production technologies has on the structure of the household. Suppose, for example, that two different technologies \((C_1(x) \text{ and } C_2(x))\) for the production of the final good allow individuals to attain the same level of effective consumption (i.e. individuals are indifferent between using either technology). This implies that the LHS of equation (2.4) is equal for both technologies, so it follows that

\[
\phi'(k_1)C'_1(x_1) = \phi'(k_2)C'_2(x_2)
\]  \hspace{1cm} (2.5)

where \( k_i \) and \( x_i \) are the number of individuals and total production of final good with technology \( i = 1, 2 \).

From equation (2.5), and using the fact that \( \phi'(k) > 0 \) and \( \phi''(k) > 0 \), it follows that the technology with lower marginal cost of production will be associated with a higher household size, and therefore with a higher scale of production (e.g. if \( C'_1(x_1) < C'_2(x_2) \), then \( \phi'(k_1) > \phi'(k_2) \), so that \( k_1 > k_2 \) and \( x_1 > x_2 \), where the last inequality follows from \( \hat{x}_1 = \hat{x}_2 \)). The intuition is as follows: if the technologies have different marginal costs but allow the individuals to achieve the same level of effective consumption, it must be that the one with lower marginal cost (\( i = 1 \) in the example) has either a higher marginal cost at lower scale of production, or simply a higher fixed cost (otherwise, individuals would strictly prefer to use the technology with lower marginal cost),
so that economies of scale are stronger for that technology.

**2.3.2 The Production Process**

In order to analyze the effects of different types of technological progress on the individuals’ decisions, I provide some structure to the process through the final consumption good is produced.

Specifically, I assume that the process of production of the final good comprises a *continuum of tasks* (or stages), that, beginning with the least elaborated good (e.g. raw material that can be found in the nature), continually increase the degree of elaboration of the good until it is fully elaborated and ready to be consumed (i.e. it is a *final* good). In this process, the good used as an input at each stage is the output of the previous one. Thus, even though there is only one final consumption good in this economy, there potentially exists an infinite number of intermediate goods that differ in their degree of elaboration.

As an example, suppose that the final consumption good consists of a pizza that an individual wants to consume at a nice place. The elaboration of the pizza entails an immense number of activities that conform the production chain. From the production of grain, milk and tomatoes, to that of flour, cheese and sauce, which can be used later for the production of pizza (other stages might still be necessary for the pizza to be ready to be consumed by the individual -such as delivery, serving the table, etc.). Each of these stages (that can be performed by different firms or individuals and in different places), increases the degree of elaboration of the output of the previous one, and produces a different good.

Formally, assume that there is a continuum of goods indexed by \( i \in [0, 1] \), ordered according to their degree of elaboration (which is represented by the same index, \( i \in [0, 1] \)). \( i = 0 \) refers to the least processed good, and \( i = 1 \) to the final consumption good. Individuals only consume the final consumption good, \( i = 1 \), so all goods \( i \in [0, 1] \) can be interpreted as intermediate goods.
with different degree of elaboration, that can be transformed into the final consumption good by adding to them the remaining part of the production process.\footnote{Even though the focus here is on the household, it could be assumed that market goods are produced by firms that use technologies of production with very low marginal costs but fixed costs high enough for households to pay (given that it would require a very large household, which would greatly reduce effective consumption). With U-shaped average costs, and a large population (so that there is a large number of firms), firm’s charge their minimum average cost and profits are zero.

A special treatment is necessary, however, for goods and services whose production is subject to decreasing marginal (and average) costs of production, or goods for which the optimal scale of production is such that, given the size of the population, there are no many firms. In these cases there could be either public provision or a special regulation for private provision, or simply monopolistic or oligopolistic competition. Abstract from this for now, even though these issues are absolutely relevant in the context studied here.

About the relationship between market size and technological innovations, see Schmookler (1966). Regarding evidence, Acemoglu & Linn (2004) document the relationship between market size and the rate of innovation for drugs of certain types.}

**Production Technologies** Individuals can produce the final good in different ways, combining in each case one intermediate market good \((i \in [0, 1])\), and the amount of *homework* that completes the process of elaboration of that particular good. I define *technology* \(i\) as the process that uses market good \(i\) as an input to produce the final consumption good\footnote{Technology \(i = 1\) is trivial, since the market good used as input is the same as the final good, and requires no additional elaboration.}. In order to produce \(x\) units of final good using technology \(i \in [0, 1]\), individuals combine \(x\) units of market good \(i\), with \(H(i, x)\) units of homework, where the function \(H\) is such that \(H_i(\cdot, x) < 0\) (i.e. more elaborated intermediate goods require less homework), \(H(1, \cdot) = 0\) (i.e. the final good requires no additional homework to be consumed), \(H_x(i, \cdot) \geq 0\) (i.e. homework is weekly increasing in the amount of final good produced), and \(H_{xx}(i, \cdot) \leq 0\) (that is, for each technology \(i \in [0, 1]\), the amount of homework is concave in the amount of final good produced).

The cost function associated with each production technology can be written as

\[
C(i, x) = p(i)x + C^H(i, x), \ i \in [0, 1]
\] (2.6)

where \(C(i, x)\) is the total cost (in terms of the numeraire) of producing \(x\) units of final good using technology \(i\), \(p(i)\) is the market price of good \(i\), and \(C^H(i, x)\) is the cost of homework necessary to
transform $x$ units of good $i$ into $x$ units of final good (the function $C^H(i, x)$ accounts for both, the opportunity cost of the individuals’ time devoted to homework, and the cost of household capital goods used in the process). The assumptions made on $H$ imply that $C^H_i(\cdot, x) < 0$, $C^H(1, \cdot) = 0$, $C^H_x(i, \cdot) \geq 0$, and $C^H_{xx}(i, \cdot) \leq 0$.

Additionally, a key assumption is that there are increasing returns to scale in homework production, which implies that the average cost of homework production, $\frac{C^H(i, x)}{x}$, is decreasing in the amount produced, $x$.

The Individual’s Problem  The problem of the individual consists in choosing the technology for home production $i \in [0, 1]$ (or, equivalently, the type of market good used as an input in the home production process), the household size $k \geq 1$, and the amount of final consumption good $x \geq 0$ that maximize his utility, subject to the household resource constraint $wk \geq C(i, x)$, $i \in [0, 1]$; given the market wage, $w$, and prices $p(i)$, $i \in [0, 1]$.

From the individual’s perspective, the problem can be solved in two stages: first, solve for $x(i)$ and $k(i)$ for each available technology $i \in [0, 1]$ and, second, pick the technology that maximizes effective consumption. The conditions for the first stage are similar to the ones from the previous case, but using the particular functional form of $C(i, x)$ of expression (2.6). For each technology $i \in [0, 1]$, the optimal household size and total production are given by equation (2.2) and by the resource constraint, whose combination allows to express (similarly to expression (2.4)) the cost per unit of effective consumption associated with each technology:

$$\frac{w}{x(i)} = \phi'(k(i))C_x(i, x(k(i))), \text{ for } i \in [0, 1]$$

(2.7)

where $x(k) = C^{-1}(wk)$ comes from the resource constraint.
Comparing Production Technologies  As in the previous case, expression (2.7) allows to compare how different technologies impact on the structure of the household. Assume that technologies $i_1$ and $i_2$, $i_1 < i_2$ (i.e. technology $i_2$ uses a more elaborated market good), are such that $\hat{x}(i_1) = \hat{x}(i_2)$, so that individuals are indifferent between using either of them. From equation (2.7), it follows that the technology with the lower marginal cost is associated with a larger household. Using expression (2.6) for the cost function, it follows that the marginal cost of production is

$$C_x(i, x) = p(i) + C^H_x(i, x)$$

which is composed by the price of the market good, $p(i)$, and the marginal increment in the amount of homework associated with that particular technology, $C^H_x(i, x)$.

The fact that $\hat{x}(i_1) = \hat{x}(i_2)$, and that technology $i_1$ uses a less elaborated market good, implies that $p(i_1) < p(i_2)$, otherwise, individuals would strictly prefer technology $i_2$ and there would be no demand for market good $i_1$. On the other hand, the fact that technology $i_1$ requires more homework to finish the elaboration of the good implies that $C^H(i_1, x(k(i_1))) > C^H(i_2, x(k(i_2)))$ and, presumably, $C^H_x(i_1, x(k(i_1))) > C^H_x(i_2, x(k(i_2)))$. Thus, the technology with the less elaborated market good, $i_1$, is associated with a higher number of individuals in the household if, and only if:

$$C_x(i_1, x(k(i_1))) < C_x(i_2, x(k(i_2)))$$

$$\iff C^H_x(i_1, x(k(i_1))) - C^H_x(i_2, x(k(i_2))) < p(i_2) - p(i_1)$$

which implies that the reduction in the marginal cost of homework associated with the use of a more elaborated market good, is less than the difference in prices of the market goods.

An Important Assumption  Condition (2.9) holds if, for example, it is assumed that within the relevant scale of operation of the household, homework production is characterized by relatively
high fixed costs, but, at the same time, very low marginal costs. Indeed, it seems very reasonable to assume that there is a range of production in which the amount of homework necessary to transform an intermediate good into a final consumption good, \( H(i, x) \), depends more on the degree of elaboration of the good, \( i \), than on the total amount produced, \( x \). In this context, switching to a technology with a more elaborated market good (say, from \( i_1 \) to \( i_2 \)) reduces the fixed cost and increases the marginal cost, increasing the final good’s average cost at a relatively high scale of production, but decreasing it at a sufficiently low scale. In the example of the pizza, for instance, the work required in the elaboration of the ingredients, shopping, cooking, and even cleaning is, within a certain range (say, to feed 1, 2, 3, or even 4 people), almost invariant to changes in the total amount of pizza produced. Depending on the specific market conditions (i.e. \( w \) and \( p(i), i \in [0, 1] \)) the average cost of pizza for a large number of people will probably be lower if less elaborated inputs (such as dough, sauce, cheese, etc.) and more homework (e.g. shopping, preparing, cooking, cleaning, etc.) are used. As the number of individuals decreases, the average cost will decrease as progressively more elaborated (and more expensive) market goods are combined with less homework (e.g. buying and heating frozen pizza, or ordering pizza to be delivered at home, or even eating at a pizzeria).

The assumption made in the previous example can be generalized as \( C_{xi}(i, x) > 0 \), or, equivalently, \( \frac{dp(i)}{di} > C_{xH}(i, x) \), which implies that the increment in the price of a marginally more elaborated good is greater than the reduction in the marginal cost of homework implied by the use of that technology. When this is the case, equation (2.7) ensures that whenever individuals are indifferent between two technologies, the optimal household size associated with each one of them is negatively related to the degree of elaboration of the market good used, \( i \).
Furthermore, the cost of homework can be expressed as

\[ C^H(i, x) = C^{HF}(i) + C^{HV}(i, x) \]  

(2.10)

where \( C^{HF}(i) \) and \( C^{HV}(i, x) \) are the fixed and variable cost functions, respectively, associated with the use of technology \( i \). Combining the first order conditions of the individual’s problem, it can be obtained

\[
k - \frac{\phi(k)}{\phi'(k)} = \frac{1}{w} \left[ C^{HF}(i) + C^{HV}(i, x) - xC^{HV}_x(i, x) \right] \\
\approx \frac{C^{HF}(i)}{w}
\]  

(2.11)

where I use the fact that, if \( C^H(i, x) \) is indeed mainly explained by the fixed cost, \( C^{HF}(i) \), and the variable component, \( C^{HV}(i, x) \), is relatively small and does not change much with \( x \) (i.e. \( C^{HF}(i) \gg C^{HV}(i, x) \geq 0, C^{HV}_x(i, x) \gg 0, \) and \( C^{HV}_xx(i, x) \approx 0 \)), then expression \( C^{HV}(i, x) - xC^{HV}_x(i, x) \approx 0 \). In this case, the LHS of expression (2.11) (which only depends on \( k \)) is determined by the ratio between the fixed cost of homework and the market wage. Moreover, it can be proved that the LHS of equation (2.11) is strictly increasing in \( k \), which implies that the optimal household size associated with a particular technology is mainly determined by, and depends positively on, \( \frac{C^{HF}(i)}{w} \), which can be interpreted as the equivalent units of time required to cover the fixed cost of homework of that technology.

Equation (2.11) also shows that \( k - \frac{\phi(k)}{\phi'(k)} \to 0 \) when \( \frac{C^{HF}(i)}{w} \to 0 \) which, using the definition of \( k^* \), implies that \( k \to k^* \) when the fixed cost in homework production approaches zero (and \( C^{HV}(i, x) - xC^{HV}_x(i, x) \approx 0 \)). This consistent with the fact that, as mentioned above, \( k = k^* \) when the marginal and the average total cost of production are equal, and there are no economies of scale.
2.3.3 Technological Progress: Introducing More Elaborated Goods and Services

The final decision of the individual about the technology to be used depends on the specific market conditions \( (w, p(i), i \in [0, 1]) \) and the assumed functional forms of \( \phi(k) \) and \( C^H(i, x) \). The previous analysis, however, allows to analyze the effect of the introduction of (or the increase in the access to) more elaborated market goods and services, which can be represented by a decline in the relative price of market goods with higher degree of elaboration.

Suppose that the price of intermediate good \( i', p(i') \), falls relative to the other prices \( p(i), i \in [0, 1], i \neq i' \). The impact that this price change has on the effective consumption of individuals that use this technology can be assessed by looking at the average cost of effective consumption, which, combining expressions (2.7), (2.8) and (2.10) can be expressed as

\[
\frac{w}{\hat{x}(i')} = \phi'(k(i'))C_x(i', x) = \phi'(k(i')) [p(i') + C^{HV}_x(i', x)] \text{ for some } i' \in [0, 1] \tag{2.12}
\]

Expression (2.12) implies that the effect of the change in \( p(i') \) on \( \frac{w}{\hat{x}(i')} \) is composed by the direct effect on the marginal cost, \( C_x(i, x) = p(i) + C^{HV}_x(i, x) \), and the indirect effect on the household size associated with the technology, \( k(i') \). As it was shown above, under the assumptions made on the structure of the cost of homework, \( k(i') \) depends mainly on \( \frac{C^{HF}(i')}{w} \) and remains almost unaffected by the change in \( p(i') \), implying that the change in \( \frac{w}{\hat{x}(i')} \) is almost completely determined by the direct change in \( C_x(i', x) \). Finally, the fact that \( C^{HV}_x(i', x) \gtrless 0 \) and \( C^{HV}_{xx}(i', x) \gtrless 0 \) imply that the fall in \( p(i') \) translates almost proportionally into a fall in \( \frac{w}{\hat{x}(i')} \), and a consequent increase in effective consumption, \( \hat{x}(i') \). Summarizing, the decline in the relative price of good \( i' \) will have little or no effect on the household size, \( k(i') \), but instead will directly increase both, the absolute \((x(i'))\) and the effective consumption \((\hat{x}(i'))\), and therefore the relative preference of individuals for
that technology. If the increase in \( \tilde{x}(i') \) is high enough, individuals will switch toward technology \( i' \) and adjust their household size to \( k(i') \). It follows then that the decline in the relative price of market goods with higher degree of elaboration will induce, *ceteris paribus*, a reduction in both, the amount of homework performed by households, and the number of individuals that compose them.

### 2.3.4 Homework Production

So far, the analysis has only considered the *cost* of homework production and has not provided details on *how* production takes place in the household. In order to analyze how changes in the market wage and in the prices of household capital goods affect individuals’ decisions, I provide now a more structured framework for the process of homework production.

It was assumed above that the entire process of production comprises a *continuum of tasks* (\( j \in [0,1] \)), that sequentially increase the degree of elaboration of the good until it is fully elaborated and ready to be consumed (i.e. it is a *final good*). While *Market production* refers to the (first) part of this process (that is performed by firms in the *market*), *homework* refers to the part of the elaboration process (that is performed by individuals in the *household*). Thus, using technology \( i, i \in [0,1] \), implies that tasks \( j \in [0,i] \) are performed in the market (so that the individuals buy market good \( i \)), and tasks \( j \in (i,1] \) are performed at home. Be \( h(j,x) \) the amount of homework required for the realization of *task* \( j, j \in (i,1] \) in the elaboration of \( x \) units of final good. The total amount of homework associated with technology \( i \) can be expressed as

\[
H(i,x) = \int_{i}^{1} h(j,x) dj
\]

Assume additionally that each task \( j, j \in [i,1] \) can be produced combining labor and domestic appliances according to the homework production function \( f(j,l,d) \), where \( j \) is the task to
be performed\(^{19}\), \(l\) the amount of labor, and \(d\) the amount of domestic goods used. As usual, assume that \(f\) is twice continuously differentiable, \(f_l, f_d > 0, f_{ll}, f_{dd} \leq 0\), and \(f_{ld}, f_{dl} \geq 0\).

Assuming that tasks can be treated separately, the optimal amounts of labor and domestic appliances used for the realization of task \(j\) when producing \(x\) units of final good can be obtained from the minimization of the cost of homework:

\[
\left\{ l(j, x, \frac{w}{p_d}), d(j, x, \frac{w}{p_d}) \right\} = \arg \min \{wl + p_d d \quad \text{s.t.} \quad f(j, l, d) \geq h(j, x) \} \tag{2.14}
\]

where the market wage, \(w\), capture the opportunity cost of labor, and \(p_d\) is the market price of domestic appliances, both in terms of the numeraire. It follows from the assumptions on \(f\) that \(l_x, d_x \geq 0, l \frac{w}{p_d} < 0\), and \(d \frac{w}{p_d} > 0\).

The total amount of labor necessary to produce \(H(i, x)\) can be expressed as

\[
L^H(i, x, \frac{w}{p_d}) = \frac{1}{i} \int l(j, x, \frac{w}{p_d}) dj \tag{2.15}
\]

where (from the assumptions on \(H\) and \(f\)) \(L^H_i < 0, L^H_x \geq 0, L^H_{\frac{w}{p_d}} < 0\). Similarly, the total amount of domestic appliances necessary to produce \(H(i, x)\) can be expressed as

\[
D^H(i, x, \frac{w}{p_d}) = \frac{1}{i} \int d(j, x, \frac{w}{p_d}) dj \tag{2.16}
\]

where \(D_i^H < 0, D^H_x \geq 0, D^H_{\frac{w}{p_d}} > 0\).

Finally, the total cost of homework necessary to produce \(x\) units of final good using technology \(i\) is

\[
C^H(i, x, w, p_d) = wL^H(i, x, \frac{w}{p_d}) + p_d D^H(i, x, \frac{w}{p_d}) \tag{2.17}
\]

\(^{19}\)In this case, task \(j\) enters as a parameter in the function \(f\). It can be assumed, instead, that there is a specific production function \(f'(l, d)\) for each task \(j \in [0,1]\).
and the total production cost of $x$ units of final good, when using technology $i$, is

$$C(i, x, w, p_d, p(i)) = p(i)x + wL^H(i, x, \frac{w}{p_d}) + p_dD^H(i, x, \frac{w}{p_d})$$

(2.18)

given market prices $w$, $p_d$, and $p(i)$, $i \in [0, 1]$. Expression (2.18) simply presents the total cost as (the sum of) the cost of the three factors of production used by the individuals, namely, intermediate good $i$, labor, and domestic appliances.

2.3.5 Technological Progress: Increase in Market Wages and Fall in Prices of Domestic Appliances

Replacing expression (2.17) in the RHS of equation (2.11), and assuming again that fixed costs\(^{20}\) are relatively large and that $C^{HF}(i, x, w, p_d) - xC^V_x(i, x, w, p_d) \approx 0$, if follows that

$$k - \frac{\phi(k)}{\phi'(k)} \approx \frac{C^{HF}(i, w, p_d)}{w} = \frac{wL^{HF}(i, \frac{w}{p_d}) + p_dD^{HF}(i, \frac{w}{p_d})}{w} = L^{HF}(i, \frac{w}{p_d}) + \frac{D^{HF}(i, \frac{w}{p_d})}{\frac{w}{p_d}}$$

(2.19)

where $L^{HF}(i, \frac{w}{p_d})$ and $D^{HF}(i, \frac{w}{p_d})$ are, respectively, the amounts of labor and domestic appliances required for the production of the fixed cost of homework of technology $i$. Expression (2.19) shows that, for each technology $i$, the ratio $C^{HF}(i, w, p_d)/w$ (and, therefore, $k$ in the RHS) is completely

\(^{20}\)The existence of fixed and variable costs in homework production can be formalized expressing $h(j, x) = h^F(j) + h^V(j, x)$, $j \in [0, 1]$, so that $H(i, x) = H^F(i) + H^V(i, x)$, where $H^F(i) = \int h^F(j) dj$ and $H^V(i, x) = \int h^V(j, x) dj$. The demand for labor and domestic appliances for the realization of task $j$ become $l(j, x, \frac{w}{p_d}) = l^F(j, \frac{w}{p_d}) + l^V(j, x, \frac{w}{p_d})$ and $d(j, x, \frac{w}{p_d}) = d^F(j, \frac{w}{p_d}) + d^V(j, x, \frac{w}{p_d})$, respectively, so the total demand functions for technology $i$ are $L^F(i, x, \frac{w}{p_d}) = L^F(i, \frac{w}{p_d}) + L^V(i, x, \frac{w}{p_d})$, and $D(i, x, \frac{w}{p_d}) = D^F(i, \frac{w}{p_d}) + D^V(i, x, \frac{w}{p_d})$. Finally, the homework fixed and variable cost functions associated with technology $i$ become $C^{HF}(i, w, p_d) = wL^{HF}(i, \frac{w}{p_d}) + p_dD^{HF}(i, \frac{w}{p_d})$ and $C^{HV}(i, x, w, p_d) = wL^{HV}(i, x, \frac{w}{p_d}) + p_dD^{HV}(i, x, \frac{w}{p_d})$, respectively.
determined by the relative price of $L$ and $D$, $\frac{w}{p_d}$. The implicit function theorem implies that

$$
\frac{\partial k(i, \frac{w}{p_d})}{\partial \frac{w}{p_d}} \approx \frac{\phi'(k)^2}{\phi(k)\phi''(k)} \frac{\partial}{\partial \frac{w}{p_d}} \left[ L^{HF}(i, \frac{w}{p_d}) + D^{HF}(i, \frac{w}{p_d}) \right]
$$

\begin{align*}
&= (> 0) \quad (< 0) \\
&< 0
\end{align*}

(2.20)

where the first factor is positive from the assumptions on $\phi$, and the second factor is negative from the assumptions made on $f$. Expression (2.20) implies that for a given technology, $i$, either an increase in the market wage or a fall in the price of domestic appliances, reduce the relative cost of homework production, and thus induce a decline in the number of individuals in the household.

**Time Devoted to Market and Home Work**

The fraction of time individuals devote to market and home work when using technology $i$ can be expressed as

$$
S^M(i, \frac{w}{p_d}) = 1 - \frac{L^H(i, x, \frac{w}{p_d})}{k} \approx 1 - \frac{L^{HF}(i, \frac{w}{p_d})}{k}
$$

\begin{align*}
&= 1 - \frac{\phi(k(i))}{k(i)\phi'(k(i))} + \frac{p_d D^{HF}(i, \frac{w}{p_d})}{wk(i)} \\
&\approx \frac{p(i)x}{wk(i)} + \frac{p_d D^{HF}(i, \frac{w}{p_d})}{wk(i)}
\end{align*}

(2.21)

where $\frac{L^H(i, x, \frac{w}{k})}{k}$ is the average time devoted to homework per individual in the household. The approximation in line 2 follows from the assumption that most of the homework associated with a particular technology is fixed (i.e. $L^{HV}(i, x, \frac{w}{p_d}) \approx 0$), the equality in line 3 is obtained from expression (2.19); and the approximation in line 4 follows from expression (2.4) and the assumption
that \( C_x(i, x, w, p_d) \approx p(i) \). It can be seen from expression (2.21) that the share of time devoted to market work is the sum of the *time-equivalent* per capita cost of the two market goods used in the process of production, namely, the cost of intermediate goods \( i, \frac{p(i)x}{w} \), and the cost of domestic appliances, \( \frac{p_d D^{HF}(i, \frac{w}{p_d})}{w} \).

How \( s^M(i, \frac{w}{p_d}) \) responds to an increase in the relative price \( \frac{w}{p_d} \) depends on two key elements: the convexity of \( \phi \), and the elasticity of substitution between labor and capital in homework production. For a given technology, \( i \), the increment in effective consumption, \( \hat{x}(x, k) = \frac{x - k}{k \phi(k)} \), can take place through either an increment in per capita consumption, \( \frac{x}{k} \), or a reduction of the "household-size adjustment factor", \( \frac{k}{\phi(k)} \), or both. The assumptions made on \( \phi \) imply that reducing \( k \) has a positive effect on the ratio \( \frac{k}{\phi(k)} \) that is decreasing as \( k \) approaches \( k^* \). Therefore,

1. **If the function \( \phi \) is strongly convex and \( k \) is relatively high** (so that \( \frac{d}{dk} \left( \frac{k}{\phi(k)} \right) \ll 0 \), even small reductions in \( k \) have strong positive effects in the ratio \( \frac{k}{\phi(k)} \) and, consequently, on \( \hat{x}(x, k) \). In this case, the increase in \( \hat{x}(x, k) \) resulting from an increase in \( \frac{w}{p_d} \) is obtained, first, through a reduction of the household size. Once \( k \) is sufficiently low (and, therefore, \( \frac{d}{dk} \left( \frac{w}{\phi(k)} \right) \approx 0 \), the increase in \( \hat{x}(x, k) \) is achieved through an increment in per capita consumption, \( \frac{x}{k} \). It follows then that:

(a) The first term in expression (2.21), \( \frac{p(i)x}{w} \frac{1}{k(i)} \), first decreases (remains constant) if the change in \( \frac{w}{p_d} \) is due to an increase in \( w \) (\( p_d \)), and then slowly increases as \( k \) approaches \( k^* \).

(b) The direction of the second term of expression (2.21), \( \frac{p_d D^{HF}(i, \frac{w}{p_d})}{w k(i)} \), as \( \frac{w}{p_d} \) rises, depends on the elasticity of substitution between \( L \) and \( D \) in the production of \( H^F(i, x) \). If the elasticity is high, the increment in \( \frac{w}{p_d} \) induces high substitution of labor for domestic appliances, which reinforces the increment in \( D^{HF}(i, \frac{w}{p_d}) \) and, combined with the decline
in $k$, makes the term $\frac{p_d}{w} \frac{D^{HF}(i, \frac{w}{p_d})}{k(i)}$ to (initially) increase. The opposite occurs if the elasticity is low. Eventually, as $k$ approaches $k^*$, the rise in $\frac{w}{p_d}$ makes the term $\frac{p_d}{w} \frac{D^{HF}(i, \frac{w}{p_d})}{k(i)}$ to decline.

(c) Which term determines the initial trend of $S^M(i, \frac{w}{p_d})$ as $\frac{w}{p_d}$ increases, depends on the technology used, $i$. If $i$ is low, the term $\frac{p_d}{w} \frac{D^{HF}(i, \frac{w}{p_d})}{k(i)}$ is initially larger and dominates the trend. The opposite occurs if $i$ is high. In any case, as $\frac{w}{p_d}$ grows, the first term becomes relatively larger and dominates the trend.

2. If, on the other hand, the convexity of the function $\phi$ is low (i.e. $\phi'' \geq 0$, so that $\frac{d}{dk} \left( \frac{k}{\phi(k)} \right) \leq 0$ even for high values of $k$) the increase in $\hat{x}(x, k)$ will take the form of a slow and steady decline in $k$, and an increase in $\frac{x}{k}$.

In summary, for a given technology $i$, the effect of an increase in the ratio $\frac{w}{p_d}$ on the share of time devoted to market work does not have a definite direction. When $\phi$ is strongly convex (i.e. individuals are very sensitive to the household size), most of the adjustment is made through a reduction $k$, and $S^M$ can initially fall if $w$ increases, or remain constant if $p_d$ decreases. In any case, if $\frac{w}{p_d}$ continues increasing, the household size will approach its minimum, $k^*$, and the share of time devoted to market work will eventually rise and converge to 1 for all $i \in [0, 1]$.

### 2.3.6 Choosing the Optimal Technology

The previous analysis considers the effects of raising market wages or decreasing prices of domestic appliances, on the household structure and the share of time devoted to market labor for each particular technology, $i \in [0, 1]$. An important question is, however, how changes in wages and prices of domestic appliances affect the relative preference of individuals for the different available technologies.
Consider, again, the case of two technologies (e.g. \(i_1\) and \(i_2\), \(i_1 < i_2\)) that initially allow individuals to attain the same level of effective consumption (i.e. \(\hat{x}(i_1, w, p_d) = \hat{x}(i_2, w, p_d)\)). Using expression (2.12), the level of effective consumption can be written as

\[
\hat{x}(i, w, p_d) = \frac{w}{\phi^\prime(k(i, w/p_d)) [p(i) + C^{HV}(i, x(i, w, p_d), w, p_d)]} \tag{2.22}
\]

As it was already explained, the denominator of expression (2.22) can be interpreted as the average unit cost of effective consumption, which must be equal for "equivalent" technologies such as \(i_1\) and \(i_2\). From expression (2.22), it can be seen that \(\hat{x}(i_1, w, p_d)\) and \(\hat{x}(i_2, w, p_d)\) will react in different ways to changes in \(w\) and \(p_d\) if, and only if, their denominators do so.

**The Effect of an Increase in the Market Wage**

Formally, the partial derivative of \(\hat{x}(i, w, p_d)\) with respect to \(w\) can be written as

\[
\frac{\partial \hat{x}(i, w, p_d)}{\partial w} \approx \frac{\hat{x}}{w} - \frac{\hat{x}}{p(i) + C^{HV}} \frac{\partial C^{HV}}{\partial w} - \frac{\hat{x}}{p_d} \phi \frac{\partial C^{HV}(i, w, p_d)}{\partial w} \tag{2.23}
\]

where the approximation follows the use of expressions (2.19) and (2.20) in the last term of the RHS. Expression (2.23) allows to analyze separately each of the three components of \(\frac{\partial \hat{x}(i, w, p_d)}{\partial w}\).

1. The first term, \(\frac{\hat{x}}{w}\), captures the *direct* effect on \(\hat{x}\) of increasing \(w\), which, given that \(\hat{x}(i_1, w, p_d) = \hat{x}(i_2, w, p_d)\), is equal for both technologies: \(0 < \frac{\hat{x}(i_1)}{w} = \frac{\hat{x}(i_2)}{w}\).

2. The second term, \(-\frac{\hat{x}}{p(i) + C^{HV}} \frac{\partial C^{HV}}{\partial w}\), captures the *indirect* effect on \(\hat{x}\) from changes in the marginal cost of \(x\). First, it was already assumed (and explained) that the marginal cost of \(x\) is positive and increasing in the degree of elaboration of the intermediate good (i.e. \(C_{xi}(i, x) > 0\)), which implies that \(0 < p(i_1) + C^{HV}(i_1) < p(i_2) + C^{HV}(i_2)\). Second, the
variable part of homework associated with technologies $i_1$ can be expressed as

$$H^V(i_1, x(i_1)) = \frac{1}{i_1} \int_{i_2}^{i_2} h^V(j, x(i_1)) dj$$

$$= \int_{i_1}^{i_2} h^V(j, x(i_1)) dj + H^V(i_2, x(i_1))$$

$$> \int_{i_1}^{i_2} h^V(j, x(i_1)) dj + H^V(i_2, x(i_2))$$

$$> H^V(i_2, x(i_2))$$ (2.24)

where the first inequality is due to the fact that $x(i_1) > x(i_2)$, and therefore $H^V(i_2, x(i_1)) > H^V(i_2, x(i_2))$. Expression (2.24) implies that the variable labor associated with technology $i_1$ is strictly higher: $L^{HV}(i_1) > L^{HV}(i_2)$, which implies that, the effect of an increase in wage in the marginal cost of homework will be higher for technology $i_1$: $0 < \frac{\partial C^{HV}(i_2)}{\partial w} < \frac{\partial C^{HV}(i_1)}{\partial w}$.

It follows, then, that the second term of expression (2.23) is negative for both technologies, and such that

$$-\frac{\hat{x}_i}{p(i_1) + C^{HV}(i_1)} \frac{\partial C^{HV}(i_1)}{\partial w} < \frac{\hat{x}_i}{p(i_2) + C^{HV}(i_2)} \frac{\partial C^{HV}(i_2)}{\partial w} < 0$$.

The intuition is simple: an increase in the market wage has a larger positive impact on the opportunity cost of the technology that uses more labor, and makes, therefore, relatively more attractive the use of technologies with more elaborated intermediate goods that require less labor for homework.

3. The third term of expression (2.23), $-\frac{\hat{x}}{p_i} \phi' \frac{\partial C^{HF}(i, w, p_d)}{\partial w}$, captures the indirect effect of $w$ on $\hat{x}$ through changes in the household size. As it was already seen, the assumptions on $f$ imply that the fixed cost of homework production measured in units of time, $C^{HF}(i, w, p_d) = L^{HF}(i, \frac{w}{p_d}) + \frac{D^{HF}(i, \frac{w}{p_d})}{p_d}$, depends negatively on the relative price $\frac{w}{p_d}$. The fact that homework production is the sum of a continuous of tasks that increase the degree of elaboration of an intermediate good, allows to show that $C^{HF}(i_1, w, p_d) = C^{HF}(i_2, w, p_d) + \tilde{C}^{HF}(i_1, i_2, w, p_d)$, where $\tilde{C}^{HF}(i_1, i_2, w, p_d) > 0$ is the fixed cost of homework necessary to transform good $i_1$
into good \( i_2 \), for any \( i_1 < i_2 \). It follows, then, that

\[
\frac{\partial C^{HF}(i_1,w,p_d)}{\partial w} = \frac{\partial C^{HF}(i_2,w,p_d)}{\partial w} + \frac{\partial C^{HF}(i_1,i_2,w,p_d)}{\partial p_d} < 0
\]

where the inequality in the second line follows that \( \frac{\partial C^{HF}(i_1,i_2,w,p_d)}{\partial p_d} < 0 \) (again, from the assumptions on \( f \)). Expression (2.25) simply shows that an increase in \( w \) or a decrease in \( p_d \) will induce a larger fall in the ratio \( C^{HF}/w \) (and, therefore, a larger fall in the household size) in technologies that are more intensive in homework (i.e. lower \( i \)).

On the other hand, it was already shown that \( k(i) \) is strictly decreasing in \( i \), which implies that \( k(i_1) > k(i_2) \). Whether the term \( \frac{\phi'}{\phi} \) is increasing or not in \( k \) depends on the convexity of \( \phi \). In particular,

\[
-\frac{d\phi'}{dk} = \frac{\phi''}{\phi} - \frac{(\phi')^2}{\phi^2} > 0 \iff \phi'' > \frac{(\phi')^2}{\phi}
\]

so that \( \frac{\phi'(k(i_1))}{\phi(k(i_1))} > \frac{\phi'(k(i_2))}{\phi(k(i_2))} \) if, and only if, \( \phi \) is sufficiently convex (i.e. \( \phi'' > \frac{(\phi')^2}{\phi} > 0 \)).

Expressions (2.25) and (2.26) imply that the third term of expression (2.23), \( -\frac{\phi'}{p_d} \frac{\partial C^{HF}(i,w,p_d)}{\partial w} \), is: first, positive for all \( i \); and, second, decreasing in \( i \) if \( \phi \) is sufficiently convex, but increasing in \( i \) otherwise (e.g. if \( 0 < \phi'' \ll \frac{(\phi')^2}{\phi} \)). The intuition is the following: the final effect on \( \phi'(k) \) from the fall in \( k \) induced by the rise in \( w \), depends on the slope of \( \phi'(k) \), namely, \( \phi''(k) \).

Specifically, while the marginal benefit of reducing the household (i.e. the fall in \( \phi'(k) \)) will in general be larger for technologies with lower \( i \) (that experience a larger fall in \( k \)), it can
happen the opposite if \( \phi''(k) \) decreases too fast with \( k \), in which case the relative fall of \( \phi'(k) \) will be larger for technologies with higher \( i \), associated with smaller households.

The final effect of an increase in the market wage, \( w \), on the relative preference of individuals for technologies that, given the market prices: \( w \), \( p_d \), and \( p(i) \forall i \in [0,1] \), allow to initially attain the same level of effective consumption, \( \hat{x} \), depends on the combined effects of the second and third terms of expression (2.23). The preference for technologies that use less elaborated goods will increase if there is a high degree of substitution between \( L \) and \( D \) in homework production (in particular for lower \( i \)'s) and the function \( \phi \) is convex enough. In this case, the high degree of substitution between \( L \) and \( D \) moderates the relative increase in the marginal cost of production for technologies more intensive in homework and induces, at the same time, a larger fall in \( k \) which, given the high convexity of \( \phi \), increases relatively more the effective consumption of individuals in larger households. The preference for technologies that use more elaborated goods, on the other hand, will increase if there is a low degree of substitution between \( L \) and \( D \) in homework production (in particular for lower \( i \)'s) and the function \( \phi \) is not sufficiently convex. In this case, the low substitution between \( L \) and \( D \) make the marginal cost of production to increase substantially, particularly for technologies relatively intensive in homework. The fall in the ratio \( \frac{C_{HF}(i_2,w,p_d)}{w} \) (and the subsequent fall in \( k \)), will be more moderated, which, given the low convexity of \( \phi \), increases relatively more the effective consumption of individuals in smaller households.

Some intuition can be gained from the fact that effective consumption can be augmented by either increasing per capita average consumption, \( \frac{\hat{x}}{k} \), decreasing the household-size adjustment factor, \( \frac{k}{\phi'(k)} \), or both. In the first case, the high convexity of \( \phi \) and the large substitutability between labor and domestic appliances, imply that it is more cost-effective to save by substituting labor by appliances in homework production, and to increase effective consumption mainly by reducing the household size (which has a high impact on \( \frac{k}{\phi'(k)} \)). In the second case, on the other hand, the
low convexity of \( \phi \) and the limited substitutability between labor and domestic appliances, imply that it is more cost-effective to save by reducing the use of homework, and to increase effective consumption mainly through increments in per capita consumption, \( \frac{\hat{\phi}}{k} \).

The Effect of a Fall in the Price of Domestic Appliances

The effect of a fall in the price of domestic appliances, \( p_d \), on the individual’s choice of technology can be analyzed by looking at the partial derivative of \( \hat{x}(i, w, p_d) \) with respect to \( p_d \):

\[
\frac{\partial \hat{x}(i, w, p_d)}{\partial p_d} \approx -\frac{\hat{x}}{p(i) + C^H^V} \frac{\partial C^H^V}{\partial p_d} + \frac{\hat{x}}{p_d^2} \frac{\phi'}{\phi} \frac{\partial C^{HF}(i, w, p_d)}{\partial p_d} \tag{2.27}
\]

where, again, the approximation follows the use of expressions (2.19) and (2.20) in the last term of the RHS of expression (2.27). The two terms in the RHS of the expression are similar to the two last terms of expression (2.23), and decompose the effect of the change in \( p_d \) on each of the factors of the average cost of effective consumption, \( [p(i) + C^H^V] \phi'(k(i)) \).

1. The term \( -\frac{\hat{x}}{p(i) + C^H^V} \frac{\partial C^H^V}{\partial p_d} \), captures the effect on \( \hat{x} \) from changes in the marginal cost of \( x \) that follow an increase in \( p_d \). Following a similar analysis as in the case of \( -\frac{\hat{x}}{p(i) + C^H^V} \frac{\partial C^H^V}{\partial p_d} \) in expression (2.23), it can be shown that 

\[
-\frac{\hat{x}}{p(i_1) + C^H^V(i_1)} \frac{\partial C^H^V(i_1)}{\partial p_d} < -\frac{\hat{x}}{p(i_2) + C^H^V(i_2)} \frac{\partial C^H^V(i_2)}{\partial p_d} < 0,
\]

for any \( i_1, i_2, 0 \leq i_1 < i_2 \leq 1 \), which implies that a decrease in \( p_d \) (that is, a negative change in \( p_d \)) generates a larger reduction in the cost of homework of the technology that uses more domestic appliances, and makes, therefore, relatively more attractive the use of technologies with less elaborated intermediate goods. (Notice that the effect is opposed to what an increase in \( w \) causes).

2. The term \( \frac{\hat{x} w \phi'}{p_d} \frac{\partial C^{HF}(i, w, p_d)}{\partial p_d} \), captures the effect of an increase in \( p_d \) on \( \hat{x} \) through changes in the household size. Similar to the term \( -\frac{\hat{x}}{p_d} \frac{\phi'}{\phi} \frac{\partial C^{HF}(i, w, p_d)}{\partial p_d} \) in expression (2.23), expression
is positive for any negative change in \( p_d \), and is decreasing in \( i \) only if \( \phi \) is strongly convex.

The combination of both terms implies that, under most circumstances, a fall in \( p_d \) increases the relative preference for technologies that use less elaborated intermediate goods and are, therefore, more intensive in homework. The effect could only be in the opposite direction (i.e. increasing the preference for more elaborated market goods) if the convexity of the function \( \phi \) is very low (e.g. if \( 0 \lesssim \phi'' \ll \left(\frac{\phi'}{\phi}\right)^2 \)) and the elasticity between \( L \) and \( D \) in homework production is extremely limited (this is less likely than in the case of an increase in \( w \), given that the change in the term \( \frac{\hat{w}}{p_l} \partial \hat{C}^{HV} / \partial p_d \) is now in favor of the use of more homework).

Intuitively, the fall in the price of domestic appliances lowers the cost of homework, which allows individuals to use more homework-intensive technologies and, at the same time, maintain or even reduce the number of individuals in the household.

### 2.3.7 Summary of Predictions

The main predictions of the model with respect to the effects of the three dimensions of technological change analyzed here can be summarized as follows:

*The decline in the relative price of more elaborated goods and services* increases the relative preference for the technologies that use those goods as inputs, and reduces the amount of homework performed by the household; reduces the size of the household and the use domestic appliances, and increases the share of time devoted to market work.

*The rise in market wage as well as the fall in the price of domestic appliances* have the same effect (through changes in the relative price \( \frac{w}{p_d} \)) on the relative use labor and domestic appliances in homework production (i.e. they increase the use of appliances and decrease the use of labor) for a given technology. They can have, however, opposite effects on the relative preference
for goods with different degree of elaboration. An increase in the market wage represents an improvement of the labor productivity in the market that makes individuals more likely to switch to technologies that are less intensive in homework and use more elaborated market goods. The fall in the price of domestic appliances, on the other hand, can be interpreted as an increase of the labor productivity in the household that, almost invariably, will make individuals to switch to technologies that use less elaborated market goods and are more intensive in homework. In both cases (though for different reasons), if the initial household size is relatively high (i.e. $k > k^*$), the adjustment will initially take the form of a reduction in the household size, and the share of time devoted to market work will remain constant or may even decrease. Eventually, as the household size approaches its minimum, $k^*$, further increments in the relative price $\frac{w}{p_d}$ will increase the share of time devoted to market work.

### 2.4 Conclusions

In this paper I analyze how different aspects of technological progress affect the structure of the household and its role as a unit of production. Specifically, I study how individuals’ decisions on the household size (understood here as the number of adults in the household), the allocation of time between market and home work, and the degree of elaboration of the purchased market goods, are affected by three key dimensions of technological progress, namely, the rise in market wages, the decline in the price of domestic appliances, and the increase in the access to more elaborated goods and services. I describe the experience of the United States during the twentieth century, which is characterized, first (1900-1960), by a large decrease in the number of adults per household, a boom in the adoption of basic housing facilities and domestic appliances, and a flat -or even decreasing- labor force participation; and, second (1960-2000), by a much slower decrease in the size of the household and a sharp increase in the labor force participation rate. When analyzed in light of the
changes experienced in the three dimensions of technological progress during the same period, the experience of the U.S. is in line with the differences currently observed in a diverse cross section of countries for which I present additional evidence.

In the proposed model, the production of the final consumption good comprises a *continuum of tasks* (or stages) that increase the degree of elaboration of the good until it is ready to be consumed. Individuals can produce the final good in different ways, using in each case a market good with a particular degree of elaboration, and adding the amount of *homework* that completes the process of elaboration. *Homework* is produced combining labor and domestic appliances. The existence of *fixed costs in homework* provides the incentives to form households for the joint production and consumption of the final good, and the fact that the marginal utility of consumption becomes decreasing in the household size once it surpasses a certain level, prevents the size of the households to increase indefinitely. According to the model, the three dimensions of technological progress induce, with different intensity, a reduction in the number of individuals per household. The effects on the degree of elaboration of purchased market goods and on the share of time devoted to market work can be completely different, though. While the increase in real wages makes individuals more likely to use more elaborated market goods that require less homework to be consumed, the decline in the price of domestic goods increases the likelihood of using less elaborated goods that require more homework. In both cases, however, the fraction of time devoted to market work will initially remain constant or decline. It is just when the household size is close enough to its minimum that the share of time in market work will increase. Finally, the decline in the price of market goods with high degree of elaboration induces, simultaneously, a fall in the household size, an increase in the use of market goods that require less homework to be consumed, and an increase in the fraction of time in market work.

Here, I present a very general version of the model with the purpose of identifying the
assumptions and conditions that generate the desired results. Future research should explore the qualitative and quantitative implications of both, assuming specific functional forms, and using a dynamic setting. The conception of the process of production as a continuum of tasks that can be performed by different agents in the market or at home, and the introduction of the degree of elaboration of the purchased goods as a new dimension of the individual’s problem are, definitely, the most novel features that the model presents. There is ample room for research exploring different implications of this approach, as well as applications of the same type of model to other contexts. Finally, the model proposed here provides a mechanism for the formation households, which is a very specific social structure. The same framework, however, based on the existence of relatively large fixed costs and economies of scale for the production of particular goods and services, can be used to explain the formation of other social structures and associations of individuals, such as the extended household, clubs, or different types of communities.
Chapter 3

Aggregate Volatility, the Composition of Investment, and the Pattern of Sectoral Production

3.1 Introduction

Are countries’ patterns of investment and production affected by aggregate volatility? During the last three decades, much attention has been paid to the study of the links between aggregate volatility, investment, and growth: many empirical works have documented a negative relationship between macroeconomic volatility and growth\(^1\), and a variety of theoretical arguments have been presented supporting the existence of both, positive as well as negative effects of volatility on economic growth\(^2\). However, while most of these works have focused on the effects of volatility on the timing and quantity of aggregate investment and output along the business cycle, little has been said about its effects on the quality (or composition) of the investment. Furthermore, while the literature that studies the relationship between sectoral production, volatility, and growth usually emphasizes the effects that relying on particular sector (such as the production of commodities based on natural resources) has on growth and the volatility of output, the effects in the opposite direction, that is, the effects of aggregate volatility on sectoral production, have not been analyzed. The focus of this work is, precisely, on the potential effects of exogenous aggregate uncertainty on both, the composition of investment, and the pattern of sectoral production of the economy.

The basic idea explored in this paper is the following: in an environment in which investors can choose from a set of investment alternatives with different degree of reversibility, aggregate

\(^1\)Kormendi & Meguire (1985), Ramey & Ramey (1995), Martin & Rogers (2000), Macri & Sinha (2000), Hnatkovska & Loayza (2004), Imbs (2007), Van der Ploeg & Poelhekke (2009), Aghion et al. (2010), and Fatás & Mihov (2013) among others. Some authors argue, however, that the evidence is still unconvincing (see, for example, Edwards & Yang (2009)).

volatility can increase the preference for the more versatile or "flexible" ones, which offer the investor more alternatives to revise his decision in response to the variable macroeconomic conditions (as opposed to the more irreversible –presumably more firm- or industry-specific– investments). When the volatility is permanently high, the bias toward more versatile investment opportunities can seriously affect the types of economic activities developed in the economy. From a macro, aggregate perspective, the key assumption is that aggregate production can be performed by combining different factors of production (other than labor), whose marginal returns are affected in different ways by aggregate uncertainty. In this context, differences in the volatility of aggregate productivity can induce differences in the relative use of production factors, and therefore in the pattern of production of the economy.

The main features of the proposed mechanism are analyzed with two simple models. The first model represents a small open economy that produces and trades two sectoral goods: a commodity whose production depends on a fixed factor (e.g. natural resources), and manufactures that are produced with capital and labor. The model predicts that, with any strictly concave utility function, higher (relative) volatility of aggregate productivity increases the reliance of the economy on the sector dependent on natural resources, and that the effect is relatively stronger in both, economies with lower average productivity, and economies with larger per capita endowments of natural resources. The second model, on the other hand, represents a closed economy that produces two sectoral goods whose production depends on the use of specific types of capital that differ in their degree of reversibility: while in one of the sectors, investment must be decided before the realization of the aggregate shock and is irreversible, the decision of investing in the other sector is made after observing aggregate productivity. The model predicts that, with sufficiently concave preferences, higher (relative) aggregate volatility increases the reliance of the economy on the sector with reversible capital. In both models, the change in the composition of investment is increasing
in the concavity of preferences, and is affected by the values used for the sectoral elasticities of substitution. Interesting, the change in the pattern of production induced by aggregate volatility is stronger in the small open economy, given that the prices of its sectoral goods (and therefore the composition of its consumption basket) are exogenously determined in the international market. This is an important and somehow controversial result, given that the literature has often emphasized that the potential negative effects of volatility on investment and growth are less important in open economies, provided that they are more likely to have access to the international financial market. In both models volatility can hurt growth in the short run by inducing an otherwise inefficient allocation resources across sectors. The negative effects on long term growth can be even higher if, for example, the sector with more irreversible capital is associated with endogenous growth à la Romer.

The paper is part of a large, broader body of literature that analyzes different aspects of the relationships between volatility, investment, and growth. In the empirical literature, an increasing number of papers has documented a negative relationship between aggregate volatility and growth for different sets of countries and periods, and using different econometric techniques. The documented negative relationship is robust in most cases to different sets of controls, including the ratio of investment to GDP. The works of Kormendi & Meguire (1985), Ramey & Ramey (1995), Martin & Rogers (2000), Macri & Sinha (2000), Hnatkovska & Loayza (2004), Imbs (2007), Van der Ploeg & Poelhekke (2009), Aghion et al. (2010), and Fatás & Mihov (2013), among others, are in this line. In an exhaustive work on the link between natural resources, volatility, and growth, Van der Ploeg & Poelhekke (2009) show that countries rich in natural resources are exposed to higher aggregate volatility. The conclusions of this paper imply that, the higher volatility faced by such countries can potentially exacerbate their dependence on natural resources, further difficulting the diversification of their productive structure.
Many theoretical works in this area provide support to both, positive and negative links between volatility and growth. Among the most related to this paper are Bernanke (1983), Pindyck (1991), and Ramey & Ramey (1991). Bernanke (1983) explains how uncertainty may increase the returns to waiting for better information, and therefore contribute with the amplification of the business cycle. Pindyck (1991), on the other hand, formalizes the equivalence between irreversible investments and options, and shows how their value depend on the specific parameterization of the model. Both papers use partial equilibrium models to analyze the role of uncertainty on the timing of irreversible investments that can be delayed. In contrast, this work focuses on the quality of investment in a general equilibrium framework. Other related papers are Ramey & Ramey (1991), who propose a model in which the decision about the technology used has to be made before the realization of the shock; Dixit & Rob (1994), who analyze the effect of uncertainty on the allocation of labor when switching costs are sunk; Jones et al. (2000), who show that in the neoclassical model of growth the relationship between aggregate volatility is positive if the curvature of preferences is at least as high as the log (through an increase in precautionary saving), and negative otherwise; and Bloom (2009), who studies the impact of uncertainty shocks on firm’s decisions of hiring and investing in a framework with convex and non-convex adjustment cost in capital and labor. Finally, closest in spirit with this paper is the work of Aghion et al. (2010), who look at the effect of uncertainty on the cyclical composition of investment (short-term and long-term investments) in an economy with financial constraints. In their model, long-term investments contribute more to productivity growth but are more exposed to liquidity shocks, which, under sufficiently tight credit constraints, can turn procyclical the share of long-term investment, eventually leading to both lower mean growth and amplified volatility. In contrast, this paper explains the differences in patterns of investment and production as a consequence of initial differences in aggregate volatility, independently of the degree of financial development. Additionally, while the type of shock used
by Aghion et al. (2010) makes their mechanism to depend critically on the "time-to-build" of the different investment projects, the mechanism proposed here is more general and simply requires that the return of the different investment projects is affected differently by aggregate volatility.

3.2 Model

In this section I analyze the effects of aggregate volatility in two different settings. The first model represents a two-sector (agriculture and manufacturing), small, open economy model, in which agricultural production requires the use of a fixed factor whose endowment is exogenously given. The model analyzes the effects of aggregate volatility on investment in manufacturing. The second model represents a closed economy, with two sectors that differ in the type of capital used for production: while capital used in one of the sectors requires "time to build", and must be decided before the realization of the aggregate shock, capital in the other sector can be decided once aggregate productivity is known. The model analyzes how the composition of total investment is affected by the volatility of the aggregate shock.

3.2.1 Two-Sector, Small, Open Economy

Consider static, small, open economy with two sectoral goods: agriculture (A) and manufactures (M), an a final good (Y), populated by a continuous of identical households of mass 1 that supply inelastically 1 unit of labor.

Production. Production in the agricultural sector is performed by a competitive firm with technology $Y^A = A^A G^A(F, L^A)$, where $F$ is a fixed factor of production (e.g. land), and $L^A$ is labor in the sector. Similarly, production of manufactures is performed by a competitive firm with technology represented by $Y^M = A^M G^M(K, L^M)$, where $K$ and $L^M$ are capital and labor in the sector,
respectively. Both firms can sell their production domestically or in the international market, where prices are \( p^A \) and \( p^M \). Sectoral productivities, \( A^T(s), T = A, M \), are stochastic across states \( s \in S \).

Investment and consumption are made in terms of final good, \( Y \), which is produced by a competitive firms according to \( Y = G(\bar{Y}^A, \bar{Y}^M) \), where \( \bar{Y}^A, \bar{Y}^M \) are the amounts of agricultural and manufacturing goods domestically used. The associated price of the final good is \( p = p(p^A, p^M) \).

Functions \( G^A, G^M, \) and \( G \) are homogenous of degree one, and strictly increasing and concave in each of their arguments.

Firms, as well as the fixed factor, \( F \), are equally owned by the households.

**Timing** There are two periods. In the first period, investment in manufacturing, \( K \), is decided, before the state of nature, \( s \), is revealed. That is, households decide how much to invest before observing the levels of productivity, \( A^A(s) \) and \( A^M(s) \). The state of nature, \( s \), is revealed at the beginning of the second period, after which, investment takes place, sectoral firms hire their inputs \( (L, F, K) \) in competitive markets, and production and consumption take place simultaneously (as is the case in any static model). Notice that investment, though decided in period 1, is performed in period 2. A key assumption is that the investment decision taken in the first period is irreversible.

**Preferences** Households’ expected utility is given by \( U = \sum_{s \in S} \Pr(s)u(c(s)) \), where \( \Pr(s) \) is the probability that state \( s \) occurs, \( u \) is a standard utility function, and \( c(s) \) is consumption of final good.

**Problem of the Firms** After the realization of \( s \) (and therefore of \( A^A(s) \) and \( A^M(s) \)), the firm in the agricultural sector chooses \( F \) and \( L^A \) that maximize its profits. Specifically,

\[
\max_{F, L^A} p^A A^A g^A(F, L^A) - p^F F - w L^A
\]
where \( p^F \) is the fixed factor’s price, and \( w \) is the wage.

Similarly, the firm in the manufacturing sector chooses \( K \) and \( L^M \) that maximize its profits. Specifically,

\[
\max_{K,L^M} p^M A^M G^M(K, L^M) - p^K K - wL^M
\]

where \( p^K \) is rental price of capital.

Finally, the firm producing the final good solves

\[
\max_{\hat{Y}^A, \hat{Y}^M} pG(\hat{Y}^A, \hat{Y}^M) - p^A \hat{Y}^A - p^M \hat{Y}^M
\]

The assumptions made on \( G^A, G^A \), and \( G \) imply that profits of each of the firms are zero for all \( s \in S \).

**Problem of the Households**  The representative household chooses the amount of capital to be invested, \( K \), and the vector of consumptions, \( \{C(s)\}_{s \in S} \) that maximize its expected utility, \( U \).

The problem can be written as

\[
\max_{K, \{C(s)\}_{s \in S}} \sum_{s \in S} \Pr(s) u(C(s))
\]

subject to the budget constraint:

\[
p[C(s) + K] \leq w(s) + p^F(s)F + p^K(s)K \quad \text{for all } s \in S
\]

where the income of the household in each state, \( s \), is composed by its labor earnings, \( w(s) \), the fixed factor’s rents, \( p^F(s)F \), and the rents from capital, \( p^K(s)K \).
Market Clearing and Trade Balance Conditions  Equilibrium in the final good market implies

\[ C(s) + K = Y(s) \text{ for all } s \in S \]

The labor market clearing condition is

\[ 1 = L^A(s) + L^M(s) \text{ for all } s \in S \]

and the trade balance condition is

\[ p^A [Y^A(s) - \bar{Y}^A(s)] + p^M [Y^M(s) - \bar{Y}^M(s)] = 0 \text{ for all } s \in S \]

Equilibrium - Labor Allocation  Equilibrium in the labor market, and the equalization of the value of marginal productivity of labor in agriculture and manufacturing (to the market wage), implies that

\[ \frac{G^M_L(K, L^M)}{G^A_L(F, 1 - L^M)} = \frac{p^A}{p^M} \frac{A^A(s)}{A^M(s)} \text{ for all } s \in S \]  

(3.2)

which, given \( F, p^A, \) and \( p^M, \) implies that the share of employment in manufacturing, \( L^M(K, s), \) depends positively on the stock of capital, \( K, \) and the ratio of sectoral productivities, \( \frac{A^M(s)}{A^A(s)}, \) realized in each particular state, \( s. \)

Aggregate Uncertainty  Assume additionally that uncertainty is aggregate, that is, that the change in absolute sectoral productivities across states, \( s \in S, \) is proportional to the change in the economy’s average productivity, so that relative sectoral productivities remain unchanged for all
\[ A^T(s) = A(s) \bar{A}^T, \quad T = A, M, \quad s \in S \quad (3.3) \]

which implies that \( \frac{A^A(s)}{A^M(s)} = \frac{\bar{A}^A}{\bar{A}^M} \) for all \( s \in S \).

It follows from this assumption and from equation (3.2), that the labor allocations, \( L^A(K) \) and \( L^M(K) \), depend only on the stock of capital chosen in period 1, and are independent of \( s \).

**Consumption** Using the budget constraint, (3.1), consumption of the households in state \( s \) can be expressed as

\[
C(K, s) = \frac{1}{p} \left[ w(s) + p^F(s)F + p^K(s)K \right] - K
= \frac{1}{p} \left[ w(s) + p^F(s)F \right] + \left[ \frac{p^K(s)}{p} - 1 \right] K
\quad (3.4)
\]

Expression (3.4) shows that the net, real return of investing one unit of capital, \( \frac{p^K(s)}{p} - 1 \), is stochastic. The reason of this is that the decision regarding the amount invested, \( K \), is irreversible, so the amount invested neither can be increased when the return is strictly positive, \( \frac{p^K(s)}{p} - 1 > 0 \) (i.e. in "good" states), nor can be decreased when the return is negative (i.e. in "bad" states).

The derivative of \( C(K, s) \) with respect to \( K \) is

\[
\frac{\partial C(K, s)}{\partial K} = \frac{p^K(s)}{p} - 1
= \frac{A(s)}{p} p^M \bar{A}^M G^M_K(K, L^M) - 1
\quad (3.5)
\]

where the first order condition of the firm, \( p^K(s) = p^M A(s) \bar{A}^M G^M_K(K, L^M) \), was used in the second
line of expression (3.5).

**Optimal Investment** Using expression (3.4) for $C(K, s)$, the problem of the household consists in choosing $K$ that maximizes its expected utility, that is

$$\max_K \sum_{s \in S} \Pr(s)u(C(K, s))$$

Using expression (3.5), the first order condition of the problem is

$$\sum_{s \in S} \Pr(s)u'(C(K, s)) \left[ \frac{A(s)}{p} p^M A^M G^M_K (K, L^M) - 1 \right] = 0 \quad (3.6)$$

which, as usual, shows that the value for households of each state’s return from capital is given by their marginal utility of consumption in that particular state. Households invest until the expected value of their marginal utility is zero. This decision depends on the volatility of the absolute return of the investment (given by the expression within the brackets in (3.6)), on the one hand, and on the responsiveness of the marginal utility to changes in consumption across states, on the other.

**The Effect of Aggregate Volatility** Rearranging the first order condition, (3.6), it follows that

$$\frac{p^M}{p} A^M G^M_K (K, L^M) \sum_{s \in S} \varphi(K, s) A(s) = 1 \quad (3.7)$$

where the term $\varphi(K, s) \equiv \frac{\Pr(s)u'(C(K, s))}{\sum_{s \in S} \Pr(s)u'(C(K, s))}$ is the (kernel-) adjusted probability of state $s$.

Equation (3.7) allows to analyze the effect that exogenous aggregate volatility (i.e. the volatility of $A(s)$) has on investment in the manufacturing sector, $K$.

---

3Equation (3.7) can be written as $\sum_{s \in S} \Pr(s)m(s)R(s) = 1$, where $R(s) \equiv \frac{p^M A(s)^M G^M_K}{p}$ is the return of capital (in units of final good) in state $s$, and $m(s) \equiv \frac{u'(C(K, s))}{\sum_{s \in S} \Pr(s)u'(C(K, s))}$ is the pricing kernel.
The assumptions made on functions $G^M$ and $G^A$ imply that the term $\tilde{A}^M G^M_K (K, L^M)$ (i.e. the marginal productivity of capital in manufacturing) is decreasing in $K$, even when considering the adjustment in $L^M(K)$ induced by changes in $K$. It follows from equation (3.7), therefore, that $K$ depends positively on the term $\sum_{s \in S} \varphi(K, s)A(s)$, which is a kernel-adjusted weighted average of the aggregate productivity, $A(s)$, across states. The effect of aggregate volatility on $K$ is determined, then, by the response of $\sum_{s \in S} \varphi(K, s)A(s)$ to the variance of $A(s)$ which, as mentioned above, is given by the way the marginal utility of consumption varies across states.

The "sensitivity" of the marginal utility of consumption to the volatility of $A(s)$ depends on two key elements: the concavity of the utility function and the level of consumption. In particular, since (as it can be seen from expression (3.4)) consumption is increasing in $A(s)$, the weight, $\varphi(K, s)$, given to a state in which aggregate productivity is high (low) is lower (higher) than its probability of occurrence if, and only if, the utility function is strictly concave, and, therefore, the marginal utility of consumption is strictly decreasing. Furthermore, the stronger the concavity of $u$, the lower the weight assigned to "good" states. It follows that, everything else constant, higher volatility of aggregate productivity, $A$ (e.g. higher variance) is associated with lower investment and size of the manufacturing sector. This negative relationship is stronger the higher the concavity of preferences.

Additionally, for a given degree of (strict) concavity of $u$, the sensitivity of the marginal utility of consumption to changes in $A(s)$ (and, therefore, in consumption) depends on the absolute level consumption. Specifically, for a given coefficient of variation (i.e. a given relative standard deviation of the productivity), if the relative risk aversion is decreasing (constant), the ratio of investment in manufacturing to total income is increasing in the average level of productivity of the economy. It follows that, for a given level of relative aggregate volatility (say, a given ratio between the standard deviation to and the mean of $A$), countries with lower average productivities will invest relatively less in the manufacturing sector.
At the aggregate level, finally, the fall in investment in manufacturing sector associated with higher aggregate volatility implies a fall in the average value of aggregate production and, therefore, in per capita production and consumption. That fall is, however, optimal from the point of view of welfare in the current context. The economy produces and exports more agricultural goods (both, in absolute and relative terms), and produces less and imports more manufacturing goods.

An Example

In this section I present an example that allows to analyze quantitatively the predictions of the model. Specifically, I assume the following functional forms:

Utility Function: \( u(c) = \frac{c^{1-\sigma}}{1-\sigma} \), with \( \sigma \geq 0 \).

Final Good Production: \( Y = G(\bar{Y}^A, \bar{Y}^M) = (\bar{Y}^A)^{1-\alpha} (\bar{Y}^M)^{1-\alpha}, \alpha \in (0,1) \). The associated price of final good is \( p = \left( \frac{\bar{p}^A}{1-\alpha} \right)^{1-\alpha} \).

Sectoral Production: \( Y^A(s) = A(s) \bar{A}^A \left[ \alpha_F F^\rho + (1 - \alpha_F) (L^A)^\rho \right] \frac{1}{\bar{A}}, \) and \( Y^M(s) = A(s) \bar{A}^M \left[ \alpha_K K^\rho + (1 - \alpha_K) (L^M)^\rho \right] \frac{1}{\bar{A}}, \alpha_F, \alpha_K \in (0,1) \) and \( \rho = \frac{\eta - 1}{\eta} \), being \( \eta \geq 0 \) the elasticity of substitution between factors of production.

Aggregate Volatility: There are two states of nature, \( s \in \{ L, H \} \), such that \( A(L) = \bar{A}(1 - \varepsilon) \), and \( A(H) = \bar{A}(1 + \varepsilon) \), with \( \bar{A} > 0 \), and where \( \varepsilon \in [0,1] \) is the coefficient of variation of \( A(s) \), as well as of \( Y(s) \) (for a given \( K \)).

Parameters: I use \( \alpha = \alpha_K = \alpha_F = 0.5 \), \( p^A = p^M = 1 \) (which implies that \( p = 2 \)), and \( \bar{A}^A = \bar{A}^B = 1 \) in all the simulations presented below.
Simulations

Figure 3.1 presents various simulations of the small open economy analyzed above. The vertical axis plots, in all cases, the ratio between the equilibrium level investment in manufacturing for the given coefficient of variation, $K(\varepsilon)$, and the level of investment when there is no volatility (i.e. $\varepsilon = 0$), $K(0)$. The horizontal axis displays the different levels of $\varepsilon$. By construction, the ratios take value 1 for $\varepsilon = 0$. They decline with different intensity as the relative volatility of the aggregate productivity increases, depending on the particular set of parameters used. The benchmark case uses $\sigma = 1$ (i.e. logarithmic preferences), $\bar{A} = 1$ (average aggregate productivity), $F = 0.02$ (fixed factor endowment), and $\eta = 1$ (so that the production functions of sectoral goods are Cobb-Douglas). The cases presented in each graph of the figure compares simulations in which only one of these parameters is varied. In all cases, the comparisons are performed for the same values of the coefficient of variation of $A(s)$ (or relative standard deviation).

In Figure 3.1 (A), different values of $\sigma$ are used. As expected, $K$ remains unaffected as $\varepsilon$ increases only when $\sigma = 0$, that is, when utility is linear in consumption. In all other cases, increasing $\varepsilon$ reduces investment in manufacturing. The "distortion" introduced by aggregate volatility is increasing in $\sigma$, and is potentially strong when the concavity of $u$ is high, as is the case when $\sigma = 5$, where a coefficient of variation of 10% reduces investment in manufacturing by almost 18%, relative to the level with no uncertainty. The example shows that the demand for risky investments (as is the case of $K$) decreases with aggregate volatility, and that the fall is particularly strong when preferences are highly concave, which make individual’s marginal utility to be very sensitive to changes in consumption.

In the simulations of Figure 3.1 (B), different values for the average aggregate productivity, $\bar{A}$, are used. The example shows that, everything else constant, the lower average of aggregate
productivity, the stronger the negative (relative) effect of higher volatility on $K$. The first consequence from different levels of productivity, are different rates of investment and allocation of labor, which is independent of the level of volatility. Thus, higher $A$ implies higher labor productivity in both sectors, and higher productivity of capital, which induces higher investment and employment in manufacturing. The negative effect on $K$ of an increase in $\varepsilon$ comes, in this context, through changes in aggregate consumption, which affects its marginal utility. As explained above, this effect is relatively stronger the lower is $A$, given that the sensitivity of the marginal utility to changes in consumption is higher at lower levels of consumption.

Figure 3.1 (C) shows the differences in investment that arise from differences in the endowment of the fixed factor, $F$. As it can be seen, the higher $F$, the stronger the negative effect of volatility on $K$. The effect is symmetric to the case of differences in $\bar{A}$. Indeed, an increase in $F$ given an average level of productivity, $\bar{A}$, has a similar as a reduction in $\bar{A}$, for a given value of $F$. In this case, the differences in the relative effect of volatility is given by the fact that the absolute level of investment is lower in economies with higher $F$ and, therefore, even small changes in $K$ induced by higher volatility imply high changes in the relative level of capital (in comparison to its original level).

Finally, the simulations in Figure 3.1 (D) consider different elasticities of substitution in the CES sectoral production functions. As it can be seen in the figure, the relationship between aggregate volatility and investment in manufacturing is affected in a non-monotonic way by changes in the elasticity of substitution between labor and $F$ or $K$. When $\eta$ is low (e.g. $\eta = 0.6$), small changes in the allocation of labor have large impact on productivity of factors. The economy is therefore very sensitive to aggregate volatility. Higher $\eta$ (say, $\eta = 1$ or $\eta = 1.3$), allows the economy to change the allocation of labor in response to changes in the macroeconomic conditions and maintain, at the same time, high levels of marginal productivity. Eventually, with a sufficiently
high elasticity of substitution (e.g. \( \eta = 1.8 \)), most labor can be used in agriculture and at a high level of productivity. In that case, investment in manufacturing is very low and, therefore, very sensitive (in relative terms) to aggregate volatility.

The example allows to see how the negative effect of aggregate volatility on investment in manufacturing varies with the specific conditions of the economy. The results show that the effect can be potentially important in economies with low levels of aggregate productivity, large endowments of land (or, more generally, of natural resources), and sectoral production (agriculture and manufactures) with low elasticity of substitution between labor and land or capital. In such cases, investment in the industrial sector can be seriously affected by high levels of aggregate
volatility. In any case, higher concavity of preferences reinforce the negative effect.

3.2.2 Two-Sector, Closed Economy

In this section I explore the implications of aggregate volatility on investment in a setting slightly different to the one previously analyzed. I consider now a closed economy with two sectors that use different types of capital: in one of the sectors, capital must be decided before the realization of the aggregate shock; in the other sector, on the other hand, investment in capital is decided after observing aggregate productivity.

In particular, consider static, closed economy with two sectoral goods \((T = 1, 2)\) an a final good \((Y)\), populated by a continuous of identical households of mass 1 that supply inelastically 1 unit of labor.

Production Production in both sectors is performed by competitive firms with technologies \(Y^T = A^T G^T(K^T, L^T)\), where \(K^T\) and \(L^T\) are capital and labor in sector \(T\), respectively. Firms sell their production domestically at prices \(p^T\), \(T = 1, 2\). Sectoral productivities, \(A^T(s)\), \(T = 1, 2\), are stochastic across states \(s \in S\).

Investment and consumption are made in terms of final good, \(Y\), which is produced by a competitive firms according to \(Y = G(\tilde{Y}^1, \tilde{Y}^2)\), where \(\tilde{Y}^1\) and \(\tilde{Y}^2\) are the amounts of sectoral goods 1 and 2 consumed domestically. The associated price of the final good is \(p = p(p^1, p^2)\).

Functions \(G^1\), \(G^2\), and \(G\) are homogenous of degree one, and strictly increasing and concave in each of their arguments.

Firms are equally owned by the households.

Timing As in the previous case, there are two periods. In the first period, investment in sector 1, \(K^1\), is decided, before the state of nature, \(s\), is revealed (that is, before observing the levels of
productivity, $A_1(s)$ and $A_2(s)$). The state of nature, $s$, is revealed at the beginning of the second period, after which, investment in sector 2, $K^2$, is decided. In period 2 investment in both sectors is performed, sectoral firms hire their inputs ($L^1, K^1, L^2, K^2$) in competitive markets, and production and consumption take place simultaneously. Notice that investment in sector 1, $K^1$, though decided in period 1, is performed in period 2. A key assumption is that the investment decision taken in the first period is irreversible.

**Aggregate Uncertainty**  Again, assume that uncertainty is aggregate, so that relative sectoral productivities remain unchanged for all $s \in S$. Formally, assume that

$$A^T(s) = A(s) \tilde{A}^T, \quad T = 1, 2, \ s \in S \quad (3.8)$$

which implies that $\frac{A_1(s)}{A_2(s)} = \frac{\tilde{A}_1}{\tilde{A}_2}$ for all $s \in S$.

**Preferences**  Similar to the previous case, households’ expected utility is given by $U = \sum_{s \in S} \Pr(s) u(c(s))$, where $\Pr(s)$ is the probability that state $s$ occurs, $u$ is a standard utility function, and $c(s)$ is consumption of final good.

**Problem of the Firms** After the realization of $s$ (and therefore observing $A_1(s)$ and $A_2(s)$) and taking factor prices as given, sectoral firms choose capital and labor, $\{K^T, L^T\}$ $T = 1, 2$, that maximize their profits. Specifically, the firm in sector 1 solves

$$\max_{K^1, L^1} p^1 A_1 G_1(K^1, L^1) - p^{K_1} K^1 - w L^1$$
where $p^{K1}$ is rental price of type-1 capital, $K^1$, and $w$ is the market wage. The firm in the sector $2$ solves

$$\max_{K^2,L^2} p^2 A^2 G^2(K^2, L^2) - pK^2 - wL^2$$

where the price of type-2 capital, $K^2$, is the price of the final good, $p$.

Finally, the firm producing the final good solves

$$\max_{\tilde{Y}^1,\tilde{Y}^2} pG(\tilde{Y}^1, \tilde{Y}^2) - p^1\tilde{Y}^1 - p^2\tilde{Y}^2$$

The assumptions made on $G^1$, $G^2$, and $G$ imply that profits of each of the firms are zero for all $s \in S$.

**Problem of the Households** The representative household chooses the amount of capital to be invested in sector 1, $K^1$ and the vector of sector-2 investments and consumptions, $\{K^2(s), C(s)\}_{s \in S}$, that maximize its expected utility, $U$. The problem can be written as

$$\max_{K^1,\{K^2(s), C(s)\}, \{s \in S\}} \sum_{s \in S} \Pr(s) u(C(s))$$

subject to the budget constraint:

$$p(s) [C(s) + K^1 + K^2(s)] \leq w(s) + p^{K1}(s) K^1 + p(s) K^2(s) \text{ for all } s \in S$$

where the income of the household in each state, $s$, is composed by its labor earnings, $w(s)$ and the rents from capital, $p^{K1}(s) K^1$ and $p(s) K^2(s)$. 

131
Market Clearing Conditions  Equilibrium in the final good market implies

\[ C(s) + K^1 + K^2(s) = Y(s) \text{ for all } s \in S \]

The labor market clearing condition is

\[ 1 = L^1(s) + L^2(s) \text{ for all } s \in S \]

and equilibrium in the sectoral goods markets imply that

\[ \bar{Y}^1(s) = Y^1(s) \text{ and } \bar{Y}^2(s) = Y^2(s) \text{ for all } s \in S \]

Optimal Consumption and Investment in Sector 1  Using the budget constraint of the households, (3.9), consumption can be written as

\[ C(K^1, s) = \frac{w(s)}{p(s)} + \left[ \frac{p^{K^1}(s)}{p(s)} - 1 \right] K^1 \text{ for all } s \in S \]  (3.10)

Expression (3.10) shows that household’s optimal consumption only depends on the market prices, \( \{w(s), p(s), p^{K^1}(s)\} \) \(_{s \in S} \), and investment in sector 1, \( K^1 \). The decisions regarding the allocation of labor, \( L^1(s) \) and \( L^2(s) \), and investment in sector 2, \( K^2(s) \), are optimally determined in period 2, given \( s \) and \( K^1 \).

Using expression (3.10), the first order condition of the problem the household for the decision on \( K^1 \) can be written as

\[ \sum_{s \in S} \Pr(s)u'(C(K^1, s)) \left[ \frac{p^{K^1}(s)}{p(s)} - 1 \right] = 0 \]  (3.11)
which is similar to expression (3.6) in the previous case. Expression (3.11) shows that households invest in $K^1$ until the expected value of the marginal utility of its return is zero.

The Effect of Aggregate Volatility

Rearranging equation (3.11), and using the first order condition of the firm in sector 1, it follows that

$$
\sum_{s \in S} \varphi(K^1, s) A(s) \frac{p^1(s)}{p(s)} \tilde{A}^1 G^1_K(K^1, L^1(s)) = 1
$$

where, as in the previous case, $\varphi(K^1, s)$ is the kernel-adjusted probability of state $s$, for all $s \in S$.

Equation (3.12) allows to analyze the effect that the volatility of $A(s)$ has on investment in sector 1, $K^1$. Similar to expression (3.7), equation (3.12) implies that the expected value (in terms of final good) of the marginal productivity of capital in sector 1 using probabilities $\{\varphi(K^1, s)\}_{s \in S}$, must be equal to its marginal cost, which is equal to one. There is a difference, however, in this case with respect to the previous one. While in expression (3.7) only the term $\sum_{s \in S} \varphi(K, s) A(s)$ depended on the aggregate productivity, $A(s)$, (given that expression $\frac{p^M}{p} \tilde{A}^M G^M_K(K, L^M)$ was completely determined by the international prices and the endowment of the fixed factor), expression $\frac{p^1(s)}{p(s)} \tilde{A}^1 G^1_K(K^1, L^1(s))$ in equation (3.12) varies across states due to the impact of $A(s)$ in both, prices and the allocation of labor. The reasons of this difference with the previous model are two: first, being a closed economy, aggregate shocks affect prices (i.e. $\frac{p^1(s)}{p(s)}$ varies with $s$); and, second, capital in sector 2, $K^2$, can be adjusted in response to the shock (unlike the fixed factor, $F$), which may induce changes in the allocation of labor and therefore in $L^1(s)$.

In particular, since the stock of capital in sector 1, $K^1$, is "fixed" by the time the state of nature, $s$, is revealed, standard assumptions on the functions $G^1$, $G^2$, and $G$ imply that the relative price of sectoral good 1, $\frac{p^1(s)}{p(s)}$, is positively associated with the level of aggregate productivity, $A(s)$ (which determines the level of aggregate demand). Additionally, if the elasticity of substitution
between labor and capital in the production of sectoral goods is sufficiently high (higher than 1),
the demand for labor in the sector, $L^1(1)$, as well as the factor $G^1_K(K^1, L^1(s))$ are also increasing
in $A(s)$. Both effects imply that the relative value of the marginal productivity of capital in sector
1, $A(s)\frac{p^1(s)}{p(s)} \tilde{A} G^1_K(K^1, L^1(s))$, is increasing and strictly convex in $A(s)$ (unlike the previous case in
which the equivalent term in expression (3.7) was linear in $A(s)$). It follows then that, in comparison
with the previous case, a higher concavity of preferences is necessary to guarantee that the effect
of aggregate volatility on $K^1$ is negative. This is mainly due to the fact that the change in prices
exacerbates the volatility of firm’s profits, making them strictly convex in aggregate productivity.
In addition to the concavity of preferences, the effect is negative link between volatility and $K^1$ is
reinforced, as in the previous case, by a low elasticity of substitution between capital and labor in
sectoral production.

Another important difference between this case and the previous one is that the mean
of aggregate productivity does not affect the composition of investment. The reason of this is
that there is no fixed factors of production, so that capital in both sectors is scaled up or down
proportionally with the average productivity of the economy.

An Example

As in the previous model, consider a particular example with the following functional forms:

**Utility Function:** $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, with $\sigma \geq 0$.

**Final Good Production:** $Y = G(Y^1, Y^2) = \left[ \alpha (Y^1)^{\rho_y} + (1 - \alpha)(Y^2)^{\rho_y} \right]^{\frac{1}{\rho_y}}$, $\alpha \in (0, 1)$, and $\rho_y = \frac{\eta_y - 1}{\eta_y}$, being $\eta_y \geq 0$ the elasticity of substitution between sectoral goods in the
production of final good.
**Sectoral Production:**  
\[ Y^T(s) = A(s)A^T \left[ \alpha_T \left(K^T\right)^\rho + (1 - \alpha_T) \left(L^T\right)^\rho \right]^{\frac{1}{\rho}}, \quad \alpha_T \in (0, 1) \]  
for \( T = 1, 2 \), and \( \rho = \frac{\eta - 1}{\eta} \), being \( \eta \geq 0 \) the elasticity of substitution between capital and labor in the production of sectoral goods.

**Aggregate Volatility:**  
There are two states of nature, \( s \in \{L, H\} \), such that \( A(L) = \bar{A}(1 - \varepsilon) \), and \( A(H) = \bar{A}(1 + \varepsilon) \), with \( \bar{A} > 0 \), and where \( \varepsilon \in [0, 1] \) is the coefficient of variation of \( A(s) \).

**Parameters:**  
I use \( \alpha = \alpha_1 = \alpha_2 = 0.5 \), and \( \bar{A} = \bar{A}^1 = \bar{A}^2 \) in all the simulations presented below.

**Simulations**

Figure 3.2 presents various simulations of the closed economy analyzed above. The benchmark case assumes \( \sigma = 1 \) (i.e. logarithmic preferences), and \( \eta_y = \eta = 1 \) (so that production of both, final and sectoral goods are Cobb-Douglas). In all cases the horizontal axis displays the different levels of \( \varepsilon \).

Figures 3.2 (A) and (B) correspond to the case in which the curvature of the utility function, \( \sigma \), is varied. Panel (A) shows the absolute value of investment in sector 1 (i.e. the irreversible capital) for different levels of aggregate volatility. It can be seen that when the curvature of \( u \) is low (e.g. \( \sigma = 0 \)) the effect of the convexity in the return of \( K^1 \) dominates, and the investment in sector 1 is increasing in \( \varepsilon \). As \( \sigma \) increases, however, the convexity of returns is "killed" by the curvature of \( u \), and the relationship between \( K^1 \) and \( \varepsilon \) becomes negative. Panel (B) displays the ratio of sectoral investments, computed as the ratio between \( K^1 \) and the average value of \( K^2(s) \), \( s = L, H \) (i.e. \( \frac{K^1}{K^2} \)). The figure shows that the (average) share of investment in sector 1 is strictly decreasing in \( \varepsilon \) as long as \( \sigma > 0 \), which shows despite the fact that the effect of aggregate volatility on the absolute level of investment in sector 1 may be increasing, the effect on the composition of
investment is in all cases against the sector with irreversible capital. The figure also shows that for high values of $\sigma$ the ratio $\frac{K^1}{K^2}$ falls less than proportionally than $K^1$, which implies that average investment in sector 2 falls as well as volatility increases.

Figure 3.2 (C) shows how the ratio $\frac{K^1}{K^2}$ is affected by $\varepsilon$ for different values of the elasticity of substitution between sectoral goods, $\eta_y$. As expected, higher substitutability between goods 1 and 2 implies that the economy will rely more on the production of good 2 when volatility increases. In the limit, as $\eta_y \to \infty$ (so that both goods are perfect substitutes), even very low levels of volatility trigger complete specialization in the production of sectoral good 2.

Finally, panel (D) of Figure 3.2 displays the evolution of $\frac{K^1}{K^2}$ for different values of the elasticity of substitution between capital and labor in the production of sectoral goods. Similar to the previous case, changes in $\eta$ have a non-monotonic effect on the (negative) relationship between volatility on $\frac{K^1}{K^2}$, which becomes stronger for both, very low and very high values of the elasticity.

### 3.3 Conclusions

This paper explores potential effects of exogenous aggregate uncertainty on both, the composition of investment, and the pattern of sectoral production of the economy. The basic idea of the paper is that when investors can chose among variety of economic activities, aggregate volatility increases the relative preference for those that use more "flexible" or versatile factors of production, whose returns are relatively less affected by the changing macroeconomic environment. The mechanism is analyzed in two models: a small open economy that produces commodities based on natural resources, and manufactures, and a closed economy that produces two types of goods that differ in the degree of reversibility of their capital. The first model predicts that higher (relative) volatility of aggregate productivity increases the reliance of the economy on the sector dependent on natural resources, and that the effect is stronger in economies with lower average productivity. The second model
Figure 3.2: The Effect of Aggregate Volatility on Irreversible Investment
Example: Two-Sector, Closed Economy

Predicts that higher volatility reduces the share of investment in the sector with more irreversible capital, and that the effect is stronger if the elasticity of substitution between sectoral goods is higher. In both cases, higher concavity of preferences strengthens the change in the patterns of investment and production induced by volatility. Aggregate volatility can negatively affect growth by inducing an otherwise inefficient allocation of resources across sectors. The negative effect on growth can be larger in the long run if, for example, there is endogenous growth associated with the expansion of a sector that uses a very specific and irreversible type of capital.

The paper contributes with the theoretical literature on volatility, investment and growth, which has mainly analyzed the effects of volatility on the quantity and timing of investment, but...
has devoted little attention to the effects on its *quality*. The work also contributes with the more specific literature on sectoral production and volatility, which emphasizes the effect that choosing a particular pattern of production has on the volatility of the economy. This paper, on the other hand, proposes a mechanism in which the causality goes precisely in the opposite direction, being aggregate volatility one of the determinants of the chosen pattern of sectoral of production.
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143


147


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