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HYPER-STRANGE HADRONIC MATTER†

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Abstract: The binding energy of hyper-strange matter in the hadronic phase is calculated as a function of strangeness fraction in the mean field approximation to a relativistic field theory of matter. This is compared to a calculation of Chin and Kerman for the quark phase.

NUCLEAR STRUCTURE  nuclear matter, hyper-strange matter, quark matter

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In a recent letter, Chin and Kerman\textsuperscript{1} have suggested the possible production of long-lived hyper-strange multi-quark objects in high-energy nuclear collisions. They studied the stability of quark matter as a function of the fraction of strange quarks. Based on the currently popular values of the MIT-bag constants they found that the energy per baryon of infinite matter becomes a minimum for a ratio of strangeness to baryon number approximately two. This minimum lies above the energy of free nucleons so that an object made of this matter is only metastable, its decay being still characterized by the weak decay lifetime of strange baryons and evaporation processes.

In this note, we investigate whether a hyper-strange hadronic object also possesses a metastable state and would have a lower or higher energy than the corresponding quark object. We do this by means of an extension of a relativistic field theory of matter solved in the mean field approximation.\textsuperscript{2} In this theory the nucleons are coupled to scalar and vector mesons. The meson fields are approximated by their mean values, which are related to the scalar and baryon densities $\bar{\psi}\psi$ and $\psi^+\psi$. The nucleon field $\psi$ then obeys a Dirac-like equation in which the nucleon mass is shifted by the scalar field and the energy eigenvalues by the
vector field. The ratio of coupling constants to masses of the mesons are
two parameters which are adjusted to reproduce the saturation properties
of infinite nuclear matter. To this system we now add the hyperon
\( \Lambda(1116), \Sigma(1193) \) and \( \Xi(1318) \) fields, each of which is coupled to the
mesons. The strength of this coupling is not well established. Universal
coupling is somewhat suggested for the coupling of these octet baryons to
the mesons. On the other hand we can look to hyper-nuclei for evidence.
A \( \Lambda \)-nucleus potential of about 1/2 of the nucleon-nucleus shell model
potential in light nuclei produces the correct binding energy.\(^3\)

Therefore, we consider two possibilities: that the hyperons have
the same coupling as the nucleons do to the mesons, 1) \( g^H = g^N \), or 2)
\( g^H = g^N/\sqrt{2} \) where the \( 1/\sqrt{2} \) corresponds to a reduction in potential
strength of 1/2.

Formulation

The interaction part of the lagrangian density is

\[
L_{\text{int}} = \sum_{\alpha} \left( g^\sigma \bar{\psi}_\alpha \psi_\alpha + g^\omega \bar{\psi}_\alpha \gamma_\mu \psi_\alpha \right)
\]

(1)

where \( \sigma \) and \( \omega^\mu \) denote the neutral scalar and vector meson fields. In
the static approximation these would give rise to attractive and repulsive
Yukawa potentials, respectively. The Dirac fields are represented by \( \psi_\alpha \)
where \( \alpha \) labels the various baryons, \( N, \Lambda, \Sigma, \Xi \). The field free parts of
the lagrangian density are

\[
L_{\text{Dirac}}^\alpha = \sum_{\alpha} \bar{\psi}_\alpha (i\gamma^\mu \partial_\mu - m_\alpha) \psi_\alpha \quad \quad \text{(2a)}
\]

\[
L_{\sigma} = \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 \right) \quad \quad \text{(2b)}
\]
\[
L_\omega = -\frac{1}{4} \varepsilon_{\mu\nu} \varepsilon_{\rho\sigma} \omega^{\mu
u} + \frac{1}{2} m^2 \omega_\mu \omega^\mu \tag{2c}
\]

\[
\omega_{\mu\nu} = \partial_{\mu} \omega_{\nu} - \partial_{\nu} \omega_{\mu} \tag{2d}
\]

We use the standard notation of Bjorken and Drell.\textsuperscript{4}

For static uniform infinite matter, the Euler Lagrange equations yield for the ground state expectation values of the meson fields:

\[
m^2_{\sigma} = \sum_{\alpha} g^2_{\omega} \langle \bar{\psi} \gamma_{\alpha} \psi \rangle \tag{3}
\]

\[
m^2_{\omega_{\mu}} = \delta_{\mu0} \sum_{\alpha} g^2_{\omega} \langle \bar{\psi} \gamma_{\alpha} \gamma_{\mu} \psi \rangle \tag{4}
\]

Substituting these mean fields back into the lagrangian, the Euler-Lagrange equations for the Dirac fields become

\[
[i \gamma^0 - g_{\omega} \bar{\psi} - (m_\alpha - \omega_{\alpha\sigma})] \psi_\alpha = 0 \tag{5}
\]

The eigenvalues are

\[
E_\alpha^\pm = g_{\omega_0} \omega_0 \pm \sqrt{p^2 + M_\alpha^2} \tag{6}
\]

where

\[
M_\alpha = m_\alpha - \omega_{\alpha\sigma} \tag{7}
\]

is the effective mass.

In the ground state (\(T = 0\)) only the positive energy solutions are relevant. These states are filled in order up to the Fermi level. We define two Fermi energies, one for nucleons and one for hyperons. The Fermi momenta \(p_\alpha\) for nucleons and hyperons satisfy the equations

\[
E_\alpha^{\text{Fermi}} - g_{\omega_0} \omega_0 = \sqrt{p_\alpha^2 + M_\alpha^2} \tag{8}
\]
Any $p_\alpha$ which cannot satisfy these relations for a real value must be set to zero, the Fermi energy lying below the effective mass $M_\alpha$ in such a case.

The scalar density $\langle \bar{\psi} \psi \rangle$ for the plane wave spinors can be evaluated as

$$\bar{\sigma} = \frac{1}{2\pi^2} \sum_\alpha \gamma_\alpha g_s \int_0^{p_\alpha} \frac{m_\alpha - g_\alpha \sigma}{[p^2 + (m_\alpha - g_\alpha \sigma)^2]^{1/2}} p^2 dp$$

(9)

(where $\gamma_\alpha$ is the degeneracy of species $\alpha$, $[\gamma_\alpha = (2I + 1)(2J + 1)]$

The baryon density $\langle \psi^+ \psi \rangle$ is simpler, namely

$$\rho = \sum_\alpha \rho_\alpha$$

(10a)

$$\rho_\alpha = \gamma_\alpha \rho_\alpha^2 / 6\pi^2$$

(10b)

and it determines the mean $\omega_0$ field through (4).

The two equations (8),(9) have to be solved self-consistently for the Fermi momenta $p_\alpha$ and the scalar field $\bar{\sigma}$ for chosen nucleon and hyperon Fermi energies. The energy density can then be evaluated from

$$\varepsilon = \frac{1}{2m_\sigma^2} \frac{2\varepsilon_{\omega_0}^2 - 1}{2m_\omega_0^2} + \frac{1}{2\pi^2} \sum_\alpha \gamma_\alpha \int_0^{p_\alpha} (g_\alpha \rho_0^2 + \sqrt{p^2 + \rho_0^2}) p^2 dp$$

This is then minimized with respect to the total baryon density as a function of the fraction of strangeness to baryon density

$$f = \sum_\alpha S_{\alpha} \rho_\alpha / \rho$$

(11)

where $S_{\alpha}$ is the strangeness quantum number of the baryon type $\alpha$. 
Results and Discussion

The coupling constants, or more precisely their ratio to meson masses, \( g^N_{o}/m_{o} \) and \( g^N_{\omega}/m_{\omega} \) were determined by Walecka\(^2\) from the saturation binding and density of symmetric nuclear matter. As discussed, the coupling of the hyperons is not well established. We consider first a universal coupling \( g^H_{o,\omega} = g^N_{o,\omega} \). The binding energy of matter as a function of strangeness fraction, \( f \), minimized with respect to density is shown in Fig. 1. No minimum develops, indicating that in the hadron phase, the interaction energy and the high degeneracy of hyperons do not overcome their larger mass. For the weaker coupling of hyperons, suggested by the evidence on hyper-nuclei, this conclusion is strengthened. Therefore, according to this estimate, we conclude that no metastable state exists for hyper-strange matter in the hadronic phase, in contrast to the estimate of Chin and Kerman for the quark phase.

One reservation can be mentioned. Both estimates of the energy are extrapolations in different models of nature. The parameters of the bag model are optimized with respect to a few hadron masses and have no known connection to larger objects. The parameters of the hadronic matter model are determined by properties of nuclear matter and hyper-nuclei, and extrapolated away from these conditions.
REFERENCES

2. J. D. Walecka, Ann Phys 83 (1974) 491

FIGURE CAPTION

1. The binding energy minimized with respect to density (marked on the curve in fm$^{-3}$) is shown as a function of strange quark fraction for the hadronic phase of matter calculated here, and the quark matter phase calculated by Chin and Walecka.
\[ B/A = \epsilon/\rho - m_N \text{ (MeV)} \]

- Quark matter (Chin, Kerman)
- Hadronic matter

Figure 1