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Publication Date
1985-08-01
#18-85
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August 1985
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A COMMENT ON
'THE EQUITY PREMIUM. A PUZZLE'
(By R. Mehra and E. C. Prescott)

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August 1985.

Abstract: The results in this paper by Mehra and Prescott conflict with other evidence on the equity premium. The reason is that they substitute the smooth per capita consumption on nondurables and services for the more variable true payment process when calculating the price of the market index. In fact, their theoretical equity premium constitutes a lower boundary for the actual premium. The data are shown to be consistent with the corresponding upper boundary also. Hence they do not reject the Arrow-Debreu equilibrium model.
In their paper in this Journal, Mehra and Prescott compare the average risk premium (i.e., the difference between the average return on the market and the risk free rate) as observed over the ninety-year period 1889-1978 to the one calculated using a general equilibrium model. Whereas the observed premium was about 6.18 percent, the theoretical risk premium was shown not to exceed 0.35 percent, a huge discrepancy indeed. These results contrast with those from a paper by Merton (1980). Merton was primarily interested in estimating the expected return on the market, and one of the equilibrium models he used to constrain estimates of the expected return is very similar to the one used in Mehra and Prescott’s article (Merton calls it Model # 1). Both models are based on a Markov-type economy with a single (representative) investor having a constant relative risk aversion utility function. Merton’s model relates the equity premium to the variance of the market return:

\[ R^e - R^f = \alpha \cdot \sigma^2 \]  

(1)

where \( R^e \) denotes the expected return on the market, \( R^f \) the risk free rate, \( \sigma^2 \) the variance of the market return and \( \alpha \) the representative’s relative risk aversion. From Merton’s investigation of monthly returns of the New York Stock Exchange Index over the period July 1926 till June 1978, one can deduce a relative risk aversion (\( \alpha \)) equal to 1.89 (see his Table 4.3). The equity premium Merton observed over this 52 year period is close to the one in Mehra and Prescott’s article (the former was equal to 8.15 percent - see Merton’s Table 4.7, while the latter amounted to 6.18 percent). However, Mehra and Prescott’s model implied at most a premium of 0.35 percent,
if one allows the relative risk aversion (\(\alpha\)) to be as high as 10. Mehra and Prescott's results clearly reject the equity premium model, whereas Merton's outcomes appear to support it.

Merton's model has the advantage of depending mainly on readily observable variables such as the variance of the market return. Mehra and Prescott's model is based on the following formula, which expresses the price of the market equity share \(P_t\) as a function, not of the market return, but of per capita consumption \(c_t\):

\[
P_t = E \left( \sum_{s=t}^{\infty} \beta^s U'(y_s)c_s/U'(y_t) \right)
\]

(This is their equation (5), where \(c_s\) and \(c_t\) are substituted for \(y_s\) and \(y_t\), the economy's output, respectively; \(d_s\) is the dividend payment at time \(s\), \(\beta\) is the subjective time discount factor and \(U'(\cdot)\) is the marginal utility of consumption). It is obvious that the dividend payment process \(\{d_t\}\) always equals the consumption process \(\{c_t\}\), at least at the level of the economy. Hence, it suffices to observe the consumption process \(\{c_t\}\) in order to test equation (2). Mehra and Prescott approximate this process by considering the time series of per capita consumption on nondurables and services, leaving out consumption on durables. But, under the very plausible assumption that the consumption on durables, on the one hand, and nondurables plus services, on the other hand, are positively correlated, it follows that Mehra and Prescott's time series will be less variable than the true payment process of the economy. Indeed, in this case:

\[
\text{var}(d_{t-1} - d_t) = \text{var}(c_{t-1} - c_t)
\]

\[
= \text{var}(c_{nt-1} - c_{nt}) + \text{var}(c_{dt-t} - c_{dt})
\]

\[
+ 2\text{cov}(c_{nt-t} - c_{nt}; c_{dt-t} - c_{dt})
\]
(c_{dt} stands for consumption on durables and c_{nt} for consumption on nondurables plus services). The difference between the variance of the true payment process and the variance of per capita consumption on nondurables and services will be larger the more variable the per capita consumption on durables is. The latter may very well be more volatile than the per capita consumption on nondurables and services, because it includes many goods which have a higher income elasticity, and, consequently, are subject to a larger extent to income (or production) volatility. Hence, Mehra and Prescott’s time series variability can be considered a lower boundary to the variance of the true payment process ((d_t) or (c_n)). At the other extreme, the variance of the true payment process will probably be less than the variance of the return on the economy because the latter includes both dividend variability (d_t) and changes in the valuation of total wealth (P_t - P_{t-1}), although these elements may be negatively correlated. If it is assumed that the possible negative correlation does not completely offset the variability of changes in the valuation of total wealth, the variance of the return on the economy can be considered an upper boundary on the variance of the true payment process.

It is important to impose the correct variability on the payment process in an empirical investigation of equation (2), because the results are very sensitive to changes in the variance. Mehra and Prescott state: 'The sensitivity to σ, the standard deviation of the consumption growth rate, is larger. The average equity premium was roughly proportional to σ squared' (p. 156). This is contrary to the
Modigliani-Miller dividend irrelevance theorem which states that the price of a security does not depend on the particular dividend stream chosen by the company. The reason is clear: the Modigliani-Miller theorem cannot hold on the level of the economy because whenever part of the total output is consumed (i.e., paid as a dividend), it is not available anymore for investment, effectively reducing total wealth (i.e., the net present value of future consumption). Mehra and Prescott generate this variance sensitivity by assuming that consumption equals output and dividends for all t \( c_t = y_t - d_t \). Notice that Merton's model implies the same variance sensitivity: the equity premium is proportional to the variance of the market return (see equation (1)).

It is then not surprising that Mehra and Prescott find the average equity premium over the period 1889-1978 to be much higher than the one implied by their model and the (smooth) per capita consumption on nondurables and services over that period. Their test investigates the lower boundary on the equity premium. Using Mehra and Prescott's model, but imposing \( \delta \) to equal the standard deviation of the market return (which equals 16.54 percent for the period 1889-1978), and remembering that the equity premium is roughly proportional to \( \delta \) squared, it follows that the equity premium should at most be 7.35 percent, well above the observed 6.18 percent. This is the corresponding upper boundary test. Hence, one can conclude that the average risk premium for the period 1887-1978 lies somewhere between a lower boundary (0.35 percent) and an upper boundary (7.35 percent). It does not violate either boundary implied by the Arrow Debreu general equilibrium model. To get a specific value for the required equity
premium, Merton's method can be used. And his results for the period 1929-1978 indicate that the average equity premium is not inconsistent with the Arrow-Debreu economy.

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