Title
THE PRESENT PHENOMENOLOGICAL STATUS OF THE REGGE POLE THEORY

Permalink
https://escholarship.org/uc/item/4p90841q

Author
Leader, Elliot.

Publication Date
1965-08-02
THE PRESENT PHENOMENOLOGICAL STATUS OF THE REGGE POLE THEORY

Berkeley, California
DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
THE PRESENT PHENOMENOLOGICAL STATUS
OF THE REGGE POLE THEORY

Elliot Leader

August 2, 1965
THE PRESENT PHENOMENOLOGICAL STATUS
OF THE REGGE POLE THEORY

E. Leader

Lawrence Radiation Laboratory
University of California
Berkeley, California

August 2, 1965

1. INTRODUCTION

Perhaps the most unified of all theories of elementary particles ever proposed was the "Regge Pole Theory" of Chew and Frautschi. Following on the brilliant analysis of Potential Scattering by Regge, in which the whole concept of complex angular momentum was introduced, Chew and Frautschi suggested that all the known particles and resonances could be grouped into families, each family being associated with a given Regge trajectory, and consisting of several members differing only in their spin quantum number. A spin-\( J \) member of a family would appear as a manifestation of the passage of the Regge trajectory \( \alpha(t) \) through an integral or half-integral value of \( J \). Actually, only alternate integer or half-integer values of \( J \) along a given trajectory would appear as particles or resonances, owing to the concept of "\( J \) parity." The square of the mass of the particle of spin \( J \) would be the value of \( t \) for which \( \text{Re} \, \alpha(t) = J \).

Moreover, a knowledge of \( \alpha(t) \) for \( t < 0 \) plus the assumption that the residue functions \( r(t) \) were slowly varying would suffice to specify the behavior of the scattering amplitude at high energies in the crossed channel.
More detailed considerations led to startling predictions: e.g., at asymptotically high energies the total cross sections for \( \pi\pi \), \( \pi N \), and \( NN \) scattering would satisfy the relation

\[
\sigma_{\pi N}^2 = \sigma_{\pi\pi} \sigma_{NN}.
\]

A further unification of the theory was afforded by the suggestion that the trajectory families themselves could be grouped together into multiplets according to some set of internal quantum numbers: e.g., into \( SU(3) \) multiplets.

The experimental successes of the Regge Pole Theory in its earliest period were very impressive. Since then a vast amount of precise experimental data has been accumulated, and the whole status of the Regge Pole Theory is much less secure. In the following we shall attempt to give an objective and coherent survey of the present phenomenological status of the theory. We shall also point out possibilities for testing some of the fundamental properties of the theory.

It will be our aim to discuss in detail only those aspects of the situation which allow reasonable quantitative tests of the validity of the theory. Thus we shall concentrate primarily upon the field of small angle, high energy scattering i.e. the region \( t \leq 0 \), and we shall not attempt to analyse the situation for \( t > 0 \). The latter region, in which one tries to find sets of particles and resonances which lie on the same trajectory, and multiplets of trajectories which fit into group theoretical symmetry schemes, abounds with interesting and exciting possibilities, but these are of a much more speculative nature and do not easily provide genuinely critical tests of the theory.
In Sec. 2 we briefly review the basic tools of the Regge Pole Theory and their origin in potential scattering.

In Sec. 3 we summarize the results of the original one-pole approach and then discuss the rules for generalizing to the case of several Regge poles.

Section 4, which is the main section of this survey, contains a detailed analysis of the present experimental situation and its relationship to the theory.

In Sec. 5 we attempt to summarize the overall situation.
2. BASIC TOOLS OF THE THEORY

Consider a scattering process

\[ A + B + A' + B' \]

described by a scattering amplitude \( f(s, \theta_s) \), where

\[
\begin{align*}
  s &= (\text{center-of-mass energy})^2 \\
  \theta_s &= \text{scattering angle in c.m.}
\end{align*}
\]

described by a scattering amplitude \( f(t, \theta_t) \), where

\[
\begin{align*}
  t &= (\text{center-of-mass energy})^2 \\
  \theta_t &= \text{scattering angle in c.m.}
\end{align*}
\]

We shall refer to this as the MAIN PROCESS. It is represented schematically in Fig. 1. Related to the main process we may also consider the CROSSED PROCESS,

\[ A + A' + B + B' \]

described by a scattering amplitude \( f(t, \theta_t) \), where

Let us digress for a moment and imagine that the process \( A + A' + B + B' \) was governed by a nonrelativistic potential. Then the results of Regge on potential scattering would apply and would tell us that if we take the limit (of course it is a formal, unphysical one) of letting \( \cos \theta_t \to -1 \), then \( f(t, \theta_t) \) has the simple form
\[ f(t, \theta) \propto \frac{B(t)}{\sin \pi \alpha(t)} \cos \theta \alpha(t) \]  \hspace{1cm} (2.3)

where \( \alpha(t) \) is the "trajectory function" associated with the Regge pole which lies farthest to the right in the complex \( J \) plane for the given \( t \) value and \( B(t) \) is the "residue function" associated with it.

The relativistic generalization of Regge's results cannot be proved rigorously. However, one can show that a reasonable heuristic attempt is to replace the formula (2.3) by

\[ f(t, \theta) \propto \xi(t) B(t) \cos \theta \alpha(t) \]  \hspace{1cm} (2.4)

where \( \xi(t) \), the "signature factor," is given by

\[ \xi(t) = \frac{1 + \text{e}^{-i\pi \alpha(t)}}{\sin \pi \alpha(t)} \]  \hspace{1cm} (2.5)

and \( \tau = \pm 1 \) is the "J parity" associated with the Regge pole. The complication introduced in (2.4) is simply due to the fact that in the relativistic case both direct and exchange forces play a role.

It will be useful to write (2.5) in a simpler form. We have

\[ \xi(t) = \frac{1}{2} \left( \cot \frac{\pi \alpha(t)}{2} - i \right) , \quad \text{Im} \xi = -\frac{1}{2} \quad \text{for} \quad \tau = +1 \]  \hspace{1cm} (2.6)

\[ \xi(t) = \frac{1}{2} \left( \tan \frac{\pi \alpha(t)}{2} + i \right) , \quad \text{Im} \xi = +\frac{1}{2} \quad \text{for} \quad \tau = -1 \]
Let us now return to our main problem. The so-called crossing theorem allows us to relate our main process to the crossed process. In fact, if all the particles were spinless it would just tell us that

\[ f(s, \theta_s) = f(t, \theta_t) \quad , \quad (2.7) \]

i.e., that one single function describes both the main and crossed processes. In the general case we will still have some simple relationship which we shall denote very schematically as

\[ f(s, \theta_s) \approx f(t, \theta_t) \quad . \quad (2.8) \]

If we now note that

\[ \cos \theta_t = -1 + \frac{2s}{4m^2 - t} \quad , \quad (2.9) \]

and that \( t = (\text{momentum transfer})^2 \) for the main process, we see that the region of high energies \( (s >> 2m^2) \) and small momentum transfers \( (|t| < 4m^2) \) for the main process just corresponds to having \( \cos \theta_t \gg 1 \).

Thus we can use the Regge asymptotic form \( (2.4) \), the crossing relation \( (2.8) \), and Eq. \( (2.9) \) to get

\[ f(s, \theta_s) \approx \xi(t) \phi(t) \left( \frac{s}{2m^2} \right)^{\alpha(t)} \quad , \quad (2.10) \]

which is valid for \( s >> 2m^2 \) and \( |t| < 4m^2 \).
To simplify the following, let us choose units in which \( m = 1 \),
and use the fact that \( \frac{s}{2m} \approx E = \text{lab kinetic energy} \). We also use \( \Theta \)
for \( \theta \) from now on. Thus our fundamental formula (2.10) becomes

\[
f(E, \Theta) \approx \xi(t) \beta(t) E^\alpha(t) \quad . \tag{2.11}
\]

Finally, to make contact with experiment, we note that by the
optical theorem,

\[
\sigma_{\text{TOT}}(E) \approx \frac{1}{E} \text{Im} f(E, \Theta = 0) \quad . \tag{2.12}
\]

Also, we have

\[
\frac{d\sigma}{dt}(E, t) \approx \frac{1}{E^2} |\beta(t) \xi(t) E^\alpha(t)|^2 \quad . \tag{2.13}
\]
3. THE ESSENTIALS OF REGGE POLOLOGY

A. The One-Pole Approach

It was originally assumed that the above formulae (2.11-2.13) might be valid with \( a(t) \) referring simply to the dominant Regge pole \( P \) (the Pomeranchuk pole). Then very powerful predictions could be made:

(i) The approximate constancy of total cross sections at high energies implied \( a_p(0) \approx 1 \). This, coupled with the Froissart result that \( a(0) \leq 1 \), led to the appealing suggestion that "forces were as strong as possible," i.e., \( a_p(0) = 1 \).

(ii) Chew and Frautschi, making the simplest possible assumption, drew straight lines representing \( a(t) \), joining particles with the same internal quantum numbers, but differing by two units of spin, and deduced the slope \( a'(t) \approx 1/(\text{BeV/c})^2 \).

(iii) Assuming this holds also for small negative \( t \), and putting 

\[
a(t) = a(0) + ta'(0) \approx 1 + t
\]

into Eq. (2.13), we have

\[
\log \left( \frac{da}{dt} \right) \approx \log \beta(t) + t \log E \quad \text{, (3.1)}
\]

indicating that \( \frac{da}{dt} \) should shrink quite rapidly with increasing \( E \).

This prediction, which was one of the most startling and characteristic results of the Regge Pole Theory, was immediately given experimental support from measurements on the pp system. However, it was soon realized that not all processes were exhibiting shrinkage, and in order to cope with this situation it was necessary to include the effect of several secondary Regge poles.
B. The Many-Pole Approach

We must now learn the rules needed for handling several Regge poles.

(i) If there are several different Regge poles that can contribute to a process, we simply replace formula (2.11) by a sum of terms, each one referring to one definite Regge pole.

(ii) Since all total cross sections look as though they will ultimately become constant, the leading Regge pole, \( P \), should have the quantum numbers of the vacuum: \( C = P = G = +1, \tau = +1, I = 0 \).

(iii) An analysis by Igi,3 in which the \( \pi N \) scattering length was calculated using a forward dispersion relation which had built into it a Regge type asymptotic behavior, indicated that there is a second pole, \( P' \), of the same type as \( P \), with \( \alpha_p(0) \sim 1/2 \). A recent, more accurate calculation by Scario4 indicates \( \alpha_{p'}(0) \sim 0.65 \).

(iv) A given Regge pole can be exchanged in any process provided that the \( t \) channel (the crossed channel) is able to communicate with systems which have the quantum numbers of the pole.

(v) The relative sign of the contribution of a given Regge pole to any process can be determined from its internal quantum numbers, as is indicated below.

For brevity we represent the amplitude arising from a given pole by the symbol for the particle (if it exists) with the quantum numbers of the pole. The relevant poles are listed below, and they seem to be more or less a minimal set.
Let us illustrate the rules by considering the systems $pp$, $pn$, and $\bar{p}p$.

Remembering (2.12) and (2.6), let us write, for the $\bar{p}p$ total cross section (the signs are arbitrary at this point),

$$\sigma_T(\bar{p}p) \approx \frac{1}{E_L} \text{Im} \left( P + P' + \rho + \omega + R \right) . \quad (3.2)$$

To change $\bar{p} + p$ reverse the signs of $\rho$ and $\omega$:

$$\sigma_T(pp) \approx \frac{1}{E_L} \text{Im} \left( P + P' - \rho - \omega + R \right) . \quad (3.3)$$

To change $p + n$ reverse the signs of $\rho$ and $R$:

$$\sigma_T(pn) \approx \frac{1}{E_L} \text{Im} \left( P + P' - \rho + \omega - R \right) . \quad (3.4)$$
The same rules will apply to \( pN \) and \( KN \) systems. They will also apply in Eq. (2.13) for \( \frac{d\sigma}{dt} \). Thus the term \( \beta(t)\xi(t)E^\alpha(t) \) is replaced by

\[
\sum_{\text{poles}} \beta_n(t)\xi_n(t)E_n^\alpha(t) \quad (3.5)
\]

We can now turn to a study of the present experimental situation and its implications for the Regge Pole Theory.
4. THE PRESENT EXPERIMENTAL SITUATION

We shall separate our analysis into two main categories, (i) $t = 0$ and (ii) $t < 0$. In (ii) we shall further subdivide into predictions which do or do not depend on "factorization" (to be discussed below). As mentioned in the introduction we shall not attempt to analyze the more speculative region in which $t > 0$.

(i) Processes for Which $t = 0$

Consider first the difference of the $\pi^- p$ and $\pi^+ p$ total cross sections. Since

$$\sigma_T(\pi^-p) \approx -\frac{1}{E} \text{Im} (P + P^* - p)$$

and

$$\sigma_T(\pi^+p) \approx -\frac{1}{E} \text{Im} (P + P^* + p)$$

the difference $\Delta(\pi p) = \sigma_T(\pi^-p) - \sigma_T(\pi^+p)$ depends only on $\rho$. Its energy dependence is $E^{-1/2}$ and determines

$$\rho(0) \approx 0.5$$

The small value of $\Delta(\pi p)$ indicates that $\rho$ couples weakly to $NN$ and $\pi\pi$. Also, from (2.6),

$$\zeta_{\rho}(0) = \frac{1}{2} (1 + i)$$

i.e., the real part of the $\rho$ amplitude relative to the imaginary part is completely determined.
Consider now the value and energy dependence of \( \frac{d\sigma}{dt} \) for the charge-exchange process \( \pi^- p + \pi^0 n \). We have

\[
\frac{d\sigma}{dt} (\pi^- p + \pi^0 n) |_{t=0} \approx |\rho|^2 \approx |1 + i|^2 E^{-1} \quad \text{(4.4)}
\]

That the imaginary part of the amplitude must have the above energy dependence follows from the optical theorem. There are two predictions in (4.4):

(a) that the real part has the same energy dependence as the imaginary part.

(b) that the real part is as large as the imaginary part, so that \( \frac{d\sigma}{dt} \) should be about twice the optical theorem value. Both these predictions are strikingly confirmed.

Using \( \alpha_p(0) = 1 \) and our values of \( \alpha_p(0) \), we can study the energy variation of \( \alpha_T(\pi^- p) \), and we shall find that a good fit is obtained with

\[
\alpha_p(0) \approx 0.5 \quad \text{(4.5)}
\]

From (2.6) follows

\[
\xi_p(0) = \frac{1}{2} (1 - i) \quad \text{(4.6)}
\]

in contrast to (4.3) for the \( \rho \).

Let us now turn to the \( NN \) system. From (3.3) and (2.6) we have
Experimentally $\Delta$ changes sign (becomes positive) at $E \approx 6$ BeV. This sign change would be impossible with just $\rho$. Thus although an $R$ particle has not yet been identified (it may be the $2^+ A_2$), it is concluded that the $R$ pole exists. This argument is somewhat dangerous, since one is talking about rather low energies. We shall see later that there are other tests for the existence of $R$.

The energy variation of $\Delta$ in (4.7) can be used to give

$$\sigma_R(0) \approx 0.3 \quad , \quad (4.8)$$

and by Eq. (2.6) follows

$$t_R(0) \approx \frac{1}{2} (1.7 - 1) \quad . \quad (4.9)$$

Consider now the charge-exchange process $np \leftrightarrow pn$, for which

$$\frac{d\sigma}{dt} \bigg|_{t=0} \approx \frac{1}{E} |\rho + R|^2 \quad . \quad (4.10)$$

From (4.9) and (4.3) we see that the real parts tend to add in (4.10), whereas the imaginary parts tend to cancel. Thus the theory predicts that the $np \leftrightarrow pn$ forward cross section should be much larger than the optical value, as is indeed found experimentally. Also the energy variation is correctly predicted.

A similar argument would predict that the process $\bar{p}p \leftrightarrow \bar{n}n$ should be about equal to the optical value, but as yet there are no experimental results.
Consider now the difference \( \Delta = \sigma_T(\overline{pp}) - \sigma_T(pp) \approx \frac{1}{E} \text{Im} (\rho + \omega) \) \quad (4.11)

The energy variation indicates

\[ \alpha_\omega(0) \approx 0.5 \] \quad (4.12)

so, that from (2.6),

\[ \xi_\omega(0) = \frac{1}{2} (1 + i) \] \quad (4.13)

The size of the difference indicates that the \( \omega \) coupling to \( NN \) is much greater than the \( \rho \) coupling.

Since we now have values of \( \alpha(0) \) for \( P, P', \rho, \omega, \) and \( R \), the energy variation of \( \sigma_T(pp) \) can be computed and the result is in good agreement with experiment. One learns also that both \( P \) and \( P' \) couple strongly to \( NN \).

Let us turn now to the \( KN \) system. A detailed analysis has recently been carried out by Phillips and Rarita. Using \( \alpha(0) \) values very close to above-mentioned ones, they are able to get very good fits to the highly precise data now available for the \( K^\pm p \) and \( K^\pm n \) total cross sections.

The forward charge-exchange processes \( K^+ n \rightarrow K^0 p \) and \( K^- p \rightarrow K^0 n \) are strictly analogous to \( np \rightarrow pn \) and \( \overline{pp} \rightarrow \overline{nn} \), respectively.

Hence the above discussion would predict
\[ \frac{\partial \sigma}{\partial t} (K^+ n \to K^0 p)|_{t=0} \gg \text{optical value} \quad (4.14) \]

and

\[ \frac{\partial \sigma}{\partial t} (K^- p \to K^0 n)|_{t=0} \sim \text{optical value} \quad (4.15) \]

There are as yet no data on (4.14). There are data on (4.15) at one energy only, 9.5 BeV, and the prediction of (4.15) is well fulfilled. The experimental uncertainty is not small, but if there were no \( R \) pole one would have \( \frac{\partial \sigma}{\partial t} \ll \) twice the optical value. Thus the KN system gives added evidence for the existence of \( R \).

Let us now attempt to summarize the situation at \( t = 0 \). The Regge Pole Theory, using a small number of parameters, is eminently successful in correlating a large amount of precise data. But what of other theories? The only other general possibility at present is to use forward dispersion relations. Assuming that the imaginary part is given at high energies by a simple combination of fractional powers of \( E \), and using the experimental data at lower energies, one uses the dispersion relation to compute the real part.

Now it is clear that the Regge pole amplitude satisfies (more or less) a Mandelstam representation and in particular, therefore, a fixed momentum-transfer dispersion relation. Thus the real part of a Regge amplitude will emerge from a dispersion-relation calculation if the imaginary part is given the Regge behavior, which it must be in order to agree with the total cross-section data. Thus for a given process there will be nothing to distinguish the two types of theory.
However, suppose that we now use the dispersion method for a series of interrelated processes. By taking various linear combinations of these processes we will discover that what we have been doing is equivalent to asserting that the exchange of a given set of internal quantum numbers always contributes a certain characteristic power behavior to the imaginary part, which is precisely what the Regge Pole Theory tells us. Thus the Regge Theory provides a simple and aesthetic underlying picture for the power law behavior used in the dispersion-relation methods.

(ii) Processes for Which \( t \neq 0 \)

(a) Processes that do not involve the factorization theorem

The most beautiful result of the original Regge Pole Theory was its prediction about the energy and momentum-transfer dependence of \( \frac{d\sigma}{dt} \). With the simplest assumptions, namely \( \alpha(t) \) linear and \( \beta(t) \) approximately constant, one had the prediction of the shrinkage phenomenon and at the same time an explanation of the exponential dropoff of \( \frac{d\sigma}{dt} \) with increasing \( |t| \) [see Eq. (3.1)].

The more precise experimental evidence that shows that only the pp (and perhaps \( K^+ p \)) differential cross sections are shrinking, and that the rate of shrinkage is so much smaller than originally supposed, has destroyed the essential beauty and simplicity of the Regge Pole Theory. This does not mean that the Regge Pole Theory fails to account for the new data. Rather, it means that the situation is much less simple and far more a contrived one. Thus by using several poles and very flat trajectories \( \alpha(t) \), Phillips and Rarita\(^9\) are able to get excellent
agreement with experiment for all the processes thus far measured. But the very use of flat trajectories means that one has lost the automatic mechanism for producing the sharp exponential dropoff of $\frac{d\sigma}{dt}$. Instead one has to put the entire burden for this into the residue functions $\beta(t)$, introducing several ad hoc parameters to describe their behavior.

There does exist, however, one class of experiment that might prove crucial in deciding the merits of the Regge Pole Theory. These are the situations in which one believes that just one Regge pole plays a role. The prime example is the charge-exchange process $\pi^- p \rightarrow \pi^0 n$, which, as discussed earlier [see Eq. (4.4)], depends solely on the $\rho$ pole. We have thus

$$\frac{d\sigma}{dt} (\pi^- p \rightarrow \pi^0 n) \propto \frac{1}{E^2} \beta^2(t) |\tan \frac{\pi}{2} a^\rho(t) + i|^2 E^2 \rho^{2\alpha^\rho(t)} .$$

Assuming a linear behavior of $a^\rho(t)$ for small $t$, one has

$$a^\rho(t) \sim 0.5 + ta^\rho(0)$$

we get from (4.16)

$$\frac{d\sigma}{dt} (\pi^- p \rightarrow \pi^0 n) \propto \frac{1}{E} \beta^2(t) |1 + \pi t a^\rho(0)| E^{2ta^\rho(0)} .$$

and the process will certainly exhibit shrinkage unless $a'(0)$ is exceedingly small. However, the detailed analysis of Phillips and Rarita indicated a value
so that a definite measurable shrinkage is predicted.

The latest experimental data indicate that there may be some shrinkage, but the measurements are not yet sufficiently accurate to make a more definitive statement.

Let us speculate a little and ask ourselves to what extent a more definitive result will influence our belief in the theory. If there is no shrinkage, then I believe that one has a clear-cut case against the theory. However, a true diehard will always be able to wiggle out of the difficulty by postulating the existence of a second Regge pole with the same quantum numbers at the \( \rho \), and in this way explain away the nonshrinkage. If, on the other hand, there is shrinkage, one has a strong case in favor of the theory.

There is one further general prediction which might eventually be used as a quantitative criterion for the validity of the Regge Pole Theory. It is generally felt by theoretical physicists that in "conventional" models of diffractive scattering the probability of a spin flip scattering in a given collision will drop off with increasing energy much more rapidly than will the nonspin flip probability. Neither the author nor any of the many colleagues whose opinion he has canvassed have been able to offer a proof of this assertion. Yet without exception the assertion seems to be accepted as fact. If it is indeed fact then the Regge Pole Theory is highly unconventional in that it predicts spin flip amplitudes which remain as large as the nonspin flip ones; i.e. spin flip and nonspin flip amplitudes have the same energy dependence. Thus it will be possible
to find certain spin dependent parameters e.g. spin correlation coefficients in NN scattering, which tend to constant, nonzero values as $E \to \infty$.

Note that this will not be true for the polarization. This is because the polarization always depends on the imaginary part of the product of a spin flip and a nonspin flip amplitude. It is therefore nonzero only if there is a phase difference between the spin flip and nonspin flip amplitudes.

In the Regge model the leading terms in these amplitudes turn out to have the same phase and therefore fail to contribute to the polarization.

Some interesting evidence that this kind of behavior occurs in practice is afforded by the charge exchange process $\pi^- p \to \pi^0 n$. Experimentally the differential cross section $\frac{d\sigma}{dt}$ first rises as we move away from $t = 0$, reaches a maximum at $t \approx -0.05$ and then decreases. This is very simply explained if there is a strong spin flip amplitude, which of course has to be identically zero at $t = 0$ to conserve angular momentum, and which then increases rapidly to a magnitude comparable with the nonspin flip amplitude. This is by no means a unique explanation of the peak in $\frac{d\sigma}{dt}$, but if it is correct then the energy variation of this peak gives some indication that both nonspin flip and spin flip amplitudes have the same energy dependence.

(b) Processes that involve the factorization theorem

In all the above, our decisions (with the possible exception of $\pi^- p \to \pi^0 n$ and the spin flip behavior) pro or contra the Regge Pole Theory were unfortunately of an aesthetic nature only. We wish now to examine a class of phenomena which we believe offer the possibility of a more fundamental decision.

Consider first the derivation of the factorization theorem.

Suppose we have a system which for a given $J$ can exist in two states
and I~} (a concrete example to keep in mind would be the two NN triplet states with \( l = J + 1 \), and suppose that \( |1\rangle \) and \( |2\rangle \) can communicate with each other and with the two-pion state \( |\pi\pi\rangle \) via the unitarity condition. Then if \( f_{ij} \) is the transition amplitude for \( |1\rangle \rightarrow |j\rangle \), one can show that the partial-wave amplitude \( f_{ij}(t,J) \) is given by

\[
f_{ij}(t,J) \approx \frac{r_{ij}(t)}{J - \alpha(t)} \quad (4.20)
\]

for \( J \) close to \( \alpha(t) \) for a given fixed \( t \) value.

The factorization theorem now asserts that

\[
r_{ij}^2 = r_{ii}r_{jj} \quad (4.21)
\]

e.g., that \( r_{11}(t)r_{22}(t) = [r_{12}(t)]^2 \).

It is this fundamental property which leads, for example, to the relationship

\[
\sigma_{\pi N}^2 = \sigma_{\pi N} \sigma_{NN} \quad (4.22)
\]

It will be some time before we can test relationships like \( (4.22) \), so we must look elsewhere to test \( (4.21) \). A good place is in NN scattering, where, however, the argument depends on a detailed consideration of the spin complications. We shall give an outline of the argument, suppressing as many inessential considerations as possible.

There are five independent helicity amplitudes \( \phi_i \) \( i = 1 \) through \( 5 \) needed to describe NN \( \rightarrow \) NN scattering in the main s-channel. By means
of very general crossing properties one can relate the \( \phi_i \) to the five amplitudes \( f_j \) needed to describe the t-channel process \( NN \rightarrow NN \).

Schematically,

\[
\phi_i = M_{ij} f_j .
\]  

These \( f_j \) are the \( NN \rightarrow NN \) transition amplitudes in a singlet-triplet spin representation.

The \( NN \rightarrow NN \) differential cross section is simply

\[
\frac{d\sigma}{dt} \sim \frac{1}{E^2} \sum_i |\phi_i|^2 .
\]

If each \( f_j \) is now replaced by its Regge asymptotic form, using just the poles discussed above, one gets

\[
\begin{align*}
 f_1 & = 0 , \\
 f_2 & = \sum_n \zeta_n r_{11}^{(n)} \alpha_n(t) , \\
 f_3 & = -t \sum_n \zeta_n \alpha_n(\alpha_n - 1) r_{22}^{(n)} \alpha_n(t)^{-2} , \\
 f_4 & = t \sum_n \zeta_n \alpha_n^2 r_{22}^{(n)} \alpha_n(t)^{-1} , \\
 f_5 & = \sum_n \zeta_n r_{n12}^{(n)} \alpha_n(t)^{-1} ,
\end{align*}
\]

where \( n \) runs over \( P, P', \rho, \omega, R \).
If we utilize the factorization theorem, Eq. (4.21), by putting

\[ r_{11}(t) = b_1^2(t) \quad r_{12}(t) = b_1(t)b_2(t) \]

and

\[ r_{22}(t) = b_2^2(t) \] \hspace{1cm} (4.26)

then, as first shown by Wagner, Eq. (4.24) takes on the simple form

\[ \frac{d\sigma}{dt} \approx \sum_{l,n,\rho,\omega} \xi^s_l \xi^s_n \left[ b_1(l)b_1(n) - a_1^k \alpha_2^k b_2(l)b_2(n) t \right]^2 E^{a_1^k + a_2^k - 2} \] \hspace{1cm} (4.27)

Remembering our rule (see Eqs. 3.2 and 3.3) for going from the pp to the pp system, we see that

\[ \frac{d\sigma}{dt} (pp + p\bar{p}) - \frac{d\sigma}{dt} (pp + pp) \approx \sum_{l\neq p,\omega} \text{Re} \left[ \xi^s_l \xi^s_\omega \right] \]

\[ \times \left[ b_1(l)b_1(\omega) - a_1^k \alpha_2^k b_2(l)b_2(\omega) t \right]^2 E^{a_1^k + a_2^k - 2} \]

\[ + (\text{term with } \omega \rightarrow p) \] \hspace{1cm} (4.28)

Now, since \( l \) in (4.28) refers only to poles of even signature, we get, from (2.6)

\[ \text{Re} \left( \xi^s_l \xi^s_\omega \right) = \frac{1}{4} \left[ \cot \frac{\pi}{2} a_1(t) \tan \frac{\pi}{2} a_2(t) + 1 \right] \] \hspace{1cm} (4.29)
and similarly for $\rho$. Thus the only factor in (4.28) that can possibly change sign are $\text{Re} \left( \zeta_\omega \zeta_\omega \right)$ and $\text{Re} \left( \zeta_\rho \zeta_\rho \right)$. But experimentally the difference in (4.28) changes sign at $t \approx -0.15$. To produce such a sign change would require enormous slopes for $\alpha(t)$, slopes many times as great as allowed by the fits to experiment discussed in Sec. (ii), part (a).

We have undertaken a detailed analysis to try to understand to what extent one can escape the above conclusion. We find that even with the inclusion of correction terms, beyond the asymptotic form, or the introduction of additional hitherto unused Regge poles, the above conclusion is inevitable. There is, however, one weak point in this conclusion. Namely, it is not really possible to show rigorously that the functions $b_1(t), b_2(t)$ introduced in (4.26) are necessarily real functions. Thus, if they suddenly become purely imaginary at $t = -0.15$, then a term like $b_1^2(t)$ could become negative. However, this is a rather unpalatable situation, and it seems very difficult to imagine a dynamical model in which this could happen.

It is thus of great importance to find a test for the factorization theorem which really has no loopholes. One such possibility is to measure the energy dependence of the spin-correlation parameter $C_{NN}$ in nucleon-nucleon scattering. It turns out that the expression for $C_{NN}$ contains terms like

$$\sum_{n,i} \left| r^{(n)}_{11} r^{(n)}_{22} - r^{(n)}_{12} r^{(n)}_{12} \right|^2 e^{i\alpha + \alpha_\Lambda - \alpha - 2}.$$(4.30)
Thus the highest power in $E$ would come from the Pomeranchuk pole, i.e. from the term with $n = l = \text{Pomeranchuk}$. Since $a_p = 1$ for $t = 0$ we would have as leading term

$$C_{NN} \sim \text{const.} \left| r_{11}^{(P)} r_{22}^{(P)} - [r_{12}^{(P)}]^2 \right|^2.$$ \hspace{1cm} (4.31)

But by the factorization theorem this is identically zero. Thus in fact, if the factorization theorem holds, we will have as leading term something like

$$C_{NN} \sim \left| r_{11}^{(P)} r_{22}^{(P)} - r_{12}^{(P)} r_{12}^{(P)} \right|^2 E^{a_p + a - 2}$$

$$\sim \frac{\text{const.}}{\sqrt{E}} \text{ at } t = 0.$$ \hspace{1cm} (4.32)

If, on the other hand, the factorization theorem does not hold, then presumably we would find,

$$C_{NN} \rightarrow \text{const. at } t = 0$$

except for possible logarithmic factors.

Thus we might expect that roughly

$$\frac{C_{NN}(E) \text{ with fact. theorem}}{C_{NN}(E) \text{ without fact. theorem}} \sim \frac{1}{\sqrt{E}}.$$ \hspace{1cm} (4.33)

A measurement of $C_{NN}$ would thus decide whether or not the factorization theorem is satisfied. 14
5. CONCLUSION

We attempt herein to summarize the rather complex situation.

On the one hand the Regge Pole Theory provides a simple and elegant explanation of the properties of cross sections, both direct and charge-exchange, in the forward direction. It also provides a neat, and probably correct, description of the relationship between the real and imaginary parts of the forward scattering amplitude. For nonforward processes it provides an acceptable but aesthetically much less pleasing description.

On the other hand, the fundamental and intrinsic property of residue factorizability appears to be completely at variance with the empirical data, and this matter deserves urgent experimental attention.

We thus conclude that what we are observing in these high-energy experiments is almost certainly the leading singularities in the complex \( J \) plane, but that the assumption that these singularities are purely poles is too naive. Of course the idea that there are branch cuts as well as poles in the complex \( J \) plane is an old one, and there is an extensive literature concerning their dynamical origin. However, none of these theoretical treatments would claim to be conclusive. Thus we feel that the direct breakdown of the factorization theorem is excellent phenomenological evidence for the relative complexity of the complex \( J \) plane.
ACKNOWLEDGMENT

The author is grateful to Dr. David Judd for suggesting that this survey talk should be circulated to a wider audience. He is also grateful to the U. S. Atomic Energy Commission for support.
This manuscript is a summary of a talk given at the Lawrence Radiation Laboratory (Research Progress Meeting), August 19, 1965.

† Present address: Physics Department, Cavendish Laboratories, University of Cambridge, Cambridge, England.


10. See, for example, P. Soding, Phys. Letters 8, 285 (1964).


14. This conclusion was reached independently by R. J. N. Phillips (Lawrence Radiation Laboratory), private communication.
Fig 1. Main and crossed processes.
Main process

\[ (s, \theta_s) \]

Crossed process

\[ (t, \theta_t) \]
This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.