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IMPLICATIONS OF MISSING MASS EXPERIMENTS
FOR REGGE CUTS

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ABSTRACT

The impact of the empirical features of missing mass spectra on Regge cuts is studied. The measured processes $a + p \rightarrow X + p$ ($a = \pi^-, p$) indicate a large diffractive component in certain reggeon-particle amplitudes which may crucially affect the structure of the corresponding cuts. This diffractive term essentially measures the possible deviation from results for cuts derived in models with resonances only, such as the Finkelstein's selection rules which are based on duality diagrams considerations.

Additional results for cuts are pointed out from the observed weak coupling of the pomeranchon to large masses.

I. INTRODUCTION

In the framework of Regge theory the natural objects to be studied after poles are Regge cuts. It is of importance therefore to investigate cuts both theoretically and phenomenologically.

Guided by field theoretic diagrams, Gribov developed an interesting approach to Regge cuts. The general structure of Gribov's reggeon calculus was later confirmed from dual models calculations. Although Gribov's theory is by no means complete, in particular because of the renormalization problem, it nevertheless provides a systematic way for exploring cuts. The reggeon-reggeon cut in this theory is determined by a new type of amplitude, namely the reggeon-particle amplitude, whose asymptotic behavior is determined by triple reggeon couplings. These new quantities are not merely mathematical objects, but do have a direct physical meaning since they are experimentally measurable in inclusive processes near the end of the spectrum.

Recently Muzinich et al. studied the relation between the pomeranchon-pomeranchon (P-P) cut and the triple pomeranchon coupling ($g_{ppp}$). It was first shown by Arbabanel et al. that $g_{ppp}$ can be measured from single and double diffraction processes. Subsequently a rough estimate for $g_{ppp}$ has been obtained from available data which was used as an input in the work of Muzinich et al.

Here we shall only discuss the contribution of double Regge (R-R) cuts to forward two-body scattering amplitudes. The restriction to "forward cuts" is needed since they are directly measurable from missing mass experiments. Hence the emerging picture is that a measurement of both the two-body and inclusive cross sections will, at
least in principle, determine both the poles and cuts. This fact strongly emphasizes the great importance of inclusive processes for understanding the structure of cuts.

We shall show that one can derive interesting results for R-R cuts by using some remarkable prominent features of the observed missing mass spectra. Explicitly it will be pointed out that the available inclusive data indicate the presence of a sizable diffractive component in the fixed pole residue of an elastic reggeon-particle scattering amplitude. Thus, in such amplitudes, one expects a considerable correction to a theory based on resonances only and exchange degeneracy. In particular, the Finkelstein's $^6$ selection rules for Regge cuts, with a nonzero reggeon-reggeon-pomeranchenon vertex, may be violated. Actually a numerical estimate will be given for the deviation from the selection rules. We shall also study the implications of the empirical missing mass spectra on absorption models and the eikonal approximation.

II. DOUBLE REGGIE CUTS AND TRIPLE REGGIEON COUPLINGS

The discussion will be limited to R-R cuts in total cross sections (forward elastic amplitudes) from which cuts in processes with exchange of quantum numbers can be obtained through iso-spin and SU(3) relations.

The $R_1 - R_2$ cut in the process

$$a + b \rightarrow a + b$$

is depicted in Fig. 1a. At high $s = (p_a + p_b)^2$ its contribution to the absorptive part of the forward elastic amplitude is given by:

$$\text{Im} F_{\text{cut}}(s,0) = \frac{1}{16\pi s} \int_{-\infty}^{0} dt \, \frac{\alpha_{R_1}(t) \alpha_{R_2}(t)}{s_{R_1} - s_{R_2}} (t),$$

(1)

where $t$ is the reggeon "mass", $s_0 = 1 \text{ GeV}^2$ and the $N$'s are fixed pole residues,$^1$ in the corresponding reggeon-particle amplitudes, defined by

$$N_{R_1 a \rightarrow R_2 a}(t) = \int_{0}^{\infty} dM^2 A_{R_1 a \rightarrow R_2 a}(M^2, t)$$

(2)

where $A_{R_1 a \rightarrow R_2 a}(M^2, t)$ is the absorptive part of the amplitude of the process $R_1 + a \rightarrow R_2 + a$ shown in Fig. 1b, $M^2 = (p_a + q_1)^2$, and $N_{R_1 b \rightarrow R_2 b}(t)$ is treated in a similar way. The factor $\tau_{R_1 R_2}(t)$
is \( \text{Re}(\xi_R(t)\xi_R^*(t)) \) for a "+" sign cut (unitarity cut) and is \( \text{Re}(\xi_R(t)\xi_R^*(t)) \) for a "-" sign cut (absorption cut or Gribov cut) with \( \xi_R \) \((i = 1,2)\) being the signature factors

\[
\xi_R(t) = -\frac{e^{-i\alpha_R(t)} + \tau_i}{\sin\pi\alpha_R(t)}
\]

\((\tau_i = +1, -1 \text{ for a positive and negative signature respectively})\). The question of the "sign" of the cut will not be discussed here as it is beyond the purpose of the present work. We shall rather focus on the fixed pole residues, given in Eq. (2), which are the basic elements for the \( R_1 - R_2 \) cut.

The direct relation between cuts and inclusive reactions is now clear from Eqs. (2) and (1) since \( A_{R_1a \rightarrow aR_2}(M^2,t) \) can be determined (at least in principle) from the interference of the \( R_1 \) and \( R_2 \) exchanges in the process \( a + b \rightarrow X + b \) as indicated in Fig. 2. Of particular importance is the asymptotic behavior of \( A_{R_1a \rightarrow aR_2} \),

\[
A_{R_1a \rightarrow aR_2}(M^2,t) \rightarrow \sum_{R_3} \beta_{R_3} R_{aR_2}(0)\xi_{R_1R_2R_3}(t)
\]

\[
\times \left( \frac{M^2}{\Lambda^2} \right)^{\alpha_R(0)\alpha_R(t)\alpha_R(t)}
\]

(3)

This defines the triple-reggeon coupling \( \xi_{R_1R_2R_3} \) (see Fig. 3) and furthermore shows the connection of \( \xi_{R_1R_2R_3} \) to the \( R_1 - R_2 \) cut.

One can see from Eq. (3) that for most cases of interest the asymptotic behavior of \( A_{R_1a \rightarrow aR_2} \) is such that the integral in Eq. (2) is divergent. However, analytically continuing the integral, by subtracting the "bad" high \( M^2 \) limit of \( A_{R_1a \rightarrow aR_2} \), a convergent expression for \( N_{R_1a \rightarrow aR_2} \) can be derived. In fact the analytic continuation procedure is automatically accomplished through the use of finite-energy-sum-rules for reggeon-particle amplitudes,

\[
\int_0^\infty dM^2 A_{R_1a \rightarrow aR_2}(M^2,t) = N_{R_1a \rightarrow aR_2}(t)
\]

\[
+ \sum_{R_3} \frac{\beta_{R_3a}(0)\xi_{R_1R_2R_3}(t)\Lambda}{\alpha_R(0) + 1 - \alpha_R(0) - \alpha_R(t)} \left( \frac{M^2}{\Lambda^2} \right)^{\alpha_R(0) + 1 - \alpha_R(t) - \alpha_R(t)}
\]

(4)

where \( \Lambda^2 \) is a properly chosen cut-off.

Later on we shall discuss in detail the case of \( R_1 = R_2 = R \) for which Eq. (4) reduces to

\[
N_{R \rightarrow R}(t) = \int_0^\infty dM^2 A_{R \rightarrow R}(M^2,t)
\]

\[
+ \sum_{R_3} \frac{\beta_{Raa}(0)\xi_{R_1R_2R_3}(t)\Lambda}{\alpha_R(0) + 1 - \alpha_R(0) - \alpha_R(t)} \left( \frac{M^2}{\Lambda^2} \right)^{\alpha_R(0) + 1 - \alpha_R(t) - \alpha_R(t)}
\]

(4')

where \( R_3 = P,R' \), \( \alpha_R(0) = 1 \), and \( \alpha_R(0) = 1/2 \). In contrast with the first term in Eq. (4') being always positive definite the terms under
the sum contribute either positively or negatively to $N_{R_a \rightarrow R_a}$ depending on whether $\alpha_R(0) + 1 - 2\alpha_R(t)$ is, respectively, negative or positive. It was first pointed out by Muzinich et al.\textsuperscript{3} that because the triple-pomeranchon coupling contributes a negative term to $N_{R_a \rightarrow R_a}$ the Gribov-Midgal\textsuperscript{11} lower bound for the P-P cut might in fact be violated.

III. RESULTS FOR TRIPLE REGGEON COUPLINGS FROM MISSING MASS DATA

We shall now turn to the empirical prominent properties of the observed inclusive spectra. The kinematics of the processes to be studied is shown in Fig. 2b where $s = (p_a + p_b)^2$, $t = (p_b - p_b)^2$, and $M^2 = (p_a + p_b - p_b)^2$. Note that in Fig. 2b the vacuum exchange is not forbidden. Such processes are $\pi^- + p \rightarrow X + p$\textsuperscript{12} (a = $\pi^-$ in Fig. 2b) and $p + p \rightarrow X + p$\textsuperscript{13,14} particularly for which interesting data have been accumulated.

The most remarkable features of these data, as observed from $(d^2\sigma/dtdM^2)$ at high $s$, small $t$, and $M^2$ above the resonance region ($s/M^2 \gtrsim 5$), are the rapid fall-off with the incident energy ($\sim 1/s$) and the flatness as a function of $M^2$.

What can one learn about Regge cuts from these experimental facts? Equivalently, what properties of reggeon-particle amplitudes are implied from the available inclusive data? To answer this the inclusive distribution will be written in the triple-Regge representation:\textsuperscript{15}

$$\frac{d^2\sigma}{dt\,dM^2}_{a+b \rightarrow X+b} = \frac{1}{16\pi s^2} \sum_{R_1, R_2, R_3} \beta_{R_1bb}(t) \beta_{R_2bb}(t) \xi_{R_1}(t) \xi^*_{R_2}(t)$$

$$\times \left( \frac{s}{s_0} \right)^{\alpha_{R_1}(t)\alpha_{R_2}(t)} \beta_{R_3aa}(0) \xi_{R_1R_2R_3}(t) \left( \frac{M^2}{s_0} \right)^{\alpha_{R_3}(0)\alpha_{R_1}(t)\alpha_{R_2}(t)}$$

(5)

From the $\sim 1/s$ fall-off at $t = 0$ one obtains that the dominant exchanges in Fig. 2a have $\alpha_{R_1}(0) = \alpha_{R_2}(0) = 1/2$. That means that the
Now to obtain information on the $R$ trajectory (see Fig. 3) we exploit the flatness property in $M^2$ at $t = 0$ which implies $\alpha_{R_3}(0) = \alpha_{R_1}(0) + \alpha_{R_2}(0)$. When this result is coupled with the result for $R_1$ and $R_2$ derived from the $s$ dependence of the spectrum, one concludes that $\alpha_{R_3}(0) \approx 1$. In other words although the pomeranchon couples weakly to large masses it is, however, dominant in the reggeon-particle amplitudes. Therefore the RRP term is stronger than the PPP and PP'P terms (only these terms contribute in the scaling limit). Moreover it is important to emphasize that the RRP term is disfavored by the energy dependence, relative to PPP and PP'P, and hence the coupling relevant for the cut, namely $\delta_{RRP}$ [see Eq. (4')] is even much stronger than $\delta_{PPP}$ and $\delta_{PP'P}$ at least for small $t$.

We can summarize, just from the salient features of present missing mass experiments, that $\delta_{RRP}$ is expected to be extremely crucial in R-R cuts and that $\delta_{PPP}$ and $\delta_{PP'P}$ will be less important in the corresponding cuts. In fact a quantitative estimate for the importance of $\delta_{RRP}$ in R-R cuts is derived below.

IV. A SUGGESTIVE ESTIMATE FOR THE $\delta_{RRP}$ EFFECT IN R-R CUTS

For definiteness we consider the processes $^{12-14}a + p \rightarrow X + p$, $a = \pi,\rho$ in Fig. 2a, where the dominant exchanges are $P,P'$ and $\omega$ (the couplings of $\rho$ and $A_2$ are small) as well as $\pi$ exchange which will not be considered in the present context as it will not affect the essential conclusions.

As discussed above the inclusive data indicates that $P'$ and $\omega$ are the dominant mechanisms for missing masses above the resonance region. Consequently for exchange degenerate $P'$ and $\omega$ one obtains from Eq. (5), for $t = 0$, the following approximate relation:

$$
\frac{d^2\sigma}{dt dM^2} \bigg|_{a+p \rightarrow X+p} \approx \frac{1}{16\pi^2} 2 \beta_{RRP}(t) |s_R(t)|^2 \left( \frac{s}{s_0} \right)^{2\alpha_R(t)} \chi \beta_{Pa}(0) \delta_{RRP}(t) \left( \frac{M^2}{s_0} \right)^{\alpha_P(0) - 2\alpha_R(t)}
$$

where $R = P'$ or $\omega$.

It will be instructive to split $N_{Ra \rightarrow Ra}$ into two pieces, the first contributed by resonances and the second by background (BG);

$$
N_{Ra \rightarrow Ra} = N_{Res} \rightarrow Ra + N_{BG} \rightarrow Ra.
$$

From Eq. (4') one obtains

$$
N_{Res \rightarrow Ra}(t) = \int_0^{M^2} dM^2 A_{Res \rightarrow Ra}(M^2, t) \beta_{Riem}(0) \delta_{RRP}(t) \left( \frac{s}{s_0} \right)^{2\alpha_R(t)} \chi \left( \frac{M^2}{s_0} \right)^{\alpha_R(0) + 1 - 2\alpha_R(t)}
$$

$$
N_{BG \rightarrow Ra}(t) = \frac{1}{16\pi^2} 2 \beta_{RRP}(t) |s_R(t)|^2 \left( \frac{s}{s_0} \right)^{2\alpha_R(t)} \chi \beta_{Pa}(0) \delta_{RRP}(t) \left( \frac{M^2}{s_0} \right)^{\alpha_P(0) - 2\alpha_R(t)}
$$
\[ \beta_{\text{pp}}(0) g_{\text{RRP}}(t) \approx -\frac{\beta_{\text{pp}}(0) g_{\text{RRP}}(t)}{\alpha_F(0) + 1 - \alpha_R(0)} \left( \frac{M^2}{s_0} \right)^{\alpha_p(0) + 1 - \alpha_R(0)} \]  

(7'')

where \( M^2 \sim 1 \text{ GeV} \) is a mass below which the background contribution is negligible relative to the terms in Eqs. (7') and (7'').

Our central goal is to study the role played by \( g_{\text{RRP}} \) in \( N \rightarrow R_\alpha \) and hence in R-R cuts. If indeed \( \beta_{\text{pp}}(0) g_{\text{RRP}}(t) \) turns out to be considerably large it might indicate a substantial correction to \( N \rightarrow R_\alpha \); the latter being constructed from models based on resonances only, e.g., dual resonance models and duality considerations.

The data of Anderson et al.\(^1\) for \( p + p \rightarrow X + p \) at \( \sqrt{s} = 15.1 \text{ GeV} \) and \( t = -0.024 \text{(GeV/c)}^2 \), will be used to estimate the coupling \( \beta_{\text{pp}}(0) g_{\text{RRP}}(t) \) which is the one relevant to the R-R cut [see Eqs. (6) and (7'')]. Using the above-mentioned data at \( M^2 \approx 4 \text{ GeV}^2 \) and the empirical Regge residue function \( \beta_{\text{pp}}(t) \),\(^1\) one obtains

\[ \beta_{\text{pp}}(0) g_{\text{RRP}}(0) \approx 53 \text{ GeV}^{-2} \]  

(8)

and from Eq. (7''), with \( M^2 \sim 1 \text{ GeV}^2 \),

\[ \beta_{\text{pp}}(0) g_{\text{RRP}}(0) \approx -53. \]  

(8')

The last result should be compared with the Born term in \( N \rightarrow R_\alpha \) which is\(^1\) \[ \beta_{\text{pp}}(0) g_{\text{RRP}}(0) \approx 50. \]

V. RESULTS FOR CUTS AND CONCLUSION

From the above estimates, based on inclusive data, one realizes that the negative term in \( N \rightarrow R_\alpha \) due to the triple-reggeon coupling \( g_{\text{RRP}} \) can be comparable with the positive resonances contribution. Therefore it is possible that the R-R cut cannot be minorized by the eikonal approximation.\(^1\)

It should be emphasized that the present discussion on R-R cuts hinges strongly on the very important role played by \( g_{\text{RRP}} \) in the structure of inclusive spectra.

An interesting rule for R-R cuts had been derived some time ago by Finkelstein.\(^6\) The rule says that the R-R cut (Fig. 1a) is negligible if there is no "s-u" duality diagram\(^2\) for \( R_1 + a \rightarrow R_2 + a \) and/or \( \bar{R}_1 + b \rightarrow \bar{R}_2 + b \). However, this rule applies only to the resonance part of the fixed pole residue. Finkelstein already remarked that the selection rule may not apply to cuts in which the reggeon-particle process has a diffractive component. However, it was not known how to estimate the diffractive contribution to the fixed pole residue. It is only recently, due to the developments in inclusive reactions, that we were able to control the vacuum exchange in the reggeon-particle amplitude. We have shown before, from inclusive data, that the \( g_{\text{RRP}} \) coupling may lead to a non-negligible deviation from the above-mentioned selection rule.

Phenomenologically the most important cuts are the so-called "absorptive cuts" namely a pomeron is exchanged with a reggeon. In most practical cases the P-R cuts appear in processes with a transfer of quantum numbers and therefore no vacuum exchange occurs in the relevant P + particle \( \rightarrow R + \) particle amplitudes. However, one
may ask about the diffractive effects in the fixed pole residue which determines the P-P' cut. Again the answer comes from the observed features of missing mass spectra indicating very small effects. Although the present inclusive data do not allow a precise determination of the various triple reggeon couplings, we can nevertheless be confident that the data imply a very small $g_{pp'p}$ relative to $g_{RRP}$. Therefore the contribution to the P-P' cut arising from $g_{pp'p}$ is much smaller than the eikonal term and the resonance approximation to the fixed pole residue is expected to be a good one [compare with Eqs. (7)].

The present work has further emphasized the important impact of missing mass experiments on reggeon-reggeon cuts. It is extremely useful to have inclusive measurements at many values of the incident energy (low as well as high) and small momentum transfer ($< 1 \text{GeV}^2$) so as to allow a more precise determination of the various reggeon-particle amplitudes and hence a better insight into the structure of cuts.

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FOOTNOTES AND REFERENCES

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9. This procedure has been first adopted in Ref. 3 in the context of the P-P cut. For a discussion of pp and A2A2 cuts see; D. P. Roy and R. G. Roberts, Phys. Letters 40B, 555 (1972).


17. The significance of M' lies in the possible difference between the extrapolated pomeron exchange in the reggeon-particle amplitude and the background in the resonance region. Strictly speaking the value of M' is not known a priori. However, for a rough estimate of \( N_{R_1}^{R_2} \) in Eq. (19) the value M' \( \sim 1 \) GeV is reasonable.

18. The normalization and the evaluation of the various Regge residue functions are as follows; the trajectory \( R \) contributes

\[
\frac{1}{16\pi^2} \beta_{pp}(t) |t_R(t)|^2 \left( \frac{s}{s_0} \right)^{\alpha_R(t)}
\]

to \( \frac{d\sigma}{dt} \) and

\[
\frac{1}{s} \beta_{pp}^2(0) \left( \frac{s}{s_0} \right)^{\alpha_R(t)}
\]

to \( q_T \).

Assuming \( P' \) and \( \omega \) to be exchange degenerate one obtains from the fall off \( q_T(pp) \) (see E. Flaminio et al., Compilation of Cross-Sections II-Antiproton Induced Reactions, CERN-HERA 70-3, August 1970);

\[
\beta_{pp}^2(0) = \frac{1}{2} \beta_{pp}^2(0) = 5.0.
\]

Furthermore from the differential cross-section data the cut-off in \( t \) for \( \beta_{pp}^2(t) \) is roughly given by \( e^{4t} \).

The result in Eq. (8) is then obtained by using the above value for \( \beta_{pp}^2(t) \) and Ref. 14

\[
\frac{d^2\sigma}{dtdw^2} \bigg|_{t=0.024, M'^2} \approx 3.5 \frac{mb}{GeV^4}
\]

at \( P_{LAB} = 15.1 \) GeV c.

19. To avoid confusion it is remarked that here the "eikonal approximation" means that the fixed pole residue is approximated by only the first intermediate state [see Eq. (2)].

Fig. 1. (a) The forward $R_1-R_2$ cut in the process $a + b \rightarrow a + b$. Note that the coupling of the external particles to the cut is through the fixed pole residues $N_{R_1a \rightarrow R_2a}^1$ and $N_{R_1b \rightarrow R_2b}^2$ defined in Eq. (2). The "mass" of the reggeons is $t = q_1^2 = q_2^2$.

(b) The reggeon-particle process $R_1 + a \rightarrow R_2 + a$ relevant to the upper blob of the diagram in Fig. 1a.

Fig. 2. (a) The contribution of the $R_1$ and $R_2$ trajectories to the amplitude of $a + b \rightarrow X + b$ in the exchange region.

(b) The $R_1$ and $R_2$ interference term in the differential inclusive cross-section which determines the amplitude $A_{R_1a \rightarrow R_2a}(M^2, t)$ [see Eq. (2)].

Fig 3. A diagram which appears in the high $M^2$ limit of the process $R_1 + a \rightarrow R_2 + a$ (Fig. 1b) and serves to define the triple reggeon coupling $g_{R_1R_2R_3}(t)$. 

FIGURE CAPTIONS
Fig. 1a

Fig. 1b
Fig. 2a

Fig. 2b
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