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Mixing of Multiple Jets With a Confined Subsonic Crossflow: Part I—Cylindrical Duct

This paper summarizes NASA-supported experimental and computational results on the mixing of a row of jets with a confined subsonic crossflow in a cylindrical duct. The studies from which these results were excerpted investigated flow and geometric variations typical of the complex three-dimensional flowfield in the combustion chambers in gas turbine engines. The principal observations were that the momentum-flux ratio and the number of orifices were significant variables. Jet penetration was critical, and jet penetration decreased as either the number of orifices increased or the momentum-flux ratio decreased. It also appeared that jet penetration remained fairly constant with variations in orifice size, shape, spacing, and momentum-flux ratio when the number of orifices was proportional to the square root of the momentum-flux ratio. In the cylindrical geometry, planar variances are very sensitive to events in the near-wall region, so planar averages must be considered in context with the distributions. The mass-flow ratios and orifices investigated were often very large (mass-flow ratio >1 and ratio of orifice area-to-mainstream cross-sectional area up to 0.5), and the axial planes of interest were sometimes near the orifice trailing edge. Three-dimensional flow was a key part of efficient mixing and was observed for all configurations. The results shown also seem to indicate that nonreacting dimensionless scalar profiles can emulate the reacting flow equivalence ratio distribution reasonably well. The results cited suggest that further study may not necessarily lead to a universal “rule of thumb” for mixer design for lowest emissions, because optimization will likely require an assessment for a specific application.

1 Introduction

Jets-in-crossflow have been extensively treated in the literature. Flows in which this is an integral constituent occur in a number of areas important in combustion and energy science and technology. In a gas turbine combustor for example, fuel and air mixing is important to combustor performance and emissions. Also, the mixing associated with arrays of jets in crossflow can play a critical role as in the dilution zone of a conventional combustor, and the mixing zone of a staged combustor such as the Rich-Burn/Quick-Mix/Lean-Burn (RQL) combustor. Although results reported to date all have contributed additional understanding of the general problem, the information obtained in them may not satisfy the specific needs of different applications.

One characteristic of jet-in-crossflow applications in gas turbine combustion chambers is that they are often confined mixing problems, with up to 80 percent of the total flow entering through the jets. The result is that the equilibrium mixing pattern and composition of the exiting flow may differ significantly from that of the entering mainstream flow.

axial direction). Because the objective in combustor applications is to identify configurations to provide a desired mixing pattern within a given downstream distance, locations of interest are identified in intervals of the duct height (radius) rather than the orifice diameter, \( d \). The primary independent flow variables are the jet-to-mainstream mass-flow (\( MR = \frac{w_j}{w_m} \)) and momentum-flux (\( J \)) ratios. These can be expressed as:

\[
J = \frac{MR^2}{(DR)(C_d)^2(A/A_o)^2} \quad (2)
\]

It was reported in Holdeman (1993) that jet penetration and centerplane profiles were similar when the orifice spacing and the square root of the momentum-flux ratio were inversely proportional, i.e.:

\[
C = \frac{(S/H)J}{\sqrt{J}} \quad (3)
\]

In a cylindrical duct, the radius, \( R \), corresponds to the channel height, \( H \), for one-side injection in a rectangular duct. For single-sided injection the centerplane profiles are approximately centered across the duct height and approach an isothermal distribution in the minimum downstream distance when \( C = 2.5 \). This appeared to be independent of orifice diameter, as shown in both calculated and experimental profiles. The similarity of the profiles with the same orifice spacing but with different orifice diameters was also shown by Holdeman et al. (1973). Values of \( C \) in Eq. (3) that are a factor of 2 or more smaller or larger than the optimum correspond to underpenetration or overpenetration, respectively.

For a can the optimum orifice spacing was specified at the radius, which divides the can into equal areas. That is, the relationship of the spacing between jet centerlines to the number of holes around the circumference of the can would be

\[
S = \frac{2\pi R_{1/2}}{n} \quad (4)
\]

It follows that the sector for each orifice would be \( 360/n \) deg.

### 3 Results and Discussion

The following paragraphs describe the results from recent investigations in the context of the effects of the primary independent variables. Both experimental and computational studies were performed, but are interspersed here. The work cited was performed by Allison Engine Company, CFD Research Corporation, United Technologies Research Center, and the University of California, Irvine. Sources are identified when results are discussed, and specifics of the calculations or experiments, as appropriate, are given in the corresponding references.

All planar nonuniformity values are expressed as a variance from the mean values. Although the definitions used in the original papers differ slightly, they are essentially rms values. All orifices considered in this paper are thin (thickness/diameter < 0.25, and are plenum-fed with no bypass air.

Investigations published prior to 1991 were primarily in a rectangular duct, and at significantly lower mass-flow ratios than in more recent studies. A schematic showing the relative orifice size is given in Fig. 2. Effects investigated in recent studies included: (1) variation in momentum-flux ratio (\( J \)) at constant geometry; (2) variation of number of orifices at constant \( J \); (3) comparison of slots and holes; (4) variation of slot aspect ratio; (5) variation of slanted slot angle; (6) results of orifice optimization; (7) effect of mixing duct size; (8) relation of mixing and emissions; and (9) effect of reaction. These are discussed in the following sections. Results from previous stud-

### Nomenclature

- \( A/A_o = \) jet-to-mainstream area ratio
- \( AC_d = (A)/(C_d) \)
- \( C = (S/H)J \)
- \( C_d = \) orifice discharge coefficient
- \( D = \) diameter of cylindrical duct
- \( d = \) orifice diameter
- \( DR = \) jet-to-mainstream density ratio
- \( H = \) duct height (rectangular)
- \( J = \) jet-to-mainstream momentum-flux ratio
- \( L = \) long dimension of orifice
- \( L/W = \) orifice aspect ratio
- \( MR = \) jet-to-mainstream mass-flow ratio
- \( n = \) number of holes around can
- \( r = \) radial coordinate
- \( R = \) can radius
- \( S = \) lateral (circular) spacing between orifice centers
- \( T = \) temperature
- \( T_{ej} = \) jet exit temperature
- \( T_m = \) mainstream temperature
- \( U = \) axial velocity
- \( U_m = \) mainstream velocity
- \( V_j = \) jet velocity
- \( W = \) short dimension of orifice
- \( x = \) downstream coordinate
- \( y = \) cross-stream (radial) coordinate
- \( z = \) lateral (circular) coordinate
- \( \theta = (T_m - T)/(T_m - T_j) \)

\[ R_{1/2} = \frac{H}{\sqrt{J}} \quad (5) \]

Substituting Eq. (5) into Eq. (4), and the resulting \( S/H \) into the spacing and momentum-flux relationship for a rectangular duct (Eq. (3)) gives the appropriate number of round holes as:

\[ n = \frac{\pi \sqrt{2}J}{C} \quad (6) \]
ies showed that the density ratio (DR) was not a significant variable at constant momentum-flux ratio ($J$).

3.1 Momentum-Flux Ratio. Based on previous studies, which reported that the most important flow variable influencing the extent of jet mixing in a crossflow was the momentum-flux ratio, Hatch et al. (1995a) performed a series of tests with eight orifices at three representative $J$ values. The results reaffirmed the importance of the momentum-flux ratio in determining the downstream flowfield.

A representative flowfield evolution for a baseline case of eight round orifices at $J$ near 25 (Hatch et al., 1995a), is shown in Fig. 3. In the first plane (bottom), the absence of jet fluid is noted by the limited near-zero mixture fraction values, while the presence of unmixed mainstream flow is apparent by the high mixture fraction values approaching 1 (unmixed jet fluid = 0 and unmixed mainstream fluid = 1). By the fifth plane downstream (up in Fig. 3), the jet and mainstream flow have mixed and have created a band of mixture fraction values that approach the equilibrium value. Mixture uniformity values calculated per plane provide the basis for the planar trend shown in Fig. 4.

A similar effect is apparent in the results of Talpallikar et al. (1990) as shown in Fig. 5. The planar mixture nonuniformity for this case is shown in Fig. 6 with a clearly defined optimum $J$ for the configurations examined, with underpenetration to the left and overpenetration to the right.

The effect of momentum-flux ratio can also be seen in the experimental results of Vranos et al. (1991) as seen in Fig. 7. A planar distribution from this study for slanted slots is shown in Fig. 8. Differences in the $J$ value of the minimum in these studies (for example Vranos et al. (1991) and Talpallikar et al. (1990) are due to slight differences in the conditions examined, and are unimportant.

The data shown in Figs. 7 and 8 are for slanted slots. The slant angle for slots is the angle between the long dimension and the axial direction. Note that slanted slots induce swirl although none is present in the main flow (Vranos et al., 1991). It is also apparent in the results that although a local minimum is identified, it is possible to achieve low values of mixture nonuniformity at higher $J$ values corresponding to overpenetration. This emphasizes that although planar-averaged values are very useful and can provide insight, one cannot rely on them alone, and must also assess the flowfield distributions as shown in Figs. 3, 5, and 7.
jets enter from the can wall at the top, and proceed toward the xIR ratios was unknown. For this configuration, 12 orifices seem to give optimum mixing planes through the orifice center that were constructed from 100 ratios, its applicability to other shapes and at higher mass-flow poor mixing and high NOx emissions (Talpallikar et al., 1990). That for 10 is overpenetrated. Furthermore, the latter case can increases as an inverse function of the number of orifices. It is whereas 11 would be predicted from Eq. (6).

thermocouple measurements in each of five planes. Here the tally for lowest values of mixture fraction downstream from the orifice. To translate adjacent vortices toward the duct wall. An additional influence of neighboring jets is to constrain the lateral spreading of the jet, and to spread it along its centerplane. This is particularly evident in the can as the lateral spread is increasingly restricted as the duct centerline is approached from the wall.

In contrast to the round jet, the slanted slot initially forms a pair of counterrotating vortices, which are of unequal size and strength. Larger vortices form downstream of the orifice leading edge and move toward the duct wall, while the smaller vortex moves away from the wall. There is considerable interaction between neighboring jets early in the injection process. In this case, unlike the round jet system, the induced velocity field is such that the vortex pair rotates about an axis connecting the vortex centers.

The bulk of the jet fluid identifies the location of the leading edge, thereby showing the direction that the slot is slanted (in Fig. 7 the upstream edge of the slot is on the clockwise side). Furthermore, the slanted slot jet experiences a lateral force that causes it to rotate about the duct axis. The greatest circumferential velocity is due to the large vortex, and is near the wall. At the same time, the flow near the center is of the opposite sense so the net angular momentum is zero.

Planar nonuniformity for round holes and 45 deg slanted slots at x/R = 1.2 is shown in Fig. 12. At this distance the two systems exhibit roughly the same average mixing, although the optimum J for round holes is less than that for slanted slots. It follows from the discussion in the previous section that the optimum spacing for slanted slots would be greater than for round holes for the same momentum-flux ratio.

### 3.2 Number of Orifices
In general, the effect of increasing the number of orifices around the perimeter of a can is similar to the effect of decreasing the momentum-flux ratio (see Eq. (6)). Although this was also evident in previous results, the optimum was recently shown in the computational study by Smith et al. (1991). Since the optimum number relation, Eq. (6), was originally developed from computational results and round-hole data obtained in rectangular ducts at low mass-flow ratios, its applicability to other shapes and at higher mass-flow ratios was unknown.

Figure 9 shows isotherms of the centerplane (radial-axial plane through the geometric center of the orifice) for a different number of 4:1 aligned slots for J = 36. The jet penetration increases as an inverse function of the number of orifices. It is obvious that the flow from 14 slots is underpenetrated, whereas that for 10 is overpenetrated. Furthermore, the latter case can lead to upstream flow near the duct center, and can cause both poor mixing and high NOx emissions (Talpallikar et al., 1990). For this configuration, 12 orifices seem to give optimum mixing whereas 11 would be predicted from Eq. (6).

The optimum number of round orifices was found experimentally for J = 52 to be between 12 and 15. This is shown in Fig. 10 from Kroll et al. (1993). These distributions are radial-axial planes through the orifice center that were constructed from 100 thermocouple measurements in each of five planes. Here the jets enter from the can wall at the top, and proceed toward the can centerline at the bottom. The mainstream flow is from left to right. The mean jet trajectory can be traced by following the lowest values of mixture fraction downstream from the orifice.

### 3.3 Slots and Holes
Representative still frames from movies of low-speed flows from Vranos et al. (1991) are shown in Fig. 11. These indicate significant differences in the jet/jet and jet/mainstream interactions between slanted slots and round hole injectors. The jet exiting a round hole forms two counterrotating vortices of equal strength. The jet penetrates directly toward the center of the duct, and the jet cross section is stretched as J increases. The connecting sheet moves closer to the duct axis, but the vortices tend to remain near the wall.

The tendency of the vortices to stay near the wall is attributed, in part, to interaction between neighboring vortices, which act to translate adjacent vortices toward the duct wall. An additional

![Figure 7](image7.png) **Figure 7** Effect of momentum-flux ratio on mixing from 6 slanted slots at x/R = 1.2 and DR = 1.0 (data from Vranos et al., 1991)

![Figure 8](image8.png) **Figure 8** Mixture nonuniformity of slanted slots (data from Vranos et al., 1991)
3.5 Slanted Slot Angle. The computational results in Fig. 15 from Oechsle et al. (1992) indicate a significant decrease in jet penetration as slot slant angle is increased. Note that the slant angle is the angular deviation from the axial direction. This effect is also shown in the experimental results of Hatch et al. (1995a) that are shown in Fig. 16.

3.6 Orifice Optimization. Sowa et al. (1994) engaged in a more comprehensive optimization scheme incorporating parameters such as the number of orifices, orifice aspect ratio, and orifice angle at a fixed momentum-flux ratio. Optimum mixing occurred when the mean trajectory lay between a radial distance of 50–65 percent from the mixer centerline at one duct radius downstream from the leading edge of the orifices. A numerical regression performed on the data yielded a nonlinear relationship between the orifice configurations and the mixture uniformity. At the optimum number of orifices, both a round hole and a 5:1 22 degree slanted slot had minima in mixing uniformity. These distributions are shown in Fig. 17.

3.7 Mixing Duct Size. Three duct sizes were reported in Smith et al. (1991) using a previously optimized 12-slot geometry as a baseline case. The three mixing section diameters were 6, 5, and 4 in. (15.24, 12.70, and 10.16 cm). As the area was reduced, the velocity of the mainstream flow in the mixing section increased proportional to the area reduction. The re-
resulting reduction in static pressure in the mixing section increases the pressure drop across the orifices, thus increasing the jet velocity. Both the jet and mainstream velocities increase, and these counterbalance such that the momentum-flux ratio remains constant as the mixing flow area is reduced.

The resultant mixing is shown in Figs. 18 and 19 from Smith et al. (1991). The slot size was adjusted according to the variation in diameter of the mixing section to ensure a constant mass-flow ratio.

Figure 18 shows the temperature distributions in radial-axial planes through the orifice centerline for all three diameters. Figure 19 shows the corresponding distributions in a radial-transverse plane at one mixing section diameter downstream of the jet inlet. In this figure a full circle is shown, although the computations were performed for a 15 deg pie section. The similarity of the plots suggests that the flow was nondimensionally identical for these cases.

However, the corresponding NOx results (Smith et al., 1991) are not identical, as shown in Fig. 20. In this figure, NOx production is plotted as a function of axial location for each of the three mixing section diameters. The NOx Emission Index (EI (as defined by ARP 1256A)) at \(x/R = 2\) for the 4" section is 70 percent less than that for the 6" section. For these cases, CO was completely depleted by \(x/R = 2\).

The formation of NOx is controlled by temperature, oxygen concentration, and residence time. Since mixing was identical, temperatures and oxygen concentrations must be identical, leaving only residence time to account for the difference. The NOx reduction apparent in Fig. 20 is related to the decrease in residence time that occurs in smaller sections both through increased velocities and shorter mixing lengths. This is discussed in Smith et al. (1991).

### 3.8 Mixing and Emissions.

The relation between mixing and NOx was investigated in Hatch et al. (1995b) using a procedure to infer NOx signatures from nonreacting experimental data. The NO formation rate corresponding to the mixing flow field in Fig. 3 is shown in Fig. 21. The mixing and NO production field for the same configuration (eight round holes) at a higher momentum-flux ratio \(J = 84.2\) are shown in Fig. 22. The Mixing Uniformity, NO Production Rate, and the Accumulated NO Produced for these configurations plus an intermediate \(J\) are shown in Fig. 23.

The majority of nitric oxide is formed early in the injection. As a result the mixing processes in the initial region are critical in the overall emissions performance of the mixer. However, as can be seen in Fig. 23, rapid early mixing due to overpenetrating jets (e.g. at \(J = 84.2\) in Figs. 22 and 23), does not necessarily lead to a minimum production of NO.

For the range of momentum-flux ratios and orifice geometries examined in Hatch et al. (1995b) the round holes and 45 deg 4:1 slanted slots at \(J \sim 55\) yielded the best mixers from a NO perspective.

The relation between NOx and mixing, for a fixed number of orifices, was also examined in the computations reported in Oechsle and Holdeman (1995). It was shown that, in general, statistical mixing parameters do not correlate with NOx production rates at downstream axial locations (e.g., \(x/R = 1\), as the planar variances lack historical information from throughout the mixing region.
NOx production is shown to be highly related to the jet penetration. Overpenetrating configurations show increased NOx production, as do underpenetrating cases. For example, at low $J$ conditions, optimum penetration is achieved with round holes, and NOx is minimum. At higher $J$'s the jets overpenetrate and NOx increases primarily due to its formation near the combustor walls. Similarly, jet penetration is optimum at higher momentum-flux ratios with large aspect ratios and slant angles, and NOx is minimum for these configurations. At lower $J$'s the jets severely underpenetrate, and NOx increases due to its formation near the combustor centerline.

Although planar parameters don't seem to correlate with NOx, one can infer relative NOx production from the radial-axial and radial-transverse distributions, namely optimum penetration will generally yield minimum NOx. One caveat is important here though: What one calls optimum depends on the axial location observed, that is, “optimum” penetration near the orifice will result in overpenetration farther downstream; and conversely “optimum” downstream penetration will look like underpenetration upstream of that location.

3.9 Reaction. The computational results reported by Oechsle et al. (1994) show that reacting flow distributions are very similar to nonreacting ones, provided that a conserved scalar is compared (which dimensionless temperature is not in a reacting flow as sources and sinks exist for this).

The dimensionless temperature distributions for the nonreacting cases are compared with normalized equivalence ratio distributions for the reacting flow cases. Radial-axial planes appear in Fig. 24, while the corresponding radial-tangential planes at $x/R = 1$ are shown in Fig. 25. These results suggest that the nonreacting temperature profiles can emulate the reacting flow equivalence ratio distribution reasonably well.

It is worth noting, however, that the nonreacting jets appear to interact more near the center of the mixer as compared to the corresponding reacting flow results. This is usually spotted by the upstream swirling flow production near where opposing jets merge. This was not observed in the 12-orifice cases investigated due to the much shallower penetration for these compared to the 8-orifice cases.

Reacting flow studies by Leong et al. (1995) were the first experimental characterization of jet mixing in a rich reacting cylindrical crossflow. Species concentration measurements were obtained for four round hole orifice configurations at a predetermined $J$. Jet penetration, as indicated by the maximum O2 trajectory, was observed to affect reaction and mixing processes.
Jet penetration toward the midradius by \( x/R = 1 \) resulted in more lateral spreading of jet fluid, which made available more fluid volume to react with the rich crossflow to produce \( CO_2 \). Figures 26 and 27 show that the 12-hole case produced an optimal jet \( (O_2) \) trajectory, which gave a more evenly dispersed \( CO_2 \) distribution that most closely matches the concentration expected for the equilibrium equivalence ratio.

4 Design Procedure

These results suggest that for a given momentum-flux ratio and downstream distance, combustor design procedure should first identify the circumferential orifice spacing required to obtain the desired penetration and profile shape. The orifice size would then be chosen to provide the required jet-to-mainstream mass-flow ratio. Some adjustments, including noncircular orifices or multiple rows, may be needed to arrive at the final design because the penetration varies slightly with orifice size and shape, and other parameters such as the combustor pressure loss; and the ratio of the orifice spacing to diameter must be monitored to insure that the suggested configuration is physically realistic.

Based on these results, the suggested procedure is, given mass-flow ratio, pressure drop, and channel height:

1. Choose desired orifice shape and \( C_d \)
2. Identify needed total orifice area
3. Calculate momentum-flux ratio \( (J) \)
4. Calculate individual orifice size
5. Select number of orifices for optimum penetration
6. Determine blockage, fit, etc.
7. Iterate to solution

Summary of Results

(A) Several results from recent studies in a cylindrical duct are consistent with previous results from investigations in rectangular ducts. These include:

Fig. 17 Predicted values of area weighted standard deviation “STD” for different orifice numbers as orifice aspect ratio and orifice angle are changed (data from Sowa et al., 1994)

Fig. 18 Predicted radial-axial plane isothermal maps (data from Smith et al., 1991)

Fig. 19 Predicted radial-tangential plane isothermal maps \( (x/R = 2.0) \) for \( J = 36 \); variation in mixing diameter (data from Smith et al., 1991)

Fig. 20 \( NO_x \) emission index for mixing diameters of 6", 5", and 4" (data from Smith et al., 1991)
Fig. 21 NO production, eight round hole configuration, $J = 26.7$ (data from Hatch et al., 1995b)

1. Variations in momentum-flux ratio and number of orifices have a significant effect on the flow distribution.
2. Optimum configurations may depend on given momentum-flux ratio, number of orifices, and orifice shape.
   (a) Optimum spacing may vary with orifice shape.
   (b) The optimum number of orifices ($n$) increases with increasing momentum-flux ratio ($J$). For most orifice shapes, $n$ is proportional to $\sqrt{J}$.
   (c) The same orifice shape may not be best for all momentum-flux ratios.
   (d) What is perceived as "optimum" depends both on the application and the downstream distance.
3. Similar distributions can be obtained, independent of orifice size and shape, when $n$ is proportional to $\sqrt{J}$. Although orifice configurations can be optimized for any $J$, a greater downstream distance is required for equivalent mixing if either $J$ and/or the optimum number of orifices is small.
4. The penetration of slanted slots is less than for aligned slots, or equal-area circular holes. Also, scalar distributions for slanted slots are rotated with respect to the injection centerplane.

Fig. 22 Equivalence ratio and NO production, eight round hole configuration, $J = 84.2$ (data from Hatch et al., 1995b)

Fig. 23 Planar results for eight round hole configuration (data from Hatch et al., 1995b)
5 For orifices that are symmetric with respect to the main flow direction, the effects of shape appear to be significant mostly in the region near the injection plane. Beyond, e.g., \( x/R = 1 \), scalar distributions are expected to be similar to those observed from equally spaced equal-area circular orifices. (B) The minimization of NO production in a quick mixer...
will often require a tradeoff between effective initial mixing and effective mixing in the wall region downstream of the plane of injection.

The results cited suggest that further study may not necessarily lead to a universal "rule of thumb" for mixer design for lowest emissions, because optimization will likely require an assessment for a specific application.

The results shown seem to indicate that nonreacting dimensionless scalar profiles can emulate the reacting flow equivalence ratio distribution reasonably well.

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