Lawrence Berkeley National Laboratory
Recent Work

Title
APERTURE OF TWO-COUNTER TELESCOPES

Permalink
https://escholarship.org/uc/item/4rr7x7v0

Author
Swanson, W.P.

Publication Date
1988-04-01
DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
APERTURE OF TWO-COUNTER TELESCOPES

William P. Swanson
Occupational Health Division
Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

ABSTRACT

Calculation of the geometrical aperture of two-counter telescopes using circular counters is discussed. A FORTRAN program OVERLAP is described that performs such calculations with convenient user input. Representative results are shown and the LBL Berklet detector is calculated as a specific example. A listing of the program is given in an appendix.

April 3, 1988
INTRODUCTION

The telescope considered consists of two coaxial circular counters (Fig. 1). The "aperture" $A_p \equiv \langle \Delta A \cdot \Delta \Omega \rangle$ is defined as the scalar quantity that satisfies the equation

$$C = A_p \times \phi_{\text{isotropic}}$$

where $C$ is the number of particles that pass through both counters ("counts"), and $\phi_{\text{isotropic}}$ is a broad uniform isotropic fluence as defined by the ICRU (1977). If $\phi_{\text{isotropic}}$ is in units of cm$^{-2}$ sr$^{-1}$, $A_p$ will have units of cm$^2$ sr. Although the author was assured by experts that this problem had been solved many times over, a specific reference showing practical results could not be found. This note solves only the simple geometrical problem; particle "loss" by scattering or interactions in the telescope are not considered.

METHOD OF CALCULATION

"Pretend you are very far away and shining a flashlight on the smaller counter. Its shadow on the plane of the larger counter will be circular. Now move the flashlight around in all possible directions ... ."* Program OVERLAP, written in FORTRAN for the VAX computer, evaluates the integral

$$\int \text{AREA} (\theta, \phi) \, d\Omega.$$  

AREA is the overlap area of the shadow on the disk of the larger counter and $d\Omega = 2\pi \sin \theta \, d\theta$, where $\theta$ is the polar angle to the telescope axis (Fig. 1).

For $\theta < \theta_1 \equiv \tan^{-1}(R_2 - R_1)/S$, $\text{AREA} = \pi R_1^2$, and

For $\theta > \theta_2 \equiv \tan^{-1}(R_2 + R_1)/S$, $\text{AREA} = 0$.

(*) D. E. Groom, SSC Central Design Group, Private communication.
where $R_1$ and $R_2$ are the radii of the smaller and larger counter, respectively, and $S$ is the counter separation. For the intermediate case, $\theta_1 < \theta < \theta_2$, the overlap area is the sum of two segments defined by the intersection of the two circles in the plane of the larger counter (Fig. 2). In this plane, the center of the smaller circle is at $(0, YC)$, where $YC = S \times \tan \theta$. The two points of intersection, $(XI, YI)$, are solved for by:

$$YI = (R_2^2 - R_1^2 + YC^2)/(2 \times YC).$$

(3a)

Then, $XI$ is found using the equation of the larger circle:

$$XI = \pm \sqrt{(R_2^2 - YI^2)}. \tag{3b}$$

The segment area for the larger circle is found from

$$AREA_2 = R_2^2 \left(\phi_2 - \sin \phi_2\right) / 2.0 \tag{4}$$

where $\phi_2 = 2 \sin^{-1} \left|XI / R_2\right| < \pi$ is the angle subtended by the segment of the larger circle at the circle's center $(0, 0)$. The same considerations are used to find $AREA_1$ (segment of the smaller circle) but in this case attention must be paid to the quadrant in determining $\phi_1$ from the FORTRAN arcsin routine because $\phi_1$ can exceed $\pi$. Then $AREA = AREA_1 + AREA_2$.

The result of the entire calculation is the summation

$$APERT = \sum (2\pi \sin \theta \Delta \theta) \times (AREA \times \cos \theta), \tag{5}$$

where APERT is the desired aperture for the counter telescope and $\cos \theta$ is the obliquity factor. As written, OVERLAP uses 100 equal intervals in $\theta$, so that $\Delta \theta = |\theta_2 - \theta_1| / 100$.

**LIMITING CASE: SEPARATION $\rightarrow$ ZERO**

For the two counters in contact ($S = 0$), we have the equivalent of a single counter of area $\pi (R_1^2)$. All particles passing through the smaller counter are registered but the relevant area will appear smaller for larger values of $\theta$ by an obliquity factor equal to $\cos \theta$. The result can be shown to be $\pi$ multiplied by the area of the smaller counter. In the results shown in Figs. 3 and 4, which involve a smaller counter of area $\pi (1 \text{ cm})^2$, all curves converge to the value $\pi^2 \text{ cm}^2 \text{ sr} = 9.87 \text{ cm}^2 \text{ sr}$ as the separation goes to zero. The counters need not be circular for this limit to hold.

**LIMITING CASE: SEPARATION $\rightarrow$ LARGE**

For very large separations the aperture is given by

$$A_p = A_1A_2 / S^2 \tag{6}$$

where $A_1$ and $A_2$ are the areas of the two counters and $S$ is the separation. Units will be
steradians times the unit of area. This is shown in Fig. 5 where the dimensionless quantity $S^2A_\phi / (A_1A_2)$ is plotted as a function of counter separation and for various ratios of $R_2 / R_1$. Unity is approached for large separations. The counters need not be circular for this limit to hold.

RUNNING THE PROGRAM OVERLAP

From any VAX terminal, invoke the procedures FORTRAN, LINK and RUN using the source file OVERLAP.FOR. During the RUN procedure OVERLAP will ask for, in turn, $R_1$ (radius of the smaller counter), $R_2$ (radius of the larger counter) and $S$ (separation of the two counters). Units are cm. A single line of answers will appear almost immediately, containing:

$$R_1, R_2, S \text{ and APERT}$$

OVERLAP will then request a new set of input parameters and so on, ad infinitum. Exit from this cycle by typing CNTL C.

An example for which this program was designed is the Berklet detector (Llacer et al., 1984). For this instrument, used to characterize beams of heavy ions, the parameters are $R_1 = 0.25 \text{ cm}, R_2 = 0.55 \text{ cm}$ and $S = 1.0 \text{ cm}$. The aperture for these parameters is calculated to be $0.138 \text{ cm}^2 \text{ sr}$.

Another file, TELE.TOP, is also generated containing pairs of values $(S, \text{APERT})$. Each pair will appear on a separate line in a form easily adapted for input to TOPDRAWER (Chaffee, 1980). To properly invoke TOPDRAWER, some editing on TELE.TOP is required.

Examples of results from OVERLAP, plotted by TOPDRAWER are shown in Figs. 3 - 5. These may be applied to other geometries by simple scaling.

REFERENCES


**FIGURE CAPTIONS**

**Fig. 1** Diagram defining parameters of a telescope consisting of two circular counters. Values shown are for the Berklet detector.

**Fig. 2** Diagram in plane of larger counter showing how the overlap area is calculated from the sum of two segments, one from each circle.

**Fig. 3** Output of the program OVERLAP showing behavior of aperture as a function of counter separation $S$ and for various radii of the larger counter. For this graph, radius of the smaller counter is $R_1 = 1$ cm; other lengths are shown in units of $R_1$. The limit for zero separation, $\pi^2$ cm$^2$ sr, is shown. The single point is the result for the Berklet detector.

**Fig. 4** Same as previous figure but with expanded scale.

**Fig. 5** The dimensionless quantity $A_p S^2/(A_1 A_2)$ as a function of counter separation $S$ and for various ratios $R_2/R_1$. For large separations the limit is 1.0.
APPENDIX: LISTING OF FORTRAN PROGRAM OVERLAP

C PROGRAM OVERLAP COMPUTES THE APERTURE OF 2-COUNTER TELESCOPES
C CONSISTING OF TWO CO-AXIAL CIRCULAR COUNTERS.
C
C S IS SEPARATION OF COUNTERS; UNITS ARE CM.
C R2, R1 ARE RADII OF LARGER, SMALLER, CIRCLE RESPECTIVELY.
C R2 MUST BE LARGER THAN R1.
C
C TO TEST THE PROGRAM, TRY THE PARAMETERS OF THE BERKLET DETECTOR:
C R1 = 0.25, R2 = 0.55, S = 1.0 CM; RESULT: APERTURE = 0.138 CM2 SR.
C
DATA NTHETA / 100 /

C THE FOLLOWING STATEMENTS CALL IN THE INPUT DATA FROM THE TERMINAL.
1 TYPE 2
2 FORMAT(/1X,'PROGRAM OVERLAP: COMPUTES APERTURE OF 2-COUNTER TELESCOPE; /
3     / 1X ' CIRCULAR COUNTERS ARE ASSUMED.' :
        ACCEPT*, R1
        TYPE 3
3 FORMAT(/ 1X, 'TYPE RADIUS OF SMALLER COUNTER [cm]' )
      ACCEPT*, R2
        TYPE 4
4 FORMAT(/ 1X, 'TYPE SEPARATION BETWEEN COUNTERS [cm]' )
      ACCEPT*, S

C THETA IS IN RADIANS
THETA  = 0.0
THETA2 = ATAN((R2 + R1) / S)
DTHETA = THETA2 / NTHETA
APERT  = 0.0

C PRINT 1902, R1, R2, S
1902 FORMAT(/ 'CONFIRM INPUT DATA: ' 12E10.3)
C PRINT 1903, THETA2, DTHETA
1903 FORMAT( / 'THETA2, DTHETA = ' 2E10.3 /)
1901 FORMAT(12E10.3 /)

DO 1000 I = 1, NTHETA
   C THETA = THETA + DTHETA
   IF (THETA . GT . THETA2) GO TO 1001

1001 FORMAT(/ 'COORDINATES OF CENTER OF SMALLER CIRCLE AS PROJECTED ONTO SECOND PLANE:
C S DENOTES SEPARATION BETWEEN PLANES OF CIRCLES

YC = S * TAN(THETA)

C WE NOW HAVE TWO CIRCLES IN THE PLANE OF THE SECOND COUNTER
C LARGER CIRCLE IS CENTERED AT (0, 0) WITH RADIUS R2.
C SMALLER CIRCLE IS CENTERED AT (0, YC) WITH RADIUS R1.

C FIND POINTS OF INTERSECTION
C AXIS OF SYMMETRY IS THE Y-AXIS
AREA1 = 0.0
AREA2 = 0.0
PHI1 = 0.0
PHI2 = 0.0

C SOLVE FOR YI:

YI = (R2**2 - R1**2 + YC**2) / (2.0 * YC)

C SOLVE FOR XI1, XI2 USING EQUATION OF LARGER CIRCLE

XI = 0.0
XSQ = R2**2 - YI**2
IF (XSQ . LE . 0.0) GO TO 899

XI = SQRT(XSQ)

C NOW WE HAVE COORDINATES OF INTERSECTION POINTS: (-XI, YI) AND (XI, YI).

C AREA OF SEGMENT BOUNDED BY LARGER CIRCLE (2):

IF (YI . LE . 0.000001) GO TO 899
PHI2 = 2.0 * ASIN(XI / R2)

AREA2 = R2 * R2 * (PHI2 - SIN(PHI2)) / 2.0

C AREA OF SEGMENT BOUNDED BY SMALLER CIRCLE (1):

AMY = YC - YI
IF (ABS (AMY) . LE . 0.000001) GO TO 899

C PHI1 FROM FORTRAN SUBROUTINE ASIN WILL ALWAYS BE POSITIVE.
PHI1 = 2.0 * ASIN(XI / R1)

C PUT PHI1 INTO CORRECT QUADRANT.
IF (AMY. LE . 0.0) PHI1 = 2.0 * 3.1415927 - PHI1

AREA1 = R1 * R1 * (PHI1 - SIN(PHI1)) / 2.0

C NOW FIND TOTAL AREA:

AREA = AREA1 + AREA2

899 IF (YC + R1 . LE . R2) AREA = 3.14159 * R1 * R1
IF (YC - R1 . GE . R2) AREA = 0.0

C USE AREA OF SMALLER CIRCLE AS SEEN BY RAYS INCIDENT AT ANGLE THETA.

999 IF (AREA . LE . 0.0) GO TO 1001

DWTAREA = 2.0 * 3.1415927 * SIN(THETA) * DTHETA * AREA
APERT = APERT + DWTAREA * COS(THETA)

C THE FOLLOWING DE-ACTIVATED STATEMENTS WERE USED FOR DEBUGGING.
C PRINT 1901, THETA, YC, YI, XI, PHI1, PHI2,
C 1 AREAl, AREA2, AREA, DWTAREA, APERT
1000 CONTINUE
1001 CONTINUE

C INPUT PARAMETERS AND RESULT ARE PRINTED ON THE TERMINAL SCREEN.
PRINT 1905, R1, R2, S, APERT
1905 FORMAT(' R1, R2, S = ' 3E10.3,
1 ' APERT = ' E12.4,' [sr cm2]' // )

C A FILE IS WRITTEN THAT CAN BE USED FOR TOPDRAWER INPUT.
OPEN (UNIT=11, NAME='TELE.TOP', ERR=1, TYPE='NEW')
WRITE(11, 1906) S, APERT
1906 FORMAT(2E12.4)

GO TO 1

END
Fig. 1  Diagram defining parameters of a telescope consisting of two circular counters. Values shown are for the Berklet detector.
Fig. 2 Diagram in plane of larger counter showing how the overlap area is calculated from the sum of two segments, one from each circle.
Fig. 3  Output of the program OVERLAP showing behavior of aperture as a function of counter separation $S$ and for various radii of the larger counter. For this graph, radius of the smaller counter is $R_1 = 1$ cm; other lengths are shown in units of $R_1$. The limit for zero separation, $\pi^2$ cm$^2$ sr, is shown. The single point is the result for the Berklet detector.
Two-counter telescope; $R_1 = 1$ cm

![Graph showing telescope aperture vs. counter separation](image)

- $R_2 / R_1 = 1$
- $\pi^2$
- $10$
- $9$
- $8$
- $7$
- $6$
- $5$
- $4$
- $3$
- $2$
- $1.5$

$S/R_1$, Counter separation in units of $R_1$

Telescope aperture (sr cm$^2$)

Fig. 4 Same as previous figure but with expanded scale.
Two-counter telescope

Fig. 5 The dimensionless quantity $A_p S^2/(A_1 A_2)$ as a function of counter separation $S$ and for various ratios $R_2/R_1$. For large separations the limit is 1.0.