Essays in Psychological and Political Economics

by

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This dissertation is composed of three unrelated chapters, all of which are theoretical.

In Chapter 1, co-authored with Kristóf Madarász, we develop a model in which people experience standard consumption utility as well as anticipatory utility, defined as the weighted sum of independently anticipated consumption “episodes” or “dimensions”. The weights on these dimensions correspond to the attention that the person pays to the dimension. We assume attention on a dimension increases when expected consumption utility in the dimension differs from expected consumption utility under the default action or the prior belief. We show that the decision maker will pay more for information about dimensions with high expected consumption utility, and the willingness to pay may be negative when expected consumption utility is low. Additionally, when expected consumption utility is sufficiently low, but not when it is high, the decision maker will follow the default action even if it is suboptimal from a consumption standpoint. Furthermore, given the decision maker’s current beliefs and preferences in a dimension, he will consume more in that dimension if he just received information. We then consider an advertisement application in which a monopolist decides whether to certifiably reveal the quality of various exogenous attributes of a good to a consumer who may choose to buy or not. There exists a sequential equilibrium for which the monopolist will not disclose information for attributes in which the consumer’s utility with the highest quality good is sufficiently worse than not buying the good. Competition increases disclosure.

The purpose of the Chapter 2 is to connect the literature on industrial self-regulation with the literature on political revolutions by showing that these seemingly different situations are, from a strategic perspective, different cases of the same basic game. I construct a simple two-player extensive game of complete information in which two players have preferences over the realization of a policy in one dimension. The first mover has a marginal cost to change the policy and the second player has a fixed cost. The first mover may placate the second mover from taking action or provoke the second mover to take action. In equilibrium, the second mover may benefit from having preferences that diverge more from the first player, and may
benefit by having higher fixed costs. The equilibria are robust even when there are multiple first movers. Many applications are discussed and incorporated into the framework.

In Chapter 3 I study the interaction where an informed party wants an uninformed party to believe that the state of the world is as high as possible. A fraction of the time the informed party exogenously “leaks” the true state of the world, and a fraction of the time the informed party can strategically choose a “decoy” that is indistinguishable from the exogenous leak. Despite preferences being similar to those in Crawford and Sobel (1982) with maximum bias, strategic senders do not babble. Instead they trade-off exaggeration for credibility. In equilibrium, all messages received below a threshold will be leaks and will be believed by the receiver, while all messages above the threshold will induce an identical expectation. The model applies to many applications. In particular we motivate the model with a political interpretation in which leaks represent transparency within a political administration.
To the memory of my grandfathers, Lloyd Tasoff and Milton Hankin, who valued hard work and education.
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Chapter 1

A Model of Attention and Anticipation

1.1 Introduction

The growing literature on anticipatory utility assumes people experience pleasure and pain in the present from future consumption. Imagining one’s future deteriorating health may cause immediate feelings of anxiety, while imagining one’s next vacation may arouse immediate feelings of excitement. Anticipatory utility can affect individual decision making in a number of economically important ways: people may delay pleasurable experiences and expedite distasteful experiences, they may behave in a time inconsistent manner [Loewenstein, 1987], and they may have intrinsic information preferences [Caplin and Leahy, 2001].

A critical open issue in this line of research is to determine which future experiences a person anticipates in the present. Although most people can expect many happy and sad episodes in their future, at any given moment a person derives anticipatory utility from only a subset of future experiences. A person need not anticipate both their future deteriorating health and their next vacation simultaneously.

In this paper we explore the economic consequences of a person’s ability to influence, through actions and information acquisition, the set of future anticipated episodes from which they derive anticipatory utility. The ability to influence this set creates a new class of unexplored incentives.

We posit that a person experiences more intense anticipatory utility from an expected future episode when the person directs more of his or her attention to that episode. Specifically, we assume that anticipatory utility is additively separable in $I$ dimensions, and we define attention on a dimension as the weight placed on anticipatory utility in that dimension. Attention increases on a dimension in two situations. First, attention increases on a dimension when a person receives information that changes expected consumption utility in that dimension. Information about one’s illness will make one think more of one’s future health, and receiving information about Hawaii will make one think more about one’s upcoming vacation. Second,
attention increases on a dimension when the chosen action differs from the habitual action and produces different expected consumption utility in that dimension. When taking non-habitual actions to combat one’s illness, one thinks about one’s future health, and when packing one’s bags for a leisurely vacation to Hawaii, one imagines being in Hawaii.

In Section 2 we review the economic literature on anticipatory utility and other related models.

In Section 3 we present a formal model of attention and anticipation. While our model requires substantial modeling judgment to apply it to specific economic environments, it does capture several important issues that are economically relevant. The model has three periods. The decision maker begins by choosing a “default”. The default is not a fully contingent strategy but an action. This default is considered to be the action that requires no additional attention to enact. We interpret the default action as a habit, or as a pre-implemented effect that would occur in the future if one were to omit action at a later date. For example, a person may have the habit of eating fatty food and not taking heart medication. This would be the person’s default. An investor’s current portfolio would be his default since he can passively keep the status quo. Choosing any other portfolio would require attention. After establishing a default, the decision maker may choose information. For instance, he may choose to learn about his heart condition. He may then continue with his default or may choose a different action. Anticipatory utility is experienced and then consumption utility is experienced.

Consumption utility is standard expected utility defined over consequences $z$, and occurs at the end of period three. The separability assumption assigns “dimensions” to qualitatively different forms of consumption, or to different points in time. For example, leisure and health, or vacation next month versus vacation next year, may be represented by different dimensions. Psychologically, we interpret these dimensions as consumption experiences that one can think about independently of other consumption experiences. For example, most people can think of their next vacation a month away without thinking of their health ten years into the future. Arguably, even contemporaneous consumption can be attended to independently. A person may think about what clothes to wear tomorrow and anticipate the experience without thinking about what will be eaten tomorrow for lunch. Total consumption utility is the sum over all the $I$ dimensions indexed by $i$, $\sum_{i=1}^{I} u_i(z)$.

Anticipatory utility is weighted expected consumption utility, and it occurs at the end of period two. Attention increases on a dimension only indirectly when information changes expected consumption utility in dimension $i$, and actions change expected consumption utility from the default expected consumption utility in dimension $i$. Total attention $A > 0$ is limited. More attention on future consumption is less attention on something else, such as less attention on the present. We assume there is a constant forgone utility of attention $u_p$. This is the hedonic experience that would have been felt if attention were not on the future consumption dimension. To illustrate, this may be the utility from paying attention to the Johnny Depp movie on television. One unit of attention on anticipation is one less unit
of attention on experiencing the present. Suppose $a_i(\cdot)$ is the amount of attention on consumption dimension $i$, and $\sum_{i=1}^{L} a_i(\cdot) \leq A$ is the total amount of attention. Then under certainty, anticipatory utility is given by $Au_p + \sum_i a_i(\cdot)(u_i(z) - u_p)$. The person accurately predicts his preferences and chooses the default, information, and action that maximizes the sum of anticipatory utility and expected consumption utility.

Section 4 presents the results. First, the decision maker will have preferences for information that are a function of the expected consumption utility of the relevant dimension. Keeping the instrumental value of information constant, the decision maker’s willingness to pay for information will be increasing in expected consumption utility. Furthermore, we show that if expected consumption utility is sufficiently low, then the decision maker will avoid information even if it would make a consumption utility maximizing agent strictly better off for all possible posteriors. An expected consumption utility maximizer and even an anticipating agent with fixed attention (e.g. of the variety found in [Eliaz and Spiegler (2006)]) both would prefer this type of information. For instance, a patient may avoid information about which procedure would be best for dealing with the patient’s heart disease. The less healthy the patient is after the proper procedure, the lower the demand for the information, even though the proper procedure may be very beneficial.

This logic applies analogously to actions. Our second result is that the decision maker will exhibit “behavioral lock-in”, in which he may follow the default action even if it fails to maximize expected consumption utility. When the expected consumption utility is low, choosing a non-default action may produce low anticipatory utility. Keeping the expected consumption utility difference between choosing a non-default action and the default constant, the value of choosing the non-default action is increasing in the expected consumption utility of the relevant dimensions.

As an example, we show that a decision maker who has access to partial insurance, but has chosen the default of not purchasing, will continue to not purchase even if new information indicates that the probability of loss is higher than previously believed. An expected consumption maximizer or an anticipating agent with fixed attention would increase insurance. Moreover, we show that if attention shifts sufficiently, and expected consumption utility is sufficiently low, then the demand for insurance is decreasing in the probability of loss. As the probability of the bad state increases, thinking about the dimension yields lower anticipatory utility, and hence changing old habits becomes more costly. The model predicts that people will underinsure when the underlying risk increases. A patient who discovers he has heart disease may neglect taking his new heart medication because it shifts attention to his poor health. An expected consumption utility maximizing agent would pay more for insurance when the risk increases.

When information increases attention on a dimension, not only is there a direct effect on anticipatory utility, but also the dimension becomes more heavily weighted relative to other dimensions. In other words, information in our model is a complement to consumption. The information-consumption complementarity can be interpreted as an “advertising effect” in the [Becker and Murphy (1993)] sense that
advertisements are complements to consumption. In an influential paper Becker and Murphy (1993) incorporate advertisement into the traditional “rational” economic framework by modeling an advertisement and a good as complements. Consumption of one increases demand for the other and advertisements may either directly increase or decrease utility. Our model gives micro-foundations for their model. The complementarity stems from anticipatory utility and shifting attention. Our model also predicts which advertisements will be goods and which will be bads as described in the second result: demand for advertisements will be higher if they provide information about dimensions with higher expected consumption utility.

We show that obtaining information can increase future consumption in a dimension, even when the information implies that marginal consumption utility is low. To illustrate, a patient with heart disease who learns that the benefit to a drug is weaker than expected may still take more of it than if he never received information about the benefit at all. An expected consumption utility maximizer would take less. The reason is that the information itself serves as a reminder of health. For the third result we show that a person who has long had full knowledge of the benefits of an activity (which we model as having a well-informed prior belief) will consume less than a person who has been recently informed about the benefits (which we model as having an uninformed prior belief but a well-informed posterior). The recency of the information increases attention on the dimension and thereby stimulates more consumption in that dimension. Moreover, this implies that two agents with identical preferences and beliefs will behave differently depending on whether information was received recently or in the more distant past. In an economic model without attention this makes no difference. This feature of preferences allows the person to be persuaded by attention-shifting information. A firm will be able to persuade consumers, that is increase their valuation of the product, by providing information that would not otherwise be persuasive to an expected consumption maximizer. This leads to the main application.

In Section 5 we present an application in which a monopolist advertises the exogenous quality of a multi-attribute good to a single consumer. Each attribute corresponds to a consumption dimension of the consumer’s utility. The good as a whole is desirable although some attributes may be bads. The standard version of this game is the “Persuasion Game” of Milgrom (1981) and Grossman (1981). In our model, the consumer is additionally influenced by the information-consumption complementarity. The monopolist can increase the price by shifting attention to dimensions in which buying the good yields higher utility than not buying. By the same logic, there will exist an equilibrium where there is no disclosure for attributes in which the utility from the highest quality good is still substantially lower than the utility of not having the good. A drug company may not disclose (without regulation) even “good news” about the side effects of their heart medication because they do not wish to draw attention to these negative attributes. Disclosure will reduce the price that the monopolist can charge.

We then discuss welfare implications and find that the firm’s decision to disclose is not affected by how disclosure affects welfare, and so there may be either too much
or too little disclosure in equilibrium. The monopolist may shroud an attribute if
the gain from separating from lower types is sufficiently less than the consumer’s
loss in that dimension from buying the good. However, shrouding increases welfare
if the utility from the good is greater than the foregone utility. There is no reason
to expect that these conditions will have any relationship with each other. Thus
the monopolist’s decision to shroud will be orthogonal to the welfare effect. We
then casually discuss a richer environment in which quality is endogenous. In such
an environment, the monopolist may undersupply quality due to an “attentional
transaction cost”: the monopolist has no means to profitably signal quality and
thus has no incentive to produce quality.

In an extension we examine the effect of competition on disclosure using a
duopoly model. When the good is sufficiently valuable, the consumers will always
buy from one of the two firms. For a given attribute, the firm with higher quality will
always do better by disclosing quality. This imposes an “attentional externality” on
the second firm who then has no incentive to shroud. The second firm will disclose
quality in equilibrium due to the standard unraveling logic. So when the good is
sufficiently valuable duopoly results in full disclosure.

In Section 6 we offer potential avenues for future research. We discuss an
extended model with multiple periods of anticipation in which the decision maker
has incentives to manipulate beliefs for commitment purposes. We also discuss ways
to extend the model to analyze risk preferences and hedonic misprediction. Then
we conclude.

1.2 Related Literature

In this section we relate our model to other economic models of anticipatory
utility and consider the small economic literature on attention.

Kreps and Porteus (1978) present a revealed preference framework which allows
for intrinsic information preferences. In their model a person may have preferences
over the timing of the resolution of uncertainty. For example, someone who gets
$1 tomorrow if a coin lands heads, may prefer to learn the outcome either now
or tomorrow. The authors assume time consistency. They show that within their
framework, if a person is indifferent to the timing of the resolution of uncertainty,
then preferences could be represented by the standard expected utility model.

Loewenstein (1987) provides the first explicit model of anticipation. In it he
assumes that anticipatory utility is time discounted consumption utility. The impli-
cation is that people will exhibit what looks like negative time discounting. A person
will pay to delay the consumption of goods and will pay to speed the consumption
of bads. Loewenstein’s model also predicts time inconsistency. A person faced with
consuming a good would want to continually delay consumption.

Caplin and Leahy (2001) extend Loewenstein’s model to uncertainty. Risk and
future expected outcomes map into anticipatory utility. The solution to their model
is analogous to subgame perfect equilibrium, and thus, unlike Kreps and Porteus
their model allows for time inconsistency. This time inconsistency could come from different preference orderings over consumption in the second period, or anticipatory risk preferences that differ from consumption risk preferences. They show that their model is capable of rationalizing the equity premium puzzle and the risk-free rate puzzle.

Kőszegi (2009) extends the work of Caplin and Leahy (2001) to allow expectations to influence utility. Preferences are defined over physical outcomes and beliefs, however unlike Caplin and Leahy (2001) these may interact in the utility function. As a result, a person’s expectations of his own actions influences his choice. Since expectations depend on actions, and actions depend on expectations, subgame perfect equilibrium is an insufficient solution concept. Instead, a new solution concept, personal equilibrium, is defined, and extends subgame perfect equilibrium with the added assumption that expectations must be consistent with the actual distribution of outcomes. Kőszegi (2009) shows that if the decision maker does not exhibit informational preferences, multiple equilibria with different utility, or time inconsistency, then preferences could be represented with standard expected utility.

Our model bears similarity with Caplin and Leahy (2001). We begin with a simplified version of their setup, but differ by adding the attention component of anticipation. Preferences that interact attention with anticipatory utility generate behavior that previous models of anticipation cannot capture. Our model relates behavior and information preferences to the level of future expected consumption utility. To our knowledge our model is also the first to micro-found an information-consumption complementarity based on anticipatory utility.

Additionally, there is a new growing literature exploring games between agents that have anticipatory utility. Caplin (2002) and Caplin and Eliaz (2003) explore mechanism design with anticipating agents. Kőszegi (2003, 2006), and Caplin and Leahy (2004) model principal-agent behavior when the principal has utility over beliefs. There may be a communication breakdown when a well-intentioned agent interacts with an anticipating principal. Our model continues in this tradition. In Section 1.5 we specify a game in which a buyer with attention-shifting anticipatory utility may purchase a good from a profit-maximizing seller.

Compared to anticipation, very little economic research has been written on attention. As far as we are aware, there are no papers except for Karlsson et al. (2008), where attention is interpreted as a component of preferences, although there are several economics papers where attention is a constraint or a component of the economic environment. Eliaz and Spiegler (2008) and Masatlioglu et al. (2008) develop models of “limited attention”, where the agent is incapable of choosing the optimum over the full choice set, but instead chooses optimally from a subset called a consideration set. Their meaning of attention differs from ours. For them, attention on an option means that the option is in the consideration set. Attention in our model is purely about anticipating different dimensions of future consumption.

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1 Caplin and Leahy (2001) are the first to discuss the interaction of attention and anticipation, and convincingly motivate it in Section IV.E.
Banerjee and Mullainathan (2008) model attention as an input available for the production of two different outputs. One can produce sellable output at their job or domestic output at their home. Gabaix et al. (2006) model attention in a Herbert Simon, satisficing sense. They develop and experimentally test a model of costly information acquisition.

Perhaps the model closest to our own, Karlsson et al. (2008) relates attention to anticipation. The authors present a model of an investor who has reference-dependent utility over money, and an attention function that discretely increases the gain-loss component upon the receipt of information. Our attention function works similarly, but we do not assume reference-dependent utility. They predict information-seeking when the market is going up and information avoidance when the market is going down. Our model makes similar predictions, but ours are based on the absolute level of consumption utility and not the change. Furthermore, our model relates actions to attention, and we predict that people will follow their default when expected consumption utility is low. We also have multiple dimensions of consumption which allows for the information-consumption complementarity.

Finally, there is a family of mostly recent papers that interacts anticipatory utility or other belief-based preferences with belief distortion. The basic concept in these models is that the decision maker gets utility directly from his beliefs and has a technology to distort his beliefs. The tradeoff in these models is between happy beliefs and accurate decisions. Starting with Akerlof and Dickens (1982), workers may choose to be fearful and appropriately use safety equipment, or blissfully ignore dangers while facing bodily harm. Brunnermeier and Parker (2005) generalize and apply this to portfolio choice and consumption-savings decisions. They show that investors will overestimate returns and consumers will overconsume. Bracha and Brown (2007) apply a similar model to insurance and show that with copayments, the worse a bad outcome is, the less insurance the agent will purchase. In Mayraz (2009), beliefs are not chosen but distorted as a function of payoffs. Unlike these models, our relevant tradeoff is not between accurate beliefs and happy beliefs, but rather consumption utility and the attention-weighted consumption utility (anticipatory utility). The driving force in the belief distortion models is the utility difference between mutually exclusive events. The driving force in our model is the difference between the expected consumption utility of a dimension and the forgone utility of attention. Since consumption in dimensions is not mutually exclusive, there is a clear substantive distinction between dimensions and events. Because of this, we address different questions. Our model directly considers informational preferences, which is an unexplored topic in the belief-distortion literature, and novelty predicts that demand for information is increasing with expected consumption utility, and that furthermore there is an information-consumption complementarity.

Psychological evidence for the assumptions of our model is desirable. There are a small handful of psychology experiments that are related (Averill and Rosem, 1972; Miller and Mangan, 1983; Wood et al., 2002), but they do not specifically test the psychological behaviors that we assume as the basis for our model. An additional benefit to specifying a formal model is to help experimentalists identify
the role of anticipatory utility in behavior.

1.3 Model

![Timeline Diagram](https://via.placeholder.com/150)

**Figure 1.1: Timeline**

In this section we present the formal model. We consider a central focus of the model to be the specification of the dependency of attention on actions and information. The timing is illustrated in Figure 1.1. There are three time periods, \( t = 0, 1, 2 \). At \( t = 0 \) the decision maker (henceforth referred to as the “DM”) chooses a default and information, at \( t = 1 \) the DM chooses an action and experiences anticipatory utility, and at \( t = 2 \) the DM experiences consumption utility.

Beginning with the economic environment, let the action set \( X \) be a nonempty finite set. Initially at the beginning of \( t = 0 \), the DM chooses a default action \( \bar{x} \in X \) to follow. A default is an action, not a strategy. It is an action that is specified in advance and cannot be made contingent on future information gathering or changes in beliefs.

The purpose of the default is to specify the action for which no attention is required. In the model, both a change in actions and a change in beliefs shift attention. Whereas the natural default belief is the prior, there does not seem to be a similarly natural default for actions. Thus as a starting point, we assume that the DM chooses his default, although in general we interpret the default action as the activity or inactivity that would naturally occur without the exertion of attention, and this need not correspond to the utility-maximizing default.

We have several interpretations of the default. The default may represent habits, automatized or routinized actions, or a plan of action: washing one’s hands after using the restroom, choosing one’s usual dish instead of considering the full menu, or a plan to work eight hours this week on a project. In other cases, the default is most appropriately the action that best corresponds to omission. Leaving one’s stock portfolio as is, an act of omission, would appropriately be the default.

\[ \text{Figure 1.1: Timeline} \]

It is unclear what an act of omission means in a formal sense. There does not exist a formal definition, to our knowledge, in game theory or decision theory. Given an action set, we interpret the act of omission to be the act that would be “chosen” by a person who is incapacitated or asleep.
We assume that the default is a non-contingent plan because it seems psychologically likely that following a full belief-contingent plan requires attention. If one discovers it necessary to eliminate salt from one’s diet for health reasons, attention is required to accomplish this, and doing so will remind oneself of one’s health. If the default were a full belief-contingent strategy, our model would have no new predictions regarding the role of actions and anticipation. The DM would be able to specify a default that he could follow in equilibrium for all beliefs, and attention would never shift through actions. In some cases our assumption that the DM may choose the default may be implausible. In this paper we will try to be clear on how the results depend on the default, and in many parts of this paper the results will not depend on the assumption that the default is ex ante optimal.

Let $\Omega$ be the non-empty finite state space. A state $\omega \in \Omega$ represents the relevant exogenous characteristics of the world that affect the physical consequence. For example, states may represent the weather in Hawaii or one’s genetic proclivity to getting heart disease. Let $\Delta(\Omega)$ represent the set of all probability distributions on the set $\Omega$. Define $b_1 \in \Delta(\Omega)$ as the beliefs at the end of period $t = 0$, and $b_0 \in \Delta(\Omega)$ is the prior.

Let $Z$ be the non-empty finite set of possible physical consequences in period 2. These represent all the outcomes that the DM cares about that can occur in the future. For example, a consequence $z \in Z$, may be having an enjoyable vacation next month, and a healthy heart ten years into the future. Another consequence may be having a stressful vacation, and heart disease ten years into the future. The default has no direct influence on physical consequences $z$. In $t = 1$, after receiving information, the DM chooses an action $x \in X$ that affects $z$. The action $x \in X$ is chosen from the same action set as the default $\bar{x}$ and need not be the same. Actions map from states to consequences, $x : \Omega \rightarrow Z$. An action is a lottery over consequences.

At $t = 1$, the DM may receive information. Let $S$ be a finite set of realized signals. A signal is a function that maps from the state to a probability distribution over realized signals $\sigma : \Omega \rightarrow \Delta(S)$. To give an example, the states of the world would be various genetic proclivities to heart disease, getting a genetic test would be a signal, and learning that one has a gene that makes one prone to heart disease would be the realized signal. At $t = 0$ the DM may choose a signal from the set of signals $\sigma \in \Sigma$. Beliefs are updated in a Bayesian way $b_1 : \Sigma \times S \rightarrow \Delta(\Omega)$.

Now we discuss preferences. Consumption utility is defined over physical consequences. Consumption utility is additively separable in $I$ dimensions. We offer two interpretations of these dimensions. First, they may represent different time periods. For example, one dimension may represent consumption of entertainment tomorrow versus consumption of entertainment 21 days from now. There is a tradition in economics of utility functions that are additively separable over time periods. If the dimensions represent non-contemporaneous consumption episodes, then the

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Specifying the omitted action then requires contextual information beyond the traditionally defined economic environment.
additive separability is consistent with standard economic models. Here the periods need not represent time periods of equal duration but rather periods that are psychologically contiguous. Under this interpretation, period 2 is composed of \( I \) sequential consumption episodes. Second, a dimension may represent contemporaneous consumption of psychologically distinct aspects of an episode. For example, tomorrow one may derive consumption from both lunch and from the clothes that one wears. These two forms of consumption may overlap in time, but as long as the taste of the food is independent of the comfort from wearing the clothes and vice versa, we can separate these forms of consumption in the utility function. Under this interpretation, \( I \) forms of consumption occur contemporaneously over the duration of period 2. In either interpretation the critical psychological assumption is that these dimensions represent forms of consumption that could be anticipated independently of each other. Finally, consumption utility in dimension \( i \) is given by the vNM utility function \( u_i : Z \to \mathbb{R} \) and total consumption utility is \( \sum_i u_i(z) \).

Since utility is defined over consequences, it is convenient to define an expected consumption utility function over actions. Let \( w_i(x; b_t) \equiv \sum_{\omega \in \Omega} \text{Pr}_b(\omega)u_i(x(\omega)) \).

Anticipatory utility is experienced at the end of \( t = 1 \). We assume that anticipatory utility is a weighted sum of expected consumption utility. Each dimension is weighted by the attention on that dimension. Let \( a_i(\bar{x}, x; b_0, b_1) \) be the attention on dimension \( i \), which depends on the default action, the realized signal, and the action. The anticipatory utility experienced in dimension \( i \) is \( a_i(\bar{x}, x; b_0, b_1)w_i(x; b_1) \).

We assume that there is a fixed amount of attention meaning that if one is not thinking about the future then one is thinking about something else (such as the Johnny Depp movie playing on TV), and deriving a flow of utility from that alternative. A critical feature of the model is that there is a tradeoff between attending to future consumption and attending to the present. Fixing the total amount of attention is the means by which we impose these tradeoffs. For simplicity we assume that there is a constant forgone utility to attention. For every unit of attention on the future, there is one less unit of attention on an experience with utility level \( u_p \), which may be interpreted as the utility from the present. The present utility \( u_p \in \mathbb{R} \) is exogenous. This assumption serves a role analogous to quasi-linear utility for money. Quasi-linear utility for money has a constant forgone utility to consumption, in our model there is a constant forgone utility to attention. Assuming a constant forgone utility is a simplifying assumption that makes the model more tractable. A richer model might have the foregone utility of attention as a function of current consumption (i.e. \( u_p : X \to \mathbb{R} \)). Total anticipatory utility is given by \( \sum_{i=1}^I a_i(\bar{x}, x; b_0, b_1)(w_i(x; b_1) - u_p) \).

\(^3\)An alternative assumption would be to fix all the attention weights \( \sum a_i(\cdot) = 1 \). The basic forces in such a model would be the same. This approach has an additional complication because increasing attention in dimension \( i \) would necessarily decrease attention in other dimensions. We would then need to assume which dimensions decrease in attention and by how much. The “constant foregone utility” assumption is simpler in this respect.

\(^4\)For clarity, as mentioned in the introduction, one can think of total attention as equal to \( A \geq \sum_{i=1}^I a_i(\bar{x}, x; b_0, b_1) \). Then anticipatory utility is equal to \( Au_p + \sum_{i=1}^I a_i(\bar{x}, x; b_0, b_1)(w_i(x; b_1) - u_p) \).
Now we define the attention function. We make two critical assumptions. First, we assume that information shifts attention to dimensions that have a different ex post expected consumption utility than ex ante expected consumption utility. When one learns about next month’s forecasted weather in Hawaii, one thinks about next month’s vacation, and not about one’s health. We formalize this by assuming that if \( w_i(x'; b_1) \neq w_i(x'; b_0) \) for some \( x' \in X \), then \( \gamma > 0 \) attention shifts to dimension \( i \). In other words, receiving a signal that changes expected consumption in \( i \) for at least one action, discretely shifts \( \gamma \) units of attention to \( i \). In some ways this assumption is pretty strong. If a person receives information about an option \( x \) that changes expected consumption utility in dimension \( i \), attention will shift even if \( x \) would never be chosen under any beliefs. For example, if the DM receives information about a vacation to Antarctica, even if the DM would never choose to go there, the DM anticipates his expected vacation to Hawaii more. However, this is somewhat more plausible than assuming that attention shifts only if the information affected utility given the chosen action. Under such an assumption, when considering a vacation to Hawaii, receiving information about Maui will not cause attention to shift to vacation if the DM ultimately decides instead to go to O‘ahu.

The second critical assumption is that choosing a non-default action shifts attention to dimensions that have a different consumption utility than with the default action. The interpretation is that deviating from habits, routines, or plans requires attention, but when following the default the DM need not think about the consequences while in the moment. If one is in the habit of eating fatty food and not taking heart medication, then changing one’s diet and taking medication makes one think more of one’s health.

We model this by assuming if \( w_i(x; b_1) \neq w_i(\bar{x}; b_1) \) or if \( w_i(x; b_1) \geq u_p \), then \( \alpha > 0 \) attention shifts to dimension \( i \). In other words, choosing an action that deviates from expected default consumption in \( i \) discretely shifts \( \alpha \) attention to \( i \). Additionally, whenever expected consumption utility is above the forgone utility, the DM will think about consumption. This second condition indicates that the DM will attend to any dimension when it is pleasurable to do so. The interpretation is

If we subtract the constant \( Au_p \), we get the desired expression. Subtracting this constant will have no effect on the preference ordering.

5 This second condition eliminates burning money problems. Without it, the DM may “burn utils” in high-level dimensions to shift attention to that dimension. For example, suppose the DM may choose between two vacations of differing quality and suppose the default is the high-quality destination. Without the second condition, the DM may choose to go to the inferior vacation for the sole purpose of shifting attention to the vacation dimension. More generally, if the DM's only means to deviate a high-level dimension’s utility from the default utility is to lower it, then the DM will frequently take this option to shift attention. This seems psychologically implausible. It seems psychologically more plausible that a person can willingly shift one’s attention to high-level dimensions. If one wants to spend more time thinking about one’s vacation, one does not need to make the vacation worse in order to think about it more. Furthermore, without this assumption the DM has an incentive to choose defaults that he has no intention of following for the sole reason of shifting attention to the dimension.

Fundamentally, this builds an asymmetry into the model. People can willingly think about high-level dimensions independent of their actions, but cannot avoid thinking about low-level dimensions.
that the DM can freely pay attention and will pay attention to pleasurable future experiences regardless of the default action. For purely expositional purposes, until we reach Section 1.5, we will assume that the anticipatory utility is always less than the forgone utility of attention, \( w_i(x; b_t) < u_p \) for all \( i, x, \) and \( b_t \). When \( w_i(x; b_t) \geq u_p \), the default action becomes irrelevant to the DM’s choice, so in this sense, behavior will be closer to an expected consumption utility maximizer.

The two critical assumptions are meant to be analogous. Deviations from a default shift attention to consumption that is affected by the deviation. In terms of information, we explicitly assume that the default beliefs are the prior. Shifting attention by way of information may be a different psychological process than shifting attention by way of actions. Both can be incorporated into our reduced form framework in a similar way, although we do not claim that the psychological process is identical. To incorporate new information, one typically needs time to think about the implications of the information. Likewise, we speculate that when one takes a non-default action, it requires time to think about the consequences of the action.

The total shift in attention comes from the attention shifted from information plus the attention shifted from actions. We interpret both of these as increasing the amount of time spent anticipating future consumption. Taking the view of Kaheman (2000) in his moment-based approach to experienced utility, more time anticipating a future episode leads to more total anticipatory utility from that episode. The attention function is given below.

**Attention Function:**

\[
a_i(\bar{x}, x; b_0, b_1) \equiv \gamma \{ w_i(x'; b_1) \neq w_i(x'; b_0) \text{ for some } x' \in X \} \\
+ \alpha \{ w_i(x; b_1) \neq w_i(\bar{x}; b_1) \text{ or } w_i(x; b_1) \geq u_p \}
\]  

We make attention discrete for two reasons. First, we do so because it is simple and we avoid issues regarding the curvature of attention. Understanding the effects of the curvature of the attention function may be a useful direction for future research. Second, in some situations discreteness is plausible. For example, if one is shopping for a camera and if “not buying” is the default, one spends a chunk of time thinking about future episodes in which one uses the camera. Our discreteness assumption implies that the amount of time thinking about the camera is independent of which camera is purchased as long as some camera is purchased. However, in some other cases, this discreteness assumption would appear to be independent of their actions.

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6When choosing a non-default action, one also requires more time to think about which action is optimal. One directs attention to a dimension in order to optimize that dimension. This interpretation, although in the spirit of the model, differs somewhat from how the model works. Suppose, \( \bar{x} = x' \) and choosing \( x = x'' \) gives a different expected consumption utility in both the vacation and health dimensions. According to the model, choosing \( x'' \) would shift attention to both dimensions. However, in the directed attention interpretation, if the person chooses \( x'' \) to optimize the vacation dimension, attention will shift only to vacation even though both vacation and health are affected by the choice.
inappropriate. It is quite likely that the more time a worker puts into a project, the more attention the worker has on the consequence of the project. Although, our model is unable to capture this effect, it would capture attention shifting when the chosen work level differs from the default work level.

We do not believe that our model exhausts all the factors that affect attention. Proximity, framing, vividness, and other psychological properties of stimuli certainly play a role. However, we focus on actions and information for two reasons. First, we think that actions and information have a strong effect on attention, so exploring how they affect attention and therefore economic decision making may yield important insights into economic behavior.

The timing is illustrated in Figure 1.1. There are three choices that are made sequentially. Going in order, at \( t = 0 \) the DM chooses a default \( \bar{x} \in X \), and then chooses information \( \sigma \in \Sigma \). At \( t = 1 \), information arrives and then the DM chooses \( x \). The objective function for each period is:

\[
V_0(\bar{x}, \sigma; x, b_0) \equiv E_{b_0}\left[\sum_{i=1}^{I}(1 + a_i(\bar{x}, x; b_0, b_1))w_i(x; b_1) - a_i(\bar{x}, x; b_0, b_1)u_p\right] \quad (1.2)
\]

\[
V_1(x; \bar{x}, b_0, b_1) \equiv \sum_{i=1}^{I}(1 + a_i(\bar{x}, x; b_0, b_1))w_i(x; b_1) - a_i(\bar{x}, x; b_0, b_1)u_p \quad (1.3)
\]

Notice that if \( \gamma = \alpha = 0 \), the DM is an expected consumption utility maximizing agent. With \( \gamma = \alpha = 0 \), attention on all dimensions \( i \) is always \( a_i(\bar{x}, x; b_0, b_1) = 0 \). The objective function then becomes \( V_1(x; \bar{x}, b_0, b_1) = \sum_{i=1}^{I} w_i(x; b_1) \), which is the objective function of an expected consumption utility maximizer with \( I \) additively separable dimensions. Thus the standard model is nested within our model. Throughout the paper we will compare an agent with \( \gamma = \alpha = 0 \) to an agent who has \( \gamma, \alpha > 0 \).

With specified objective functions we can now define equilibrium. Since actions will be a function of beliefs, we must define strategies. A period 1 strategy \( \psi(b_0, b_1) \) is a function that assigns a default action \( \bar{x} \in X \) and a signal \( \sigma \in \Sigma \) to the first two decision nodes respectively, and an action \( x \in X \) for every subsequent information set. We denote a strategy with the triple \((\bar{x}(b_0), \sigma(b_0), x(b_0, b_1))\). The model is solved backwards: \( x^*(b_0, b_1) \equiv \arg\max_{x \in X} V_1(x; \bar{x}, b_0, b_1) \) is the optimal action. The optimal signal is \( \sigma^*(b_0) \equiv \arg\max_{\sigma \in \Sigma} V_0(\bar{x}, \sigma; x, b_0) \). And the optimal default action is \( \bar{x}^*(b_0) \equiv \arg\max_{\bar{x} \in X} V_0(\bar{x}, \sigma; x, b_0) \).

Preferences in this model are time inconsistent as they are in other models of anticipatory utility [Loewenstein 1987; Caplin and Leahy 2001; Kőszegi 2009]. Total expected utility at the beginning of period two before the resolution of uncertainty is given by \( V_2 \equiv \sum_{i=1}^{I} w_i(x; b_1) \). The time inconsistency arises because the \( t = 0 \) and \( t = 1 \) selves have different preferences than the \( t = 2 \) self for \( t = 2 \) consumption. For our simple setup this does not matter, since no decisions are made at \( t = 2 \). If we were to have actions at \( t = 2 \) the analogous solution concept for the model would be subgame perfect equilibrium. A similar structure is used in [Caplin and Leahy 2001]. Their model is two periods with time inconsistent preferences, and their solution concept is analogous to subgame perfect equilibrium.
**Definition** The *optimal strategy* is defined as

\[ \psi^*(b_0, b_1) = (\bar{x}^*(b_0), \sigma^*(b_0), x^*(b_0, b_1)). \]

When we take a decision problem in which the DM receives no new information the DM behaves just like a consumption-maximizing agent.

**Proposition 1** If for all \( \sigma \in \Sigma \), \( b_1 = b_0 \) for all realizations of the signal, then \( x^*(b_0, b_1) \) maximizes expected consumption utility.

When beliefs do not change, the DM will choose a consumption utility maximizing default and follow it through. Since we assumed \( w_i(x; b_t) < u_p \) for all \( i \) and \( x \), it is optimal to choose \( \bar{x} = x \) otherwise attention will shift to low-level dimensions. Since all dimensions are weighted equally, the consumption utility maximizing bundle will maximize total utility.

More simply put, when the DM does not receive any new information and expected consumption utility is low in all dimensions, the DM maximizes expected consumption utility. No attention will be on future consumption. New information will be the key component of the environment that induces novel behavior.

When there is new information, but some dimensions have high expected consumption utility and some have low, the DM will have \( \alpha \) attention shifted to only the high dimensions. Since these dimensions are weighted more heavily, the DM will consume more in these dimensions than a consumption utility maximizer would. To an observer, when there is no action in \( t = 2 \), the DM’s behavior would be indistinguishable from a consumption maximizer. However, if there were actions in \( t = 2 \) then the DM may behave in a time inconsistent manner (i.e. the DM may pay for commitment in the absence of uncertainty).

### 1.4 Results

#### 1.4.1 The Level of Expected Consumption Utility

The DM prefers to pay attention to dimensions that have high expected consumption utility. Taking a non-habitual action such as planning for one’s vacation is far more enjoyable than doing similar planning tasks for a dentist’s appointment. An agent with \( \gamma = \alpha = 0 \) consumes as long as the marginal benefit to consumption utility exceeds the marginal cost. Our attention-sensitive anticipatory agent cares both about the marginal benefit of consumption and the *level* of consumption utility. Lower level dimensions offer a less pleasurable anticipatory experience. It follows that the DM is incentivized to shift attention away from low-level dimensions through the information and action that he chooses.

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*If \( w_i(x; b_t) \geq u_p \), or for all \( i \) and \( x \), the proposition would remain true. If all the dimensions are greater than \( u_p \), then \( \alpha \) attention will shift to all of those dimensions and so dimensions will be weighted equally. It follows that the consumption utility maximizing bundle will maximize total utility.*
Information

An agent with $\gamma = \alpha = 0$, without strategic concerns would never avoid information. Our model generates intrinsic information preferences that are a function of the level of expected consumption utility. Receiving information about a dimension increases attention and hence anticipatory utility from that dimension. If the expected consumption utility in a dimension is sufficiently low, the DM will prefer to avoid information that is relevant to the dimension. Unlike some other models with anticipatory utility, information avoidance may occur in our model even when information is purely “how” information; as in “how to choose the right action”. When we use the term “how” information, we refer to a signal that with any realization would increase consumption utility by a fixed amount given consumption-maximizing behavior.

**Definition** Let $x^c(b_t) \equiv \arg\max_{x \in X} \sum_{i=1}^I w_i(x; b_t)$. A signal $\sigma$ contains purely “how” information if for all realizations of the signal $s$ in the support of $\sigma$ give, $\sum_{i=1}^I w_i(x^c(b_1); b_1(s)) > \sum_{i=1}^I w_i(x^c(b_0); b_0)$ and for any two realizations of the signal in the support $s, s' \in S$, $\sum_{i=1}^I w_i(x^c(b_1); b_1(s)) = \sum_{i=1}^I w_i(x^c(b_1); b_1(s'))$.

After receiving a purely “how” signal, a consumption utility maximizer will have a fixed higher utility regardless of the realization of the signal. In other words, before receiving the signal, the DM knows exactly what her expected consumption utility could be. An example of purely “how” information would be receiving instructions on how to bake grandma’s well-known cake. A person with the proper ingredients and the wrong instructions will make a mess, but a person with grandma’s recipe will bake a good cake of known quality. The person’s utility is independent of the procedure, as long as following the instructions produces an instance of grandma’s cake.

In other models with intrinsic information preferences (Kreps and Porteus [1978], Kőszegi [2003], Caplin and Leahy [2004], Eliaz and Spiegler [2006]), the agent always prefers purely “how” information. In several models (Kőszegi [2003], Caplin and Leahy [2004]), anticipatory utility is either a concave or convex function of expected consumption utility. Information that has no instrumental benefit adds a random variable with mean zero to expected utility. As a result, anticipatory utility that is concave in expected consumption utility produces information-averse preferences, and anticipatory utility that is convex in expected consumption utility produces information-seeking preferences. To the contrary, purely “how” information is purely instrumental. Ex ante, obtaining purely “how” information yields a known expected consumption utility. In other words, expected consumption utility becomes a degenerate random variable. With purely “how” information, information preferences are not relevant, and since the frontier of expected consumption utility increases with purely “how” information, the agents in these models will always prefer the information.

**Example:** Avoiding Purely “How” Information
In this example we show that an anticipating agent with attention will avoid purely “how” information when expected consumption utility is low.

The DM has two actions available to him $X = \{j, k\}$. The state denotes which option is better, $\Omega = \{j, k\}$. The action and the state affect only dimension $i$. The DM has access to two signals, one that reveals the state, and one that provides no information, $\Sigma = \{r, n\}$. Notice that the signal $r$ contains purely “how” information. The optimal default $\bar{x}^*$ will depend on which signal is chosen, but the main conclusion will be true for any default.

Total utility from the two signals is given by:

$$V_0(\bar{x}^*(b_0), n; x^*(b_0, b_0), b_0) = E_{b_0} \left[ \left( 1 + a_i \left( \bar{x}(b_0), x^*(b_0, b_0) ; b_0, b_0 \right) \right) w_i(x^*(b_0, b_0); b_0) \right.$$

$$\left. - a_i \left( \bar{x}(b_0), x^*(b_0, b_0) ; b_0, b_0 \right) u_p \right]$$

$$V_0(\bar{x}^*(b_0), r; x^*(b_0, b_1), b_0) = E_{b_0} \left[ \left( 1 + a_i \left( \bar{x}(b_0), x^*(b_0, b_1) ; b_0, b_1 \right) \right) w_i(x^*(b_0, b_1); b_1) \right.$$

$$\left. - a_i \left( \bar{x}(b_0), x^*(b_0, b_0) ; b_0, b_0 \right) u_p \right]$$

This can be simplified because in the absence of new information, the optimal default and optimal action will be the same and so attention will not shift to dimension $i$, $a_i \left( \bar{x}(b_0), x^*(b_0, b_0) ; b_0, b_0 \right) = 0$. We wish to find conditions on the parameters such that no information is preferred

$$V_0(\bar{x}^*(b_0), n; x^*(b_0, b_0), b_0) > V_0(\bar{x}^*(b_0), r; x^*(b_0, b_1), b_0),$$

which implies

$$u_p > \frac{E_{b_0} \left[ \left( 1 + a_i \left( \bar{x}(b_0), x^*(b_0, b_1) ; b_0, b_1 \right) \right) w_i(x^*(b_0, b_1); b_1) \right] - w_i(x^*(b_0, b_0); b_0)}{E_{b_0} \left[ a_i \left( \bar{x}(b_0), x^*(b_0, b_1) ; b_0, b_1 \right) \right]}. \]

The denominator of the right-hand side term $E_{b_0} \left[ a_i \left( \bar{x}(b_0), x^*(b_0, b_1) ; b_0, b_1 \right) \right] \geq \gamma$ since information changes expected consumption, which makes this term positive. This means that if $u_p$ is relatively large compared to the consumption utility, it will never be optimal to receive information. Receiving the information shifts attention to dimension $i$ which is undesirable to think about. An agent with $\gamma = 0$ and even an agent with “standard” anticipatory utility with fixed attention would always choose $\sigma = r$.

As long as utility from the dimension is sufficiently low, the DM will avoid information. In fact, as the utility from the dimension decreases, the total benefit from information goes down. We can see this looking at the numerator of the expression above. Keeping the instrumental benefit of information constant

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9There are two consequences, matching the state and mismatching, $Z = \{ma, mi\}$. Matching is preferred $u_i(ma) > u_i(mi)$. 
Information avoidance plausibly occurs in various important facets of life. Our prediction is that it will occur more when expected consumption utility is low. Health, may be a particularly dread-provoking topic. Upon finding early evidence of a breast tumor many women delay seeking help (Nosarti et al., 2000; Bish et al., 2005). Rather than getting a diagnosis and treatment, many women avoid confronting the issue until the situation is substantially worse. Lindberg and Wellisch (2001) find that breast self exams are negatively correlated with anxiety. Smith and Croyle (1995) find that 16.1% of their sample would not want to get a genetic test for susceptibility to colon cancer. Many at risk individuals avoid STD tests, despite the benefits of diagnosis and treatment. For example, more than half of those diagnosed with AIDS between January 1990 and December 1992 first tested for HIV within one year of their AIDS diagnosis (Chesney and Smith, 1999), even though the onset of AIDS typically occurs a decade after HIV infection. This suggests that many patients had HIV for nearly a decade before testing despite the benefits of diagnosis.

The proposition below formalizes this logic.

Proposition 2 Let σ induce beliefs $b_1$ such that there is a positive probability that $w_j(x; b_1) \neq w_j(x; b_0)$ for some $x$, and $\sigma_n$ induces beliefs $b_1 = b_0$ for all realizations of the signal. Then keeping $E_{b_0}[w_i(x; b_1)] - w_i(x; b_0)$ constant for all $i$, the difference $V_0(\bar{x}, \sigma; x^*(b_0, b_1), b_0) - V_0(\bar{x}, \sigma_n; x^*(b_0, b_0), b_0)$ is strictly increasing in $E_{b_0}[w_j(x^*(b_0, b_1); b_1)]$.

The proposition says that increasing the expected consumption utility of a dimension that is affected by the information, while keeping the instrumental value of information constant, increases the willingness to pay for the information. The reasoning is straightforward. The signal shifts $\gamma$ attention to the dimension. Information that shifts attention to high-level dimensions is more desirable.

Corollary 1 Suppose the DM has a degenerate action set $X = \{x\}$. Let σ induce beliefs $b_1$ such that there is a positive probability that $w_j(x; b_1) \neq w_j(x; b_0)$. Then the difference $V_0(\bar{x}, \sigma; x^*(b_0, b_1), b_0) - V_0(\bar{x}, \sigma_n; x^*(b_0, b_1), b_0)$ is strictly increasing in $w_j(x; b_0)$.

The corollary simply restates Proposition 2 when the action set is degenerate. If there is no choice of action, then information that changes expected consumption utility in only dimension $j$ is more desirable the higher the expected consumption

\[w_i(x^*(b_0, b_1); b_0) - w_i(x^*(b_0, b_0); b_0),\] reducing the level of both $w_i(x^*(b_0, b_1); b_0)$ and $w_i(x^*(b_0, b_0); b_0)$ decreases the numerator. This increases the range of $u_p$ for which no information is preferred.

\[\text{According to the Center for Disease Control: “Before antiretroviral therapy became available in 1996, scientists estimated that AIDS would develop within 10 years in about half the people with HIV.” http://www.cdc.gov/hiv/resources/brochures/livingwithhiv.htm#q3}\]
utility of dimension $j$. This formulation can be used to identify the expected consumption utility of a dimension. If the dimensions of consumption are known, then a person’s information preferences for a dimension will correspond to the expected consumption utility of that dimension. The preference ordering of signals that are only informative about a single dimension, will correspond with the same ordering of expected consumption utility levels. Thus it is possible, under these special conditions, for an observer to infer relative expected consumption utility levels.

**Actions**

In this section we show that the DM will exhibit “behavioral lock-in” by not fully adjusting to new information when the expected consumption utility is low. The DM wishes to follow the default when consumption utility is low to avoid negative anticipatory feelings. A patient, recently diagnosed with heart disease, is advised to stick to a strict low-fat diet. Adjusting habits are naturally costly. But adjustment is even more challenging when the new diet is a constant reminder of one’s illness. The important factor for behavioral lock-in is that the optimal deviation still produces a low consumption utility. If the new diet could completely cure heart disease, then the individual will adjust.

The model predicts that as the achievable expected consumption utility declines in a dimension, ceteris paribus, the DM has a stronger preference to choose the default. If expected consumption utility in a dimension becomes sufficiently low, the anticipatory cost to deviating from the default may exceed the benefit to consumption, and as a consequence the DM will choose the default. We illustrate this in an example below.

**Example : Partial Insurance**

In this example the DM faces a risk in a low-level dimension. We show that under plausible conditions, the DM will avoid partial insurance. Moreover, the DM’s willingness to pay for insurance will be decreasing in the probability of the low state.

The action set has two elements, partial insurance and no insurance, $X = \{p, n\}$. The partial insurance could be a precautionary action that reduces the risk of a bad consequence, like taking medication to reduce the risk of heart attack, or using protection to reduce the risk of getting an STD. There are two states, high and low, $\Omega = \{h, l\}$. Only dimension $i$ is affected by this choice and state. The DM has the prior that the low state occurs with probability $\Pr_0(l) \in (0, 1)$. The choice set for information is degenerate with only one signal $\Sigma = \{\sigma\}$ which provides information about the probability of the low state. Given the prior, there will be an optimal $\bar{x}$. We will consider both $\bar{x}^* = p$ and $\bar{x}^* = n$.

This is a reduced form approach to insurance. Let $\hat{b}$ be the beliefs at which the DM is indifferent between the two actions $w_i(p; \hat{b}) = w_i(n; \hat{b})$. Furthermore, since $\frac{\partial}{\partial \Pr_0(l)} w_i(x; b_t) < 0$, expected consumption utility is decreasing in the probability of the low state but the relative benefit of insurance is increasing in the probability bad state $\frac{\partial}{\partial \Pr_0(l)} (w_i(p; b_t) - w_i(n; b_t)) > 0$. The timing goes: the DM chooses a default
\( \bar{x} \in X \), the DM receives \( \sigma \) which updates the probability of the low state to \( \Pr_{b_1}(l) \), the DM chooses \( x \in X \), experiences anticipatory utility, and then consumption utility.

We solve backwards. First, suppose the default is \( \bar{x} = p \). Then
\[
V_1(p; p, b_0, b_1) = (1 + \gamma)w_i(p; b_1) - \gamma p
\]
\[
V_1(n; p, b_0, b_1) = (1 + \alpha + \gamma)w_i(n; b_1) - (\alpha + \gamma)p
\]
Here we can see that if \( b_1 = \hat{b} \), then \( V_1(p; p, b_0, b_1) > V_1(n; p, b_0, b_1) \). So when the default is partial insurance, partial insurance will be chosen for lower levels of \( \Pr_{b_1}(l) \) relative to an expected consumption utility maximizer. Deviating from the default of buying insurance induces low anticipatory utility because the attention shifts to a dimension with low expected consumption utility. This additional cost incentivizes the DM to stay with the default.

Now suppose the DM chooses \( \bar{x} = n \), then the utility of the two options are
\[
V_1(p; n, b_0, b_1) = (1 + \alpha + \gamma)w_i(p; b_1) - (\alpha + \gamma)p
\]
\[
V_1(n; n, b_0, b_1) = (1 + \gamma)w_i(n; b_1) - \gamma p.
\]
Here we can see that when \( b_1 = \hat{b} \), then \( V_1(n; n, b_0, b_1) > V_1(p; n, b_0, b_1) \). So when the default is no insurance, no insurance will be chosen for lower levels of \( \Pr_{b_1}(l) \) relative to what an expected consumption utility maximizer would choose. To summarize, when the affected dimension has low expected consumption utility, and a large amount of attention shifts from actions, the DM will stick to old plans more often than an agent with \( \gamma = \alpha = 0 \) would. If he initially plans no insurance, he will get no insurance more often than a \( \gamma = \alpha = 0 \) agent, and if he initially plans to get insurance then he will get insurance more often than a \( \gamma = \alpha = 0 \) agent.

Furthermore, under a plausible condition the utility difference between choosing \( p \) versus \( n \) when the default is \( n \), \( V_1(p; n, b_0, b_1) - V_1(n; n, b_0, b_1) \), may decrease with \( \Pr_{b_1}(l) \). If
\[
\alpha > \frac{(1 + \gamma)\frac{\partial}{\partial \Pr_{b_1}(l)}(w_i(p; b_1) - w_i(n; b_1))}{\frac{\partial}{\partial \Pr_{b_1}(l)}w_i(p; b_1)}
\]
\[
\Rightarrow \frac{\partial}{\partial \Pr_{b_1}(l)}(V_1(p; n, b_0, b_1) - V_1(n; n, b_0, b_1)) < 0
\]
This expression says that when \( \alpha \) is sufficiently large, the DM will pay less for insurance when the probability of the bad state increases. The explanation is that the anticipatory cost of deviating from the plan is increasing with the probability of the bad state. When anticipatory utility is large, this effect exceeds the consumption benefit of becoming partially insured.\[\text{[11]}\]

\[\text{[11]}\]We should note that this comparative static is driven by the fact that the person does not want to pay attention to the decision. If for some reason the person’s attention were shifted to the dimension (i.e. there are multiple levels of coverage and the person is required to get some
Will the DM ever choose $\bar{x} = n$ in equilibrium? If $Pr_{b0}(l)$ is sufficiently low, it may be optimal to set $\bar{x} = n$. If the DM is surprised by learning that the probability of the low state is very high, the DM may continue to avoid insurance and will actually pay more to avoid it as the bad state becomes more likely.\footnote{There are three forces at work here. The first force is the one emphasized: the DM does not want to deviate in dimensions with low expected consumption utility. The second force is the information-consumption complementarity described in greater detail later in the paper. Receiving information about the risk shifts attention to $i$ which gives the DM more incentive to buy insurance. Depending on the relative magnitude of $\alpha$ versus $\gamma$, receiving the information could either induce greater demand for insurance or not. The third force is the standard incentive — the DM wants to maximize consumption utility. Either way, the cost of deviating is decreasing in the expected consumption utility under the action to which the DM deviates.}

What would happen if insurance were full? It depends on whether the utility in the good state is high $u_i(x(\omega)) \geq u_p$. If so, then with full insurance there would be no associated anticipatory cost to deviating from the default. A higher probability of the bad state could only increase the willingness to pay for insurance, just like for an agent with $\gamma = \alpha = 0$. However, if full insurance still left consumption utility low $u_i(x(\omega)) < u_p$, then even if $p$ provided full insurance, the analysis above would apply.

We speculate that the logic presented in the example above may be a cause for a variety of real-world situations in which people underinsure. A person with heart disease may take medication which lowers the risk of a heart attack. However, using the medication may be a constant reminder of one’s illness. Compliance rates with physician-recommended health regimens are often quite low \cite{Becker and Maiman, Sherbourne et al., Lindberg and Wellisch, DiMatteo et al.}. Greater fear, lower expectation of health outcome, and lower self-reported health has been correlated with less adherence \cite{Sherbourne et al., DiMatteo et al.}. Moreover, in more serious illnesses such as cancer, HIV, and heart failure those with objectively poorer health are less likely to be adherent \cite{DiMatteo et al.}.

Condom use can be considered a form of partial insurance against sexually transmitted diseases. The standard direction of causation is that less condom use will result in greater HIV transmission. However, our model suggests that reverse causation may confound standard interpretations. If it is a cultural norm not to use condoms, then deviating from this default may be more dread provoking when the prevalence of HIV is higher. In South Africa condom use is rare and HIV infection rates are high \cite{MacPhaila and Campbell}. It may be worth investigating whether anticipatory fear is a factor in condom avoidance.

Another issue that is plagued by inadequate preparation and widespread ignorance is planning for end-of-life care. This is a form of insurance that specifies the type of care one wants in the event of becoming terminally ill. Despite the benefits and professional support for the use of living wills in stating one’s preferences for insurance, then demand for insurance would be increasing with risk as standard theory would predict.
end-of-life care, less than 50% of adults have one (Carr and Khodyakov 2007). If the default is to not prepare for end-of-life care then the model predicts that doing so would cause low anticipatory utility.

Bracha and Brown (2007) have a similar result in their model in which the decision maker simultaneously chooses beliefs and insurance. They show that an agent who buys more than full insurance will buy even more if the bad state worsens. An agent who buys less than full insurance will buy less if the bad state worsens. Our model makes a similar prediction. In the insurance example above, it is possible that increasing the probability of the bad state will decrease the amount an agent is willing to pay if he were planning on not buying insurance. Keeping the change in expected consumption utility fixed, our model makes the same prediction for decreasing the utility of the bad state, and increasing the probability of the bad state. These are different mechanisms in Bracha and Brown (2007). In their model there is a cost to distorting beliefs from the true probability, so updating about the probability of an event may incur costs, but updating about the utility conditional on the event is costless.

Mayraz (2009) has a model in which beliefs are distorted from the true probability by the payoffs. An “optimist” in his model will believe that the worse the state, the less likely the event. However, in his model, beliefs are monotonic in the true probability. Thus as the probability of the bad state increases, the agent will increase his belief and his insurance, although both will be biased downward.

The logic in this example and the information avoidance example is analogous. In terms of actions, deviating from the default is undesirable if future consumption is low. In terms of information, deviating from current beliefs is undesirable if future consumption is low. The result is generalized below. The anticipatory benefit to choosing a non-default action is decreasing in the expected consumption level.

**Proposition 3** Let \( x \neq \bar{x} \). Keeping \( w_i(x; b_1) - w_i(\bar{x}; b_1) \) constant for all \( i \), the difference \( V_1(x; \bar{x}, b_0, b_1) - V_1(\bar{x}; \bar{x}, b_0, b_1) \) is strictly increasing in \( w_j(x; b_1) \).

Proposition 3 says that, keeping the instrumental benefit of a deviating action fixed, the benefit to deviating is increasing in the expected consumption utility.\(^{13}\) This implies that when expected consumption utility gets too low, the equilibrium action must be one in which attention is not shifted to the dimension. The intuition for this result is that people want to follow their default when even the best deviation produces a low expected consumption utility.

The “behavioral lock-in” result depends on knowing the level of expected consumption utility. Expected consumption utility in a dimension is not observable directly. If expected consumption utility in a dimension is strictly increasing in an observable variable, changes in this expected consumption may be observable. Moreover, as discussed in relation to Corollary 4, expected consumption utility may be indirectly observed via information preferences.

\(^{13}\)If \( w_j(x; b_1) \geq u_p \), then the difference is constant in \( w_j(x; b_1) \). Once the level is above \( u_p \), the DM wants to attend to the consumption and can freely do so without deviating from his plan.
1.4.2 Information-Consumption Complementarity

The model predicts that information is a complement to consumption. When
the DM receives information relevant to dimension $i$, he thinks about this dimension
more, weights it more heavily in the utility function, and consequently trades off
more consumption in other dimensions for consumption in dimension $i$. We can
think of this as an “advertisement effect” in the sense of Becker and Murphy (1993)
where advertisements are complements to consumption. Independent of the induced
beliefs, a signal makes a person think more about the dimensions of consumption
for which it is informative. Since the person is more intensely anticipating these
dimensions of consumption, the person has a stronger preference for improving these
dimensions.

An agent with $\gamma = \alpha = 0$ would also change his behavior upon obtaining
information. However, what we show is that if the information-consumption com-
plementarity is sufficiently strong, the DM may consume more in a dimension even
after receiving information that suggests that the marginal benefit to consumption
is lower than expected. An agent with $\gamma = 0$ would not behave in this manner. We
illustrate with the following example.

Example: Consuming More When the Benefit Decreases

In this example we show that the DM increases consumption in a dimension
after learning that the marginal benefit is low. Let $x \in \{0, 1\}$ be future consumption
in dimension $i$. There are two states. The consumption yields high consumption
utility or low consumption utility $\Omega = \{h, l\}$. Initially, beliefs are that both states
are equally likely. If the good is very enjoyable, then the benefit $\omega$ is $h$, otherwise
the benefit is $l$, where $h > l$. There are two signals: revealing the state, and nothing,
$\Sigma = \{r, n\}$. Choosing $\sigma = r$ reveals the true benefit of consuming the good. Both
the state and the action only affect utility in dimension $i$. Expected consumption
utility is $w_i(x; b_1) = E_{b_1}[\omega|x]$.

First, we wish to know the relative benefit of consumption in the absence
of information. Since there is no new information, the DM will choose $\bar{x} = x$. Utility
will be $V_1(x; x, n, b_1) = \frac{h+l}{2} x$. The difference between consuming and not is $(h+l)/2$.

With information, $\sigma = r$, there are two possible defaults. Let us consider each
in turn. If $\bar{x} = 1$, then $V_1(1; 1, r, b_1) - V_1(0; 1, r, b_1) = (1 + \gamma)\omega + \alpha u_p$. Doing the
same for $\bar{x} = 0$, gives $V_1(1; 0, r, b_1) - V_1(0; 0, r, b_1) = (1 + \alpha + \gamma)\omega - \alpha u_p$.

In both of these, the DM’s willingness to pay in utils is increasing in $\gamma$. If
$\gamma > \frac{h+l+2\alpha u_p}{2} - (1 + \alpha)$, then the DM’s willingness to pay in utils is greater after
learning that the marginal benefit is low than when receiving no information. The
intuition is that the information itself increases the person’s attention on dimension
$i$. In this particular case, the attention-shifting effect of the signal overwhelms the
actual content of the signal and so the person adjusts in the direction opposite to
the realization of the signal.

The example illustrates the information-consumption complementarity. Loosely
stated, information relevant to a dimension of consumption $i$ increases demand for
consumption in $i$. The complementarity works in both directions as indicated by Proposition 2 (when consumption increases in $i$, the willingness to pay for information about $i$ increases). In order for behavior to actually exhibit increased consumption in $i$, there must be a tradeoff between consumption utility in $i$ and another dimension.

One potential application of the information-consumption complementarity is that information can influence time preferences. If dimensions correspond to different time periods, then the complementarity will influence the weighting of different periods. Information about distant future episodes would cause the DM to behave as if he had a large discount factor. Information about the near future may make the DM appear to behave as if he were impatient.

The example illustrates that it is possible for the information-consumption complementarity to be observed. However, if the complementarity is weak, then it may be swamped out by the standard instrumental effect of information. In order to identify the information-consumption complementarity, we examine the behavior of the DM based on the timing of the receipt of information. To proceed, we define two different agents with identical preferences and beliefs, a prior-informed (PI) agent and a recently-informed (RI) agent. The PI agent begins with beliefs $b_{0}^{PI}$ and receives a signal $\sigma_n$ with no information, so his posterior $b_{1} = b_{0}^{PI}$ for all realizations of the signal. The RI agent begins with beliefs $b_{0}^{RI}$ and receives a signal $\sigma$ and realization $s$ such that his posterior is also $b_{1} \neq b_{0}^{RI}$. Receiving the signal shifts attention to relevant dimensions. As a consequence, the two agents with identical preferences and identical beliefs, will behave differently. The two agents have attention shifted in different ways which will affect their anticipatory utility, and their behavior. The agent who became recently informed cares more about future consumption relevant to the information. To better identify the information-consumption complementarity, we compare a PI agent to an RI agent.

**Proposition 4** Let $x', x'' \in X$ such that $w_i(x'; b_t) > w_i(x''; b_t)$ and $w_j(x'; b_t) = w_j(x''; b_t)$ for all $b_t$ and $j \neq i$. Furthermore let the signal $\sigma$ only affect dimension $i$: $w_i(x; b_1) \neq w_i(x; b_0^{RI})$ for some $x \in X$, and $w_j(x; b_1) = w_j(x; b_0^{RI})$ for all $j \neq i$ and all $x \in X$. Then for a fixed $\bar{x}$, $V_1(x'; \bar{x}, b_1, b_0^{RI}) - V_1(x''; \bar{x}, b_1, b_0^{RI}) > V_1(x'; \bar{x}, b_1, b_0^{PI}) - V_1(x''; \bar{x}, b_1, b_0^{PI})$

Proposition 4 says that an RI agent who received information that affected utility in dimension $i$ will pay more to increase consumption in dimension $i$ than the PI agent. The caveat is that the comparison is made when both agents have the same default. More generally, it may not be the case that these two agents choose the same default.

The timing of information here is critical. What is the difference between the PI and RI agents if both receive information before taking actions? We interpret the difference to be a matter of degree. A PI agent was informed long before the RI agent. This is admittedly a somewhat subjective matter. However, the model still does make a clear observable prediction. A person who was informed more recently
will consume relatively more in the relevant dimensions than one who was informed less recently.

It is important to note that the information-consumption complementarity exists only when the receipt of information and the choice of consumption are contemporaneous. It is not the beliefs themselves that causes the complementarity, but the change in beliefs. Presumably, the PI agent at some point before period 1 received information and also had attention shifted to future consumption. However, by the time period 1 arrives with the consumption choice, attention had already returned to baseline and so the PI agent is unable to exploit the information-consumption complementarity.

An implication is that information campaigns will only have a temporary effect. The model predicts that there are two effects from an information campaign, the standard informational effect and the complementarity. Since the complementarity is temporary, the model predicts that the effect of a fixed-duration information campaign should decline over time. Murry, Jr. et al. (1993) study the effect of a paid advertising campaign to discourage drinking and driving targeted at young drivers. They found that the advertisement reduced fatal accidents for young drivers during the duration of the campaign and a few months later but the effect was only temporary.

1.5 Application: Advertising

In this section we apply the model to advertising. Advertisement as information has a long history in economics. See Bagwell (2007) for a broad survey. We model advertisement as certifiable information about the exogenous quality of an attribute of a multi-attribute good. In the language of Milgrom (1981), we consider a variant of the “Persuasion Game”. According to the famous unraveling logic of Grossman (1981) and Milgrom (1981), the seller with highest quality will reveal the quality in order to separate from lower types and thus command a higher price. Consequently, any seller not revealing must be lower than highest quality. But then the seller with slightly lower quality will do strictly better by revealing. The logic continues until only the lowest type is left indifferent between revealing and not revealing. In the standard version of this game, the only sequential equilibrium is one in which all sellers with quality above the worst reveal their quality.

In our variant of this game, revealing the quality of an attribute will have two effects. First, it will have the standard effect to update the consumers’ beliefs. If the quality is above average then disclosing the quality can increase the consumer’s willingness to pay for the good. Second, through the information-consumption complementarity, the consumer will pay more attention to the attribute. More weight will be placed on the dimensions that have different expected consumption utility from expected consumption utility under the prior. As a result, the consumer will exhibit greater demand for attributes for which new information was provided. This effect is in the spirit of Becker and Murphy (1993) where advertisements are treated
as complements with their respective good. Here, revealing the quality of an attribute of the good will increase the importance of that attribute to the consumer. The firm can use this complementarity to increase the price whenever the consumption utility in the dimension associated with the revealed attribute is greater than the consumption utility in the same dimension from not buying the good.

By the same logic, advertising attributes in which the consumption utility is greater from not buying can lower the price. The good in its entirety may be desirable but there may be undesirable attributes such as safety, maintenance, or inconveniences associated with using the good. We show that there exists a sequential equilibrium in which the firm will not reveal attributes and expected consumption utility in the dimension is substantially greater from not buying. In other words, the firm will shroud even “good news” about the attributes of their good that make the consumer worse off. A drug company will avoid advertising side effects of their drugs even if the side effects are better than average. The standard motivation to disclose high quality in this dimension is still present, but the information-consumption complementarity reduces the consumer’s willingness to pay. The full disclosure equilibrium will also exist, but if the monopolist can choose which equilibrium to play, then the monopolist would prefer the shrouding equilibrium with any level of quality.

We then consider welfare in this framework. The firm’s decision to shroud an attribute depends on whether the quality of the good for that attribute is sufficiently below the quality of not buying. However, welfare depends on whether the quality of the good is above the forgone utility of attention. These two comparisons are unrelated to each other and so in general, the firm will not be maximizing social welfare. There may be both too much or too little disclosure. In a richer framework there are two additional issues. First, consumers have access to outside information when there is too little advertisement. Second, if quality were endogenous, lack of disclosure may lead to an under-provision of quality.

When a second firm is added to the market we show that competition will increase the amount of disclosure. If the value of the goods are sufficiently high then there will be full disclosure in equilibrium. Each firm can increase its profit if it can increase the consumers’ beliefs that its quality is above the competitor’s. Since there is always a firm that has higher quality, there will always be one firm that has an incentive to disclose.

1.5.1 Setup

There is a monopolist and a single consumer. We begin with the timing followed by the monopolist’s choices and preferences, and then the consumer’s choices and preferences. The timing is illustrated in Figure 1.2 First, the consumer enters with beliefs $b_0$ about the distribution of quality and then decides his default action, then Nature chooses the quality of the good which is only observable to the monopolist, the monopoly chooses a reporting strategy and a price, the consumer buys the good or not, anticipatory utility and profit are realized, and finally quality is revealed and consumption utility is experienced in $t = 2$. 
The monopolist possesses a single good which contains exogenous quality in $I$ attributes. Each of these attributes corresponds to a dimension in the consumer’s preferences. For example, if the good is heart medication, one attribute may be heart health, and another attribute might be the unintended side effect of bad digestion. An attribute in dimension $i$ has quality $q_i \in \{q_{i1}, ..., q_{im}\}$ ordered in increasing quality, and is known only to the monopolist. Each attribute may be either a good or a bad. We assume that not buying is equivalent to possessing a good with zero quality in every attribute, $q_i = 0 \quad \forall i$. Let $q$ be the vector of quality. Quality in this game is the state, and all potential quality levels are the state space. Nature determines $q$ from a commonly known distribution $b_0$. A critical assumption that we make is that $q_i$ and $q_j$ are independent for all dimensions $i$ and $j$. As we explain later, independence is necessary for our results. The monopolist’s strategy consists of two components, a report and a price. The monopolist may choose to certifiably report the exact quality of any number of attributes or not. Let the report in dimension $i$ be to disclose or to shroud, $r_i \in \{d, s\}$. The full report conditional on quality is $r(q) \in \{\{d, s\}_{i=1}^I\}$. A price is any non-negative number $p(q) \in \mathbb{R}_+$. The monopolist has zero cost to producing the good, so the monopolist’s profit is simply equal to the price, $\Pi = p$.

The consumer begins by choosing a default action, to buy or not buy, $\bar{x} \in \{B, N\}$ which we assume is observable by the monopolist. The consumer has at most unit demand and would always purchase the good if $p = 0$ (for both defaults). The consumer has a degenerate choice of only one signal and the realization is determined by the monopolist’s report. The consumer then chooses to buy or not buy $x \in \{B, N\}$ as a function of expected quality and price. We consider the consequence $z$ to be possession of quality $q$. The action “buying” $B$ given the state $q$ produces the consequence, possession of quality $q$. The action “not buying” $N$ given the state $q$ produces the consequence, possession of quality $0$. Technically, utility in dimension $i$ takes the whole consequence vector $q$ as an argument. Since utility

---

\[\text{Figure 1.2: Game Timeline}\]
in dimension $i$ only depends on the quality of attribute $i$, we will switch notation slightly using the shorthand $u_i(q_i)$. Loosening the assumption from Section 2.2, we now allow both $u_i(q_i) \geq u_p$ and $u_i(q_i) < u_p$. Our welfare analysis will depend on these inequalities.

We assume that utility is quasi-linear in a numeraire good. If the price equals the consumer’s expectation of the price, then less attention will be on the numeraire than if the price did not equal the consumer’s expectation. This attention-shifting to the numeraire via information about the price substantially complicates the analysis. To simplify, if the price of numeraire is $1+\nu$, where $\nu$ is a zero-mean random variable with arbitrarily small variance and revealed before the monopolist moves, then as long as $\nu$ never equals 0, $\gamma$ attention will always shift to the numeraire dimension. This is admittedly an ancillary assumption about the environment but it simplifies the results and crisply helps to illustrate the intuition. Without this assumption, the basic result that the firm will shroud attributes remains true. Let $a_n(\tilde{x}, x; b_0, b_1)$ be attention on the numeraire dimension, and let $-p$ be utility in the numeraire dimension. Total utility from buying and not buying given price $p$ and report $r$ are given by:

\[
V_1(B; \tilde{x}, b_0, b_1, p) = -p + a_n(\tilde{x}, B; b_0, b_1)(-p - u_p) + \sum_{i=1}^{I} E_{b_1}[u_i(q_i)] + a_i(\tilde{x}, B; b_0, b_1)(E_{b_1}[u_i(q_i)] - u_p)
\]

(1.4)

\[
V_1(N; \tilde{x}, b_0, b_1, p) = -a_n(\tilde{x}, N; b_0, b_1)u_p + \sum_{i=1}^{I} u_i(0) + a_i(\tilde{x}, N; b_0, b_1)(u_i(0) - u_p)
\]

(1.5)

### 1.5.2 Solution

The solution concept we use for the game is sequential equilibrium (Milgrom and Roberts, 1986; Okuno-Fujiwara et al., 1990; Osborne and Rubinstein, 1994). In this particular game, the sequential equilibrium will be equivalent to a perfect Bayesian equilibrium (Osborne and Rubinstein, 1994). A triple $((\tilde{x}^*, x^*), (r^*, p^*), b_1(r^*))$ is a sequential equilibrium in pure strategies if it satisfies the following:

1. Monopolist maximizes profit:
   \[
   \Pi((\tilde{x}^*, x^*), (r^*, p^*), b_1(r^*)) \geq \Pi((\tilde{x}^*, x^*), (r, p), b_1(r)) \quad \forall (r, p).
   \]

2. Consumer maximizes expected utility in both periods:
   \[
   V_1(x^*; \tilde{x}, b_0, b_1(r^*), p^*) \geq V_1(x; \tilde{x}, b_0, b_1(r^*), p^*) \quad \forall x \in X,
   \]

\[\text{It is worth mentioning that uncertainty over numeraire consumption is a function of the monopolist’s choice. In all other parts of the model, both in this section and previous sections, uncertainty over consumption in a dimension is a function of the state and not choice variables.}\]
\[ V_0(\bar{x}^*, \sigma; x^*, b_0, p^*) \geq V_0(\bar{x}, \sigma; x^*, b_0, p^*) \quad \forall \bar{x} \in X. \]

3. The consumer has rational beliefs: If \( r_i = s \), 
\[ \Pr_{b_1}(q_i | r) = \frac{\Pr_{b_0}(r = s | q_i) \Pr_{b_0}(q_i)}{\sum_{q} \Pr_{b_0}(r = s | q) \Pr_{b_0}(q)}. \]

4. The consumer has consistent beliefs: If \( r_i = d \) and realized quality in \( i \) is \( q_i' \), then 
\[ \Pr_{b_1}(q_i' | r) = 1. \]

We begin by solving backwards. The consumer will buy if and only if
\[ V_1(B; \bar{x}, B_0, B_1, p) \geq V_1(N; \bar{x}, B_0, B_1, p). \]

The monopolist, knowing this will set prices to extract all expected surplus from the purchase,
\[ p^*(q) = \frac{1}{1 + a_n(\bar{x}, B; B_0, B_1)} \left( \left( a_n(\bar{x}, N; B_0, B_1) - a_n(\bar{x}, B; B_0, B_1) \right) u_p \right. \]
\[ + \left. \sum_{i=1}^I E_{b_1}[u_i(q_i)] + a_i(\bar{x}, B; B_0, B_1) (E_{b_1}[u_i(q_i)] - u_p) - u_i(0) \right) \]
\[ - a_i(\bar{x}, N; B_0, B_1) (u_i(0) - u_p). \]

The multiplier on the left is the inverse of how much the consumer weights the numeraire consumption. The first term in the parentheses represents the relative foregone attention from attending to numeraire consumption. The first two terms in the summation are the utility from purchasing the good, and the second two terms are the utility from not purchasing the good. This leaves the consumer with the utility from not buying, \( V_1(N; \bar{x}, B_0, B_1, p) \). For this reason, the consumer’s utility is always weakly greater if the consumer plans not to buy, \( \bar{x} = N \). To remind the reader, attention will not shift to low level attributes when \( \bar{x} = x \) and attention always shifts to high level attributes independent of \( \bar{x} \). Therefore, if the consumer comes to the marketplace expecting to get \( V_1(N; \bar{x}, B_0, B_1, p) \), then choosing the habit of not buying \( \bar{x} = N \) is optimal. The interpretation is that relative to forming the habit of buying, this increases the consumer’s reservation utility. “Non-habitual” consumers have a higher outside option so they command better deals.\(^{16}\)

Remaining to be solved is the monopolist’s report and the consumer’s beliefs about quality. With standard preferences the only sequential equilibrium is full disclosure (Grossman 1981; Milgrom 1981; Okuno-Fujiwara et al. 1990). We now construct a sequential equilibrium in which the firm shrouds some attributes from the consumer. We begin by considering the consumer’s side of the problem. Suppose

\(^{16}\)If \( \bar{x} \) is unobservable to the monopolist, there will be another set of pure strategy equilibria in which the consumer chooses the default \( \bar{x} = B \), the monopolist will set prices to leave the consumer indifferent between buying and not, and the consumer will buy. In this set of equilibria, the monopolist will obtain a greater fraction of the surplus. More pertinent, the monopolist’s choice to shroud or reveal will remain unaffected.
the monopolist will shroud attribute $i$, $r_i = s$ for any realization of $q_i$. If this is the case then the consumer can infer nothing about the quality from observing $r_i = s$, because all types are shrouding, and so $E_{b_1}[q_i] = E_{b_0}[q_i]$. Since expected quality does not change in the dimension, no attention will shift to the dimension. The monopolist’s incentive to deviate is based on its quality. If the monopolist has the highest quality, $q_i = q_m$, then the incentive to separate from the lower types is greatest. The benefit from deviating for this monopolist is

$$p^*_r = p^* - q^*_i = \frac{1}{\alpha + \gamma} \left( (1 + \alpha) \left( u_i(q^m_i) - E_{b_0}[u_i(q_i)] \right) + \gamma \left( q^m_i - u_i(0) \right) \right)$$

(1.7)

If the consumption utility from not buying in dimension $i$ is substantially higher than buying $\gamma(u_i(0) - u_i(q^m_i)) > (1 + \alpha)(u_i(q^m_i) - E_{b_0}[u_i(q_i)])$ then the firm will shroud, even with the highest quality.

**Proposition 5** There exists a sequential equilibrium in which the monopolist shrouds in dimensions $j$ if and only if $\gamma(u_j(0) - u_j(q^m_j)) \geq (1 + \alpha)(u_j(q^m_j) - E_{b_0}[u_j(q_j)])$.

The left hand side of the expression is the utility difference between buying and not buying in dimension $i$ weighted by the shift of attention from information, and it represents the loss to the firm from advertising a low attribute. The right hand side of the expression is the utility difference from the best quality and the unconditional average quality weighted by the shift of attention from actions, and it represents the benefit to the highest-quality monopolist from separating from types who have low quality. Notice that if there is no information-consumption complementarity, that is $\gamma = 0$, the inequality is never satisfied, and so the only equilibrium is full disclosure for all dimensions.

The intuition is that by disclosing, the monopolist shifts attention to the attribute. This can be effective at increasing the price when the consumption utility from buying is much higher than not buying. However, the firm does not wish to draw attention when the attribute is a drawback to consumption. For example, a drug company may advertise the primary effect of their drug, an attribute for which quality is high relative to no drug, but the firm may not wish to advertise the side effect of the drug, an attribute for which quality is worse than no drug. If the quality in several dimensions are correlated, then disclosing quality in one dimension will change expected quality in the other dimensions, and thereby shift attention to these dimensions.

Equilibria in which there is full disclosure in dimension $i$ also exist. To see this suppose the consumer believes the monopolist will disclose attribute $i$ for all $q_i$ and that any shrouding comes from types possessing the lowest quality $q_1^1$. Under these beliefs, shrouding will still change beliefs about quality from the prior beliefs, and so $w_i(B; b_1) \neq w_i(B; b_0)$ and thus attention will shift to dimension $i$. Since attention shifts to dimension $i$ for either disclosing or shrouding, the monopolist has
no incentive to shroud. In other words there is an “attentional externality”. As long as at least one type discloses, attention will shift for all types, even if they do not disclose. Shrouding will cause the consumer to infer quality is the lowest $q_i = q_1$. The standard unraveling logic applies. Whenever there exist two sets of equilibria, one in which $q_i$ is shrouded and one in which $q_i$ is disclosed, the monopolist will prefer the shrouding equilibria for all realized $q_i$. The condition for preference is the same condition that allows shrouding in equilibrium, equation (1.7).

While our model predicts one reason that the unraveling argument may fail, the existing literature has uncovered several others (Okuno-Fujiwara et al., 1990). We consider these and contrast them with the predictions of our model. There are several conditions in which full disclosure will break down into partial disclosure, in which only types with quality above a threshold will disclose. First off, the unraveling argument fails if disclosure is costly. If costs are sufficiently high there will be partial disclosure or no disclosure. The tradeoff for the firm is between the benefit of separating and the cost of disclosure. All types below a threshold will have little benefit to separating and thus prefer to pool. In the psychological model of Eyster and Rabin (2005) agents under-appreciate the correlation of other agents’ behaviors with their types. It is as if the observer mistakenly believes that some types who disclose shroud, and some who shroud disclose. The observer infers less from shrouding than a rational agent would infer, and thus the consumer’s expected utility after observing pooling is mistakenly higher than the actual expected utility from pooling. Types with low quality will exploit this bias by pooling. Hirshleifer et al. (2002) present a psychologically motivated model in which observers behave in two non-standard ways which the authors collectively label as “limited attention”. There is some probability that observers ignore the disclosure of the sender, and there is some probability that observers fail to update their beliefs conditional on no disclosure. Under these assumptions the observer will over-estimate quality conditional on no disclosure, and thus there will only be partial disclosure since senders with low quality can do better by being silent. A breakdown of common knowledge about quality could prevent unraveling too. For example, if the monopolist only knew the quality of the product with some probability, then silence in equilibrium may imply that the firm does not know the quality. The consumer’s expected utility after observing pooling will be higher than the expected utility from those who choose to pool, because there will be some high types who are forced to pool since they are unable to signal. As a result, types with low quality will choose to pool.

We can distinguish from all four of these possibilities since our model will not allow for partial disclosure. The assumption that attention shifts discretely drives our result. If the shift in attention were increasing in the change of expected consumption utility, then partial disclosure may occur in equilibrium. But more importantly, our model predicts that the monopolist will simultaneously disclose

\[q_i = q_1\]
some attributes and shroud others. These other theories cannot make this prediction unless there are different costs for revealing different attributes, different degrees of mis-prediction for different attributes, or different degrees of knowledge for different attributes, respectively. Furthermore, there are some situations where common knowledge would seem appropriate (e.g., drug companies are required by law to conduct clinical trials for the FDA before they can go to market).

Finally, the result can breakdown if utility is not monotonic in quality, or tastes in the population are heterogenous. These are reasonable concerns but we do not think it is too much of a stretch to assume that there are some attributes for which all individuals would prefer higher quality. It is hard to imagine a consumer who would prefer a drug to cause more nausea.

1.5.3 Welfare

We now analyze welfare effects of the advertisement. Care should be made in interpreting these results. There are many facets to advertising and we are looking at one, the effect of advertisements on directing attention to future consumption. Advertisements may have additional benefit such as providing useful information and they may have additional costs such as stealing consumers away from competitors (Bagwell, 2007). In the following analysis we ignore all other potential concerns. We take the social welfare function to be the sum of consumer utility and profit converted from money to utils. Since one numeraire equals $1 + \alpha + \gamma$ util, $SW = V_1 + (1 + \alpha + \gamma)\Pi$. In this subsection, we will compare welfare under mandatory disclosure and prohibited advertisement to welfare under equilibrium.

We begin by comparing utility in an equilibrium in which there is full shrouding in dimension $i$, to a regime in which the monopolist is required to disclose. The reporting strategy is forced to have $r_i(q) = d$ for all $q$. First we analyze the effect on the consumer. Disclosing shifts attention to dimension $i$. In equilibrium, the consumer buys but the price is such that he is left indifferent between buying and not buying and so gets the utility $V_1(N; N, b_0, b_1, p^*)$. The effect of learning about quality on the consumer is $\Delta V_1 = \gamma(u_i(0) - u_p)$. This can be either negative or positive. The effect of disclosing quality for the firm will be the gain from separating plus the gain or loss from the information-consumption complementarity $\Delta \Pi = \frac{1}{1+\alpha+\gamma}(1+\alpha)(u_i(q_i) - E_{b_0}[u_i(q_i)]) + \gamma(u_i(q_i) - u_i(0))$. Thus the change in social welfare will be $\Delta SW = \Delta V_1 + (1 + \alpha + \gamma)\Delta \Pi = (1 + \alpha)(u_i(q_i) - E_{b_0}[u_i(q_i)]) + \gamma(u_i(q_i) - u_p)$. If we take the average over all quality levels we get $E_{b_0}[\Delta SW] = \gamma(E_{b_0}[u_i(q_i)] - u_p)$. To perform the reverse comparison, that is comparing utility in an equilibrium in which there is full disclosure in dimension $i$, to a regime in which the monopolist is prohibited from disclosing quality in dimension $i$, we simply reverse the sign of $\Delta SW$.

So whenever the attribute’s average expected consumption utility is greater

\[18\text{Information available at http://www.fda.gov/Drugs/DevelopmentApprovalProcess/HowDrugsareDevelopedandApproved/default.htm}\]
than the forgone utility, full disclosure of the dimension on average produces higher welfare than full shrouding. Intuitively, shifting the consumer’s attention to an attribute that is pleasurable to anticipate increases welfare. For example, receiving an advertisement about one’s hobby may be pleasurable, because even if the quality of the product is low, one thinks about an enjoyable topic. When expected consumption utility is less than the forgone utility, but the firm finds it profitable to disclose, welfare is decreased. The firm draws attention to attributes that may be better than not buying the good, but that are unpleasant to think about. For example, a monopolist advertising the quality of heart medication will draw attention to a consumer’s debilitating health, something which the consumer may not enjoy thinking about. In the language of Becker-Murphy, this advertisement is a bad. Even though the advertisement increases demand for the drug, the consumer would prefer not to think about such matters.

There is little reason to expect that there is a systematic relationship between the difference of the utilities of buying and not buying \((u_i(q_i) - u_i(0))\), and whether the dimension is pleasant to anticipate \((u_i(q_i) - u_p > 0)\). So the firm’s advertising decisions are essentially orthogonal to consumer welfare. It is then difficult to say whether there is too much or too little advertisement. When there is too much advertisement, consumers are being “forced” to attend to dimensions that they would rather not think about. A typical consumer is barraged with advertisements that he did not choose: from billboards, to advertisements on buses. If a consumer with a heart condition receives information about a heart drug, this draws the consumer’s attention to his health which may be depressing. When there is too little advertisement, consumers want to think about positive things but do not have any information to shift attention. For example, a person living in Hawaii wants to receive more information about beaches and tropical wilderness, but a travel agency based in Hawaii cannot profit from this so they instead advertise the skyline of New York City.

In a natural variation of the model in which consumers have cheap sources of information other than advertisers, a resident of Hawaii could freely acquire information about beaches. The consumer could get information from alternative non-advertisement sources. If information that can shift attention to any dimension is freely available to the consumer, then it must be the case that there is too much advertisement. The consumer obtains information for all dimensions in which \(E_{b_i}[u_i(q_i)] \geq u_p\). Only the undesirable advertisements for which \(E_{b_i}[u_i(q_i)] < u_p\) will have an additional effect.

However, in this simple world of exogenous quality and zero costs the only role of disclosure is for the firm to command a higher price. Efficiency requires that the consumer buys, and that the firm advertises an attribute if and only if \(u_i(q_i) \geq u_p\). However, when quality is endogenous and costly to produce there may be efficiency lost from too little advertisement. Suppose nature exogenously chooses the cost function (instead of quality) that the monopolist uses to produce quality. For dimensions in which the utility from buying is substantially less than the utility from not buying, not only will the monopolist shroud, but because the
monopolist shrouds there will be zero investment in quality for that dimension. A monopolist that invests a positive amount may reveal or not, but revealing will make the price lower. So the monopolist has no profitable way to reveal quality and hence no incentive to produce quality. This can lead to a market inefficiency. The consumer benefit to higher quality may exceed the costs but does not want his attention drawn to it. A consumer may want the drug company to reduce the side effects, but does not want to have to “think about it”. We can call this an “attentional transaction cost”. The monopolist will be unwilling to communicate information that draws attention to displeasurable attributes (unless the expected consumption utility difference between investing optimally and zero investment is large, in which case the firm will choose to invest optimally and disclose). This suggests a role for third parties to monitor and ensure quality within an industry without directly informing the consumer about quality. The consumer will infer that quality is higher by the presence of the monitor but need not learn the details. The firm would benefit as well because quality and hence higher prices would be higher.

There is empirical evidence that mandatory disclosure increases quality. Jin and Leslie (2003) find that disclosure of restaurant hygiene reports in Los Angeles increases hygiene. In 1997 Los Angeles passed an ordinance that required all restaurants to prominently display A, B, or C hygiene grade cards in their window. After implementation, inspection scores increased by 5.3%, grade A restaurants made 5.7% more revenue (average revenue increased as well), and there was a 20% decrease in food-borne illness related hospitalizations. Additionally, Bennear and Olmstead (2008) found that mandatory disclosure of drinking water violations in Massachusetts reduced total violations by between 30–44% and 40–57% for severe violations. Our analysis suggests that, while quality investment is good for both consumer and firm, disclosure may also hurt both because consumers do not want to think about restaurant hygiene violations when they are eating their dinner. Hypothetically, verification by a third party may both insure high quality without concerning consumers.

1.5.4 Extension: Duopoly

In this extension, we consider the role of competition on unraveling. As in Milgrom and Roberts (1986), one may suspect that competition between informed parties will elicit all information. To address this issue, we use a Hotelling model in which two firms compete for a continuum of consumers with transportation costs. We show that competition can completely eliminate shrouding. Firms obtain more customers by revealing attributes for which their product is better than their opponent’s product. If the goods are valuable enough so that all consumers buy, there can only be full disclosure in equilibrium.

We extend the basic setup in two major ways. There is now a continuum of consumers, each of whom lives at a point \( \rho \in [0, 1] \) on a linear city with uniform density. As previously, consumers have at most unit demand. There are now two
firms, firm 0 and firm 1 located at each end of the city. The consumers must pay a transportation cost to purchase the good. One can also interpret $\rho$ as a taste parameter. The transportation cost for purchasing from the first firm for a consumer at $\rho$ is $pt$ and the cost for purchasing from the second firm is $(1 - \rho)t$. We assume these costs are dimensionless with no anticipatory utility. Nature chooses quality for each firm, and quality of the firms $q$ and $k$ respectively, are independent. The firms have common knowledge about each other’s quality, but the consumer only knows the distribution from which quality is drawn. The timing is the same, and the firms move simultaneously. Denote $B^0$ and $B^1$ to indicate that the consumer purchases the good from firm 0 or 1 respectively.

Preferences and profit are slightly different now. To be explicit, a consumer’s utility is

$$V_1(B^0; \bar{x}, b_0, b_1, p, \rho) = -\rho t - p^0 + a_n(\bar{x}, B^0; b_0, b_1)(-p^0 - u_p)$$

$$+ \sum_{i=1}^I E_{b_1}[u_i(q_i)] + a_i(\bar{x}, B^0; b_0, b_1)(E_{b_1}[u_i(q_i)] - u_p)$$

$$V_1(B^1; \bar{x}, b_0, b_1, p, \rho) = -(1 - \rho)t - p^1 + a_n(\bar{x}, B^1; b_0, b_1)(-p^1 - u_p)$$

$$+ \sum_{i=1}^I E_{b_1}[u_i(k_i)] + a_i(\bar{x}, B^1; b_0, b_1)(E_{b_1}[u_i(k_i)] - u_p)$$

$$V_1(N; \bar{x}, b_0, b_1, p, \rho) = -a_n(\bar{x}, N; b_0, b_1)u_p$$

$$+ \sum_{i=1}^I u_i(0) + a_i(\bar{x}, N; b_0, b_1)(u_i(0) - u_p)$$

The firms’ profits are $\Pi^0 = p^0\rho^0$ and $\Pi^1 = p^1(1 - \rho^1)$ where $\rho^0$ is the rightmost consumer that purchases from firm 0 and $\rho^1$ is the leftmost firm that purchases from firm 1.

To solve, when all consumers buy, then $\rho^0 = \rho^1$. Let us call this point $\rho^* \equiv \rho^0 = \rho^1$. This means a consumer with $\rho^*$ is indifferent between the two goods $V_1(B^0; \bar{x}, b_0, b_1, p, \rho^*) = V_1(B^1; \bar{x}, b_0, b_1, p, \rho^*)$. We will assume the default is $\bar{x} = N$. If we do not, there will be a discontinuity in the consumers’ willingness to pay at $\rho = 1/2$.\footnote{The default of $\bar{x} = N$ will not be optimal. A consumer would prefer to set the default to buy from the firm from which they expect to buy. But a consumer does not know which firm they will buy from at the time at which they choose the default. If consumers choose the optimal default, all $\rho \leq 1/2$, will choose $\bar{x} = B^0$ and and the rest choose $\bar{x} = B^1$. This generates the discontinuity.} This assumption may not be consistent with the setup in which the default is exogenous, but it is plausible to assume that the good in question is not a routine or habitual purchase.
First we find $\rho^* = \frac{1}{2t}(1 + \alpha + \gamma)(p^1 - p^0) + \frac{1}{2} + Y$, where

$$Y \equiv \sum_{i=1}^{I} \frac{1}{2t} \left( \left( 1 + a_i(N, B^0; b_0, b_1) \right) E_{b_1}[u_i(q_i)] - \left( 1 + a_i(N, B^1; b_0, b_1) \right) E_{b_1}[u_i(k_i)] \right).$$

The next step is to solve for the profit-maximizing price for each firm. Taking the first-order condition for firm 0 with respect to price yields the best-response function for price $p^0 = \frac{t}{1 + \alpha + \gamma}(\frac{1}{2} + Y) + \frac{1}{2}p^1$. Similarly, $p^1 = \frac{t}{1 + \alpha + \gamma}(\frac{1}{2} - Y) + \frac{1}{2}p^0$. Solving gives $p^0 = \frac{t}{1 + \alpha + \gamma}(1 + \frac{2}{3}Y)$ and $p^1 = \frac{t}{1 + \alpha + \gamma}(1 - \frac{2}{3}Y)$.

The point of this exercise is to see how duopoly affects disclosure. The key term in these expressions is $Y$ which is a weighted quality difference between firm 0 and firm 1's goods. Notice that a firm that has higher quality in dimension $i$ than its competitor can only increase its price by disclosing. By disclosing, more attention shifts to this attribute via the information-consumption complementarity. This magnifies the effect of the difference in quality on the price. The higher quality firm imposes an “attentional externality” on the lower quality firm. The weakly lower quality firm cannot avoid attention being drawn to its attribute even if it shrouds. The firm with weakly lower quality then has no attention-based incentive to shroud. The standard unraveling logic will apply. A firm with weakly lower quality equal to the competitor would disclose, so a firm with weakly lower quality that is slightly lower than the competitor would disclose, and so on.

**Proposition 6**

There exists a constant $\chi > 0$ such that if $\sum_{i=1}^{I} u_i(q_i) - u_i(0) \geq \chi$ and $\sum_{i=1}^{I} u_i(k_i) - u_i(0) \geq \chi$, then $r^0_i(q_i) = d$ and $r^1_i(k_i) = d$ for all $i$, all $q_i > q^1_i$, and all $k_i > k^1_i$ in any equilibrium.

Proposition 6 says that if the consumption value of the goods are sufficiently high, then any equilibrium will result in full disclosure. This is a possibility result, meaning that it is possible for the equilibria to have only full disclosure although this need not be the case. The critical assumption that leads to unraveling is that the good is sufficiently valuable such that the two best choices for all consumers are $B^0$ and $B^1$, as opposed to buying nothing at all. When the quality of the two goods is high enough so that all consumers are assured to buy, the firm with higher quality has an incentive to disclose and the firm with lower quality must disclose in order to not be thought lowest quality. The key difference from the monopoly case is that there is always a firm that can profit by disclosing since one firm always has higher quality than the other.

If there are some consumers in the middle of the city that do not buy, then the model reduces to two local monopolies. Disclosing attributes for which the utility from not buying is sufficiently larger than buying increases the appeal of not buying, and thus can induce some consumers near the center to not buy. In other words, it is possible that if some attributes are shrouded, all consumers buy, but if one of those shrouded attributes unravels, some consumers will not buy. Under this circumstance, shrouding may be preferred by both firms.
Even though it is possible to have only full disclosure equilibria, it is not guaranteed by the presence of competition alone. Casually speaking, the competitive effect of winning over consumers from the other firm increases disclosure, but the need to keep the consumer interested in buying the product at all may limit disclosure.

This prediction contrasts with the prediction of Hotz and Xiao (2006). They present a duopoly Hotelling model with quality disclosure in which the consumer’s preferences are monotonic but heterogeneous for quality. When preferences for quality and location on the line are correlated in such a way that those who have a greater demand for quality are located near the lower quality firm, disclosure may lead to a price competition that lowers profits for both firms. As a result, they make the opposite prediction that we make: they predict there will be less disclosure under duopoly.

1.6 Conclusion

Our goal in this paper has been to introduce a formal model of attention and anticipatory utility that can resolve puzzling human behavior. In Sections 4 and 5 we showed that the model produces plausible behavior that differs from standard theory in economically important ways. We conclude by discussing possible extensions to the model.

When there are two rounds of anticipatory utility, the decision maker will exhibit interesting time-inconsistent behaviors. First, information can be used as a commitment device. The first period self can use information to incentivize the second period self through the information-consumption complementarity. Second, the interaction between attention and anticipation also leads to motivated belief (Carrillo and Mariotti, 2000; Bénabou and Tirole, 2002). This manifests in two ways. First, when the expected consumption utility of a dimension is below a threshold, there is a demotivation effect: the decision maker will avoid information, and defer to the default in order to not think about a depressing dimension. To avoid this demotivation effect, the decision maker will prefer information that keeps expected consumption above the threshold. This results in subtle information preferences where the decision maker cares about the type and timing of information. For example, if the default for a dissertation-writing graduate student is to spend time teaching instead of research, the student may avoid information about the job market in order to avoid possible future low morale and low research productivity following a low signal. The student will also work hard to keep utility above the threshold.

Modeling mutually exclusive events as different consumption dimensions may be a fruitful extension to risk preferences. Receiving information about the likelihood of surviving a plane crash while standing in the airport security line makes one feel uneasy. One can think of this as attention shifting to the event “crashing”. Information about low-probability events would cause them to be overweighted. As a consequence, the DM may buy lottery tickets and still purchase excessive insurance.
Furthermore, it is plausible, in a dynamic setting, that the DM may systematically mispredict his future utility with a bias towards his current attention-shifted preferences. This would combine projection bias (Loewenstein et al., 2003) with our model. A DM with projection bias would incorrectly believe that the information-consumption complementarity is permanent. This would produce an effect similar to the focusing illusion (Kahneman et al., 2006) where “Nothing in life is quite as important as you think it is while you are thinking about it.”
Chapter 2

Placation and Provocation

2.1 Introduction

2.1.1 Overview

The primary purpose of this paper is to present a model that captures many disparate strategic interactions and unites them all under a simple conceptual framework. The hope is that this paper stands as a bridge between the separate strands of literature, thereby allowing for transfer of knowledge across these strands. Specifically, this paper connects the literature on the self-regulation of industries with the literature on political revolutions. The placation logic of self-regulation is closely related to the provocation logic of revolt. With this common framework it is then shown that the equilibrium remains robust for multiple first movers and that equilibrium utility of the first player is non-monotonic in the fixed cost.

Many industries regulate themselves. The story of the Motion Picture Association of America (MPAA) in the 1920’s is a typical example. Films during this era became more morally ambiguous relative to the social values of the time. In 1915, the Supreme Court case Mutual Film Corporation v. Industrial Commission of Ohio ruled that motion pictures were not covered by the First Amendment and that the government had the power to censor film (Ellis and Wexman, 2002). The threat of federal regulation loomed over the industry.

Hollywood’s solution to this problem was to form their own censorship entity. By self-regulating, they could restrict themselves minimally while still placating the government. A film history textbook explains (Mast and Kawin, 1992):

Such notoriety brought the film business to the attention of the United States Congress and to the edge of federal censorship – the last thing any producer wanted. The industry decided once again to clean its own house, to serve as its own censorship body . . . The loose informal advising of the Hays Office in the 1920s was another in a series of successful Hollywood attempts to keep films out of the hands of government censors.
I model this interaction as a game with two agents who have single-peaked preferences over a single dimension. The agents play an extensive-form game where the first agent has a marginal cost associated with moving the policy and the second agent has a fixed cost. Under the right conditions, this structure provides strategic incentives for the first player to move the policy such that it barely placates the second player from taking action, or barely provokes the second player to take action.

In the previous example, Hollywood moves first and the government follows. Congress has fixed costs to establishing a censorship bureau. Both care about the level of “iniquity” in film. A high level of “iniquity” means more profits for the industry (since sex and violence sell) but upsets the sensibilities of those in Congress. Hollywood discourages Congress from taking any action by voluntarily lowering the level of “iniquity”. Ultimately, this leaves the total level of violence and sex higher than if Congress had taken action.

A very similar interaction can be observed between states and political agitators. It is often said that revolutionaries want to make the situation for the people worse in order to spur them to action. There are tremendous fixed costs to organizing and starting a revolution because there is a major collective action problem. The people will not revolt unless the situation is sufficiently grim to outweigh the costs of organizing. Thus, it may be in the best interest of the vanguard to make the condition for the people worse in order to start a revolt.

This was the ideology of several Latin-American Communist Revolutionaries. According to historian Richard Gott, Che Guevara believed in this revolutionary philosophy: Marighela 1972:

Guevara . . . grasped the importance of an American invasion. In an apathetic continent, only direct engagement on the part of the Americans would stir up the necessary nationalism that could lead to a successful revolutionary war . . . Consequently, Guevara began a rural guerilla foco in Bolivia that was designed to bring American intervention and to spark off a continental war.

Perhaps the best known theorist on provocation was Carlos Marighela who tried to start a revolution in Brazil. Wardlaw 1982. In Marighela 1972:

The basic principle of revolutionary strategy in a country of permanent political crisis is to unleash, in urban and rural areas, a volume of revolutionary activities which will oblige the enemy to transform the country’s political situation into a military one. Then discontent will spread to all social groups and the military will be held exclusively responsible for failures.

This quote foreshadows the revolution in Nicaragua, where the communist Sandinistas ousted the Somoza family from power in 1979 Nolan 1984. In the time leading up to the revolution, the Somoza regime engaged in a repressive campaign
against the peasants, embezzled international aid money following the 1972 earthquake, and possibly assassinated a well known critic of the regime, Pedro Joaquín Chamorro. The ranks of the Sandinistas rose as a consequence and they soon had sufficient strength to gain control of the country.

This interaction is captured in the same model where the revolutionaries move first and the people follow. Both care about the socio-economic condition of the people. Contrary to the Hollywood example, the revolutionaries make the situation worse for people, not to placate the people to dissuade action but to provoke action. Both Hollywood and the revolutionaries move the policy in the opposite direction of their interest for strategic reasons.

The main results of this paper are as follows. It is shown that the second player may be better off when the players’ preferences diverge more. Similar preferences encourage Player 1 to free ride on Player 2. When preferences are sufficiently distant, Player 1 (the vanguard) cannot rely on Player 2 (the populace) to induce the desired outcome (revolution). Furthermore, it is shown that higher fixed costs for Player 2 may be a strategic advantage because it may discourage provocation on the part of the first player. Tying one’s hands discourages provocation. Lastly, it is shown that if Player 1 factions into multiple first movers, the placation and provocation equilibria become even more robust.

2.1.2 Literature Review

The self-regulation literature is relatively new, but has grown in the last few years. Maxwell et al. (2000) present a model where polluting firms reduce their emissions to preempt political lobbying on the part of consumers. In DeMarzo et al. (2005) the industry forms a self-regulating organization that monitors and enforces policies against fraud in order to mitigate the deadweight loss from asymmetric information. Stefanadis (2003) examines the tradeoff between nimble flexible regulation via self-regulation, and more sluggish but more consumer-favoring government regulation. Volden and Wiseman (2008) develop a model of unenforceable government regulation where consumer activism and legal action induce compliance with the government mandate or induce self-regulation above the mandate. Although the setup of these models differs from ours, all produce the same qualitative result where self-regulation “placates” or “pre-empts” government regulation. Beyond this, the direction of our paper is different. We generate provocation from the same model, and examine how fixed costs, bias, and default all determine the equilibrium regime. More fundamentally, the focus of our paper is to relate the logic of placation with the logic of provocation.

There have been several empirical studies on the effect of government threat to self-regulation. Maxwell et al. (2000) show that the presence of strong environmental group membership in a state correlates with greater pollution abatement over time. Antweiler (2003) finds mixed evidence on the effectiveness of green regulatory threat on emissions. Stango (2003) discovers that credit card companies made abnormally low returns and exhibited more rate cutting when they were under regulatory threat.
These studies support the hypothesis that self-regulation is motivated in part to placate the government.  

Baniak and Grajzl (2007) and Grajzl and Murrell (2007) also explore government regulation and self-regulation, but do not base their analysis on the pre-emption logic. Baniak and Grajzl (2007) model self-regulation in environments where corruption in the courts is a major issue. Self-regulation versus government regulation is a tradeoff between control versus enforcement. The tradeoff in Grajzl and Murrell (2007) is between the superior information and lower transaction costs under self-regulation versus greater consumer bargaining power under government regulation. These two studies are conceptually very different from this paper and address different questions.

The economic literature on revolutions is fairly small. Tullock (1971) emphasizes that most of the benefit of a revolution comes in the form of externalities. Roemer (1985) models the iconic struggle between Lenin and the Tsar. In his model Lenin and the Tsar propose a set of payoffs for the populace for which coalitions then form and fight to probabilistically determine who’s policy is implemented. Along similar lines, Grossman (1991) models insurrections as a cost for an autocrat. A very high tax rate will reduce revenue via the Laffer curve and via encouraging more insurrections. Acemoglu and Robinson (2001) develop a model of class clash where either the poor or the rich choose the tax rate. The party not in power may revolt, and the party in power may decide to placate the party not in power to deter a revolution or a coup. None of these papers address the provocation logic in this paper.

The next section presents the model. The interaction is represented as an extensive-form game with two players who have single-peaked preferences over a single dimension. The first player has a marginal cost to move the policy and the second player has a fixed cost.

Section 2.3 characterizes the equilibrium regimes and applies them to diverse situations. The subgame perfect equilibrium is generically unique, but the parameter space can be divided into regions that induce qualitatively different equilibrium regimes. Each of these regions is characterized and the logic of each regime is illustrated with an anecdote. In Section 2.4 we explore the optimal fixed cost for the fixed-cost player, Player 2. It is shown that equilibrium utility is not monotonic in the fixed cost. For some regions of the parameter space, Player 2 would prefer greater fixed costs in order to dissuade Player 1 from provocation. Section 2.5 presents a psychological interpretation of the model. The logic of placation and provocation is illustrated in the context of a time inconsistent agent. Section 2.6 extends the model to include multiple first-movers that all move simultaneously. It is shown that both the placation and provocation equilibrium regimes become more robust with many first movers. Section 2.7 concludes.
2.2 Model

The game is an extensive-form game with two players, \( i = \{1, 2\} \), who have a single-peaked strictly concave felicity function, \( f_i(\cdot) \), over a single policy dimension \( q \). At the beginning of the game the default policy is \( \bar{q} \). Player 1 acts first and can choose to move the policy an amount, \( m_1 \in \mathbb{R} \). Then Player 2 may move the policy \( m_2 \in \mathbb{R} \). The realized policy \( q = \bar{q} + m_1 + m_2 \) determines the utility of both players.

The bliss point is defined as the realized policy, \( q \), that maximizes the felicity function. Without loss of generality, Player 2 has bliss point 0 and Player 1 has bliss point \( b \), where it is assumed, \( b \geq 0 \). The parameter \( b \) is the bias of Player 1 relative to Player 2. When \( b \) is small, the bliss points of both players are near, and when \( b \) is large the bliss points are distant.

Both players have a weakly convex cost function, that is positive and monotonically increasing in the magnitude of Player i’s action, \( |m_i| \), and where initial costs and initial marginal costs are zero, \( C_i(0) = 0 \) and \( C'_i(0) = 0 \). Furthermore, we assume the cost function is symmetric so that \( C_i(x) = C_i(-x) \). The main difference between the two utility functions is that Player 2 has a fixed cost of \( e \) to moving the policy a nonzero distance, \( m_2 \neq 0 \). Both felicity and cost functions are assumed to be differentiable as many times as necessary. The utility functions are given by:

\[
U_1(m_1, m_2) = f_1(\bar{q} + m_1 + m_2) - C_1(m_1) \tag{2.1}
\]

\[
U_2(m_1, m_2) = f_2(\bar{q} + m_1 + m_2) - C_2(m_2) - e \ast 1\{m_2 \neq 0\} \tag{2.2}
\]

In order to guarantee a unique solution, we have an additional restriction on the third derivative of the felicity and cost functions. There is nothing particularly intuitive or interesting about this condition, and thus it is relegated to the appendix under Proof of Lemma 4, expression (A.2).

Notice that if Player 2 chooses to pay the fixed costs associated with taking an action, then Player 2 will move the realized policy \( q \) such that it maximizes \( U_2(m_1, m_2) \). However, if this gain in utility is not worth the fixed cost of taking an action, then no action will be taken. This fixed cost produces an inaction zone around Player 2’s bliss point. Let \( q_L \) and \( q_U \) denote the lower and upper bounds of the inaction zone respectively. Define the inaction zone as the interval \([q_L, q_U] \) where if \( m_1 + \bar{q} \in [q_L, q_U] \), then Player 2’s optimal action is \( m_2 = 0 \). Thus, Player 2’s bliss point in the inaction zone, \( 0 \in [q_L, q_U] \). The subgame perfect equilibrium will depend on the location of the two bliss points, the span and location of the inaction zone, and the location of the default policy.

A key assumption in the model is that Player 2 does not have the power to directly penalize Player 1. The government could change the level of “iniquity” in the media, but they cannot directly penalize the industry in this model. This is a realistic assumption in some cases. In the context of self-regulation, the government cannot legally prosecute the industry before they legislate regulations, hence they cannot directly punish firms. In the context of revolution, the populace may wish to
punish the vanguard for making the situation worse, but they are unorganized and unarmed. If they did have the power to punish the vanguard, they would likely have the power to directly change the policy by pressuring the oppressive government. The oppressive government (who is not modeled as a player) may want to directly punish the vanguard, but it may not have the means to do so without oppressing the whole populace. The vanguard may hide amongst the people and wage protracted guerilla warfare using civilians as a shield.

Another issue that we should address is the origin of the default policy, $\bar{q}$. In many political economy models the default policy is considered exogenous. This paper takes the same approach. Our interpretation is that the location $\bar{q}$ was determined as the outcome of some other unrelated interaction. The fact that movies had sex and violence in the 1930s may be the result of many socioeconomic factors. The fact that the populace are oppressed may be the result of the long political economic history of the state. Both of these causes are complex and beyond the scope of the model and so we assume $\bar{q}$ is exogenous.

2.3 Equilibrium and Applications

2.3.1 The Equilibrium Regimes

Since this is an extensive game with perfect information, the equilibrium concept we employ is subgame perfect equilibrium. Generically, the parameters uniquely determine the pure-strategy subgame perfect equilibrium. We categorize the equilibria into five equilibrium regimes, each of which have an equilibrium path that is qualitatively similar. These regimes each exist in a contiguous region of the $\bar{q}$, $b$, and $e$ parameter space. The five regimes are named placation, gravity, provocation by extremists, provocation by moderates, and improvement and are graphed in $\bar{q}$-$b$ space in Figure 2.1. Placation is where Player 1 discourages Player 2 to act. Gravity is where both players move the policy toward their bliss points. The provocation regimes are where Player 1 encourages Player 2 to act. And the improvement regime is where Player 1 moves the policy within Player 2’s inaction zone. These equilibria will be explored in the context of examples in the following sections.

Let $m_2^*(m_1)$ be Player 2’s optimal strategy. Let $m_1^{I^u}$ be the solution to Player 1’s first-order condition, if the solution is within the interior of the inaction zone, and $m_1^{O^u}$ be the solution if the solution is not an element of the inaction zone. These are like Stackleberg solutions because they take into consideration Player 2’s move (for more precise definitions, see the appendix).

**Proposition 7** If $C_2''(\cdot) > 0$, the profile $(m_1^*, m_2^*(m_1))$ is the generically unique subgame perfect equilibrium if it is the profile from the set

$$(m_1^*, m_2^*(m_1)) \in \{(m_1^{O^u}, m_2^*(m_1)), (m_1^{I^u}, m_2^*(m_1)),
(q_L - \bar{q}, m_2^*(m_1)), (q_U - \bar{q}, m_2^*(m_1))\}$$
that maximizes Player 1’s utility.

**Corollary 2** If \( C''_2(\cdot) = 0 \), the profile \((m_1^*, m_2^*(m_1))\) is the generically unique sub-game perfect equilibrium if it is the profile from the set
\[
(m_1^*, m_2^*(m_1)) \in \{(0, m_2^*(m_1)), (m_1^{ln}, m_2^*(m_1)), (q_L - \bar{q}, m_2^*(m_1)), (q_U - \bar{q}, m_2^*(m_1))\}
\]
that uniquely maximizes Player 1’s utility.

Proposition 7 characterizes the subgame perfect equilibrium regimes of the model. The set contains the five possible equilibrium profiles. The last term \((q_U - \bar{q}, m_2^*(m_1))\) actually represents two profiles because Player 2’s optimal action, \(m_2^*(q_U - \bar{q})\), has two elements, \(m_2 = 0\) and a nonzero \(m_2\) that satisfies Player 2’s first-order condition. Corollary 2 characterizes the subgame perfect equilibrium for the special case when Player 2 has no marginal costs. The remainder of this section will explore each of these equilibrium regimes in greater detail.

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1Remember \( C''_2(\cdot) = 0 \) and \( C'_2(0) = 0 \) \( \Rightarrow C'_2(\cdot) = 0. \)
2.3.2 Self-Regulation

Placation

We begin by considering the equilibrium that occurs when Player 1’s bias is rightward of the inaction zone, \( b > q_U \), and the default policy is rightward of the inaction zone, \( \bar{q} \geq q_U \). This is the region in Figure 2.1 labelled “Sacrificial Placation”. This region results in an equilibrium that captures the logic of self-regulation. The industry moves the policy away from its bliss, reducing the policy to a level that is acceptable to the government. This leaves the government indifferent between regulating and taking no action. This is illustrated in Figure 2.2.a. The solid arrow indicates where Player 1 moves the policy.

A second region of the parameter space that induces the same equilibrium profile exists when the bias of Player 1 is very large (at least \( b > q_U \)) and the default policy begins either within the inaction zone or leftward of the inaction zone, \( \bar{q} < q_U \). This is the center shaded region in Figure 2.1 that is labeled “Extraction”. This region fits the story of self-regulation in the video game industry. Early video games with simple geometric figures like *Pac-Man* (1980) or *Tetris* (1985) were innocuous, but once technology allowed for more realism, games soon incorporated vivid.

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\footnote{Wolf, ed (2008) has dates available on p. xviii–xix.}
Video game violence is profitable, and so the industry increased the level of realistic violence, as soon as technology permitted, up to the point where the government was indifferent between censorship and no action. This is the same equilibrium regime as placate because the equilibrium path is similar (Player 1 brings the policy to the upper bound of the inaction zone and Player 2 does nothing). But since we begin in a different region of the parameter space, low sex and violence (low \( q \)) instead of high sex and violence (high \( q \)), the interpretation is that the industry is extracting as much profit as they can from the government’s immobility. Figure 2.2.c illustrates this last interpretation labeled as the “extraction” equilibrium. The utility conditions to induce this equilibrium are exactly the same as the conditions in the willing-placation equilibrium.

Now consider the case when the players’ preferences are more closely aligned, when the bias is within the inaction zone, \( b \leq q_U \), and the default policy remains rightward of the inaction zone, \( q > q_U \). This is the region in Figure 2.1 labelled “Willing Placation”. This region induces the same equilibrium path as the sacrificial placation, but the interpretation is slightly different. The following anecdote illustrates.

The Rainforest Action Network (RAN) is an activist organization whose mission is to preserve the rainforests of the world. In 2000, RAN targeted Citigroup for a publicity campaign to discourage them from lending money to firms that destroy rainforests (Gunther, 2004; Baron and Diermeier, 2007). The publicity campaign included protests, an internet campaign, a 60 foot banner unveiled across from Citigroup headquarters in Manhattan, advertisements and sit-ins. In the end, Citigroup conceded to RAN’s demands and established a whole new environmental policy. Citigroup claimed that it wanted to have a good environmental policy and that it was already meeting with environmentalists (Gunther, 2004). However it is unlikely that the bank cares as much about the environment as the environmentalists.

We model this as Player 1 (firm) and Player 2 (activist) have preferences over the environment-profit tradeoff (\( q \)). The activists have a fixed cost (\( e \)) to organizing and running a publicity campaign. They can use the publicity campaign to shift (\( m_2 \)) consumers away from using the product of the offending firm. The firm wants to reduce the damage it does to the environment, but cares more about profit (prefers a higher \( q \)) than do the activists, and does not want to incur the bureaucratic costs (\( C(m_1) \)) of redefining their environmental policy. The firm must have an even more pro-environment policy than they would otherwise choose, in order to deter the activists from running their publicity campaign. The firm moves the policy toward

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3 On pages 243 and 278. Chapters were written by Carl Therrien and Dominic Arsenault respectively.

4 The mission statement is available at the Rainforest Action Network website under the “Rainforest Action Network: Our Mission & History” page, available at http://ran.org/who_we_are/our_mission_history/. The page was referenced on June 23, 2008.

5 In this particular example, the publicity campaign started before the firm placated the activists. However, when RAN targeted Lowe’s, the firm conceded to their demands before the campaign started (Baron and Diermeier, 2007).
its bliss, however, it moves the policy so far that its marginal benefit is less than its marginal cost. This action is still a best response because it dissuades Player 2, the activists. We call this a “willing placation” equilibrium because Player 1 is moving the policy in the direction of its bliss but must do so more than it optimally desires in order to placate Player 2. This is illustrated in Figure 2.2.b.

Gravity

The previous three examples of sacrificial placation, willing placation, and extraction were variations of the same equilibrium regime. Now, we consider a second equilibrium regime. The regions of extreme default policies, far to the left and far to the right where $\bar{q} \not\in (q_L, q_U)$, and where the bias, $b$, is fairly low. These regions are labeled in Figure 2.1 as “Gravity”. The parameters might fall into this region for several reasons. If self-regulation were very costly because individual firms have commitment problems, or alternatively because the fixed costs for the government are small, the parameters may fall into this region. If the default policy begins as an extreme and the players preferences do not diverge too much the parameters may fall into this region as well. In either case, it may not be worthwhile for the industry to self-regulate. Self-regulation would be too costly relative to letting the government take control. If Congress had strong support for censorship, and if the fixed costs for establishing a monitoring bureau were cheap, then Hollywood would let Congress regulate.

When Player 2’s (the government’s) marginal costs are nonzero, both players move the policy toward their bliss points. The lower the marginal costs of a player, the more that player will move the policy toward their bliss (the more gravity that player exerts). In the special case when Player 2’s marginal costs are zero, Player 1 takes no action, $m_1 = 0$, and Player 2 moves the policy to its bliss, $m_2 = -\bar{q}$. This special case is depicted in Figure 2.2.d. In Figure 2.2.d. the dotted arrow indicates where Player 2 moves the policy.

There is a second region of the parameter space that will induce this same equilibrium. We can have a gravity equilibrium where the default policy begins within the inaction zone, $\bar{q} \in [q_L, q_U]$. The interpretation is that the current level of iniquity is not enough to encourage regulation, but even if it were, the government would be ineffectual at regulating. Hollywood best responds by producing blockbuster films filled with violence and sex. The industry knows that there will be only minor repercussions for raising the ire of Congress. This can be seen in Figure 1.e. For this to be an equilibrium, rather than having the bias small, it must be very large and the marginal costs of Player 2 must be fairly large as well.

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6This region of $\bar{q}$ cannot result in gravity if Player 2’s marginal costs are zero. This equilibrium is not depicted in Figure 2.1 because the functions used to generate the graph have $C_2(\cdot) = 0$. 
2.3.3 Revolutionaries

... All experience hath shown that mankind are more disposed to suffer, while evils are sufferable, than to right themselves by abolishing the forms to which they are accustomed.

-Thomas Jefferson, The Declaration of Independence

![Figure 2.3: Provocation and Improvement Equilibria. Note: The solid line represents Player 1's action and the dotted line represents Player 2's action.](image-url)

Provocation

The provocation equilibrium occurs when the bias of Player 1 is not too large, and the default policy begins in the inaction zone of Player 2, \( \bar{q} \in [q_L, q_U] \), and near one of the two bounds, \( q_L \) or \( q_U \). If \( \bar{q} \) is near \( q_L \) then this parameter space gives rise to a “Provocation by Extremists”. If \( \bar{q} \) is near \( q_U \) then this parameter space gives rise to a “Provocation by Moderates”.

The anecdote that accompanies these two equilibrium regimes is the story of the vanguard and the populace. The populace (Player 2) is dissatisfied with the oppressive regime (not a strategic player), but there is a fixed cost to revolt. The cost of organization and the cost of violent clash with the government are fixed and independent of the political goals of the people. As long as the “evils are sufferable”, the people will not revolt. The vanguard (Player 1) of the revolution wishes to exacerbate the situation so that the evils are beyond sufferable, and the populace does revolt. Both Players want the policy to move in the same direction.
However, Player 1 moves the policy in the opposite direction in order to induce action on the part of Player 2 thereby free-riding on Player 2’s efforts.

As presented in the introduction, this was the strategy of Latin-American Communist revolutionaries such as Carlos Marighela. It also closely parallels the strategy of al-Zarqawi who aimed to start a civil war in Iraq following the U.S. invasion. By attacking the Shia, al-Zarqawi expected a violent response which would exacerbate the condition of the Sunni (Hashim [2006]). Al-Zarqawi’s strategy was to make the conditions for the Sunni so dire that they would revolt against the Shia and the United States.

There are two different provocation equilibrium regimes. In the first, the provocation by extremists equilibrium, the default policy is less than the populace’s bliss point but within the inaction zone. The vanguard pushes the policy lower, away from their preferred policy in order to induce action on the part of the people. In this scenario the vanguard has more extreme preferences than the populace relative to the default policy. The vanguard moves the policy specifically to the lower bound \(q_L\) of the inaction zone. The people respond by moving the policy toward their bliss. This is illustrated in Figure 2.3.a in the case where the marginal cost of the people is zero, \(C_2(\cdot) = 0\). The solid arrow shows where Player 1 moves the policy and the dotted arrow shows where Player 2 moves the policy. When Player 2’s marginal costs are zero, Player 2 moves the policy directly to its bliss.

In the second provocation equilibrium regime, the vanguard moves the policy to the upper bound, \(q_U\), of the inaction zone. Now the default policy is greater than the populace’s bliss. The vanguard actually has more moderate preferences than the populace relative to the default but still pushes the policy higher, away from their bliss in order to induce the populace to action. This is illustrated in Figure 2.3.b. The vanguard knows that the populace will take the revolution too far, from their perspective, but they would rather free ride on the uprising than to do nothing, or to try and make the policy better directly. Perhaps the Girondin of the French Revolution fit this description. They were at the forefront at the beginning of the French Revolution even though they were much more moderate than many of the other participating political groups.

**Improvement**

Another option available to the vanguard is to directly improve the condition of the populace through their own actions. The vanguard sets up a soup kitchen rather than igniting a revolt. In this equilibrium regime, the direct improvement does nothing to inspire the populace to take action. This occurs when the preferences of the vanguard and the populace are closely aligned (\(b\) is sufficiently small), and the default policy begins within the inaction zone, \(q \in [q_L, q_U]\), near Player 1’s bliss (0). This region is labelled as “Improvement” in Figure 2.1. In this equilibrium, Player 1 moves the policy within the inaction zone toward its bliss, specifically to the point where the marginal cost of moving the policy equals the marginal benefit.

When does the vanguard decide that they should set up a soup kitchen rather
than organize a revolt? When is direct action to improve the situation more effective than shocking the people to action? Let us define \( \hat{q}_E \) (and \( \hat{q}_M \)) as the cutoff values for \( \bar{q} \) at which Player 1, in equilibrium, is indifferent between moving the policy closer to its bliss or moving the policy to the lower bound (or upper bound) of the inaction zone and inducing a “provocation by extremists (moderates)”. When \( \bar{q} = \hat{q}_E \) (or \( \bar{q} = \hat{q}_M \)) both provocation by extremists (moderates) and improvement are subgame perfect equilibria. This curve is depicted in Figure [2.1] in \( q-b \) space separating the provocation and improvement regions.

We already have a lower bound of \( \bar{q} \) for which provocation by extremists is possible and that is \( \bar{q} = q_L \). Thus the region of \( \bar{q} \) in which the subgame perfect equilibrium is a provocation by extremists is \([q_L, \hat{q}_E]\). The equivalent expression for a provocation by moderates is \([\hat{q}_M, q_U]\). How do these regions change as a function of the divergence in preferences \( (b) \) between the two players?

Consider the provocation by extremists where Player 1 prefers to push the policy to \( q_L \). As the vanguard’s interests become more extreme relative to the people’s, the vanguard trust the people less. If there is a revolution, the people are too moderate to push it far enough. This force makes revolution less attractive to the vanguard. On the other hand, as the vanguard becomes more extreme, the status quo appears worse. Thus, the situation gets so bad that establishing a thousand soup kitchens would be unsatisfactory. It would take too much effort, so it is better to ignite the revolution. This force makes revolution more likely. For the provocation by moderates regime, as the vanguard’s interests become more moderate relative to the people \( (b \text{ increases}) \), the lack of common goal makes revolution less likely.

When the preferences of the two players are very different, there will be no default values that lead to provocation. When preferences between the populace and the vanguard are far apart, revolution is no longer an appealing option to the vanguard. The vanguard will not trust that the populace will push the revolution to the extent that the vanguard would find acceptable.

**Proposition 8** There exists a bias, \( \hat{b} \), such that \( \forall \ b > \hat{b}, \) provocation no longer exists as an equilibrium for any \( \bar{q} \).

Keep in mind that a revolution is an undesirable outcome for the people. A provocation equilibrium presumes that the default policy begins within the inaction zone and that the vanguard pushes it out of the zone. The people always prefer the vanguard to leave the policy within the inaction zone. In other words, the people always prefer soup kitchens because they do not want to pay the costs of revolution. Thus, there are regions of the default space for which the people are better off when the vanguard are more extreme because the vanguard switches from agitation to soup kitchens.

For some parameter values increasing the fixed costs of revolting increases the people’s utility. Figure [2.4] shows a graph of Player 2’s utility for some fixed \( \bar{q} < 0 \) and a fixed \( b \). As the fixed costs for Player 2 increase, the equilibrium regime changes from gravity, to provocation by extremists, to improvement. Initially, increasing the
fixed costs results in a loss. The intuition is that as the fixed costs grow, there are greater costs for Player 2 to move the policy in a gravity equilibrium. As the regime changes, increasing $e$ implies that more exacerbation is required by the vanguard to induce a revolution. When these costs become prohibitive, the vanguard decides instead to improve the situation directly. If the populace is ineffective at organizing a revolt, then the vanguard might as well put away their guns and open a soup kitchen. As the fixed costs grow further, the vanguard’s optimal action is unaltered. Once the equilibrium is an improvement regime, increasing the fixed costs further has no effect on the equilibrium nor the utility of either player. We will explore the optimal fixed costs in Section 4 of this paper.

### 2.3.4 Marginal and Fixed Costs

In the examples of government regulation, most of the costs that the government must incur are fixed costs rather than marginal costs. In the optimal fines literature starting with Becker (1968), it is less costly for regulators to impose strict fines rather than to invest in monitoring. The level of the fine has no effect on the regulator’s costs. Thus the regulator can set up a more strict fine structure and move the policy a great deal farther without incurring extra costs. Perhaps some additional monitoring is also required to reach the desired policy and so this implies some marginal cost. The important point is that most of the costs are fixed costs incurred from setting up the regulating entity in the first place.

Likewise, it is likely that most of the costs to revolution are fixed. The majority of costs are in overcoming the collective action problem: organizing and arming the
people. Once this has been achieved, the strength of the organized masses easily dominates the strength of a small tyrannical regime. Storming the Bastille is the easy part. Rallying the citizens to march to the stone walls of the fortress is the hard part.

If it is the case that the marginal costs of the populace are high relative to the vanguard’s marginal costs, then the vanguard has little incentive to initiate the revolution. If the marginal costs of the populace are high, then the populace will not push the revolution very far and so the vanguard is better off setting up a soup kitchen.

What happens when the fixed costs approach zero? When fixed costs are sufficiently small, the default policy and Player 1’s bliss will inevitably fall outside the inaction zone for any \( q \neq 0 \). This implies that the two choices facing the industry are a placation or a gravity equilibrium. In the gravity equilibrium, both players move the policy toward their blisses (which may or may not be in the same direction relative to the default). As the fixed cost decreases, placation becomes more and more costly until placation implies moving the policy to Player 2’s bliss. When the fixed costs become arbitrarily small, the only equilibrium is gravity.

**Proposition 9** If \( C''_1(\cdot) > 0 \) and \( \bar{q} \neq 0 \), there exists an \( \hat{e} \) such that for all \( e \in [0, \hat{e}) \), the only subgame perfect equilibrium is gravity.

Thus a model that ignores fixed costs would also ignore the diversity of all the other equilibrium regimes. Fixed costs are necessary in this model to generate placation and provocation equilibria.

### 2.3.5 Summary

In summary, the outcome of the interaction depends on the initial parameters. Placation (sacrificial, willing, extraction), gravity, provocation by extremists, provocation by moderates, and improvement are all potentially equilibria. However, generically, a vector of parameters will uniquely determine one of these equilibria. Figure 2.1 shows all these equilibrium regimes in \( \bar{q} \)-b space. The curves that separate the regimes represent parameter values for which there are multiple equilibria. These curves are the knife-edge cases where Player 1 is indifferent between inducing either of the neighboring regimes. When the default \( \bar{q} \) begins near Player 2’s bliss, a large bias will result in Player 1 pulling the policy higher since Player 1’s bliss is also higher. Thus the regime moves from improvement to extraction. When the default begins near the bounds of the inaction zone and the bias is small, Player 1 induces a provocation equilibrium to free ride on Player 2’s actions. When Player 1’s bias is larger, free-riding no longer makes sense since the final policy will be undesirable for Player 1. As long as the bias is not too large, and the default is slightly larger than \( q_U \), the solution is to placate thereby sacrificing a little distance to deter Player 2. When the default is very far away from the inaction zone, placation efforts are too costly so Player 1 allows for a gravity equilibrium.
An alternative perspective is presented in Figure 2.5 which depicts the equilibrium regimes in $b$-$e$ space. In this figure, a $\hat{q}$ was arbitrarily chosen. Here we can see that when fixed costs are high, Player 2 will not take action, and so placation is not worthwhile and provocation is too costly. The only potential equilibrium is improvement. When the bias is high and Player 2’s fixed costs are low, Player 1 moves the policy as far to the right causing either placation, extraction, or gravity. When the bias and fixed costs are low, Player 1 either free rides in a provocation or gravity equilibrium.

Figure 2.5: Equilibrium Regions in $b$-$e$ Space. Note: In this graph $C_2(\cdot) = 0$.

### 2.4 Optimal Fixed Costs

What is the optimal fixed cost for Player 2? In the provocation equilibrium regimes Player 1 engages in Pareto damaging behavior that lowers Player 2’s utility, and in a gravity equilibrium Player 1 may shade down its action and free ride on Player 2’s action. It may be in Player 2’s interest to have fixed costs that are large in order to deter Player 1 from revolting or free riding. This would serve as a credible commitment to Player 1 that Player 2 will take no action.

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$^7$If $\hat{q} < 0$ then the graph would look slightly different, most notably the provocation by moderates region would be replaced by a provocation by extremists region.
Figure 2.6 plots the equilibrium regimes in $q$-$e$ space for a fixed positive bias, $b$. Notice that as Player 2’s fixed cost ($e$) increases, the region of $\bar{q}$ that results in an improvement equilibrium expands because, the bounds of the inaction zone are too far away to warrant placation or provocation. The region of $\bar{q}$ that results in provocation by extremists expands into the gravity equilibrium region since the bounds mover farther away from Player 2’s bliss point.

In this section we explore situations in which Player 2 can choose its own fixed costs, and in which the default policy, is determined stochastically. First Player 2 chooses $e$, then nature determines $\bar{q}$, and then the game continues as in the basic model. The interpretation is that Player 2 faces a sequence of the basic game against many different first movers. All of these first movers have identical and known preferences. However, across these interactions, the default policies vary according to some distribution. The main result is that there always exists an interval of $e$ in which the expected utility of Player 2 is increasing in the fixed cost $e$. In other words, there are situations in which Player 2 may prefer higher fixed costs.

### 2.4.1 Setup

Let the utility of the two players be given by (2.1) and (2.2). Player 2 may choose a fixed cost $e$, from a menu of fixed costs, $[e_L, e_U]$. We assume that $e_L$
is sufficiently large such that \( m_1(0) \leq q_u(e_L) \), where \( m_1(0) \) satisfies \( f'_1(m_1(0)) = C'(m_1(0)) \). Let \( \bar{q} \) be distributed according to the CDF \( H(\bar{q}) \), with continuous and atomless PDF \( h(\bar{q}) \) over the support \([s_L, 0] \). Define \( \hat{q}(e) \) to be the value of \( \bar{q} \) where both provocation by extremists and improvement are equilibria. This curve is graphed in Figure 2.6. To limit our attention to the relevant region of Figure 2.6 we set the upper bound of the support to 0, and the lower bound of the support, \( s_L \), must satisfy \( s_L < \hat{q}(e_L) \). The timing of the game begins with Player 2 choosing the fixed cost \( e \), then nature determines the default state \( \bar{q} \) and then the basic game continues as described in Section 2.

When \( e \) increases, Player 2 faces a tradeoff. Increasing \( e \) expands the improvement regime into the provocation and gravity regimes. Keeping \( \bar{q} \) constant benefits Player 2 because Player 2’s utility in an improvement equilibrium is higher than in either a gravity equilibrium or a provocation equilibrium. Over the region in consideration, in an improvement equilibrium Player 1 will move the policy toward Player 2’s bliss, while in a provocation equilibrium Player 1 moves the policy away from Player 2’s bliss. In a gravity equilibrium Player 1 moves the policy towards Player 2’s bliss, but shades this movement down (relative to an improvement equilibrium) in order to induce Player 2 to assume more of the costs. Thus expanding the improvement equilibrium relative to provocation and gravity, benefits Player 2. However, the expected cost of increasing \( e \) is that when \( \bar{q} \) does result in a gravity or a provocation equilibrium, the payoffs will be lower.

**Proposition 10** There always exists a menu of fixed costs \([e_L, e_U]\), where \( e_U \) maximizes expected utility. Moreover, there exists an \( e^* \) for which all \( e \in [e^*, \hat{q}^{-1}(s_L)] \), \( \frac{d}{de} EU_2(e) > 0 \).

Proposition 10 states that there exists a range of \( e \) in which the expected utility maximizer is the highest fixed cost. The intuition is that once \( e \geq \hat{q}^{-1}(s_L) \), the only equilibrium is improvement. By decreasing \( e \) slightly, Player 2 is made strictly worse off. The provocation equilibrium expands into the improvement region which lowers Player 2’s utility, and there is no benefit by lessening the costs of provocation or gravity, since there was zero probability that those equilibria would occur. This is illustrated in Figure 2.7.

### 2.4.2 Examples

**Settlers and the Military**

History is rife with conflicts between settlers and natives. In a PBS Frontline documentary, an Israeli settler says (Setton, 2005):

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\[ \text{This ensures that the only equilibria that can result from the distribution are gravity, provocation by extremists, and improvement. To focus on the relevant region we ignore the equilibria that result from large values of } \bar{q}: \text{ extraction, placation, provocation of moderates and gravity in the positive region of } \bar{q}. \]

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And it’s only a matter of time until the war, with God’s help, will begin, and it will begin with us. And in the end, we’ll win. We’ll inherit the land and expel the people who are in it.

The settler wants to provoke a conflict with the Palestinians. In this sort of conflict, the military must decide how much assistance they will grant to the settlers. The settlers are Player 1 and the military is Player 2.

In a general sense, settlers can choose to pursue relations with the natives in either a cordial manner or an aggressive manner. In our simplified abstraction, settlers care only about access to land and its resources. This may include land ownership, access to water sources, use of pasture land, etc. Let us model this interaction as we have done throughout the paper. Map the settlers’ access to the land onto a single policy dimension. A low value of \( \bar{\bar{q}} \) can be interpreted as natives who grant limited access to the settlers, whereas a high value of \( \bar{\bar{q}} \) can be interpreted as natives who grant wide access to the settlers. Moving the policy rightward, toward greater access, can be interpreted as having a cordial relation with the natives, with a marginal cost of giving tribute. Moving the policy leftward, toward lesser access, can be interpreted as engaging in aggressive relations, with a marginal cost of instigating skirmishes. Thus it is costly for the settlers to move the default policy.
The military wants the settlers to have a high level of access to the land, but not as high as the settlers want for themselves. The military can attack the natives to improve the settlers’ access but doing so incurs a fixed cost. It requires logistics, recruitment, reconnaissance, etc. Thus the military will only get involved if the settlers’ access to resources is very poor. The settlers are Player 1 and the military is Player 2. If the initial access is horrible, the military will come in to help the settlers by fighting off the natives (gravity equilibrium). If the initial access is bad, this gives the settlers incentive to challenge the natives in order to draw the military into the conflict and to aid them (provocation equilibrium). If the initial access is good, this gives the settlers incentive to have good relations with the natives (improvement equilibrium).

The military faces a large frontier full of many settlements interacting with different native groups. Each interaction is drawn from a distribution. In some cases, the settlers might have good access (high \( \bar{q} \)), and in other cases the access might be poor (low \( \bar{q} \)). What if the military could control the size of the fixed costs (\( e \)) to aiding the settlers? The military might construct bases closer to the frontier to decrease the fixed costs of intervention, or they could choose to construct bases far from the frontier in order to keep the fixed costs high.

The military will choose the location of the base depending on the distribution. If the distribution is heavily weighted toward defaults with very bad access, the military would like to place the base near the frontier in order to easily intervene. If the distribution is weighted towards poor to mediocre access, then the military should place the base far from the frontier in order to encourage the settlers to extend relations to the natives.

**Supervisor and Subordinates**

Consider a firm with a hierarchical structure. The supervisor oversees many subordinates, each of whom works on his own project. A subordinate is Player 1 and the supervisor is Player 2. There is an inherent tradeoff between the quality of the final outcome of the project and the time spent on the project. Map this quality-time tradeoff into a single dimension \( q \). The subordinates have a bliss point that differs from the supervisor. The supervisor might encourage the subordinates to spend less time in order to complete more projects (or vice-versa). Projects differ in the amount of time it requires to reach a certain level of quality. This translates to a distribution of \( \bar{q} \). For example, a low \( \bar{q} \) can be interpreted as a project that requires a lot of time to complete at a decent level of quality, whereas a high \( \bar{q} \) can be interpreted as a project that requires little time to complete at a decent level of quality.

The supervisor may assist the subordinates on their projects. If the supervisor has expertise that the subordinates lack, we would expect that her marginal costs to working on a project are much lower than the subordinates’ marginal costs. However, we may also expect that the supervisor has fixed costs to aiding a subordinate. It may take a large chunk of time for the supervisor to familiarize herself with the
necessary details of a project in order to be of any assistance.

When a project is very difficult ($\bar{q}$ is very low), it is best for the supervisor to aid the subordinate (gravity equilibrium). And if a project is of easy difficulty (high $\bar{q}$) the subordinate finishes the project solo (improvement equilibrium). However, if the project is of medium difficulty (intermediate $\bar{q}$), the subordinate may neglect necessary maintenance in order to allow the quality to become sufficiently low to elicit the aid of the supervisor (provocation equilibrium). If many projects fall under this category, the supervisor would wish to tie her own hands in order to deter this pareto-damaging behavior.

Suppose upper management can choose what kind of supervisor to hire and their payoff is equal to the supervisor’s payoff. Upper management can promote someone from within the organization who has detailed knowledge of the projects (low $e$), or they can hire an outsider who is competent but has little specific knowledge of any projects (high $e$). If there are many projects of intermediate difficulty, they might hire the outsider in order to discourage subordinates from sabotaging the projects. When the subordinates know that the supervisor is incapable of helping them, they choose to solve their problems on their own.

2.5 Psychological Interpretations of the Model

We model a dynamically-inconsistent agent with multiple selves over time. The first self has marginal costs to affect some change in the world while the second self has fixed costs. We explore two examples.

2.5.1 The Diet

Suppose we have a dynamically inconsistent person, George, who has present-biased preferences and is on a diet. On day one, he goes food shopping at the market, and on the second day he eats whatever he buys. On day one, George is Player 1 and on day two George is Player 2. George wants to lose weight but he has problems committing to his diet. George knows from past experience that if his meal does not include a dessert, his cravings overwhelm him and he will walk down the block to the local convenience store and buy his favorite candy bar. Ideally, he would like to only eat fresh vegetables and tofu for his meal the next day. However, he is aware that if he does not provide a dessert for himself, he will buy an unhealthy candy bar. We assume there is a fixed cost of time and effort associated with the walk down to the local store. If George buys a small square of chocolate, this will

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9In situations it may be plausible for these strategic problems to be contracted away. However, in other situations, the costs to drafting, monitoring, and enforcing the contract may be more costly than no contract. For example, think of the advisor-advisee relationship in academia where the advisor is the supervisor and the advisee is the worker. A formal contract in this situation seems implausible.
probably be sufficient to discourage himself from going to the local store the next day.

This is a placation equilibrium analogous to the regulation example and illustrated in Figure 2.2a. In this example, Player 1 has zero costs \( C_1(m_1) = 0 \). The example assumes that George only cares about calories and not about money, so choosing what calorie level to consume is costless to day-one George. But on day two there is a fixed cost to changing his meal plan.

2.5.2 Work and Leisure

I never get enough sleep. I stay up late at night, ‘cause I’m Night Guy. Night Guy wants to stay up late. What about getting up after five hours of sleep? Oh, that’s Morning Guy’s problem. That’s not my problem, I’m Night Guy. I stay up as late as I want. So you get up in the morning, you’re exhausted, groggy—ooh, I hate that Night Guy! See, Night Guy always screws Morning Guy. There’s nothing Morning Guy can do. The only thing Morning Guy can do is to try and oversleep often enough so that Day Guy looses his job, and Night Guy has no money to go out anymore.

- *Seinfeld*[^10]

George aims to have an ideal balance between work and leisure over the course of his Fridays and Saturdays. George only cares about the displeasure of effort on the day that he exerts the effort. However, there is an amount of work that George would like to accomplish. Map these preferences onto a single dimension of the amount of work that needs to be finished. George has the same bliss points on both days. A low \( \bar{q} \) means that more work needs to be accomplished to give George the same felicity as a higher \( \bar{q} \). George has a fixed cost to work on Saturday: he must go out of his way to drive to the office, whereas on a typical weekday this is psychologically a sunk cost. On Friday George is Player 1, and on Saturday George is Player 2.

Various exogenous circumstances affect how much effort is required to get the ideal level of work done. On tough weeks (low \( \bar{q} \)) George works a fair amount on Friday and goes to the office on Saturday (gravity equilibrium). He knows he will go to the office on Saturday and so he shades down his effort on Friday. On easy weeks (high \( \bar{q} \)), George works on Friday and takes Saturday off (improvement). On weeks of moderate difficulty (intermediate \( \bar{q} \)), George knows that he will have trouble committing to going to the office on Saturday due to the commute. He will only go if there is enough work to warrant it. So on Friday, rather than working very hard, George procrastinates and allows the work to pile up. George procrastinates on

Friday because he is present-biased and he knows it is the only way to force himself to work on Saturday. George needs the stress and pressure in order to overcome the fixed cost of commuting to the office on the weekend (provocation).

Over the course of George’s job tenure, the work load of the week will be drawn from a distribution. On some weeks George will have to put more effort to get the same felicity as he does in other weeks. Before he starts his job, George can decide how far from work he will live and consequently determine his fixed costs \( e \) to attending the office on the weekend. He must balance out the costs of commuting on Saturday (gravity and provocation) and the downward shading of work on Friday (gravity), against having a good commitment device to not work on Saturday (improvement). When the typical work load is moderate, George wants a commitment device to restrict work on Saturday, thereby encouraging himself to be more productive on Friday. Here, George may optimize by separating his work from his leisure. He may choose to live farther away from the office in order to keep fixed costs high.

This anecdote is not here to suggest that present-biased preferences and self-provocation is an important issue for housing choice. Rather, this is an explanation of why a person might want costly barriers between their work and their leisure (such as not having a home office). These costly barriers make the time at work more productive.

### 2.6 Multiple Simultaneous First Movers

In the previous sections, we modeled a whole industry and a group of revolutionaries as single agents. In this section we extend the model to allow for multiple first movers. An industry can now be modeled as an arbitrary number of firms, and a revolutionary vanguard can be modeled as an arbitrary number of factions.

One of the main concepts in this paper is that there are equilibria where the first mover moves a policy away from its bliss (i.e. placation and provocation equilibrium regimes). In this section we find that these equilibria remain robust when allowing for an arbitrary number of first movers. Furthermore, these equilibria actually become more favorable relative to the equilibria where first movers move the policy toward their bliss points. The intuition is that the latter equilibrium regimes, specifically gravity and improvement, exhibit free-riding effects while the former equilibrium regimes, specifically placation and provocation, involve a “provision point” that eliminates any free riding. With the provision point all the first movers are pivotal.

#### 2.6.1 Many Firms

In the self-regulation example, the industry was modeled as a single agent. Alternatively, we might imagine that the industry is composed of \( N \) identical firms,
with the same bliss point, that move simultaneously. How does the number of firms, \( N \), affect equilibrium behavior?

Suppose Hollywood is composed of \( N \) identical firms and the federal government is considering censorship. We construct the model to keep the total costs and benefits of the industry invariant to the number of studios. Let \( M = \sum_{k=1}^{N} m_k \) be the total movement of the policy, and \( M_{-i} = M - m_i \) the movement of the policy excluding that induced by player \( i \). The new utility functions are:

\[
U_i(m_i, m_{-i}) = \frac{1}{N} f_F(\bar{q} + M + m_G) - m_i * c_F \quad (2.3)
\]
\[
U_G(m_{-G}, m_G) = f_G(\bar{q} + M + m_G) - C_G(m_G) - \epsilon * \mathbf{1}\{m_G \neq 0\} \quad (2.4)
\]

Notice that \( \sum_{i=1}^{N} U_i(m_i, m_{-i}) \) is equal to the original utility function in (2.1) except now individual firm cost functions have constant marginal cost. This cost function scales to keep total industry costs invariant to \( N \). Even though constant marginal cost is a break from the basic model, it is considerably more tractable and the analysis does not differ substantially from a model with increasing marginal costs.

For the following analysis, we assume that marginal costs \( (c_F) \) are sufficiently small so that some action on the part of a firm is optimal when \( N = 1 \). Otherwise the firm will do nothing and the analysis ends there. Furthermore we assume \( C''_G(m_G) > 0 \).

In our original model, we compared the placation equilibrium to the gravity equilibrium. When the default policy is very large and outside the inaction zone, self-regulation is too costly, and so a gravity equilibrium results. If the default policy were rightward (outside) of the inaction zone but sufficiently low, the industry best responds by self-regulating, thereby deterring government entry. \textit{Ceteris paribus}, there exists a single value of \( \bar{q} \) for which both are equilibria as \( b \) changes this single point becomes a curve which is shown separating the gravity region from the placation region in Figure 2.1.

By introducing multiple firms in this extended model coordination becomes an issue. Equilibria are no longer generically unique. There may be a region where placation and gravity are equilibria depending on what firms coordinate on. As \( N \) increases, all equilibria become more robust. The intuition is that as the number of firms increases (and the total utility of the industry is kept constant), each firm has a smaller influence on the market. A small firm will incur exorbitant costs relative to its size if it deviates in an attempt to substantially change the final policy.

First, we consider the multiple player equivalent of the placation equilibrium. Total action by the first movers moves the default policy to the upper bound of the inaction zone, \( q_U \). Let us consider only a symmetric placation equilibrium. This is the placation equilibrium that can be induced from the largest interval of \( \bar{q} \). Asymmetric placation equilibria can only be induced from a strict subset of the interval. Define the region \([q_U, \hat{q}_P(N)]\) as the region of \( \bar{q} \) where a symmetric placation equilibrium can be coordinated upon.
Now consider the multiple player equivalent of the gravity equilibrium. Define the region \([\hat{q}_G(N), \infty)\) as the region of \(\hat{q}\) where a gravity equilibrium can be coordinated upon. We wish to know how \(\hat{q}_P(N)\) and \(\hat{q}_G(N)\) change as a function of \(N\). Notice that when there is one firm \(\hat{q}_P(1) = \hat{q}_G(1)\).

**Lemma 1** There exists an \(N'\) such that for or all \(N > N'\), the optimal firm action in a gravity equilibrium is \(m_i^*((N) = 0, \text{ and } \hat{q}_G(N) < \hat{q}_P(N)\).

Lemma 1 states that when the industry is composed of many firms, the region of \(\bar{q}\) that induces a placation equilibrium, \([q_U, \hat{q}_P]\), overlaps with the region of \(\hat{q}\) that induces a gravity equilibrium, \([\hat{q}_G, \infty)\). The intuition for this is stated above: as \(N\) increases and the total industry utility remains constant, each firm has less influence to affect the final policy. Both equilibrium regimes become more robust as \(N\) grows. The lemma also says that the optimal firm action in a gravity equilibrium drops down to zero. The intuition here is that there is a free-rider effect. When there are many firms, each firm will do less. When there are very many firms, each firm will do nothing.

Suppose \(\bar{q} \in [\hat{q}_G, \hat{q}_P]\) and the firms could communicate in advance and coordinate on a subgame perfect equilibrium. Which equilibrium regime would the firms choose? This leads us to our next proposition.

**Definition**
- \(U_{plac}(N; \bar{q}) \equiv \sum_{i=1}^{N} U_i(m_i, m_{-i}; \bar{q})\) in a symmetric placation equilibrium.
- \(U_{grav}(N; \bar{q}) \equiv \sum_{i=1}^{N} U_i(m_i, m_{-i}; \bar{q})\) in a gravity equilibrium.

**Proposition 11** Let \(N_1 < N_2\) and \(\bar{q} \in [q_U, \hat{q}_P(N_1)]\) then \(U_{plac}(N_1; \bar{q}) = U_{plac}(N_2; \bar{q})\). Furthermore, there exists an \(N'\) such that for all \(N > N'\) and \(\bar{q} \in [\hat{q}_G(1), \infty)\), \(U_{grav}(1; \bar{q}) > U_{grav}(N; \bar{q})\).

Proposition 11 states that equilibrium utility in a symmetric placation equilibrium is invariant to \(N\) while the equilibrium utility in a gravity equilibrium goes down when \(N\) is large. Specifically we can say that if \(N = 1\) and \(\bar{q} = \hat{q}_P(1) = \hat{q}_G(1)\), then as \(N\) increases, the symmetric placation equilibrium becomes preferred by all firms to the gravity equilibrium. Note also that the government does better in this equilibrium as well. The firms and the government would want coordination on a placation equilibrium.

The intuition is that there are positive externalities when a firm moves the policy in a gravity equilibrium. Moving the policy benefits the whole industry, but each firm only internalizes their individual returns. However, when we examine the placation equilibrium, the key is that all players are pivotal and thus if any one firm shirks, the policy will not reach the edge of the inaction zone and then the government will regulate. In the public goods literature we would call this target a provision point.
As $N$ increases, the firms internalize less of the benefit of their actions, and hence the positive externality problem worsens in the gravity equilibrium. Since the provision point in the placation equilibrium solves the positive externalities problem, as $N$ increases the industry has a larger incentive to coordinate on a placation strategy. This is Pareto Optimal since no firm can do better without the government or another firm doing worse, and the government can do no better without a firm doing worse.

In the special case where the marginal costs of the government are zero ($C_G(m_g) = 0$) this result no longer holds. In a gravity equilibrium the government moves the policy all the way to their bliss and the firms take no action. The resulting utility is invariant to $N$. Thus as $N$ increases, there is no effect on the region of $\bar{q}$ that results in a gravity equilibrium.

2.6.2 Split Revolutionary Factions

In this subsection we consider analysis similar to the preceding section. We re-examine the question, will the vanguard improve or revolt? However, this time we posit that there are $N$ identical revolutionary factions, each with the same bliss point and each of which can act independently. They have utility functions given by (2.3). They face a single populace that has a utility function given by (2.4). The default policy, $\bar{q}$ begins within the inaction zone and it is assumed without loss of generality that $b > \bar{q}$. We show that the region of $\bar{q}$ that results in both improvement and provocation equilibrium regimes expands as $N$ increases. We also show that the relative equilibrium utility of the factions in a symmetric provocation equilibrium increases relative to the equilibrium utility of an improvement equilibrium.

As in the previous section, when $N > 1$ the equilibrium requires coordination between the factions. There may be regions of the default space ($\bar{q}$) where both improvement and provocation are equilibria. Define $[q_L, \bar{q}_R(N)]$ as the region of $\bar{q}$ where a symmetric provocation by extremists is an equilibrium and define $[\hat{q}_{sl}(N), \hat{q}_{su}(N)]$ as the region of $\bar{q}$ where improvement is an equilibrium. Notice that $\hat{q}_R(1) = \hat{q}_{sl}(1)$.

**Lemma 2** The total optimal faction action in an improvement equilibrium is $N \ast m_i^*(N)$ is monotonically decreasing in $N$. There exists an $N'$ such that for all $N > N'$, $m_i^*(N) = 0$ and $\hat{q}_{sl}(N) < \hat{q}_R(N)$.

This means that as the vanguard splinters into more factions, the region of $\bar{q}$ that results in a symmetric provocation of extremists expands and the region of $\bar{q}$ that results in an improvement equilibrium expands as well. The intuition is the same as in the previous subsection. If we keep the total vanguard utility constant, splintering the vanguard into more factions gives each faction less power to affect the final outcome. When $N$ is large, $\bar{q}$ must be extreme for a single deviation to be profitable. Lemma 2 also states that the optimal faction action in an improvement equilibrium will reduce to zero.
Suppose \( \bar{q} \in [\hat{q}_d(N), \hat{q}_R(N)] \) and the firms could communicate in advance and coordinate on a subgame perfect equilibrium. Which equilibrium regime would the firms choose?

**Definition**
- \( U_{rev}(N; \bar{q}) \equiv \sum_{i=1}^{N} U_i(m_i, m_{-i}; \bar{q}) \) in a symmetric provocation by extremists equilibrium.
- \( U_{soup}(N; \bar{q}) \equiv \sum_{i=1}^{N} U_i(m_i, m_{-i}; \bar{q}) \) in an improvement equilibrium.

**Proposition 12** Let \( N_1 < N_2 \) and \( \bar{q} \in [q_L, \hat{q}_R(N_1)] \) then \( U_{rev}(N_1; \bar{q}) = U_{rev}(N_2; \bar{q}) \). Furthermore, there exists an \( N' \) such that for all \( N > N' \) and \( \bar{q} \in [\hat{q}_d(1), \hat{q}_{su}(1)] \), \( U_{soup}(1; \bar{q}) > U_{soup}(N; \bar{q}) \).

Proposition 12 states that equilibrium utility in a symmetric provocation by extremists equilibrium is constant while the utility in an improvement equilibrium decreases when \( N \) is large. Specifically, we can say that if \( N = 1 \) and \( \bar{q} = \hat{q}_R(1) = \hat{q}_d(1) \), then as \( N \) increases the provocation equilibrium becomes preferred by all factions. The factions would prefer to coordinate on revolution. However, this provocation equilibrium is not Pareto Optimal because a provocation equilibrium always makes the people (Player 2) worse off than in an improvement equilibrium.

The intuition for this result is that the edge of the inaction zone \( (q_L) \) provides a provision point for the factions in the provocation by extremists equilibrium. There is no free riding since all factions are pivotal. A single deviation would result in a failed provocation. Whereas in an improvement equilibrium, the free-riding problem gets worse as \( N \) increases.

In both this subsection and in the previous subsection (on multiple firms), the analysis is predicated on the assumption that there is perfect coordination. There also exist mixed strategy equilibria in which all \( N \) firms or factions mix their amount of effort. In this type of equilibrium, there may be a high probability that the effort exerted by the firms or factions falls short of the provision point; provocation rarely occurs and placation is insufficient to deter government regulation. If we assume a mixed equilibrium and compare the default space that maps into equilibria as a function of \( N \) we will not get the same result as in the previous analysis.

### 2.7 Conclusion

#### 2.7.1 Robustness

The model is admittedly simplistic. Naturally, one may wonder if the same logic works if we allow for some typical complications. In this subsection, we informally explain that the basic logic of placation and provocation will remain robust under multidimensional policies, repetition, and asymmetric information.

Suppose we modify the model such that the players have preferences over \( n \) policy dimensions rather than just a single dimension. This is a reasonable extension.
Perhaps Hollywood and the government care about violence and sex in different ways. The vanguard and the populous care about numerous political issues. In other words, the policy $q$ is now an $n$-dimensional vector. Assume that preferences remain single peaked, and that the marginal cost of moving the policy is a function of the distance moved. Observe that Player 2 will still have an inaction zone around its bliss point. The logic of placation and provocation follows naturally. If the policy, $\bar{q} + m_1$, is near Player 2’s bliss point on Player 2’s turn, then Player 2 will take no action, just as in our basic model. If Player 2’s preferences are symmetric (i.e. $f_2(q) = f_2(-q)$), then the inaction zone will take the form of an $n$-dimensional ellipsoid. If the default policy ($\bar{q}$) is outside the inaction zone but very near it, and Player 1’s bliss is fairly close, then it may be optimal for Player 1 to placate Player 2 by pushing the policy to the bound of the inaction zone. Likewise, if Player 1’s bliss point is very close to Player 2’s bliss point (small $b$), and Player 2 has low marginal costs relative to Player 1, and the default policy ($\bar{q}$) is inside but near the edge of the inaction zone, then it may be optimal for Player 1 to provoke Player 2 by pushing the policy to the edge of the inaction zone. The intuition remains the same.

What if the game is repeated? The vanguard and the populace may have a long strategic relationship after a revolution. Suppose the game is repeated an infinite number of times where future utility is exponentially discounted, and the equilibrium policy from the previous round ($q^{t-1} = \bar{q}^{t-1} + m_1^{t-1} + m_2^{t-1}$) becomes the default policy in the current round ($\bar{q}^t$). For games like this, it is usually helpful to restrict attention only to Markov strategies, where players condition their strategy solely on the state $\bar{q}^t$, and not on the history. Assume that a stage-game outcome that places the policy closer to Player $i$’s bliss in period $t$, makes Player $i$ better off in the infinitely repeated game. As long as this is true, Player 2’s fixed costs will create an inaction zone, (although it will not be the same inaction zone from the basic game). If Player 2 takes action, not only does it produce benefit in the current period but also in all subsequent periods. Nonetheless, for policies that are very close to Player 2’s bliss, the net-present value of taking action is insufficient to outweigh the fixed costs. Thus Player 2 will have an inaction zone. This allows the logic of placation and provocation to follow.

Finally, consider the case where Player 2 knows its fixed costs with certainty but Player 1 only knows the distribution from which the fixed cost ($e$) is drawn. Perhaps Hollywood knows that it is costly for the government to establish a censorship bureau but they do not know exactly how much. Thus the location of the bounds of the inaction zone will be uncertain for Player 1. This uncertainty does not fundamentally change anything. Imagine that under perfect information, placation is the equilibrium. With a little bit of noise, Player 1 will not know exactly where $q_U$ is, but if Player 1 pushes the policy far enough to the left, the probability that it is within the inaction zone will approach one quickly. Thus the qualitative aspect of placation remains, Player 1 pushes the policy toward Player 2’s bliss point in order to deter action. The policy will not hit exactly the upper bound but will be somewhere close, quite possibly it will be even closer to Player 2’s bliss. The
logic is exactly the same for the provocation equilibrium. If the variance of the fixed cost increases, since Player 1 is risk averse (due to concave felicity), then it is likely that Player 1 will push the policy even more toward Player 2’s bliss in the placation equilibrium. This has the effect of reducing Player 1’s expected utility. Thus, if the variance becomes too large, Player 1 may instead opt for a gravity equilibrium. From Player 2’s point of view, a little noise introduced to a placation may be good because it will cause Player 1 to overshoot, but too much noise may backfire inducing the gravity regime. On the other hand, too little noise introduced to a provocation regime may cause Player 1 to over-shoot the policy which is costly to Player 2 if Player 2 has marginal costs, but a lot of noise may be good because it induces the improvement regime.

A full characterization of these extensions remains for future research, but the basic logic of placation and provocation is robust.

2.7.2 Summary

A general phenomenon arises from the interaction between a first mover with marginal costs to affect a policy, and a second mover who has fixed costs to affect this policy. The purpose of this paper is to relate the dissimilar interactions of placation and provocation to each other. By providing a common simple platform for understanding many applications of placation and provocation, discoveries made in one application could more easily be transferred to the others.

The analysis has shown that the second mover may be made better off when preferences diverge from the first mover. The intuition is that when preferences are aligned the first mover may exacerbate the policy in order to free ride off of the second mover. When preferences are divergent, free riding will not result in an outcome that is favorable to the first mover, and thus the first mover instead chooses to directly improve the policy. The implication is that revolutions are less likely when the vanguard have extreme preferences relative to the population.

We show that the second mover may actually choose to increase its fixed costs in order to strategically dissuade the first player from aggravating the policy. There always exists a menu of fixed costs where the optimal fixed cost is the highest fixed cost.

Finally, the paper examines comparative statics as the number of first movers increases. As an industry breaks up into more and more firms, self-regulation becomes more likely than resistance. The intuition is that resistance has a free-rider effect. If one firm resists, other firms can free ride from the first firm’s effort. However, self-regulation requires a provision point to be reached which makes all firms pivotal. The same logic also implies that as revolutionaries break up into more and more factions, they collectively become more likely to incite violence and less likely to improve things directly.
Chapter 3

Leaks of Information

3.1 Introduction

3.1.1 Overview

Vice President Dick Cheney on *Meet the Press*, September 8, 2002:

There’s a story in *The New York Times* this morning, this is – and I want to attribute this to the *Times* – I don’t want to talk about, obviously specific intelligence sources – but it’s now public that in fact he [Saddam Hussein] has been seeking to acquire . . . the kinds of tubes that are necessary to build a centrifuge, and the centrifuge is required to take low grade uranium and enhance it into highly enriched uranium which is what you have to have in order to build a bomb.


*Bush administration officials* say the quest for thousands of high-strength aluminum tubes is one of several signs that Mr. Hussein is seeking to revamp and accelerate Iraq’s nuclear weapons program.

Officials say the aluminum tubes were intended as casing for rotors in centrifuges, which are one means of producing highly enriched uranium.

In the quote above Dick Cheney cites *The New York Times* article as supporting evidence, when in fact it was his own office that was the original source for the *The New York Times* article.\(^1\) Often there are news leaks from government officials to the press. Are these leaks coming from idealistic analysts who believe the public has a right to be informed? Or are these leaks fabricated by politicians in order to promote a political agenda?

\(^1\)Scooter Libby leaked the aluminum tube story to Judith Miller according to “Hardball with Chris Matthews” aired on November 1, 2005, available at: http://www.msnbc.msn.com/id/9896575/. The page was referenced on May 1, 2010.
This paper models leaks of information. We think of a leak as a truthful costless non-verifiable message from an informed party to an uninformed party. There are two interpretations to a leak. The first interpretation is that the informed party has imperfect secret-keeping technology. In other words, there is a probability that accurate information is exogenously sent to the receiver. The second interpretation is that there is some probability that the sender's preferences are such that he chooses to send truthful information. In either case, when the sender does not leak, he can instead send a “decoy”. Decoys are non-verifiable messages that are indistinguishable from exogenous leaks of information. Under both interpretations, there is some probability that the sender sends accurate messages called leaks, and some probability that the sender sends completely false messages called decoys.

The implicit assumption in the literature, that secret-keeping technology is perfect, is strong, and in many real-world situations where one finds secrets one finds leaks. But the presence of leaks does more than just reduce the information differential between the two parties. Rather, the presence of leaks allows the informed party to create decoys that will influence the sender's behavior. Only with leaks are decoys believed.

This interaction where information leaks from the informed party to the uninformed is widespread. Political leaks, as exemplified by the quotes, are a particularly salient and socially important example. However, the importance of leaks extends beyond the political arena. Leaks from the Federal Reserve may cause macroeconomic market reactions which gives the Fed Chairmen incentives to spread rumors. User-submitted content online, for such websites as the encyclopedia Wikipedia.com, and online review websites such as Yelp.com, leak product quality to the consumer. The non-verifiable nature of these reviews allows for shilling – a form of decoy. Likewise, “whisper numbers”, the unofficial forecasts of stock prices, are leaks about firm earnings. On occasion, individuals have been caught trying to manipulate whisper numbers for private financial gain. In IO, industry rumors inform competitors about a firm’s R&D progress and this allows the firm to start rumors about their own products in order to deter entry. Warfare is filled with examples about critical leaks leading to victory and successful and unsuccessful attempts to mislead the enemy with the use of decoys. Finally, as Robert [Frank (1988)] suggests, body language and facial expressions leak to others our intentions. That ability to credibly communicate our emotions and convictions allows con artists and actors to deceive.

In the model there are two actors: an informed sender (S) and an uninformed receiver (R). We assume the underlying preferences of S are strictly increasing in R's belief about the state. A politician wants the voters to believe that she is of the highest ability to maximize her vote share, a seller wants the buyer to believe that the quality of a good is the highest so that the buyer pays the maximum price, and a monopolist wants an entrant to believe that the monopolist has total market control to deter entry. Some proportion of the time S leaks true information about the state and some proportion of the time S is strategic and has the technology to craft a decoy that is indistinguishable from the exogenous leak.
In equilibrium, a strategic sender will choose a mixed strategy where she sends messages in the upper tail of the distribution. She does not always report the highest state because that would make the message less credible. The strategic sender has a trade-off between strong claims and credibility. In equilibrium the receiver will always believe low messages but will be skeptical about all messages above a threshold and will be completely unresponsive to increases in the message above this threshold. I show that decreasing the proportion of strategic senders and increasing the rate of leaks changes the distribution of R’s beliefs, specifically it increases the variance. Although the proportion of strategic senders and the leak rate have no effect on the mean of R’s beliefs, affecting the distribution of beliefs can have considerable impact on behavior depending on the economic environment. Additionally, I explore the case when leaks are caused by senders who have a preference for telling the truth whom I refer to as whistleblowers. First, I establish that when S has the same preference as R, S will be a whistleblower. Then I show that leaks by whistleblowers have a multiplier effect in equilibrium. An additional leak by a whistleblower will encourage other whistleblowers to leak.

In the next subsection I briefly review the literature. In Section 3.2 I develop the model and solve it. In Section 3.3 I describe potential applications, and in Section 3.4 I conclude with extensions for future research.

### 3.1.2 Literature Review

This paper intersects two literatures: communication with costless non-verifiable information – also known as cheap talk, and the small but growing literature on deception.

In a seminal paper, Crawford and Sobel (1982) (hereafter ‘CS’) begin the cheap talk literature with a model of costless communication between an informed sender and an uninformed receiver. Both players have single-peaked preferences over a single dimension. The sender knows with certainty the state and can send a costless message to the receiver that has no direct effect on either player’s utility. The receiver takes an action that affects both player’s utility. Whenever the preferences differ, full revelation is not possible. CS characterize the set of equilibria and show that they take the form of “partition equilibria”. As preferences diverge, the amount of information transmitted decreases and when preferences are sufficiently different, no information is communicated – the only equilibria are “babble equilibria”.

The CS model is incredibly rich and hence there have been numerous extensions to the original model. These extensions include but are not limited to multiple senders, multi-dimensional preferences, multiple audiences, multiple rounds, and costly and costless communication, (Krishna and Morgan, 2001; Battaglini, 2002, 2004; Farrell and Gibbons, 1989; Stocken, 2000; Krishna and Morgan, 2004; Austen-Smith and Banks, 2000; Kartik, 2007). Our model bears some similarities to Ottoviani (2000), and Kartik et al. (2007) who allow for some receivers to be “naïve” in the sense that they always believe the message that they receive. When the state space is unbounded above, they prove that there is full revelation. In comparison, I
allow for some senders to be “naïve” in the sense that they always tell the truth.

There has also been a whole line of research exploring the case when the sender’s preferences are unknown. These extensions come closer to our model. The literature begins with Sobel (1985) who models a sender who over many periods sends information to the receiver. The sender may have preferences that are perfectly aligned with the receiver or they may be exactly opposite. Benabou and Laroque (1992) extend this model to noisy signals and many receivers and apply it to insider trading. Unlike the previous two papers, in Morris (2001) even senders who have the same preferences as the receiver lie in order to gain a better reputation and thus have greater influence in future periods. In all three models reputation dynamics are an important focus of the model and the results. Morgan and Stocken (2003) also develop a model in which the bias of the sender is unknown but rather than exploring reputation and dynamics, they look at the responsiveness of the receiver to the messages and apply this to the context of analyst stock reports. A different approach is taken by Dimitrakas and Sarafidis (2005) who characterize the equilibria when the bias of the agent is drawn from a smooth distribution rather than when bias is one of two types. The research of Li and Madarász (2008) specifically looks at the role of disclosing the bias of the senders on welfare. They show that under certain conditions, disclosure reduces the effectiveness of communication.

Our model differs from all the CS inspired models mentioned above in that the sender’s preferences in our model are strictly increasing in the receiver’s action. Thus the model is similar to CS when bias is maximal. In CS, when bias is maximal, the only equilibrium is a babble equilibrium. In contrast, in our model with exogenous leaks of information, strategic senders send messages that influence the receiver’s behavior in equilibrium because a fraction of the senders “leak” the true state. Our model comes closest to Morgan and Stocken (2003) but I look at a case they do not consider, that is when the biased sender has very large bias. Later in Section 3.2.4 we show that strategic senders with the same preferences as the receiver tell the truth.

The second literature that this paper fits into is the small but growing research on deception. In this literature, contrary to models based on CS, the game played is zero sum. In Hendricks and McAfee (2004) the sender and receiver play a zero-sum game similar to “matching pennies”. They model a feint as a costly investment in one of the two actions. The signal sent is information about which action the first player invested in. Inspired by this paper but taking a very different approach Crawford (2003) models the signals as cheap talk and the receiver as probabilistically being one of several different boundedly rational types. Ettinger and Jehiel (2004) also model the receiver as being boundedly rational. Contrary to Hendricks and McAfee, we assume messages are free, and contrary to Ettinger and Jehiel and Crawford we assume receivers are rational. Furthermore, our game is not zero sum so our applications are very different.

Finally, there are a few other papers that relate to ours. Solan and Yariv (2004) introduce games of espionage. They allow one player to spy on the other and learn which action the other player chooses. In their paper spying reveals actions,
whereas in this paper a leak reveals the state. To our knowledge, the only paper that explicitly mentions “leaks” is Matsui (1989). He looks at the ability of a player to revise his strategy after observing the other player’s action in an infinitely repeated game. This is also quite different in the sense that our model is a one-shot game and our sender’s action is a message that does not directly affect payoffs.

3.2 The Model

3.2.1 Setup

There are two players, the Sender (S) and the Receiver (R). There exists a state $\theta \in [0,1]$ that is randomly distributed according to the CDF, $F_{\theta}(\cdot)$. The PDF will be denoted by $f_{\theta}(\cdot)$ and is assumed to be continuous and differentiable. S knows the realization of $\theta$ but R has no information.

S cannot convey information to R in any verifiable manner. There are two types of senders, simple and strategic. The simple types, who occur with probability $\alpha$, are unable to send any messages. However, their secret-keeping technology is imperfect. There is a chance $\epsilon$ that the true value of $\theta$ is leaked from S to R. In other words, R receives a message with the true value of $\theta$. We refer to this as an exogenous leak. Strategic types, on the other hand, have the technology to construct strategic leaks (decoys). These strategic leaks are messages sent from S to R that are indistinguishable from an exogenous leak. It is assumed that this message is costless and can be chosen from the interval $[0,1]$. A strategic S may also choose to send no message, $m = \emptyset$.

S’s utility, $U_S(y)$ is assumed to be strictly increasing in R’s action, $y$, and not directly a function of any messages S sends or even the state $\theta$. It is assumed that R’s best-response function is simply $y = g(E[\theta|m])$ where $g(\cdot)$ is a differentiable strictly increasing function. For example, the utility function $U_R(y;\theta) = -(y - \theta)$ is a function that is maximal when $y = \theta$.

The timing of the game is as follows: (1) Nature determines if the sender is simple with probability $\alpha$ or strategic with probability $(1 - \alpha)$; (2) Nature determines the realization of $\theta$; (3) S observes $\theta$ and if S is simple there is a probability $\epsilon$ of an exogenous leak but if S is strategic S chooses a message $m$ to send to R that is indistinguishable from an exogenous leak; (4) R takes action $y$.

3.2.2 Equilibrium Solution

Simple senders are automata since they have no decision to make in this basic game. We only need to consider the behavior of the strategic types who are assumed to have the technology to create decoys. S’s best response is to send a message that induces the highest action from R. As mentioned before, R’s best response function $BR_R(m)$ is a strictly increasing in the conditional expectation $E[\theta|m]$. Consequently, S will send the message that induces the highest $E[\theta|m]$. Notice that
since S’s preferences are independent of the state, a strategic S’s equilibrium strategy may be independent of the state (although it need not be). For the moment, let us assume that S’s strategy is independent of $\theta$.

First observe that since the distribution of $\theta$ is continuous and atomless, if S sends any message $m$ with probability one, R will know that any such message must be coming from a strategic S and thus $E[\theta|m] = E[\theta]$. For the same reason, if S mixes over messages and sends according to the distribution $q(m)$ with atoms, R can infer that any message sent with positive measure emanates from a strategic S. Therefore we look for a solution in which strategic S sends decoys according to an atomless distribution given by the PDF $q(m)$. R’s conditional expectation is given below by Baye’s Rule:

$$E[\theta|m] = \frac{\alpha \epsilon f_\theta(m)}{\alpha \epsilon f_\theta(m) + (1 - \alpha)q(m)} m + \frac{(1 - \alpha)q(m)}{\alpha \epsilon f_\theta(m) + (1 - \alpha)q(m)} E[\theta] \quad (3.1)$$

Since S is mixing, it must be the case that S’s utility is constant for any message in the support of $q(m)$. Also, for this to be a best response of S, all messages in the support of $q(m)$ must maximize $E[\theta|m]$. Thus we have the following two conditions:

1. Let $M$ be the support of $q(m)$. Then for any $m' \in M$, $m' \in \text{argmax}_m E[\theta|m]$.  
2. For all $m \in M$, $E[\theta|m] = k$ where the $k$ is a constant.

Using Condition 2, we solve for $q(m)$.

$$q(m) = \frac{(m - k)\alpha \epsilon f_\theta(m)}{(k - E[\theta])(1 - \alpha)} \quad (3.2)$$

In order for $q(m)$ to be a proper PDF, it must integrate to one over the support.

$$\int_M \frac{(m - k)\alpha \epsilon f_\theta(m)}{(k - E[\theta])(1 - \alpha)} dm = 1 \quad (3.3)$$

Now we must determine the support. We claim that the support must be the interval $M = [m, 1]$. First notice that the upper bound of the support must be the upper bound of the distribution of 1. Suppose that it is not. Then whenever R gets the non-verifiable $m = 1$ she can assume that it is an exogenous leak and hence the true value is $\theta = 1$. But if R always believes the non-verifiable $m = 1$ is an exogenous leak, then S can profitably deviate by always sending $m = 1$. This is a contradiction.

Now we must prove that there cannot be any gaps in the support. A gap is an interval where S sends no messages but does send messages above and below the gap. Suppose there is a gap. Whenever R receives a non-verifiable message from this gap then R will believe that it is the truth. Since S is sending a message below the
gap this can only induce an expectation that is a weighted average with the message and the unconditional expectation. If the unconditional expectation is also below the gap then the induced expectation is below the gap and S can profitably deviate by sending a message in the gap because the gap is higher. If the unconditional expectation is above the gap then S can only do worse by sending a message below the unconditional expectation rather than sending no non-verifiable at all so this never happens. Thus there are no gaps.

We are left with an interval as the support and we must compute the lower bound of this interval. Notice that it must be the case that \( k \geq E[\theta] \), otherwise strategic types would send no message at all. This along with equation 3.2 implies that \( m \geq k \); otherwise the PDF of S’s action is negative. Finally, notice that \( k \geq m \). Assume otherwise, this implies that there exists a \( \delta \) such that \( k < m - \delta \). S can profitably deviate by sending \( m - \delta \). R will interpret this as an exogenous leak, and since it is higher than \( k \), which is the conditional expectation, this action is preferred by S. Thus \( k = m \). We now have two equations (\( k = m \) and the PDF must integrate to one) and two unknowns (\( k \) and \( m \)).

**Lemma 3** In an equilibrium where the messages sent by strategic senders are independent of the state, strategic senders will mix over the support \([m, 1]\) where \( m \) is uniquely determined by

\[
\int_m^1 \frac{(m - m)\alpha f_\theta(m)}{(m - E[\theta])(1 - \alpha)} dm = 1
\]

**Proof** The explanation is given above. The uniqueness of \( m \) comes from the fact that the LHS of the expression is strictly decreasing in \( m \), thus there can only be one intersection.

Now we characterize a property of any mixed strategy equilibrium – that the utility from each action in the support produces the same utility.

**Proposition 13** In an equilibrium where the messages sent by strategic senders are independent of the state, strategic senders will send messages from the support \([m, 1]\) with PDF given by

\[
q(m) = \frac{(m - m)\alpha f_\theta(m)}{(m - E[\theta])(1 - \alpha)}.
\]

The intuition behind Proposition 13 is intuitive. The strategic sender faces the tradeoff between exaggeration and credibility. If S puts too much weight on sending high messages then R will find high messages to be less credible. When S sends lower messages, they are more credible but the exaggeration is less. In equilibrium, S mixes in such a way that R’s expectation is constant for all messages that a strategic S sends. As an example, suppose \( \theta \sim U[0, 1] \) then \( m = \frac{1 - \alpha + \sqrt{1 + \alpha(\epsilon - 2) + \alpha^2(1 - \epsilon)}}{\alpha} \). This implies that \( q(m) \) is linear in \( m \) with \( q(m) = 0 \) and rising to \( q(1) = 2 + \frac{2\sqrt{(1 - \alpha)(\alpha(\epsilon - 1) + 1)}}{1 - \alpha} \).
Technically the equilibrium is not unique. There are also an infinite number of pure-strategy equilibria. S could condition her strategy on the state such that R’s expectation will be constant for all messages that a strategic S sends. Thus the support and the expectation will be the same as in Proposition 13, even though the strategy will be pure. In these pure-strategy equilibria, the distribution of strategic messages need not be the same as in the mixed equilibrium. For example, if the pure strategy entails sending the messages near $m = 1$ only when $\theta$ is very low, then less of these high messages will be sent than in the state-invariant strategy, because R knows that only very low strategic types or simple senders send these high messages. In the state-invariant strategy, all strategic types are sending messages over the full support. Additionally, combinations of the pure-strategy equilibria could be combined to produce mixed-strategy equilibria. The point is, there are many equilibria but R’s behavior is unchanged in all of them.

Now we summarize R’s beliefs as a function of the message in equilibrium. Since all messages $m < m$ are sent only by simple S, in expectation beliefs must be $E[\theta| m, m < m] = m$. In order for S to mix it must be the case that for $m \leq m \leq 1$, in which the message may be either a leak or a decoy, $E[\theta| m, m \geq m] = m$. So R’s beliefs are accurate whenever he receives a message $m < m$.

3.2.3 Comparative Statics

From Lemma 3, we can directly see how the support of the messages changes as a function of the population ($\alpha$), the leakiness of the technology ($\epsilon$), and the distribution of the state.

We interpret changes in $\alpha$ in many ways. One interpretation is that $\alpha$ is the fraction of the population that does not have the technology to craft decoys. Another interpretation is that $\alpha$ is the proportion of time that S does not notice that she leaks information. A third interpretation is that $\alpha$ is the fraction of the population that has the same preferences as the receiver. We can see that as a greater portion of the population becomes truth-telling ($\alpha$ increases) the support of the messages chosen by a strategic sender becomes more and more concentrated amongst the upper tail of the distribution. Since less decoys will be sent, those that can send decoys choose to exaggerate more. Intuitively, when there are less liars, their lies are bigger. Clearly as the portion of strategic senders decreases, those that are strategic do better. When almost the whole population can create decoys, strategic senders send messages from the unconditional mean of $\theta$ and above and do only slightly better than truth-tellers in expectation. When there are no truth-tellers then the expression collapses. The only equilibria are babbling where S sends any message and R ignores all messages and expects $\theta$ to be the unconditional mean. No information is communicated.

The leakiness or transparency $\epsilon$, has a similar effect on equilibrium behavior. As $\epsilon$ increases there are more leaks. As $\epsilon$ goes to zero no truthful information is sent to R and hence decoys have no effect on R’s beliefs. Again, all equilibria are babbling. Therefore the presence of leaks is necessary for decoys. As $\epsilon$ increases,
decays become more concentrated in the upper tail of the support of $\theta$. Strategic senders exaggerate more since there are more truth-tellers.

The distribution of the state also has an effect on the equilibrium. Greater mass on a state within the support causes strategic senders to send more messages indicating that state. Intuitively, strategic senders are trying to mimic exogenous leaks. I conjecture that a stochastically dominant distribution of $\theta$ will shrink the support and concentrate messages in the upper tail. When there are more high types, there will be more high type decoys.

### 3.2.4 Strategic Whistleblowers

Now we consider the possibility that both types of senders are strategic. The whistleblowers compose $\alpha$ of the distribution. The remaining $1 - \alpha$ of the senders are strategic as before. Suppose whistleblowers have the same preferences as R. The quote at the beginning of the paper motivates this formulation. The news reports information from an anonymous government official (S) about some important issue. The voter (R) does not know if the government official is an honest whistleblower who has a desire to inform the voter, or if the official is a politician who has ulterior motives and wishes to persuade the voter.

First we show that whistleblowers will choose to tell the truth and that the equilibria conditions we found in Section 3.2.2 still hold. The proof is simple. Suppose that $\theta \in [0, m]$. Then from Section 3.2.2, R’s expectation is given by $E[\theta|m] = m$. Since R believes all messages in this range, a whistleblower can do no better than to report the truth. Now suppose that $\theta \in [m, 1]$. Then from Section 3.2.2 we know that $E[\theta|m] = m$. If the whistleblower wants to minimize the error (i.e. $|\theta - E[\theta|m]|$), she can do no better than tell the truth. Any messages $m \in [m, 1]$ will induce the same belief, but messages that are lower will induce lower beliefs thereby increasing the error.

We can allow greater realism by assuming that whistleblowers face a cost to sending a message. This cost may either be a subsequent punishment or costs to acquiring the information. We assume the cost is incurred before the state is known and that only the whistleblower knows the cost. Assume that the cost is random but drawn from the distribution with support $[c, \bar{c}]$, where $c \geq 0$, CDF $F_c(\cdot)$ and with smooth differentiable PDF $f_c(\cdot)$, all of which are common knowledge. Thus R knows the distribution of costs but does not know whether she faces a strategic type or a whistleblower, nor the cost to the whistleblower for leaking. Now we can endogenize the leak rate $\epsilon$ by characterizing the fraction of whistleblowers who are willing to send a message. Let the whistleblower’s utility function be additively separable in the cost. For example, with squared loss utility, $U_{wb}(y; \theta, c) = -(y - \theta)^2 - c$. The whistleblower will only send a message if the cost is sufficiently low. The inequality below characterizes this condition.

\[ \text{This makes the acquisition-costs interpretation fit better. If the cost is incurred after the state is known, more like an expected punishment, the endogenous leak rate becomes a function of the state, } \epsilon(\theta). \]
\[
F_\theta(m) \cdot \frac{m}{0} U_{wb}(\theta; \theta)f_\theta(\theta)d\theta + (1 - F_\theta(m)) \cdot \frac{1}{m} \int U_{wb}(m; \theta)f_\theta(\theta)d\theta \\
- \int U_{wb}(E[\theta]; \theta)f_\theta(\theta)d\theta \geq c
\]  

The first term is the probability of getting a state below \( m \) times the expected utility conditional on this state. This is where R will believe any message. The second term is the probability of getting a state above \( m \) times the expected utility conditional on this state. This is where all messages induce the same expectation. The third term is the whistleblower’s expected utility if he does not send a message.

Now we wish to determine the equilibrium leak rate. Any whistleblower who satisfies (3.4) will send the message. Thus the equilibrium leak rate will be given by,

\[
\epsilon = \int_{\xi}^{x} f_c(c)dc
\]  

where \( x \) is given by

\[
x = F_\theta(m) \cdot \frac{m}{0} U_{wb}(\theta; \theta)f_\theta(\theta)d\theta + (1 - F_\theta(m)) \cdot \frac{1}{m} \int U_{wb}(m; \theta)f_\theta(\theta)d\theta \\
- \int U_{wb}(E[\theta]; \theta)f_\theta(\theta)d\theta
\]  

Keep in mind that \( m \) is a strictly increasing function of \( \epsilon \). This further implies that \( x(\epsilon) \) is a strictly increasing function as well. Thus equation (3.5) indicates that there is a multiplier effect to whistle blowing. Suppose costs go down for some marginal whistleblowers. This shifts the RHS of equation (3.5) up and since the LHS is an increasing function in epsilon there is a multiplier effect. The intuition is that as costs go down for a few whistleblowers, they will send more information, and this expands the region of the message space \([0, m]\), where \( S \) is honest. This increases the whistleblower’s expected utility to sending a message, and thus the marginal whistleblowers will send messages.

The same logic applies to increases in \( \alpha \). As whistleblowers become more common, the RHS of Equation (3.5) shifts up. More whistleblowers increases the range and the frequency of truthful messages. This increases R’s credulity thereby increasing the whistleblower’s expected benefit to sending a message.
3.3 Applications

The model has numerous applications. We do not mean for the model to be a complete description of each of these applications, but the model captures some key intuition in all examples.

3.3.1 Political Elections

Suppose there is a political candidate who wants to be re-elected. The voters care only about competency ($\theta$). The greater the voters $E[\theta]$ of the candidate’s quality, the higher the probability that she will be re-elected. Suppose there is some major ongoing project where success will indicate the competency of the politician and the politician has private information regarding the progress of the project. An idealistic whistleblower who works in the bureaucracy under the politician might choose to anonymously share this information with the people. This would be an exogenous leak ($\epsilon$).

But because exogenous leaks exist, the politician can have his aide anonymously decoy how well the project is going. The question for the voter ($R$) is, does the information come from a whistleblower or the Vice President’s aide?

Here, the interpretation is that simple politicians are unable to prevent whistleblowers in their staff from leaking information to the public. Strategic politicians can actually create leaks of information to the public. A question for future research is to ask what is the optimal level of transparency $\epsilon$ for the politician? High transparency will result in more leaks but also result in more effective decoys.

3.3.2 User-Submitted Content

More recently there appears to be a growing trend in people’s reliance on user-submitted content. This category is broad and is motivated by three specific examples: online product reviews, wikipedia articles, and whisper numbers. In all cases, honest users submit information to websites because they have a sincere desire to inform the world. However, there are special interests who wish to send biased submissions in order to bias upward the receiver’s belief. This is a model of shills.

Online Product Reviews

Often after purchasing an item online, customers can review the transaction. There are also whole websites dedicated to accumulating ratings of various products and services. Websites like Yelp! catalogue numerous reviews of small businesses. The vast majority of reviewers probably only have honest intentions. However, the seller of the good or service has a strong incentive to fabricate and post positive reviews. These shill reviews can be made to look authentic.

To illustrate, restaurants sometimes give free meals to frequent Yelp reviewers.\(^3\)

\(^3\)Quoted from McLaughlin, Katy. “The Price of a Four-Star Rating”, The Wall Street Journal,
How did Dine garner such favorable reviews? One thing that probably didn’t hurt: It fed many of the reviewers free. Last August, Dine spent about $1,500 on an event for members of Yelp, a Web site where consumers post reviews and rate restaurants. The nearly 100 members were treated to an open bar, duck roulade appetizers and red velvet cupcakes for dessert. As a bonus, they all received certificates for discounts on subsequent meals. The result: a torrent of favorable reviews on Yelp. Most reviewers mentioned that they attended a Yelp event, though few highlighted that the food and drink was free.

As it becomes more costly to shill reviews ($\alpha$ increases) the information becomes more reliable but one still expects positive bias in reporting. As the costs of reviewing decrease ($\epsilon$ increases), the information becomes more accurate.

Wikipedia

A recent computer program revealed that many wikipedia articles were edited by special interest groups. According to BBC News\(^4\) “Earlier this year, Microsoft was revealed to have offered money to trawl through entries about document standards it and other companies employ.” The IPs affiliated with the CIA have edited numerous entries. Diebold was caught eliminating a passage that criticized their voting machines.

Stocks and Whisper Forecasts

Similar to Shin\(^{1994}\), suppose there is a manager of a firm that is risk averse and owns stock in the company (S). His incentive is to make the company look as profitable as possible. That way he can sell his stock and maximize profit. R is an investor who offers a price. R wishes to choose the price closest to the value of $\theta$ as possible. If he prices too high he will own too much stock and if he prices too low, he will own too little stock.

Analysts post anonymous predictions on the value of a stock online called whisper forecasts. According to Bagnoli et al.\(^{1999}\), whisper forecasts are actually more accurate than First Call analyst forecasts. Some of these whispering analysts might have private information about $\theta$. When these analysts whisper it is equivalent to a leak. However, it might also be the case that the analysts have a private incentive to increase the beliefs about the $\theta$ of the company because they own stock. This is in the same vein as Benabou and Laroque\(^{1992}\). For these analysts with ulterior motives, the whispers are decoys. Since whispers are anonymous the leaks and decoys are indistinguishable from each other.

October 6, 2007. Available at http://online.wsj.com/article/SB119162341176250617.html. The page was referenced on May 1, 2010.


The head of the organic food giant Whole Foods has been caught touting his company and trashing a competitor in anonymous writings on the Internet. Using a pseudonym, Whole Foods CEO John Mackey said a competitor’s stock price was too high and that it had no future. Now he’s trying to buy the company.

### 3.3.3 Leaks and Firms

There are two firms racing to release a new product to capture a niche market. This is a war of attrition. The firm wants to signal to the competitor that she is very far ahead in the race in order to deter entry. In practice, there is a phenomenon in the literature called vaporware in which a company (usually software) announces the release date for a product and then never delivers the good or severely delays it.

There may also be industry rumors that can serve to deter entry. The rumors may be exogenous leaks started by careless employees or perhaps they are planted by the firm to deter competitors. The leak rate ($\epsilon$) may be a function of the firm’s trade-secret contracts or its secrecy. A firm might prefer to have more lax secrecy (higher $\epsilon$) in order to have the ability to send decoys to deter competitors.

### 3.3.4 Warfare

Much of the small literature on deception is modeled in the context of war. Armies are incapable of completely blocking all information about their status to the enemy. Inevitably there will be some leaks of information. The opponent has spies and surveillance. Some information about the army will leak to the enemy. Given this fact, the military uses these leaks to create decoys. The military may want to signal strength ($\theta$) in order to deter the enemy like in a hawk-dove game. Greater strength results in greater deterrence\footnote{We could also model the militaries’ preferences like matching pennies. $S$ wants $R$ to believe she is weak when she is strong and wants $R$ to believe she is strong when she is weak. This requires a modification to the model but I conjecture that an analogous equilibrium would result.}

### 3.3.5 Evolution of Emotional Expression

Frank\footnote{Frank, 1988} presents a fascinating explanation of the role of emotions. He suggests that emotions are commitment devices. Humans engage in far more cooperation than game theory predicts. This is because feelings such as vengeance, honor, reciprocity, etc. keep us committed to certain courses of action that encourage pro-social behavior. Moreover, we signal the emotions we are experiencing via
facial expressions and body language that is mostly involuntary. Most people have no ability to fake a blush and fake smiles are easy to spot.

Thus expressions can be thought as leaks of our emotional state. There are some talented actors and con artists who are quite capable of faking these expressions (decoys). What is the optimal ability to lie ($\alpha$) if people can slowly learn each other’s ability through repeated interaction? A very low $\alpha$ makes one the “boy who cried wolf”. Such a person is such a good liar that they soon are never believed because they can deceive so well. Someone who is very honest, a very high $\alpha$, cannot lie well but everyone knows it so at least such a person will always be trusted.

### 3.4 Conclusion

In this paper I presented a simple game-theoretic model and I characterized the equilibrium. The model has potentially wide applicability to many areas of economics.

In the future there are several extensions I would like to explore. The first is to endogenize the leak rate $\epsilon$ by making it a choice variable of $S$ before he knows the state. The motivation of this extension would be to determine the effect of transparency on an incumbent’s re-election. I conjecture that when the challenger is expected to be relatively strong, the incumbent will want high transparency to increase the variance of the voter’s beliefs. When the challenger is expected to be relatively weak, the incumbent will want low transparency to minimize the variance of the voter’s beliefs.

Other extensions to explore include looking at the effect of certifiable information. What happens if a strategic $S$ has a limited ability to craft a decoy? Another interesting question would be to ask how cheap talk interacts with the leak channel. A politician may make cheap announcements but lose utility the more he lies (due to punishment or loss of credibility) and the presence of a leak channel may affect what he announces. Furthermore, we can explore what would happen when information leaks from more than one outlet. A journalist may question several officials in different parts of the administration. How should a strategic official react when he knows that other officials may also leak or send decoys?
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Appendix A

Appendix

A.1 Chapter 1 Proofs

Proof of Proposition 1

Define $x^c = \arg\max_{x \in X} \sum_{i=1}^{I} w_i(x; b_1)$. Remember, $w_i(x; b_1) < u_p$ for all $i, x$. First we show $w_i(\bar{x}^*(b_0); b_1) = w_i(x^*; b_1)$. We backwards induct to the decision node for choosing $\bar{x}$. When $\bar{x} = x^*$, then $V_0(x^*, \sigma; x, b_0) = \sum_{i=1}^{I} w_i(x^*; b_0)$. When $\bar{x} \neq x^*$, then for dimensions $j \in J$ that have $w_j(\bar{x}) \neq w_j(x^*)$ and dimensions $k \in K$ that have $w_k(\bar{x}) = w_k(x^*)$,

$$V_0(\bar{x}, \sigma; x, b_0) = \sum_{j \in J} (1 + \alpha)w_j(x^*; b_0) - \alpha u_p + \sum_{k \in K} w_k(x^*; b_0).$$

Since $w_j(x^*; b_0) < u_p$, choosing $\bar{x}^*(b_0)$ such that $w_i(\bar{x}^*(b_0); b_0) = w_i(x^*(b_0, b_0); b_0)$ for all $i$, is preferred. No attention shifts and so $V_0(\bar{x}^*(b_0), \sigma; x, b_0) = \sum_{i=1}^{I} w_i(x^*; b_0)$ and by definition the maximizer is $x^c$.

Proof of Proposition 2

Let $\bar{x}_1^*$ and $x_1^*$ denote optimal default and actions conditional on receiving the information $\sigma$ and let $\bar{x}_2^*$ and $x_2^*$ be optimal default and actions conditional on receiving no information $\sigma_n$.

$$V_0(\bar{x}_1^*, \sigma; x_1^*, b_0) - V_0(\bar{x}_2^*, \sigma_n; x_2^*, b_0) =$$

$$\sum_{i=1}^{I} \left( (1 + a_i(\bar{x}_1^*, x_1^*; b_0, b_1))w_i(x_1^*; b_1) - a_i(\bar{x}_1^*, x_1^*; b_0, b_1)u_p \right)$$

$$- \sum_{i=1}^{I} \left( (1 + a_i(\bar{x}_2^*, x_2^*; b_0, b_0))w_i(x_2^*; b_0) + a_i(\bar{x}_2^*, x_2^*; b_0, b_0)u_p \right).$$
We rewrite this expression putting all the constant terms in $K$.

$$E_{b_0} [(1 + a_i(\bar{x}_1^*, x_1^*; b_1, b_0))w_i(x_1^*; b_1)] - (1 + a_i(\bar{x}_2^*, x_2^*; b_0, b_0))w_i(x_2^*; b_0) + K.$$ 

Since $w_i(x_2^*; b_0) < u_p$, $a_i(\bar{x}_1^*, x_1^*; b_1, b_0) = \gamma$ or $a_i(\bar{x}_1^*, x_1^*; b_1, b_0) = \alpha + \gamma$ and $a_i(\bar{x}_2^*, x_2^*, b_0, b_0) = 0$. In this case the difference is increasing $w_i(x_1^*; b_1)$.

**Proof of Corollary 1**

$$V_0(\bar{x}, \sigma; x, b_0) - V_0(\bar{x}, \sigma_n; x, b_0)$$

$$= E_{b_0} \left[ \sum_{i=1}^{I} (1 + \gamma)w_i(x; b_1) - \gamma u_p \right] - \sum_{i=1}^{I} w_i(x; b_0)$$

$$= \sum_{i=1}^{I} \gamma(w_i(x; b_0) - u_p)$$

This expression is strictly increasing in $w_j(x; b_0)$.

**Proof of Proposition 3**

Utility from $x$ and $\bar{x}$ are respectively

$$V_1(x; \bar{x}, \sigma, b_1) = \sum_{i=1}^{I} (1 + a_i(\bar{x}, x; b_0, b_1))w_i(x; b_1) - a_i(\bar{x}, x; b_0, b_1)u_p$$

and

$$V_1(\bar{x}; \bar{x}, \sigma, b_1) = \sum_{i=1}^{I} (1 + a_i(\bar{x}, \bar{x}; b_0, b_1))w_i(\bar{x}; b_1) - a_i(\bar{x}, \bar{x}; b_0, b_1)u_p.$$ 

The difference is

$$(1 + a_j(\bar{x}, x; b_0, b_1))w_j(x; b_1) - (1 + a_j(\bar{x}, \bar{x}; b_0, b_1))w_j(\bar{x}; b_1) + K$$

where $K$ is a constant (for the purposes of varying $w_j$).

Since $w_j(\bar{x}; b_1) < u_p$, the difference can be rewritten as $\alpha w_j(x; b_1) + (1 + a_j(\bar{x}, x; b_0, b_1))w_j(x; b_1) - w_j(\bar{x}; b_1) + K$ because $a_j(\bar{x}, x; b_0, b_1) = \alpha + a_j(\bar{x}, \bar{x}; b_0, b_1)$. One can see that the difference is increasing in $w_j(x; b_1)$ while keeping $w_j(x; b_1) - w_j(\bar{x}; b_1)$ constant.
Proof of Proposition 4

\[
V_1(x'; \bar{x}, b_1, b_0^R) - V_1(x''; \bar{x}, b_1, b_0^R) - V_1(x'; \bar{x}, b_1, b_0^P) + V_1(x''; \bar{x}, b_1, b_0^P) \\
= \gamma(w_i(x'; b_1) - u_p) - \gamma(w_i(x''; b_1) - u_p) \\
= \gamma(w_i(x'; b_1) - w_i(x''; b_1)) > 0
\]

Proof of Proposition 5

Assume \( u_j(0) - u_j(q_j^l) \geq \frac{1+\alpha}{\gamma}(u_j(q_j^l) - E_{b_0}[u_j(q_j)]) \) for at least one dimension \( j \in J \). We construct a strategy profile and show it is a sequential equilibrium. The monopolist chooses \( r_i(q_i) = d \) for all \( i \notin J \) and all \( q_i \), and \( r_j(q_j) = s \) for all \( j \in J \) and all \( q_j \). The price as given in the text

\[
p^*(q) = \frac{1}{1 + a_n(\bar{x}, B; b_0, b_1)} \left( (a_n(\bar{x}, N; b_0, b_1) - a_n(\bar{x}, B; b_0, b_1))u_p + \sum_{i=1}^J E_{b_0}[u_i(q_i)] + a_i(\bar{x}, B; b_0, b_1)(E_{b_1}[u_i(q_i)] - u_p) - u_i(0) \\
- a_i(\bar{x}, N; b_0, b_1)(u_i(0) - u_p) \right).
\]

The consumer chooses \( \bar{x} = N \), and at his second information set chooses \( x = B \) if and only if \( V_1(B; N; b_0, b_1, p) \geq V_1(N; N; b_0, b_1, p) \), otherwise \( x = N \). The consumer’s equilibrium beliefs are \( \Pr_{b_1}(q_j|r_j = s) = \Pr_{b_0}(q_j) \) for dimensions \( j \in J \), and if the realized value is \( q_j^l \) then \( \Pr_{b_1}(q_j^l|r_i = d) = 1 \) and \( \Pr_{b_1}(q_j|r_i = d) = 0 \) for all other \( q_i \neq q_j^l \). The off-the-equilibrium-path beliefs for dimensions are \( j \in J \), \( \Pr_{b_1}(q_j^l|r_j = d) = 1 \) if the realized value is \( q_j^l \) and \( \Pr_{b_1}(q_j|r_i = d) = 0 \) for all other \( q_j \neq q_j^l \). The consumer has skeptical beliefs for dimensions \( i \notin J \), believing the quality to be the lowest possible after observing shrouding, \( \Pr_{b_1}(q_i^1|r_i = s) = 1 \), and \( \Pr_{b_1}(q_i|r_i = s) = 0 \) for all \( q_i \neq q_i^1 \).

Now we check the four conditions for sequential equilibrium. This price maximizes the firm’s profit. Notice charging a higher price would prevent sale of the good. Disclosing any attribute \( j \in J \) would lower the price, by construction. Shrouding any attribute \( i \notin J \) would also lower the price because the consumer would believe it to be lowest quality. So the firm is profit maximizing. The consumer is indifferent between buying and not buying. If the consumer switched the default to buying, equilibrium utility would go down, so that would be suboptimal. So the consumer is maximizing. The consumer’s equilibrium beliefs and out-of-equilibrium beliefs are rational and consistent by construction.
Proof of Proposition 6

First we show existence of a full disclosure equilibrium. Then we show that for all such equilibria shrouding is not possible when the value of the goods are sufficiently large. When the value of the good is sufficiently large so that all consumers buy, the equilibrium prices as given in the text are $p^0 = \frac{t}{1+\alpha+\gamma}(1 + \frac{2}{3}Y)$ and $p^1 = \frac{t}{1+\alpha+\gamma}(1 - \frac{3}{2}Y)$. Since we are constructing a full disclosure equilibrium $r^0_i = r^1_i = d$ for all $i$, all quality levels. A consumer will choose $B^0$ if and only if $V_1(B^0; N, b_0, b_1, p^0, p_1, \rho) > V_1(B^1; N, b_0, b_1, p^0, p_1, \rho)$, otherwise $B^1$. The consumer believes the quality is the realized quality upon disclosure, and believes quality is the lowest if shrouded. Thus equilibrium beliefs are $Pr_b(q^0_i| r_i = d) = 1$ if the realized value is $q^0_i$ and $Pr_b(q^1_i| r_i = d) = 0$ for all other $q_i \neq q^0_i$ and likewise for Firm 1. The off-equilibrium beliefs are $Pr_b(q^0_i| r_i = s) = 1$, and $Pr_b(q_i| r_i = s) = 0$ for all $q_i \neq q^1_i$ and likewise for Firm 1. This is an equilibrium because the prices are optimal by construction, the buying strategy is optimal by construction, and the beliefs are both rational and consistent.

Now we will consider different cases of strategy profiles with shrouding and we will show that these cannot be equilibria. Observe that Firm 0’s equilibrium profit is strictly increasing in $Y$ and Firm 1’s equilibrium profit is strictly decreasing in $Y$.

Case 1 (Partial Shrouding): Without loss of generality, let us consider Firm 0 partially shrouds meaning that there is a non-empty strict subset of quality for which if $q_i \in \tilde{Q}_i \subset \{q^0_i, \ldots, q^{m}_i\}$ then $r^0_i(q^0_i) = s$ but if $q_i \notin \tilde{Q}_i$ then $r^0_i(q^0_i) = d$.

Subcase A (Partial Shrouding With Exactly One Type): Now consider the case where there is exactly one quality in $q^0_i \in \tilde{Q}_i$ and $q^0_i \neq q_i$. The consumers will believe $E[q_i| r^0_i = s] = q^0_i$. But if the firm has quality $q^1_i$ it could profitably deviate by shrouding $r^0_i(q^1_i) = s$ which would increase $Y$. But then $\tilde{Q}_i$ would have more than one quality type which is a contradiction.

Subcase B (Partial Shrouding With At Least Two Types): Under partial shrouding, due to rational beliefs, with the exception of knife-edge cases, the consumer’s posterior expected quality will always differ from the prior expected quality: $E_{b_1}[q_i] \neq E_{b_0}[q_i]$. Consequently, attention will shift to this dimension. The consumers’ expected quality given shrouding is

$$E[q_i| r^0_i = s] = \frac{\sum_{k=1}^l Pr(r^0_i = s| q^k_i) Pr(q^k_i) q^k_i}{\sum_{k=1}^l Pr(r^0_i = s| q^k_i) Pr(q^k_i)}.$$

If Firm 0 has $q_i = \max \{\tilde{Q}_i\}$, by disclosing it can increase $Y$ and get a strictly higher profit, so this cannot be part of an equilibrium.

Case 2 (Full Shrouding): Now, without loss of generality, let us consider Firm 0 fully shrouds, meaning that $r^0_i(q_i) = s$ for all $q_i$. Then the consumers’ beliefs will be $E_{b_1}[q_i] = E_{b_0}[q_i]$. We now consider two subcases, first where the other firm does not fully shroud.
Subcase A (Firm 1 Does Not Fully Shroud) If the other firm does not fully shroud then attention will be shifted to the dimension, because except for knife-edge cases, \( w_i(B^1; b_1) \neq w_i(B^1; b_0) \). If \( q_i > E_{b_0}[q_i] \) then disclosing can only increase \( Y \) and hence profit so this cannot be an equilibrium.

Subcase B (Firm 1 Fully Shrouds) Now suppose Firm 1 fully shrouds as well so attention will not be shifted to the dimension. If \( q_i > E_{b_0}[q_i] \) and \( q_i > E_{b_0}[k_i] \) then disclosing can only increase \( Y \). So this cannot be an equilibrium either.

We have covered all cases where Firm 0 shrouds. Since the firms are symmetric the same logic applies to Firm 1.

A.2 Chapter 2 Proofs

Proof of Proposition 7

We shall introduce some notation. Define the set of Player 1’s actions that leave the policy within Player 2’s inaction zone, \( M_1^{In} = \{ m_1 : m_1 + \tilde{q} \in [q_1, q_i] \} \), and define the set of Player 1’s actions that leave the policy outside of Player 2’s inaction zone to be \( M_1^{Out} = \{ m_1 : m_1 + \tilde{q} \notin (q_L, q_U) \} \). Define \( m_1^{Out} \in M_1^{Out} \) as an action for Player 1 that satisfies \( f_1'(\tilde{q} + m_1^{Out} + m_2^{Out}) * (1 + m_2'(\tilde{m}_2^{Out})) = C_1'(m_1^{Out}) \) and define \( m_1^{In} \in M_1^{In} \) as an action for Player 1 that satisfies \( f_1'(\tilde{q} + m_1^{In}) = C_1'(m_1^{In}) \).

Lemma 4 If \( C_2''(\cdot) > 0 \), in subgame perfect equilibrium, the \( m_1^* \) that maximizes \( U_1(m_1, m_2) \) must be \( m_1^* \in \{ m_1^{Out}, m_1^{In}, q_L - \tilde{q}, q_U - \tilde{q} \} \).

Lemma 4 says that Player 1’s strategy in any subgame perfect equilibrium is limited to four possible strategies.

Proof We begin using backwards induction by finding Player 2’s optimal strategy as a function of Player 1’s strategy. If \( m_1 \in M_1^{In} \), then by construction, Player 2’s fixed cost exceeds any benefit from taking a nonzero action. Thus if \( m_1 \in M_1^{In} \), then \( m_2^*(m_1) = 0 \). If \( m_1 \in M_1^{Out} \), then by construction, Player 2’s benefit from moving the policy optimally exceeds the fixed cost. It must be the case that the optimal action satisfies Player 2’s first order condition. Thus \( m_2^*(m_1) \) satisfies \( f_2'(\tilde{q} + m_1 + m_2^*(m_1)) = C_2'(m_2^*(m_1)) \). This is a maximum because the second derivative of the utility function is always negative. We now need to show existence of an \( m_2 \) that satisfies this condition. From single-peakedness we know that, for a constant \( a \), \( f_2'(x + a) > 0 \) for \( x < -a \), \( f_2'(x + a) = 0 \) for \( x = -a \), and \( f_2'(x + a) < 0 \) for \( x > -a \). From symmetry and the other conditions on the cost function we have \( C_2''(x) \geq 0 \) for \( x \geq 0 \), \( C_2'(x) = 0 \) for \( x = 0 \), and \( C_2'(x) \leq 0 \) for \( x \leq 0 \). Thus \( f_2'(x + a) > 0 \geq C_2''(x) \) for \( x < -a \) and \( x \leq 0 \), and \( f_2'(y + a) < 0 \leq C_2''(y) \) for \( y > -a \) and \( y \geq 0 \). By the Intermediate Value Theorem, there exists a \( z \in [x, y] \) such that \( f_2'(z + a) = C_2''(z) \). This gives us existence. This point is unique by virtue of the fact that both functions are monotonic, specifically \( f_2''(\cdot) < 0 \) and \( C_2''(\cdot) > 0 \).

Now that we’ve characterized Player 2’s optimal strategy, we can solve for Player 1’s optimal strategy. First we solve for the optimal \( m_1 \) conditional on \( m_1 \in M_1^{In} \).
Assume, for the moment, that there exists a maximum in the interior of the domain. At this maximum $m_1$ must satisfy the first-order condition and so the optimal strategy will be $m_1^*$. Assuming that there is a maximum in the interior, this $m_1^*$ exists and is unique (the logic is the same as in the preceding paragraph). If there is no maximum in the interior, it implies that the maximum must be on one of the bounds, $q_L$, or $q_U$.

Second, we wish to solve the optimal $m_1$ conditional on $m_1 \in M_1^{Out}$. Assume, for the moment, that there exists a maximum in the interior of the domain $m_1 \in (-\infty, q_L - \tilde{q}) \cup (q_U - \tilde{q}, \infty)$. At this maximum $m_1$ must satisfy Player 1’s first-order condition and so the optimal strategy will be $m_1^{Out}$ which is defined as satisfying $f_1'(\tilde{q} + m_1^{Out} + m_2^*(m_1^{Out})) = C_1'(m_1^{Out})$. We now wish to show existence of an intersection conditional on a maximum in the interior. As long as we show that $m_2^*(m_1)$ is differentiable and $-1 < m_2''(m_1^{Out}) < 0$, we can apply the Intermediate Value Theorem to the first-order condition to guarantee an intersection. Differentiating Player 2’s first-order condition gives us

$$m_2''(m_1) = \frac{f_2''(\tilde{q} + m_1 + m_2^*(m_1))}{C_2''(m_2^*(m_1)) - f_2''(\tilde{q} + m_1 + m_2^*(m_1))},$$

which implies that $-1 < m_2''(m_1^{Out}) < 0$ from concavity of $f_2(\cdot)$ and convexity of $C_2(\cdot)$. This allows us to apply the Intermediate Value Theorem guaranteeing existence conditional on the maximum being in the interior.

Now we must show uniqueness. It is sufficient to show that for any critical point, the second derivative of Player 1’s utility is negative which would imply a single crossing. The condition is given by:

$$f_1''(\tilde{q} + m_1^* + m_2^*(m_1^*)) (1 + m_2^*(m_1^*))^2 + f_1'(%(\tilde{q} + m_1^* + m_2^*(m_1^*)) * m_2''(m_1^*) C_1''(m_1^*) < 0. \tag{A.1}$$

The first term is negative from concavity and the last term is negative from convexity. In order for the second order condition to be satisfied the magnitude of $m_2''(m_1^*)$ must be sufficiently low.

We assume this condition is satisfied. The exact condition required places restrictions on the second and third derivatives of the felicity and cost functions. Switching notation slightly, allow $f_i^{(n)}$ and $C_i^{(n)}$ to represent the $n$th derivative of Player $i$’s felicity and cost functions with the equilibrium policy and actions as arguments respectively. Substituting $m_2''(m_1^*)$ and $m_2''(m_1^*)$ into the inequality above, the condition becomes

$$f_1^{(1)} \left( \frac{f_2^{(3)}}{(f_2^{(2)})^2} - \frac{C_2^{(3)}}{(C_2^{(2)})^2} \right) < \frac{(C_2^{(2)} - f_2^{(2)}) (C_2^{(2)} - f_2^{(2)})^2 - f_1^{(2)} (C_2^{(2)})^2}{(f_2^{(2)} C_2^{(2)})^2} \tag{A.2}$$

Notice that the right-hand side is positive but $f_i^{(1)}$ may be positive or negative. The difference of the third derivatives of the felicity and cost functions is bounded by
However, it is still possible that the maximizer is not in the interior. If this is the case then one of the bounds is a maximizer. This gives us all the candidates for our lemma.

Now we prove Proposition 7. From Lemma 4, there are only four candidates for Player 1’s optimal action $m^*_1$. Generically, one of these four values will give Player 1 higher utility than the others. Player 1 will simply choose the $m_1$ that maximizes $U_1(m_1, m_2)$.

Finally, we prove Corollary 2. If $C''_2(\cdot) = 0$ then $C'_2(\cdot) = 0$ and $C_2(\cdot) = 0$ (because $C'_2(0) = 0$ and $C_2(0) = 0$) so Player 2 has no marginal costs. If $m_1 \in M_1^{\text{Out}}$, then Player 2’s optimal action is to move the policy directly to her bliss, $m^*_2(m_1) = -\bar{q} - m_1$. If $m_1 \in M_1^{\text{In}}$ then $m^*_2(m_1) = 0$. The rest of the proof follows the logic from Lemma 4 and Proposition 7.

Proof of Proposition 8

The utility of Player 1 in provocation by extremists and extraction equilibria respectively are given by

$$U_1(\text{provocation}) = f_1(q_L + m^*_2) + C_1(\bar{q} - q_L)$$

$$U_1(\text{extraction}) = f_1(q_U) + C_1(q_U - \bar{q}).$$

First notice that the costs are invariant to $b$. The difference in felicities, given by $f(q_U) - f_1(q_L + m^*_2)$, is positive when $b$ is large since $q_U > q_L + m^*_2$. Since the felicity function is concave, increasing $b$ increases the difference between the felicities. Thus this difference can be made arbitrarily large indicating that $U_1(\text{extraction}) > U_1(\text{provocation})$. The same argument can be made for provocation by moderates. Thus for large enough $b$, provocation cannot be an equilibrium.

Proof of Proposition 9

When $e$ decreases the inaction zone shrinks. When $e = 0$, the inaction zone collapses to Player 2’s bliss point, $q_L = 0 = q_U$ and the only equilibrium is gravity. When $\bar{q} \neq 0$, there always exists an $e$ sufficiently small such that $\bar{q} \notin [q_L, q_U]$. This immediately eliminates the provocation by extremists, and provocation by moderates equilibria since these can only occur if the default begins within the inaction zone.

We now wish to eliminate the improvement equilibrium. In an improvement equilibrium, $m_1 = m_1^{\text{In}}$. However when $e$ is sufficiently small the $m_1$ that satisfies the first order condition is generically not in $M_1^{\text{In}}$ and so $m_1^{\text{In}}$ generically does not exist. This eliminates the improvement equilibrium.

It remains to be shown that the utility in a gravity equilibrium is greater than
the utility in a placation equilibrium. The utility of Player 1 in a gravity equilibrium is given by

\[ U_1(\text{gravity}) = f_1(m_1^{Out} + m_2^*(m_1^{Out}) + \bar{q}) - C_1(m_1^{Out}). \] (A.5)

When \( e \) is arbitrarily small, the utility in a placation equilibrium is given by

\[ U_1(\text{placation}) = f_1(0) - C_1(\bar{q}) + \delta \] (A.6)

where \( \delta \leq 0 \). As \( e \) decreases toward zero, \(|\delta|\) decreases and can be made arbitrarily small.

First, consider that when \( C_2(\cdot) = 0 \), Player 2 will bring the policy to its bliss whenever she takes action. Thus under gravity \( m_1 = 0 \) so \( U_1(\text{gravity}) = f_1(0) \). Clearly this exceeds \( U_1(\text{placation}) \), and so the proof is complete for \( C_2(\cdot) = 0 \).

Now consider when \( C''_2(\cdot) > 0 \). When \( e = 0 \), then \((m_1, m_2(m_1)) = (-\bar{q}, 0)\) is in the choice set of Player 1 under a gravity equilibrium. It is generically true that \( f_1(0) + C_1(\bar{q}) < U_1(\text{gravity}) \), unless it just so happens to be the unique solution to Player 1’s first order condition. Since \(|\delta|\) can be made arbitrarily small and is decreasing in \( e \), it follows that, for some \( \hat{e} \) all \( e < \hat{e} \), \( U_1(\text{placation}) < U_1(\text{gravity}) \).

**Proof of Proposition 10**

We will prove Proposition 10 by solving for the expected utility of Player 2 and taking the limit as fixed costs get high.

\[
EU_2(e) = \text{Prob}(\text{gravity}) \times EU_2(e \mid \text{gravity}) + \text{Prob}(\text{provocation}) \times EU_2(e \mid \text{provocation}) + \text{Prob}(\text{soup}) \times EU_2(e \mid \text{soup})
\]

\[
= \int_{\hat{q}(e)}^{\hat{q}(e)} \left[ f_2(\bar{q} + m_1^*(e, \bar{q}) + m_2^*(e, \bar{q})) - e - C_2(m_2^*(e, \bar{q})) \right] h(\bar{q}) d\bar{q} + \int_{\hat{q}(e)}^{\hat{q}(e)} f_2(q_L(e)) h(\bar{q}) d\bar{q} + \int_{\hat{q}(e)}^{\hat{q}(e)} f_2(\bar{q} + m_1^*(\bar{q})) h(\bar{q}) d\bar{q}
\]

The first term, which is the expected utility when the policy induces a gravity equilibrium, can be broken down into the sum of two expressions. Define \( x(e) \) as the lowest value of \( \bar{q} \) where Player 1 induces gravity and does so by bringing the

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\(^1\)Placation includes Sacrificial, Willing, and Extraction. Unlike the others, in extraction \( \bar{q} < q_U \).
policy to the edge of the inaction zone $q_L(e)$.

$$\text{Prob}(\text{gravity}) \ast EU_2(e \mid \text{gravity}) =$$

$$\int_{s_L}^{x(e)} \left[ f_2(\bar{q} + m_1^*(\bar{q}) + m_2^*(\bar{q})) - e - C_2(m_2^*(\bar{q})) \right] h(\bar{q}) d\bar{q}$$

$$+ \int_{x(e)}^{q_L(e)} f_2(q_L(e)) h(\bar{q}) d\bar{q}$$

(A.8)

The first term is where both players move the policy such that marginal cost equals marginal benefit. The second term is where Player 1 has a corner solution; she moves the policy to $q_L(e)$ and Player 2 moves the policy such that marginal cost equals marginal benefit. The region between $[x(e), q_L(e)]$ is where Player 1 has a corner solution. Define

$$U_{\text{grav}}(\bar{q}) \equiv f_2(\bar{q} + m_1^*(\bar{q}) + m_2^*(\bar{q})) - C_2(m_2^*(\bar{q}))$$

$$U_{\text{bound}}(e) \equiv f_2(q_L(e))$$

$$U_{\text{soup}}(\bar{q}) \equiv f_2(\bar{q} + m_1^*(\bar{q})).$$

Then expected utility can be rewritten as

$$EU_2(e) = \int_{s_L}^{x(e)} -e \ast h(\bar{q}) d\bar{q} + \int_{s_L}^{x(e)} U_{\text{grav}}(\bar{q}) \ast h(\bar{q}) d\bar{q}$$

$$+ \int_{x(e)}^{q_L(e)} U_{\text{bound}}(e) \ast h(\bar{q}) d\bar{q} + \int_{0}^{\hat{q}(e)} U_{\text{soup}}(\bar{q}) \ast h(\bar{q}) d\bar{q}.$$  

(A.9)

Now we wish to differentiate the expected utility to see how it varies as a function of $e$. Using Leibnitz’s Rule we find that,

$$\frac{d}{de} EU_2(e) = -H(x(e)) - \left( U_{\text{bound}}(e) - U_{\text{grav}}(x(e)) + e \right) h(x(e)) x'(e)$$

$$- \left( U_{\text{soup}}(\hat{q}(e)) - U_{\text{bound}}(e) \right) h(\hat{q}(e)) \hat{q}'(e)$$

$$+ f'(q_L(e)) \ast q_L'(e) \left( H(\hat{q}(e)) - H(x(e)) \right).$$  

(A.10)

This is a sum of four terms. First we show that both the first and fourth terms can be made arbitrarily small when $e$ is large. Then we show that the second term is zero and the third term is always positive, even when $e$ is large.

In the first and fourth terms, as $e$ increases, $x(e)$ and $\hat{q}(e)$ decrease and move toward the lower bound of the support $s_L$. When $e \geq x^{-1}(s_L)$, then $H(x(e)) = 0$.
and likewise when $e \geq \tilde{q}^{-1}(s_L)$, then $H(\tilde{q}(e)) = 0$. Thus for large $e$ we can make the first and fourth terms arbitrarily small.

In the second term, $U_{grav}(x(e)) - e = U_{bound}(e)$ by construction, and so the whole term is zero. The difference, $U_{soup}(\tilde{q}(e)) - U_{bound}(e)$, is the difference between Player 2’s utility in an improvement equilibrium minus Player 2’s utility in an equilibrium in which Player 1 leaves the policy at the lower bound of the inaction zone. This difference is positive because in an improvement equilibrium Player 1 leaves the policy within the inaction zone, which is generically preferred for Player 2. The term $\tilde{q}(e)$ is negative and so the whole term is positive. It remains to be shown that for large $e$, $U_{soup}(\tilde{q}(e)) - U_{bound}(e)$ does not go to zero. The difference can be written as $f_2(\tilde{q}(e) + m_1^*(\tilde{q}(e))) - f_2(q_L(e))$. Notice that always $\tilde{q}(e) > q_L(e)$ and $m_1^*(\tilde{q}(e))$ is increasing in $e$ and thus the term will not go to zero for large $e$. Consequently, for large $e$, the derivative of Player 2’s expected utility is always positive.

**Proof of Lemma [1]**

First we observe that $\tilde{q}_P(1) = \tilde{q}_G(1)$. When there is only one firm, there is only one default policy in which the firm is indifferent between placation and gravity. If the default policy is just slightly more to the left, placation is preferred, and if slightly more to the right, gravity is preferred. We wish to show that when $N$ is large, the equilibrium regions overlap for more than just a single point.

In order to show this, we must show that $\tilde{q}_P(1) < \tilde{q}_P(N)$ and $\tilde{q}_G(N) < \tilde{q}_G(1)$ for all $N > N'$. To prove the first inequality, we specify the condition in which no firm has an incentive to deviate from a placation equilibrium. This condition is given by

$$f_F(q_U) - (\tilde{q} - q_U) * c_F \geq f_F \left( \frac{N-1}{N} q_U + \frac{1}{N} \tilde{q} + \tilde{m}_i(N) + m_G \left( \frac{N-1}{N} q_U + \frac{1}{N} \tilde{q} + \tilde{m}_i(N) \right) \right) - N * \tilde{m}_i(N) * c_F \tag{A.11}$$

where $\tilde{m}_i(N)$ is the optimal deviation. Notice that the optimal deviation will be pulling the policy upward where marginal benefit equals marginal cost.

Let $\tilde{q} = \tilde{q}_P(1)$ so that expression $[\text{A.11}]$ holds with equality when $N = 1$. When we allow $N > 1$, the left-hand side of equation $[\text{A.11}]$ does not change. The righthand side of the equation must decrease for two reasons. First, the cost function is now multiplied by $N$. Second, the other firms’ actions place the default policy farther away from the bliss point and closer to $q_U$. Thus for $[\text{A.11}]$ to continue to hold with equality, $\tilde{q}$ must increase. This implies that $\tilde{q}_P(N)$ is monotonically increasing in $N$.

Now we show that $\tilde{q}_G(1) > \tilde{q}_G(N)$ for all $N > N'$. The no deviation condition

\footnote{For $[\text{A.11}]$ and $[\text{A.12}]$, both sides have been multiplied by $N$.}
for a gravity equilibrium is given by
\[
\begin{align*}
&f_F(\bar{q} + N \ast m_i^*(N) + m_G(\bar{q} + N \ast m_i^*(N))) - N \ast m_i^*(N) \ast c_F \ge \\
&f_F(q_U) - N \ast (\bar{q} - q_U + (N - 1)m_i^*(N)) \ast c_F.
\end{align*}
\] (A.12)

Notice that the best deviation is to placate. The equilibrium action already specifies
the optimal action if the action moves the policy weakly higher. The only way that
moving the policy lower could be optimal is if it placates.

When \( \bar{q} = \hat{q}_G(N) \), then \( \text{(A.12)} \) holds with equality. Let \( \bar{q} = \hat{q}_G(1) \). When \( N \) is
very large, we will show that the right-hand side exceeds the left-hand side.

First, we show that \( m_i^*(N) = 0 \) for large \( N \). The optimal action in gravity,
\( m_i^*(N) \) will satisfy Player \( i \)'s first order condition:
\[
\frac{1}{N} f'_F(\bar{q} + N \ast m_i^*(N) + m_G(\bar{q} + N \ast m_i^*(N))) \left(1 + \frac{\partial m_G}{\partial m_i}\right) = c_F.
\]

Since the marginal benefit is decreasing by \( \frac{1}{N} \) but the marginal cost is constant,
there will be an \( N \) sufficiently large such that the marginal cost always exceeds
the marginal benefit. This implies a corner solution to the first-order condition, hence
\( m_i^*(N) = 0 \).

This means as \( N \) gets large, the left-hand side of \( \text{(A.12)} \) converges to \( f_F(\bar{q} + m_G(0)) \). The right-hand side explodes to negative infinity. Thus in order for this
equation to hold with equality, it must be the case that \( \bar{q} \) decreases, lowering the
equilibrium utility on the left-hand side and increasing the deviation utility on the
right hand side. This implies that \( \hat{q}_G(N) < \hat{q}_G(1) \) when \( N \) is large.

Proof of Proposition 11

Let us more carefully inspect the placation and gravity equilibrium regimes as \( N \)
increases so that we can determine how the equilibrium utility of these two equilibria
change. First let us look at equilibrium utility of a single firm in a symmetric
placation equilibrium,
\[
U_i(m_i, m_{-i}) = \frac{1}{N} f_F(q_U) - \frac{1}{N}(\bar{q} - q_U) \ast c_F.
\]

If we sum over all \( N \) firms we find that the total industry utility is invariant in \( N \).
This means that \( U_{plac}(N; \bar{q}) \) is invariant in \( N \).

Remember from Lemma 1 that when \( N > N' \) then \( m_i^*(N) = 0 \). This implies
for \( N > N' \), \( U_{grav}(N; \bar{q}) = f_F(\bar{q} + m_G(\bar{q})) \). But clearly this expression is less than
\( U_{grav}(1; \bar{q}) = f_F(\bar{q} + m_1^*(1) + m_G(\bar{q} + m_1^*(1))) - m_1^*(1) \ast c_F \) since \( m_1^*(1) = 0 \) does not
satisfy the firm’s first-order condition. Thus from when \( N = 1 \) to when \( N \) is large,
the industry equilibrium utility decreases.
Proof of Lemma 2

First we observe that $\hat{q}_R(1) = \hat{q}_{sl}(1)$. When there is only one firm, there is only one default policy in which the firm is indifferent between provocation and improvement. If the default policy is just slightly more to the left, provocation is preferred, and if slightly more to the right, improvement is preferred. We wish to show that when $N > 1$ the equilibrium regions overlap for more than just a single point.

In order to show this, we must show that $\hat{q}_R(1) < \hat{q}_R(N)$ and $\hat{q}_{sl}(1) > \hat{q}_{sl}(N)$ for all $N > 1$. To prove the first inequality, we specify the condition in which no firm has an incentive to deviate from provocation by extremists equilibrium (the logic is the same as in the placation equilibrium)$^3$

$$f_F(q_L - m_G(q_L)) - (\bar{q} - q_L) * c_F \geq f_F \left( \frac{N - 1}{N} q_L + \frac{1}{N} \bar{q} + \tilde{m}_i(N) \right) - N * \tilde{m}_i(N) * c_F$$

where $\tilde{m}_i(N)$ is the optimal deviation. This deviation will move the policy upward toward the faction’s bliss such that marginal costs equal marginal benefit.

Let $\bar{q} = \hat{q}_R(1)$ so that (A.13) holds with equality when $N = 1$. The left-hand side is invariant to $N$. The right-hand side of the equation, must decrease for two reasons. First, the cost function is now multiplied by $N$. Second, the other firms’ actions place the default policy farther away from the bliss point and closer to $q_L$. Thus for (A.13), it must be the case that $\bar{q}$ increases, which decreases the equilibrium utility on the left-hand side and increases the deviation utility on the right-hand side. This implies that $\hat{q}_R(N)$ is monotonically increasing in $N$.

The logic we use here to show that $\hat{q}_{sl}(N) < \hat{q}_{sl}(1)$ is similar to the logic we used when examining the gravity equilibrium. The condition for there to be no deviation from an improvement equilibrium is given by

$$f_F (\bar{q} + N * m^*_i(N)) - N * m^*_i(N) * c_F \geq f_F(q_L + m_G(q_L)) - N * (\bar{q} - q_L + (N - 1) m^*_i(N)) * c_F$$

where $m^*_i(N)$ is the firm’s equilibrium action in an improvement equilibrium. Notice that the best deviation is to induce a provocation by extremists equilibrium. The equilibrium action already specifies the optimal action if the action moves the policy weakly higher. The only way that moving the policy lower can be optimal is if it induces provocation.

When $\bar{q} = \hat{q}_{sl}(N)$ then expression (A.14) holds with equality. When $N > N'$ we will show the right-hand side exceeds the left-hand side. But first, we show the total optimal action $N * m^*_i(N)$ decreases when $N$ is small. The optimal action in

$^3$For (A.13) and (A.14), both sides have been multiplied by $N$. 
improvement, $m_i^*(N)$ will satisfy Player $i$’s first-order condition:

$$\frac{1}{N} f_F'(\bar{q} + N \ast m_i^*(N)) = c_F \Rightarrow m_i^*(N) = \max \left\{ \frac{f_F^{-1}(Nc_F) - \bar{q}}{N}, 0 \right\}.$$

Thus $m_i^*(N)$ strictly decreases with $N$ until $m_i^*(N) = 0$ for all $N > N'$.

This means as $N$ gets large, the left-hand side of (A.14) converges to $f_F(\bar{q})$ and the right-hand side explodes to negative infinity. This implies that $\hat{q}_{sl}(N) < \hat{q}_{sl}(1)$ when $N$ is large.

**Proof of Proposition 12**

The following analysis parallels the proof for Proposition 11. First let us look at equilibrium utility of a single faction in the symmetric provocation by extremists equilibrium:

$$U_i(m_i, m_{-i}) = \frac{1}{N} f_F(q_L - m_G(q_L)) - \frac{1}{N} (q_L - \bar{q}) \ast c_F.$$

If we sum over all $N$ factions we find that $U_{rev}(N; \bar{q})$ is invariant to $N$.

Remember from Lemma 2 that when $N > N'$ then $m_i^*(N) = 0$. This implies for $N > N'$, $U_{soup}(N; \bar{q}) = f_F(\bar{q})$. But clearly this expression is less than $U_{soup}(1; \bar{q}) = f_F(\bar{q} + m_1^*(1)) - m_1^*(1) \ast c_F$ since $m_1^*(1) = 0$ does not generically satisfy the faction’s first-order condition. Thus total faction utility decreases when $N$ is large.