Lawrence Berkeley National Laboratory
Recent Work

Title
THE CONVECTIVE HEATING & COOLING OF A BODY

Permalink
https://escholarship.org/uc/item/4v6034bj

Author
Warren, R.

Publication Date
1981-10-01
LEGAL NOTICE

This book was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Lawrence Berkeley Laboratory is an equal opportunity employer.
DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
THE CONVECTIVE HEATING & COOLING OF A BODY

SUMMARY & INTRODUCTION

Equations are developed which allow estimates of the time for the convective heating or cooling of a body. Three cases are considered, constant mass flow of coolant and constant heat capacities, variable solid heat capacity ($C_s = CT$) and both variable heat capacity of the solid and variable coolant mass flow (constant pressure drop). In all cases heat exchanger effectiveness is assumed constant thru the process. The variable solid heat capacity gives rise to a non-symmetry in the heating and cooling times. For equal pressure drops, cooling times will be substantially less than the heating times between the same temperature limits. The expressions obtained are evaluated for several sets of temperature extremes for both heating and cooling and the results applied to an example problem. The solid temperature is assumed uniform in this development. However the results obtained should be useful for making first cut estimates of the temperature response of a distributed system where the solid temperature may vary over the length of the gas or liquid path.

*The term solid really has no significance here. It could just as well pertain to a liquid with an immersed heat exchanger.
Consider a body at temperature $T_1$, which exchanges heat with a gas stream which approaches at temperature $T_{g1}$ and which leaves at $T_{g2}$. An energy balance for the time interval $dt$ yields,

$$m_g C_p (T_{g1} - T_{g2}) dt = m_s C_s dt$$

To account for imperfect heat transfer we define a heat exchanger effectiveness $\varepsilon$

$$\varepsilon = \frac{T_{g1} - T_{g2}}{T_{g1} - T_1}$$

Combining these expressions, separating variables and integrating from $0$ to $\Theta$ and $T_{g1}$ to $T_{g2}$, yields

$$\varepsilon m_g C_p \int_0^\Theta \frac{d\Theta}{m_s C_s} = \int_{T_{g1}}^{T_{g2}} \frac{dT_1}{T_{g1} - T_1}$$

or

$$\frac{m_g C_p \Theta}{m_s C_s} = \ln \left( \frac{T_{g1} - T_{g2}}{T_{g1} - T_1} \right)$$

If the solid heat capacity is allowed to vary with temperature according to $C_s = C_0 \times T_1$, the result is

$$\frac{m_g C_p \Theta}{m_s C} = T_{g1} - T_{g2} + T_1 \ln \left( \frac{T_{g1} - T_1}{T_{g1} - T_{g2}} \right)$$

Note that for the same end temperatures, equation (4) will predict identical heating and cooling times whereas there is no such symmetry in equation (5). The reason for this is that as the end point of the cooling or heating period is approached the temperature difference between the gas stream and solid is
decreasing, and therefore the amount of refrigeration available in the gas stream or heating is likewise decreasing. This, however, is compensated in the cooling case by the diminished heat capacity whereas in the heating case the solid heat capacity is approaching a maximum. Cooling times will therefore be less than heating times for equal mass flows and temperature differences.

If now, rather than assuming a constant mass flow a constant pressure drop across the load is assumed. The mass flow will vary inversely as the square root of the gas temperature, i.e.,

\[ m_g = \frac{k}{\sqrt{T_g}} \]  \hspace{1cm} (6)

where \( T_g \) is the mean value of the gas temperature and for simplicity we use the arithmetic average

\[ \bar{T}_g = \frac{T_{g1} + T_{g2}}{2} = \left(1 - \frac{\Delta P}{\Delta T}ight)T_{g1} + \frac{\Delta P}{\Delta T}T_s \]  \hspace{1cm} (7)

The form of (6) does not lead to an easy integration. Therefore we approximate the mass flow with

\[ m_g \approx A + \frac{B}{\bar{T}_g} \]  \hspace{1cm} (6a)

which really is not too bad over most of the temperature range. Using (2), (6a), (7) and \( C_s = C_T S \) in (1) and integrating from \( T_{g1} \) to \( T_{g2} \) and \( 0 \) to \( \bar{\omega} \) we get

\[ \int_{T_{g1}}^{T_{g2}} \frac{T_g \, dT_g}{(A + \frac{B}{\bar{T}_g})(T_{g1} - T_g)} = \frac{C_p g E}{m_g C} \int_0^{\bar{\omega}} d\bar{\omega} \]  \hspace{1cm} (8)
Which after some algebra yields

\[
\frac{ACp g \Theta}{m_c} = F(T_s, T_s, T_g)
\]  \hspace{1cm} (9)

where

\[
F(T_s, T_s, T_g) = T_s - T_s + \left[ \frac{E}{A} \right] \ln \left\{ \frac{\frac{E}{2} T_s^2 + \left[ (1-E) T_g + \frac{B}{A} \right] T_s + \left[ (1-E) T_g + \frac{B}{A} \right] T_g}{\frac{E}{2} T_s^2 + \left[ (1-E) T_g + \frac{B}{A} \right] T_s + \left[ (1-E) T_g + \frac{B}{A} \right] T_g} \right\} 
\]

\[
+ \frac{T_g^2 - \left( \frac{E}{A} \right) \frac{B}{A} T_g + \frac{E}{A} \frac{B^2}{A}}{T_g + \frac{B}{A}} \ln \left\{ \frac{E T_s^2 + (2-E) T_g + \frac{B}{A} T_g}{E T_s^2 + (2-E) T_g + \frac{B}{A} T_g} \right\} 
\]

To determine the flow constants \( A \) and \( B \) we enforce the following conditions.

at \( T_g = T_g^* \)

\[
K = A + \frac{B}{T_g^*}
\]  \hspace{1cm} (10)

and

\[
\int_{T_g^*}^{T_g^*} K \sqrt{T_g} \, dT_g = \int_{T_g^*}^{T_g^*} (A + \frac{B}{T_g^*}) \sqrt{T_g} 
\]

We find

\[
\frac{B}{A} = \frac{2(T_g^*)^2 (T_g^* - T_g)}{(T_g^* - T_g) - \ln \left( \frac{T_g}{T_g^*} \right)} - \left( \frac{T_g}{T_g^*} \right)^2 
\]

\hspace{1cm} (12)

So we see that \( \frac{B}{A} \) is a function only of temperature and

\[
\frac{A}{K} = \frac{T_g^*}{K} - \frac{2(T_g^* - T_g^*) \left( \frac{T_g^*}{T_g} \right)^{1/2} \left( T_g - T_g \right)}{\left[ \ln \left( \frac{T_g}{T_g^*} \right) - \frac{T_g^*}{T_g^*} \right]} 
\]

\hspace{1cm} (13)
For \( T_g^* = 200 \, K \), \( T_0 = 4 \, K \), \( T_3 = 290 \, K \) we get
\[
\frac{B}{A} = 64.45 \, K
\]
\[
\frac{A}{K} = 0.0536 \, K^{-1/2}
\]

For the case of constant flow eq (9) reduces to eq (5) on the substitution of \( B = 0 \).

The function \( F(T_s_1, T_s_2, T_g) \) is plotted vs the final solid temperature, \( T_s_2 \), for the case of cooling from 290 K with 4.5, 20 & 80 K gas in Fig 1. Cooling from 30K with 4.5K gas in Fig 2 and heating from 4K with 300K gas in Fig 3.

The flow constants \( A \) & \( K \) may be evaluated as follows. For a compressible, constant area flow with friction (See for instance, Venard, Elementary Fluid Mechanics 3rd Ed pg 209)

\[
P_1^2 - P_2^2 = \frac{mg^2 R T_g}{S^2} \left[ 2 \ln \frac{u_1}{u_4} + \frac{fl}{dh} \right]
\]

(14)

where \( R \) is the gas constant, \( S \) the flow area, \( u \) the flow velocity. If \( fl/dh \gg 2 \ln (u_2/u_4) \) as is generally the case we get

\[
\dot{m}_g = \sqrt{\frac{(P_1^2 - P_2^2) S^2}{(fl/dh)^2 R T_g}}
\]

(14a)

(Which is identical to the result obtained by evaluating the density at the arithmetic average pressure in the incompressible approximation.)

Therefore

\[
A = 0.0536 \, K = 0.0536 \, \dot{m}_g \sqrt{T_g}
\]

(15)
FIG 1 TEMPERATURE FUNCTION FOR COOLING
FROM AN INITIAL TEMPERATURE OF 290K
**Fig. 2** Temperature function for cooling from 30K with \( T_f = 4.5K \)
for helium
\[
\frac{A}{S} = 11.91 P_2 \sqrt{\left(\frac{P_2^2}{P_1^2} - 1\right) \frac{d_2}{d_h}} = 9/3 \text{ cm}^2 \quad (P \text{ in atm}) \quad (16)
\]

The use of Figures 1-2 & 3 along with eq (9) & (16) are demonstrated for a constant pressure drop case in the following example.

**EXAMPLE**

Determine the cooling time from 290 to 65K using 4.5K gas and the heating time from 4 to 280K using 300K gas for a 5 m long, 7800 kg cold iron magnet. Eight, 1/2 inch diameter coolant passages are provided thru the iron laminations. In each case gas is supplied at 1.3 atm and exhausted to compressor suction at 1.07 atm.

**SOLUTION**

The iron laminations are assumed to be aligned to ±0.002 inch. Therefore, the coolant passages can be treated as a rough pipe with relative roughness
\[
\frac{h}{d} = \frac{(2)(0.002)}{0.5} = 0.008
\]

which results in a friction factor of 0.035 and which is very nearly independent of Reynolds number. Further, we will ignore the velocity head loss at the exit. Then
\[
\frac{dL}{d} = \frac{(0.035)(5)(3.28)(12)}{0.5} = 13.8
\]

From (16)
\[
\frac{A}{S} = (11.91)(1.07) \sqrt{\left(\frac{1.27^2}{1.07^2} - 1\right)/13.8} = 2.37
\]

and
\[
A = (2.37)(\frac{4}{1})(1.27^2) = 24.0 9/3
\]
FIG. 3 TEMPERATURE FUNCTION FOR HEATING FROM AN INITIAL TEMPERATURE OF 4 K WITH 300 K GAS 0.1 < E < 1.0
The iron heat capacity is plotted below vs temperature.

The heat capacity curve is approximated by the dashed line which is determined so that the two curves have equal areas (enthalpy) in the interval 0 - 300 K. Therefore \( C = 1.72 \times 10^{-3} \text{ J/}^\circ\text{K}^2 \)

From Eq 1
\[ E = 0.5 \quad F_{\text{cooling}} = 106K \]
\[ E = 1.0 \quad F_{\text{cooling}} = 148K \]

From Fig 3
\[ 0.1 < E < 1.0 \quad F_{\text{heating}} = 430K \]

From Eq (9) we find for cooling with a heat exchanger effectiveness of 0.5
\[ \theta_{\text{cooling}} = \frac{106 \times (7500 \times 10^3) \times (1.72 \times 10^3)}{(24.0)(5.2)(1.5)} = 21900s = 6.1 \text{ hr} \]
or for $\epsilon = 1$.

$$\theta_{\text{cooling}} = \frac{(14.8)(7500 \times 10^{-3})(1.72 \times 10^{-3})}{(24)(5.2)(1.)} = 15300 \text{ s} = 4.3 \text{ hr}$$

For the heating case, since $F_{\text{heating}}$ is nearly independent of $\epsilon$, the heating time is inversely proportional to $\epsilon$, so from eq (9)

$$\epsilon \theta_{\text{heating}} = \frac{(1.30)(7500 \times 10^{-3})(1.72 \times 10^{-3})}{(24.0)(5.2)} = 44,450 \text{ s} = 12.4 \text{ hr}$$

So that

$$\epsilon = 0.5, \quad \theta_{\text{heating}} = 24.8 \text{ hr}$$

$$\epsilon = 1.0, \quad \theta_{\text{heating}} = 12.4 \text{ hr}$$

For the same pressure drop the heating time is seen to be much greater than the cooling time. However, much higher flows are generally available with warm gas taken for the cold gas supplied by a refrigerator. The pressures chosen for this example are typical of that available downstream of the JT valve on a refrigerator. If the warm up time is critical pressures up to typically 13 atm are available from the refrigerator compressor. For example assume that $P_i$ is 3 atm in the heating case above, then $A$ becomes 91.0 g/s and the heating times become 6.5 and 3.3 hr for $\epsilon = 0.5$ or 1.0 respectively.

Another example in the use of this method is given in M587.
This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Department of Energy to the exclusion of others that may be suitable.