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Neutral Current Phenomenology
of
Supersymmetric SU(2) \times U(1) \times \tilde{U}(1) Models

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Abstract

The neutral current sector of a class of supersymmetric
SU(2) \times U(1) \times \tilde{U}(1) models is parametrized. Bounds on the neutral
boson masses are obtained from the low energy data, and the
implications of future experimental findings for these models
are discussed.

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In some models with spontaneously broken supersymmetry, a new $\hat{U}(1)$ gauge symmetry is introduced to give large masses to the unseen scalar partners of leptons and quarks [1, 2]. In addition to the requirements on theoretical consistency such as the correct vacuum structure and anomaly-freedom, model-building of this sort is subject to phenomenological constraints, as the modified neutral current structure may spoil the success of the standard model.

It is the purpose of this paper to perform the same type of phenomenological analysis on a class of supersymmetric $SU(2) \times U(1) \times \hat{U}(1)$ models as the variously motivated multi-boson electroweak models of the past [3]. Bounds on the masses of the neutral bosons are obtained from the data for neutrino scattering, electron-nucleon interactions, and electron-positron annihilation.

Any supersymmetric $\hat{U}(1)$ model with a realistic mass spectrum should possess the following minimum features:

(I) The axial part of the $\hat{U}(1)$ charge of a 4-component fermion is greater in magnitude than its vector part. This allows the fermion to be lighter than both of its scalar partners.

(II) There are at least two Higgs isodoublets acquiring non-zero VEV's to give mass to both charge $\frac{2}{3}$ and $-\frac{1}{3}$ quarks through Yukawa terms at the tree level.

A simple supersymmetric generalization of the standard model particle assignment satisfies (I); left-handed matter supermultiplets consist of $Q_L = (U_L, D_L)$, $\overline{U}_R$, $\overline{D}_R$, $L_L = (N_L, E_L)$, $\overline{E}_R$, all of which have the same $\hat{U}(1)$ charge, $\frac{y}{2}$. Quarks and charged leptons then have the identical, purely axial $\hat{U}(1)$ charge, $\tilde{y}$, and $\nu$ has a V-A coupling, $\frac{\tilde{y}}{2}$, to $\hat{U}(1)$. Any extension of this minimal fermionic structure, leading to a proliferation of parameters, finds little theoretical motivation in most
models, and is not considered here [F1].

The simplest Higgs structure satisfying (II) consists of two Higgs isodoublets \( \phi_1 \) and \( \phi_2 \), whose \( U(1) \) charges are \( \frac{1}{2} \) and \( -\frac{1}{2} \), and whose neutral components obtain the same VEV. The \( U(1) \) sector then remains detached from \( SU(2) \times U(1) \). It was observed by Fayet [6], however, that given the fermionic \( U(1) \) couplings outlined above, this minimal Higgs structure leads to unacceptable values for axial parameters in neutrino scattering.

We seek extension of this minimal model in two ways [F2]. First, we allow mixing between \( U(1) \) and \( SU(2) \times U(1) \) by having \( \langle \phi_1 \rangle = \begin{pmatrix} 0 \\ h_1 \end{pmatrix} \), \( \langle \phi_2 \rangle = \begin{pmatrix} h_2 \\ 0 \end{pmatrix} \) with \( h_1 \neq h_2 \) in general. This defines the Minimal Mixing Model (MMM) which is probably the most economical supersymmetric \( U(1) \) model that can be made consistent with the present data. Secondly, we allow Higgs fields \( \phi_i^0 \) \( (i = 1 \to N) \) which are singlets under the standard gauge group and have \( U(1) \) charges \( \gamma^i_0 \), to obtain VEV's \( h_i^0 \).

This is the situation often found in actual models [2]. We call the class of models with both mixing and \( \phi_i^0 \) the Extended Mixing Model (EMM).

Now we parametrize the neutral current sector of EMM, of which MMM is a special case. The neutral gauge boson mass squared matrix is:

\[
M^2 = \frac{1}{4} \begin{pmatrix}
\bar{g}^2 & -g g_2 & -g g e \\
-g g_2 & g_2^2 + g^2 e^2 & -g g e \\
-g g e & -g g e & g^2 t
\end{pmatrix}
\]

where \( \bar{g} = 2\gamma^0 \), \( \epsilon = (h_1^2 - h_2^2)/(h_1^2 + h_2^2) \), \( t = 1 + \frac{\gamma^0 h_0^0}{\gamma^2 (h_1^2 + h_2^2)} \), and \( g, g_2, \bar{g} \) are the couplings and \( B^\mu, W^\mu_3, \tilde{B}^\mu \) the boson fields associated with \( U(1), SU(2), \tilde{U}(1) \), respectively.

The mass eigenstates are written as follows:
\[
\begin{pmatrix}
A^\mu \\
Z^\mu \\
\gamma^\mu
\end{pmatrix}
=
\begin{pmatrix}
\cos \theta & \sin \theta & 0 \\
-c\cos \alpha \sin \theta & c\cos \alpha \cos \theta & -s\sin \alpha \\
s\sin \alpha \sin \theta & -s\sin \alpha \cos \theta & c\cos \alpha
\end{pmatrix}
\begin{pmatrix}
B^\mu \\
W_3^\mu \\
B_\gamma^\mu
\end{pmatrix}
\]  

where \( \sin^2 \theta = g^2 / (g_1^2 + g_2^2) \) is the standard model mixing parameter, and \( \cos^2 \alpha = \frac{1}{2} \{ 1 + [ 1 + 4A \epsilon^2 (1 - A \tau)^2 ]^{-1/2} \} \), \( A = -g^2 / (g_1^2 + g_2^2) \).

One of the eigenvalues of the matrix (1) is of course zero, corresponding to the photon, and the other two are given as follows:

\[
\begin{pmatrix}
\tilde{\rho} \\
\tilde{\gamma}
\end{pmatrix} = \frac{1}{2} \{ 1 + A \tau \mp (1 - A \tau) [ 1 + 4A \epsilon^2 (1 - A \tau)^2 ]^{1/2} \}
\]

where \( \rho = \frac{m_2^2 \cos^2 \theta}{m_w^2} \), \( \gamma = \frac{m_2^2 \cos^2 \theta}{m_w^2} \), i.e., mass squared in units of the standard model Z mass squared. Notice that \( (1 - \rho)(1 - \gamma) \leq 0 \), which satisfies the Georgi-Weinberg theorem [7], although its premise is not met.

There are six independent parameters for EMM: \( g, g_2, \gamma, h_1, h_2, t \). Apart from \( \epsilon = gg_2 / (g_1^2 + g_2^2)^{1/2} \), \( G_F = 1 / [\sqrt{2} (h_1^2 + h_2^2)] \), we choose to work with the following four free parameters: \( \sin^2 \theta \), as in the standard model, and \( A \geq 0, |\epsilon| < 1, t > 1 \), which measure the relative strength of the \( \mathcal{U}(1) \) coupling, the degree of mixing between the two massive neutral bosons, and the isosinglet Higgs contribution to boson masses, respectively.

In order to compare the predictions of these models with the data, we make use, as usual, of low energy model independent parameters [8] defined in the following processes.

(i) Neutrino interactions:

\[
\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \bar{\nu} \gamma^\alpha (1 - \gamma_5) \nu \tilde{\nu}_\alpha (g_1^f + g_2^f \gamma_5) f,
\]

where \( f = e, u, \text{ or } d \).
(ii) Asymmetry ($A_{eD}$) in electron-deuteron scattering:

$$\frac{A_{eD}}{Q^2} = -\frac{9 \cdot G_F}{20\sqrt{2} \pi \alpha_{EM}} \left[ a + \frac{1 - (1 - y)^2}{1 + (1 - y)^2}, b \right].$$

Atomic experiments provide further, if not complete, information on the four parameters describing parity-violating electron-quark reactions, but are not considered here.

The expressions for the following eight combinations of the above model independent parameters have been derived in terms of the model parameters, and listed in Table 1:

$$g^e_V, g^e_A, \alpha \equiv g^u - g^d, \beta \equiv g^u - g^d, \gamma \equiv g^u + g^d, \delta \equiv g^u + g^d, a, c \equiv a + b.$$ 

Also shown are the corresponding standard model expressions, and the data from various neutrino interactions and SLAC eD experiments.

We notice from Table 1 that $A$ does not enter into these expressions, since they are, in the zero momentum transfer limit, functions of $\frac{g^2}{m^2}$ which is independent of $\gamma$. Information on the strength of the $U(1)$ coupling will come from $e^+e^- \rightarrow \mu^+\mu^-$ at PETRA energies.

In fitting the model parameters $\sin^2 \theta$, $\epsilon$, $t$ to data, we first consider MMM ($t = 1$). That non-zero mixing is needed in this case as mentioned earlier is seen immediately from the expressions for $g^e_A$ and $\delta$ (the isoscalar axial coupling); for $\epsilon = 0$, they are more than 4 $\sigma$'s off. The other neutrino parameters (vector, and isovector axial) and the eD parameters in this non-mixing case are the same as in the standard model, since the $e$, $u$, $d$ $U(1)$ neutral currents are purely axial.

It is not a priori clear that there should exist a region in the parameter space consistent with the data, since for no value of $(A, \sin^2 \theta, \epsilon)$ MMM reduces to the standard model. However, the best fit gives $\epsilon = -.44$, $\sin^2 \theta = .295$, for which agreement with the data is good.
for all the eight parameters, as shown in the last column of Table 1.

The location of the best fit is not surprising, since for $\varepsilon = -\frac{1}{2}$ all $\nu$ parameters are reduced to their standard model expressions, and the change in $\nu$D parameters is compensated by a larger value for $\sin^2 \theta$ than in the standard model, as seen in Fig. 1. This value of $\sin^2 \theta$ may not be a problem in view of the absence of a realistic supersymmetric GUT including $\tilde{U}(1)$.

If some isosinglet Higgs obtain VEV's (EMM), for large $t$ the standard model is asymptotically recovered, as in models where the scale of the $\tilde{U}(1)$ breaking is made large compared to $O(100)$ GeV. The best fit for EMM parameters to data gives $t$ of the order of a few hundred, which corresponds to $h_0^i \sim h_1,2 \times O(10)$, unless $N$ is large.

This value of $t$ may seem too large in view of the theoretical prejudice regarding naturalness. However, the value of $t$ is very sensitive to data, and it turns out that the current data can be accommodated for a wide range of $t$ (including $t \sim 5$) as well as or better than in the standard model. The present experimental accuracy thus places virtually no useful constraint on the allowed range of $t$. It should be noted that for smaller $t$ the allowed range of $\varepsilon$ becomes more restricted, which has implications for the upper bound on the lower boson mass.

As an illustration of these points, we consider the case where $\varepsilon = -\frac{1}{2}$ for which all the $\nu$ parameters are the same as in the standard model, whereas $\nu$D parameters depend on an additional parameter $t$. In this respect, this model resembles the model of Deshpande and Iskandar [9]. Taking the one-parameter fit to the $\nu$ data from previous analyses [8], $\sin^2 \theta = 0.239 \pm 0.010$, we find $t = 7.85 \pm 12.9$ from the best fit to SLAC $\nu$D data. Fig. 2 shows the sensitivity of $t$ to data.
Since $e$ and $\mu$ have axial $\mathbb{U}(1)$ charges, the exchange of a new gauge boson contributes to the forward-backward asymmetry ($A^{\mu\mu}$) in $e^+e^-\rightarrow \mu^+\mu^-$. The interference terms between the photon and the massive boson exchanges give, to lowest order,

$$A^{\mu\mu} = -\frac{3\sigma}{32\sin^2\theta\cos^2\theta} \left\{ \frac{(1 + t + 2\epsilon) - \sigma}{t(1 - \sigma) - \epsilon^2} + \frac{1}{[t(1 - \sigma) - \epsilon^2]A - (\sigma - \sigma^2)} \right\} ,$$

where $\sigma = \frac{s\cos^2\theta}{m_w^2}$. 

The dependence on $A$ is isolated in the second term in braces, which is always positive for the CM energy ($\sqrt{s}$) less than the smaller boson mass, including where the above approximation is valid. The minimum of $|A^{\mu\mu}|$ (corresponding to $A \rightarrow \infty$) is larger than $|A^{\mu\mu}|$ in the standard model for the same value of $\sin^2\theta$. For the values of the MMM parameters determined earlier, and for $\sqrt{s} = 34$ GeV, (4) becomes

$$A^{\mu\mu} = -(.117 + \frac{.005}{A - .226}) .$$

While some experimental groups (CELLO [10], MARK J [11]) reported data consistent with the standard model prediction, other groups (TASSO [12], JADE [13]) find the asymmetry considerably more negative. In the former case, the errors are not small enough to rule out MMM, and $A$ will have a lower bound. In the latter, if we take the TASSO data ($A^{\mu\mu} = -.16 \pm .03$), for example, the bounds on $A$ are:

$.29 \leq A \leq .61$, which corresponds to $43$ GeV $\leq m_z \leq 53$ GeV, and $85$ GeV $\leq m_z \leq 90$ GeV. Fig. 3 shows the minimum $|A^{\mu\mu}|$ for MMM, along with the data from various groups.

For MMM, any appreciable deviation ($\Delta$) from the standard model pre-
prediction for \(A^{\mu \nu}\) favors small \(t\) \((< 5)\). This is because for large \(t\),
\[
A \sim \frac{\text{const.}}{t^2 \Delta} + \frac{\sigma}{t},
\]
so that unless \(\Delta\) is very small, \(A \sim \frac{\sigma}{t}\),
which implies that the current CM energy is very close to the lower
boson mass. Thus the version of EMM given before \((\varepsilon = -\frac{1}{2})\), for example, would be ruled out by the TASSO or JADE data. If small \(t\) is
indeed preferred, large mixing \((\varepsilon)\) could put a nontrivial upper bound
on the lower boson mass, since \(\rho < 1 - \frac{\varepsilon^2}{t}\).

The contribution to \((g - 2)/2\) of the muon from the two neutral
bosons in EMM is given by:
\[
\frac{m^2_{\mu} G_F}{24\sqrt{2} \pi^2} \frac{1}{t - \varepsilon^2} [(1 - 4 \sin^2 \theta) t - 5(1 + t - 2\varepsilon)].
\]  
(6)

For EMM no useful bound on \(t\) is obtained, again because of the sen-
sitivity of \(t\) to experimental errors. For MMM, in a wide range of \(\varepsilon\)
including the best fit from low energy data quoted earlier, (6) is
more negative and cancels more of the charged weak current contribution
(which has the opposite sign) than in the standard model. Thus no useful
constraints on the model parameters are obtained from the current data
for the muon anomalous magnetic moment.

In conclusion, the large mixing (both \(\varepsilon\) and \(\sin^2 \theta\)) required in
MMM places the upper bound of 74 GeV on the mass of the lighter boson.
Depending on which \(e^+e^- \rightarrow \mu^+\mu^-\) asymmetry data we take, the prediction
for the lower boson mass can be as low as 43 GeV. If a neutral
boson does not turn up in the upcoming pp experiments until around
90 GeV, MMM is probably ruled out.

As for EMM, if the asymmetry is indeed considerably larger than
predicted in the standard model, the isosinglet contribution to the
boson mass does not dominate. This further implies that the mixing (\(\varepsilon\))
would have to be small if a low-mass boson is not found. On the other hand, if the asymmetry is not large, EMM survives for a wide range of $t$, regardless of the location of the lightest boson.

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REFERENCES


Note our axial parameters have the opposite sign from theirs.


FOOTNOTES

F1: In fact, there are arguments suggesting that the scalar partners of u and d (pairwise) \[4\], and also of different generation copies \[5\] are almost degenerate in mass. They support in part the minimal fermionic structure considered here.

F2: Fayet \[6\] considered isosinglets and vector fermionic U(1) couplings, and made observations relevant to the case with no mixing, based on the neutrino and eD data.
Table 1.

<table>
<thead>
<tr>
<th>Model Independent Parameters</th>
<th>SU(2) x U(1) x U(1)</th>
<th>Standard Model</th>
<th>Data [8]</th>
<th>MMM Best Fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g^e_V$</td>
<td>$-\frac{1}{2}(1-4x)\frac{t+\epsilon/2}{t-\epsilon^2}$</td>
<td>$-\frac{1}{2}(1-4x)$</td>
<td>0.06 ± 0.08</td>
<td>0.09</td>
</tr>
<tr>
<td>$g^e_A$</td>
<td>$\frac{1}{2} \frac{t-(1+\epsilon)/2}{t-\epsilon^2}$</td>
<td>$\frac{1}{2}$</td>
<td>0.52 ± 0.06</td>
<td>0.45</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$(1-2x)\frac{t+\epsilon/2}{t-\epsilon^2}$</td>
<td>$1-2x$</td>
<td>0.589 ± 0.067</td>
<td>0.396</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$-\frac{t+\epsilon/2}{t-\epsilon^2}$</td>
<td>$-1$</td>
<td>-0.937 ± 0.062</td>
<td>-0.967</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$-\frac{2}{3}x\frac{t+\epsilon/2}{t-\epsilon^2}$</td>
<td>$-\frac{2}{3}x$</td>
<td>-0.273 ± 0.081</td>
<td>-0.190</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$-\frac{1/2+\epsilon}{t-\epsilon^2}$</td>
<td>0</td>
<td>-0.101 ± 0.093</td>
<td>-0.074</td>
</tr>
<tr>
<td>$a$</td>
<td>$(1-\frac{20}{9}x)\frac{t-\epsilon}{t-\epsilon^2}$</td>
<td>$1-\frac{20}{9}x$</td>
<td>0.60 ± 0.16</td>
<td>0.62</td>
</tr>
<tr>
<td>$c$</td>
<td>$(1-\frac{20}{9}x)\frac{t-\epsilon}{t-\epsilon^2}$ + $(\frac{1}{4}-x)\frac{t+\epsilon/3}{t-\epsilon^2}$</td>
<td>$\frac{5}{4}-\frac{29}{9}x$</td>
<td>0.53 ± 0.05</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Note: $x \equiv \sin^2 \theta$. 
FIGURE CAPTIONS

Fig. 1: Regions in the $c - \sin^2 \theta$ plane consistent with data for $\nu$ scattering (I) and SLAC eD experiments (II). The best fit to MMM is also shown.

Fig. 2: The shaded region in the $\sin^2 \theta - t$ plane is allowed by eD data to within one standard deviation for EMM with $c = -\frac{1}{2}$. The best fit for $t$ as a function of $\sin^2 \theta$ is also shown.

Fig. 3: Predictions for the asymmetry in the standard model and in MMM. The shaded region is allowed by MMM, and the solid line corresponds to the minimum $|A^{1\mu}|$. Also shown are the data from various groups.
Fig. 1
Fig. 2
Fig. 3
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