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SOFT-GLUON EFFECTS IN SEMILEPTONIC DECAYS OF CHARMED F MESONS

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ABSTRACT

The QCD multipole expansion is applied to the study of nonperturbative soft-gluon effects in semileptonic decays of charmed F mesons. For reasonable values of F-meson bound-state parameters, the soft-gluon effects activate quark-annihilation processes, leading to a significant enhancement of semileptonic decay rates.

Recent experimental observations [1], such as \( \Gamma(D^0) < \Gamma(D^+) \) and \( \text{BR}(D^0 \to K^\pm \pi^\mp) \gtrsim \text{BR}(D^0 \to K^0\pi^0) \), indicate significant enhancements of nonleptonic decays of \( D^0 \) mesons and require a revision of the simple picture of charmed-meson decays based on charm-quark decay mechanisms alone. Several explanations for the nonleptonic enhancements [2] have been proposed within the framework of quantum chromodynamics (QCD). In particular, in D- and F-meson decays, QCD effects are considered to activate W-boson exchange processes ("quark-annihilation" processes), as shown in Fig. 1b, which by themselves are strongly suppressed because of helicity mismatch and the small probability for quark-pair annihilation in the meson. Hard-gluon emission (short-distance effect) [3-5] and soft-gluon emission (long-distance effect) [4,6] from \( D^0 \) and \( F^+ \) mesons indeed remove helicity suppression factors and enhance the total decay rates of these mesons; the soft-gluon effect seems to substantially exceed the hard-gluon effect [6].

Unlike the \( D^0 \) meson, the \( F^+ \) meson possesses annihilation channels into leptons. Semileptonic decays of the \( F^+ \) meson therefore provide an important clue to the dynamical mechanisms that govern charmed-meson decays, especially to the working of quark-annihilation processes. Accordingly it is of importance to ask how large the semileptonic enhancements will be on the basis of QCD. The purpose of this paper is to examine this problem by a dynamical calculation of the soft-gluon effect in \( F^+ \)-meson decays. We evaluate the nonperturbative soft-gluon effect in terms of the gluonic vacuum condensate

\[ \mathcal{V} = (0 | (a_s/\pi) F_{\mu\nu}^2 [A] | 0) \approx 0.012 \text{ GeV}^4 \] (1)

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known phenomenologically from the charmonium sum rules of Shifman, Vainshtein and Zakharov [7].

As illustrated in Fig. 2, $\phi^+$ annihilation into simileptonic channels needs at least two gluons forming a color singlet. This is in sharp contrast with the nonleptonic process in Fig. 1b, where the hard-gluon correction to the $W$-exchange induces a color-octet $\bar{c}s$-annihilation channel [8]. The emitted gluons may be soft and/or hard gluons. In what follows, we consider only the soft-gluon case, which, as indicated by the previous analysis of nonleptonic decays [6], is expected to be the dominant one.

We treat the $\phi^+$ meson as a nonrelativistic bound state of $c$ and $\bar{s}$ quarks of mass $M = m_c \approx 1.65$ GeV and $m = m_\bar{s} \approx 0.54$ GeV. This $\bar{c}s$ pair is surrounded by a cloud of gluons (as well as light quarks) to form the $\phi^+$ meson. [We call these gluons soft gluons.] The spatial spread of this soft-gluon cloud will be of the order of $1/(100 - 200 \text{ MeV})$, typical spatial spread of ordinary hadrons (or a scale characterized by color confinement). On interaction with the soft-gluon cloud, the $\bar{c}s$ pair changes its color (i.e., $1\leftrightarrow 8$ or $8\leftrightarrow 8$). With this picture in mind, we describe the $\phi^+$-meson state $|\phi^+\rangle$ as a color-singlet, $1S_\text{o}$ $\bar{c}s$-pair $|\bar{c}s\rangle$ surrounded by a color-singlet, $0^+$ soft-gluon cloud (of lowest energy) $|0\rangle$:

$$|\phi^+\rangle = |\bar{c}s\rangle \otimes |0\rangle,$$  \hspace{1cm} (2)

as illustrated in Fig. 3. To a good approximation the gluon cloud $|0\rangle$ may be regarded as the gluonic vacuum, since $|0\rangle$ consists of soft-gluon fluctuations (of vacuum quantum numbers) extending over the typical hadronic size. The soft-gluon cloud is governed by long-distance dynamics of QCD. In contrast, $c\bar{s}$-pair annihilation by the weak current is a short-distance phenomenon characterized by the $c$-quark Compton wavelength $1/M$. Correspondingly, it is a reasonable approximation to assume that the soft gluons emitted (or absorbed) by the $\phi^+$ meson are predominantly coupled to the gluon cloud $|0\rangle$ while the $c\bar{s}$ pair is coupled to the weak current. Through this factorization procedure, the soft-gluon effects are related to the vacuum matrix elements of gluon (and light-quark) operators. The emitted soft gluons eventually turn into light hadrons.

The idea of the multipole expansion [10-12] is useful for the description of soft-gluon interactions. As an illustration, let us consider Fig. 2a. The gluon fields at $\bar{y}_1$ and $\bar{y}_2$, being soft, can be expanded in multipoles around the $\bar{c}s$-annihilation position $\bar{z}$. The $c$-quark field at $\bar{y}_1$ may also be expanded in multipoles around $\bar{z}$, since the quark motion is assumed to be nonrelativistic. By this procedure, soft-gluon emission combined with quark-annihilation is cast into a series of local interactions at $\bar{z}$. It is necessary to sum a certain set of diagrams to bring the multipole interactions into manifestly gauge-invariant form. Alternatively, this resummation is done at the Lagrangian level by means of a suitable gauge transformation [11,12], which is defined as [12]

$$(A_\mu(x), c(x), s(x)) \rightarrow (A'_\mu(x), c'(x), s'(x))$$.
\[ A_0' (\tilde{x}) = A_0 (\tilde{u}) + \sum_{n=0}^{\infty} \frac{1}{(n+1)!} (r \cdot \gamma)^n r^n F_{k0} \{ A (\tilde{u}) \} , \]

\[ A_k' (\tilde{x}) = \sum_{n=0}^{\infty} \frac{1}{n! (n+1)!} (r \cdot \gamma)^n r^n F_{k0} \{ A (\tilde{u}) \} , \]

\[ c' (\tilde{x}) = [1 + r \cdot v + \frac{1}{2} (r \cdot v)^2 + \ldots ] c (\tilde{u}) , \text{ etc.} \]

where \( \tilde{x} = \tilde{r} + \tilde{u} \), \( \tilde{u} \) is a fixed position, \( F_{\mu \nu} \{ A \} = \partial_\mu A_\nu - \partial_\nu A_\mu + g [ A_\mu \times A_\nu ] \), \( v_k = v_k [ A ] \) and \( D_k = D_k [ A ] \) are the ordinary covariant derivatives for the gluon and the quark, respectively, defined at position \( \tilde{u} \), and \( r \cdot v = r^k v_k \), etc. Here and in what follows, all the fields are defined at common time \( t \), which is suppressed.

We evaluate the processes in Fig. 2 using the old-fashioned perturbation theory with \( \tilde{u} \) chosen to be the \( c \bar{s} \)-annihilation position \( \tilde{x} \) in the gauge transformed interaction Hamiltonian. Let us go to momentum space to perform the integration over each soft-gluon position: A momentum carried by a gluon field \( A_{\mu}' \) corresponds to a spatial derivative acting on it in \( x \)-space; the derivatives of \( A_{\mu}' \) are then rewritten in terms of \( F_{\mu \nu} \{ A \} \) and \( v \{ A \} \) at \( \tilde{x} \) by use of eq. (3).

The color-Coulomb interaction does not act on color-singlet \( c \bar{s} \) systems. Terms linear in the momentum of each gluon are converted into color-\( E \) or color-\( M \) interactions. We ignore higher multipoles as well as the momenta of the initial \( c \) and \( \bar{s} \) quarks. The resulting effective Hamiltonian is given by

\[ H_{SL} = \int d^3 z \frac{1}{4} \left( \frac{2^2}{\Delta t_1 \Delta t_2} \right) \left[ (H_1^2 (z) + H_2^2 (z)) \{ L, \overline{L} \} + \text{h.c.} \right] , \]

\[ H_{1}^2 = \bar{s} \cdot [ H^2, L \cdot \bar{L} \cdot 2 + m_1^2 \bar{A}^2 ] + 2 \alpha m_0 \bar{A} \bar{L} , \]

\[ H_{2}^2 = \bar{s} \cdot [ H^2, L \cdot \bar{L} \cdot 2 \alpha \bar{A} \bar{L} + m_1^2 \bar{A}^2 ] , \]

\[ L^\mu = \cos \theta (4G/\sqrt{2}) \bar{v} v_{\mu \bar{L}} , \]

where \( \gamma^\mu \bar{L} = \frac{1}{2} \bar{L} (1 - \gamma_5) \). In the leptonic weak current \( L^\mu \), \( L^D \) is helicity-suppressed. The energy denominators associated with the first and the second intermediate states have been replaced by their typical average values \( \Delta t_1 \) and \( \Delta t_2 \), respectively; we discuss this point later. Spatial derivatives \( \tilde{k} \) acting on \( A_{\mu}' (\tilde{x}) \) are involved in \( \Lambda \) and \( \phi^k \):

\[ \Lambda = (\tilde{k} A') \cdot T = \gamma_0 (E + i \gamma_5 \bar{L}) \cdot T , \]

\[ \phi^k = (\tilde{k} A') \cdot T = \gamma_0 \bar{E}^k + \frac{1}{2} \gamma_5 \bar{E}^k \cdot T , \]

where \( \bar{E} = \frac{1}{2} \gamma (A (z)) \), \( \bar{E}^k = - \frac{1}{2} \gamma \bar{E} = \frac{1}{2} \gamma \bar{E} \), etc, and \( T^a = \frac{1}{2} \lambda^a \) is the color matrix for the quark.

The hadronic weak current \( H_2 \) derives from the quark-energy terms in the second intermediate states in Fig. 2a and 2b. Its physical interpretation is the following: On emission of a color-\( E \) gluon, the initial \( S \)-wave \( c \bar{s} \) state is turned into a \( P \)-wave state, which subsequently emits another gluon through the \( L \)-dependent
part of the color-Ml interaction \( \frac{1}{2} (g/\Lambda)(\hat{L} + 2\hat{S})\cdot\hat{n}a\hat{L} \), etc., with \( \hat{L} \) being the angular momentum of the quark motion.

As explained earlier, we write the decay amplitude in factorized form, e.g.,

\[
\langle 0 | H_{1}^{1}(\hat{O}) + H_{2}^{1}(\hat{O}) | f^{+} \rangle > = \frac{1}{2} \frac{h_{F}^{f}}{m_{F}} \left[ 1 + \frac{1}{4} \left( \frac{N}{m} + \frac{M}{N} \right) \right] \langle 0 | (\hat{n}^{a}(\hat{O}) \cdot \hat{n}^{b}(\hat{O})) | f^{+} \rangle 0 ,
\]

where \( \langle 0 \rangle \) denotes the soft-gluon final state; and, as usual, c\( \bar{c} \)-pair annihilation into the short-distance vacuum \( | 0 \rangle \) has been expressed in terms of the F-meson decay constant \( f_{F} \) (defined in the same convention as \( f_{\pi} \approx 140 \) MeV for the charged pion). The first term in the square bracket comes from \( H_{1}^{1} \). Note the structure of the soft-gluon matrix element.\(^{(2)}\) Color-El and color-Ml gluons combine to form a color-singlet, vector soft-gluon final state which is distinct from the gluonic vacuum \( | 0 \rangle \).

The decay rate involves a sum over the final-state phase space. As done before \([6]\), we approximate the sum over soft gluons as follows

\[
\sum_{G} \langle G | (\Delta \epsilon_{1} \Delta \epsilon_{2})^{-2} | G \rangle \sim (\Delta \epsilon_{1} \Delta \epsilon_{2})^{-2} ,
\]

where the right-hand side is understood to be inserted between soft-gluon states. Namely, the matrix elements of soft-gluon operators are assumed to be predominantly saturated by soft-gluon states.

(Note that the energy denominator \( \Delta \epsilon_{1} \Delta \epsilon_{2} \) tends to suppress hard-gluon states.) Thus the soft-gluon sum leads to the vacuum matrix element

\[
\langle f^{+} | (\hat{n}^{a}(\hat{O}) \cdot \hat{n}^{b}(\hat{O})) | f^{+} \rangle 0 .
\]

This matrix element is not known phenomenologically. However, with the usual factorization hypothesis \([7]\) (or equivalently, with the saturation of \( \langle f^{+} \rangle \) by the vacuum state only), \( \langle f^{+} \rangle \) is related to \( \langle 0 \rangle \) in eq. (1) so that\(^{(3)}\)

\[
\langle 0 | \langle f^{+} | (\hat{n}^{a}(\hat{O}) \cdot \hat{n}^{b}(\hat{O})) | f^{+} \rangle 0 ,
\]

\[
\langle f^{+} | (\hat{n}^{a}(\hat{O}) \cdot \hat{n}^{b}(\hat{O})) | f^{+} \rangle 0 ,
\]

\[
\langle f^{+} | (\hat{n}^{a}(\hat{O}) \cdot \hat{n}^{b}(\hat{O})) | f^{+} \rangle 0 .
\]

The \( \bar{c} \) pair is a vector or an axial-vector \( (1_{F}^{1} \text{ or } 3_{F}^{1}) \) in the intermediate states; it is a color octet (color singlet) in the first (second) intermediate state. This spin and color structure indicates that these intermediate states have higher energy than the initial \( F^{+} \) meson state. Estimates of the energy differences \( \Delta \epsilon_{1} \) and \( \Delta \epsilon_{2} \) may be made, e.g., from the \( 3_{F}^{1} - 1_{F}^{0} \) fine structure of the \( F \)-meson system:

\[
\Delta \epsilon_{1} - \Delta \epsilon_{2} \sim M_{F}^{a} - M_{F}^{b} \sim 110 \text{ MeV},
\]

or from the "binding energy" of the \( F^{+} \) meson:

\[
\Delta \epsilon_{1} - \Delta \epsilon_{2} \sim M + m - M_{F} \approx 160 \text{ MeV}.
\]

These are crude guesses. It will, however, be certain that \( \Delta \epsilon_{1} \) and \( \Delta \epsilon_{2} \) are of the order of \( 100 \sim 200 \text{ MeV} \). This is because the \( F^{+} \) meson,
being a bound state of light and heavy quarks, has roughly the same spatial spread as ordinary hadrons. On the other hand, low-lying heavy-quark bound states such as $\psi$ and $\Upsilon$ have spatial spread substantially smaller than the size of the soft-gluon cloud surrounding them; color fluctuations of such tightly-bound states will therefore be associated with large energy differences (e.g. of the order of $2M_D - M_\psi \sim 600$ MeV for $\psi$).

The c-quark decay process Fig. 1a leads to equal semileptonic decay rates for $F^+, D^0$ and $D^+$ mesons:

$$r^C(\ell) = \cos^2 \theta_C \frac{G_F^2 M^5}{192\pi^3}.$$  \hspace{1cm} (12)

The QCD corrections [13, 14] and the phase-space correction reduce $r^C(\ell)$ considerably. On the other hand, the present soft-gluon effect activates $F^+$-meson annihilation into a semileptonic channel (e or $\mu$), with the decay rate

$$r^{SB}(\ell) = \cos^2 \theta_C \frac{G_F^2 M^3}{64\pi} \left( \frac{\pi}{6} \right)^4 \left[ 1 + 4 \frac{m}{M} + \frac{m^2}{M^2} \right] \frac{V}{m^2 \Delta \ell_1 \Delta \ell_2}.$$  \hspace{1cm} (13)

where $V$ stands for the vacuum condensate eq. (1). Therefore,

$$\frac{r^{SB}(\ell)}{r^C(\ell)} = \frac{n^6}{432} \left( \frac{f_F}{M} \right)^3 \left( \frac{\Delta \ell_1 \Delta \ell_2}{\Delta \ell_1 \Delta \ell_2} \right)^2 \left[ 1 + 4 \frac{m}{M} + \frac{m^2}{M^2} \right] \frac{V}{m^2 \Delta \ell_1 \Delta \ell_2}.$$  \hspace{1cm} (14)

An empirical scaling law for the observed decay rates of vector mesons into a lepton pair gives an estimate of $f_F$ [5]:

$$f_F/\sqrt{2} \sim 200 \text{ MeV}.$$  \hspace{1cm} (15)

With this value for $f_F$ and $(\Delta \ell_1 \Delta \ell_2)^{1/2} \sim 140$ MeV(200 MeV), one gets

$$\frac{r^{SB}(\ell)}{r^C(\ell)} \sim 3 \left( 0.8 \right).$$  \hspace{1cm} (16)

This result indicates that the soft-gluon effect significantly enhances the semileptonic decays of the $F^+$-meson.

The numbers in (16), being sensitive to the unknown parameters $f_F$, $\Delta \ell_1 \Delta \ell_2$, $W$, etc., should be taken to give an order-of-magnitude estimate. They, nevertheless, are generally sizable for a reasonable range of these parameters. The above qualitative result will therefore, we believe, survive more elaborate analyses. It would be interesting to estimate $f_F$, $\Delta \ell_1 \Delta \ell_2$, etc, by use of a phenomenological quark-binding potential.

For the $D^+$ meson, the annihilation process is Cabbibo suppressed so that the soft-gluon effect $r^{SB}(\ell)$ for $D^+$ is smaller than $r^{SB}(\ell)$ for $F^+$ by a factor $\sim 1/10$, where $m_d \approx 0.34$ GeV and $f_F/f_D \sim 1.4$ have been used.

The annihilation of the $F^+$ meson, enhanced by the soft-gluon effect, gives rise to energetic leptons. Measurement of the lepton-energy spectrum will therefore clarify the working of the activated quark-annihilation process.
For heavy mesons containing b or t quarks, the soft-gluon effect leads to much smaller semileptonic enhancements than in $F$-meson decays. The ratio $\Gamma^{SG}(i)/\Gamma^{C}(i)$ in (14) decreases rapidly like $1/M^3$ as $M \to \infty$ for fixed $m$ (since $f_F = M^{-1/2}$). There are further sources of suppression: Annihilation of the $B^-(bu)$ meson is Cabbibo suppressed (at least) and that of the $B^-(bc)$ meson is expected to have a larger energy denominator $\Delta\epsilon_1\Delta\epsilon_2$, as noted earlier.

It will be useful to summarize, for comparison, the result for the nonleptonic process in Fig. 1b. The effective nonleptonic Hamiltonian is given by

$$H^{NL} = \int d^3 z \frac{1}{2} \left( \frac{G}{\Delta\epsilon} \right) [H^{ja} \Sigma^a_j + \text{h.c.}],$$

$$H^{ja} = \frac{i}{\bar{\sigma} \cdot [M^{-1}\gamma_\nu^a \delta^{\nu}_j + m^{-1}\gamma_\nu^a \delta^{\nu}_j]c],$$

$$L^{ja} = (\frac{\Delta\epsilon}{\sqrt{2}}) \frac{1}{2} f_2 \cos^2 \theta_{CP} \gamma^{\nu} \gamma^{\nu} \gamma^{\nu} \gamma^{\nu} d, \quad (17)$$

where $\Delta\epsilon$ stands for the energy denominator and $f_2 \approx -0.74$. This reproduces, in a simpler manner, the corresponding expression in Ref. [6] where only the light quark contribution is calculated.

The nonleptonic annihilation rate $\Gamma^{SG}(NL)$ of the $F^+$ meson with the soft-gluon effect is given by [6]

$$\Gamma^{SG}(NL)/\Gamma^{C}(NL) \approx 0.1 \times (f_F/\Delta\epsilon)^2, \quad (19)$$

where $\Gamma^{C}(NL)$ is the nonleptonic decay rate of the $c$ quark. Owing to the hard-gluon exchange correction,

$$\Gamma^{C}(NL) \approx 3 \times (1.8) \Gamma^{C}(i), \quad (20)$$

where $\Gamma^{C}(i)$ is given by eq. (12). The semileptonic branching ratio is calculated from eqs. (14), (19) and (20); as an estimate, using eq. (15) and $\Delta\epsilon \sim \Delta\epsilon_1 \sim \Delta\epsilon_2 \sim 140$ MeV (200 MeV), one obtains

$$BR(i) \sim 0.3(0.2), \quad (21)$$

where $i = e$ or $\mu$. 

This reproduces, in a simpler manner, the corresponding expression in Ref. [6] where only the light quark contribution is calculated.
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REFERENCES


FOOTNOTES

[F1]. The nonrelativistic description of charmed mesons has certain phenomenological success; see, e.g., Ref. [9].

[F2]. In eq. (6), the $c\bar{s}$ pair is annihilated by the axial-vector part of the hadronic weak current.

[F3]. Here Lorentz invariance of the QCD vacuum has been assumed such that [7] \[ (0|\bar{f}_j Q_j^a|0) = -(0|\bar{f}_j E_j^a|0) = \frac{1}{12} \delta_{ij}^k (0|\gamma_\mu F^a_{\mu\nu}|0) \]. Note that $E_k$ is antihermitian in the nonperturbative QCD vacuum.

[F4]. The sign of $E$ in eq. (15) of Ref. [6] should be reversed.

FIGURE CAPTIONS

Fig. 1. $F^+$ decays. (a) charm-quark process. (b) annihilation of $F^+$ into nonleptonic channels with emission of a (hard or soft) gluon. Shaded blobs represent $W$-boson exchanges with hard-gluon exchange corrections.

Fig. 2. Annihilation of $F^+$ into semileptonic channels with emission of (soft) gluons. Dashed lines refer to intermediate states.

Fig. 3. $F^+$-meson state regarded as a $c\bar{s}$-pair surrounded by a soft-gluon cloud whose size characterizes spatial spread of the $F^+$ meson.
\[ \approx |c\tilde{S}\rangle \otimes \langle 0| \]

Figure 3
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