Lawrence Berkeley National Laboratory
Recent Work

Title
ENERGY CONTINUATION OF AN ENERGY-INDEPENDENT PION-NUCLEON PHASE-SHIFT ANALYSIS FROM 385 TO 1700 MeV

Permalink
https://escholarship.org/uc/item/4wf9m6mw

Author
Field, R.D.

Publication Date
1970-02-01
ENERGY CONTINUATION OF AN ENERGY-INDEPENDENT PION-NUCLEON PHASE-SHIFT ANALYSIS FROM 385 TO 1700 MeV/c

R. D. Field, Jr.

February 10, 1971

AEC Contract No. W-7405-eng-48
DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.
ENERGY CONTINUATION OF AN ENERGY-INDEPENDENT
PION-NUCLEON PHASE-SHIFT ANALYSIS FROM 385 TO 1700 MeV/c *

R. D. Field, Jr.
Lawrence Radiation Laboratory
University of California
Berkeley, California 94720

February 10, 1971

ABSTRACT

Starting from an energy-independent phase-shift analysis carried out at 26 momenta between 385 and 1700 MeV/c, we attempt to find the proper energy continuation through these momenta. We compare several path-finding schemes which are designed to take into consideration both the continuity and the smoothness of the partial-wave amplitudes as viewed on an Argand diagram. While differing in detail, the various methods yield essentially the same results for the important partial waves. These results agree in general with paths found by other methods, but contain some evidence for additional resonances between 1500 and 1700 MeV.
I. INTRODUCTION

In a typical energy-independent phase-shift analysis there exist many possible solutions at a given energy. The hope is that the tremendous ambiguities which result when each energy is considered separately will be reduced or removed by imposing certain theoretical assumptions on the behavior of each partial-wave amplitude as a function of energy. We assume that each partial-wave amplitude is continuous and maintains a certain amount of "smoothness" when plotted on an Argand diagram. We investigate several criteria for continuity, and in addition impose certain demands for smoothness. We hope that these added theoretical assumptions will be enough to enable us to select one solution at each energy which will make up the proper energy-continued path. This work is a continuation of earlier work on pion-nucleon scattering carried out at Berkeley.¹

In Section II we discuss briefly the methods used in obtaining the energy-independent phase-shift solutions. In Section III we examine various methods for selecting an energy continuation and compare these paths with the one found by Lovelace² using a more complicated method in which continuity is applied through partial-wave dispersion relations.
II. PION-NUCLEON PHASE-SHIFT ANALYSIS

An energy-independent phase-shift analysis of pion-nucleon scattering was carried out at Berkeley several years ago. Solutions were found at some 26 momenta from $P_{\text{lab}} = 385$ to $P_{\text{lab}} = 1700$ MeV/c. For completeness we shall briefly mention the method used to obtain the solutions.

At each of the 26 momenta initial guesses for the parameters $\eta_{I,j,\ell}$ and $\delta_{I,j,\ell}$ are made. Then by use of a variable-metric minimization scheme called ORPHEUS these parameters are varied in an attempt to get a good fit to the data by minimizing $\chi^2$. The data include the $\pi^+, \pi^-$ and charge-exchange differential cross sections as well as the $\pi^+$ and $\pi^-$ polarizations. The parameters $\eta$ and $\delta$ are related to the transition matrix in the usual way:

$$T_{I,j,\ell} = [\eta_{I,j,\ell} \exp(2i \delta_{I,j,\ell}) - 1]/2i$$

where $I$ is the isospin, $j$ the total angular momentum, and $\ell$ the orbital angular momentum. The analysis includes waves through G waves ($\ell = 4$) and involves both $I = 1/2$ and $I = 3/2$.

The initial guesses for the $\eta$'s and $\delta$'s are obtained by one of three methods. One consists of starting with a rather coarse survey conducted with a ravine-following minimization method using starting points chosen randomly in the general vicinity of the solutions published by other groups, and then to use the minima found here as initial guesses in ORPHEUS. A second method used to obtain initial guesses is to use the first method to obtain solutions at momenta
k_{n-1} or k_{n+1} and then to use these as initial guesses at momenta k_n. This method is particularly useful when one is trying to continue to k_n a path that stops at k_{n-1}. The third method consists of simply using solutions obtained by other groups as starting points.

At each momenta solutions with intolerable $\chi^2$ are removed. In the earlier work done at Berkeley¹ solutions that were approximately equal were also edited out. In this work, however, we have included all solutions that are not exactly equal. In Table I we list at each momentum the degrees of freedom (number of data points), the best $\chi^2$, the worst $\chi^2$, and the number of solutions remaining after editing.

We feel it is important when carrying out energy-continuation procedures to have many solutions at each energy. We must require, however, that the $\chi^2$ of each solution remain reasonably good. The $\chi^2$ value alone is not a very good way of deciding which solution at each energy is the right one. For example, an energy continuation made up of the best $\chi^2$ at each energy has very discontinuous and rough behavior and hence is unsatisfactory. On the other hand, a method that imposes smoothness while ignoring $\chi^2$ may give a poor fit to the data. The best method is one that keeps only reasonable $\chi^2$, but has enough solutions at each energy to allow the smoothing program freedom to find a smooth and continuous path.
III. ENERGY CONTINUATION

The problem is to define a procedure by which a computer can pick out one solution at each energy in such a way that the resultant path is both continuous and smooth when viewed on an Argand diagram. We attempt to find a proper energy continuation through the 26 momenta listed in Table I, using a criterion of continuity based on a method of "minimal path." We define a "distance" $D_i$ for the path $i$ as

$$D_i^{\text{sum}} = \sum_{\ell,j,I} \sum_{k=k_{\text{min}}}^{k_{\text{max}}} d(\ell,j,I,k,i),$$

where a path consists of one solution at each momentum $k$, and where $\ell$ is the orbital angular momentum, $j$ the total angular momentum, and $I$ the isospin. We investigate various choices for the function $d(\ell,j,I,k,i)$, the simplest being just the geometric distance between two points on the Argand diagram,

$$d_0(\ell,j,I,k,i) = |T(\ell,j,I,k,i) - T(\ell,j,I,k-1,i)|,$$

where $T$ is the appropriate partial-wave amplitude (see Fig. 1). The proper energy-continued path is assumed to be that path $i_{\text{min}}$ for which the "distance" $D$ is minimum. Another definition of "distance" we shall investigate is

$$D_i^{\text{Euc}} = \left( \sum_{k=k_{\text{min}}}^{k_{\text{max}}} \left( \sum_{\ell,j,I} d^2(\ell,j,I,k,i) \right) \right)^{1/2}.$$

In this definition of distance we are considering the $\ell$, $j$, and $I$ variables as making up a many-dimensional Euclidian space, whereas $D_{\text{sum}}^i$ is just the sum over $\ell$, $j$, and $I$ of each path length

$$\sum_{k=K_{\text{min}}}^{K_{\text{max}}} d(\ell, j, I, k, i),$$

which is the length of a path viewed on an Argand diagram for a given $\ell$, $j$, and $I$.

The problem with using just the geometric distance [Eq. (2)] for $d$ is that although it does incorporate the idea of continuity (i.e., the solution does not change much when the energy is changed slightly), it does not produce paths that are smooth. Indeed, the computer would select path 1,2,3,4b over path 1,2,3,4a in Fig. 2. To correct this we define

$$d(\ell, j, I, k, i) = \left[1/(a + \cos \Theta)\right] d_0(\ell, j, I, k, i),$$

where $\Theta$ is the angle between the vectors

$$A = T(\ell, j, I, k-1, i) - T(\ell, j, I, k-2, i)$$

and

$$B = T(\ell, j, I, k, i) - T(\ell, j, I, k-1, i),$$

and where $d_0(\ell, j, I, k, i)$ is defined in Eq. (2) (see Fig. 3). For fixed $d_0$ the parameter $a$ determines the ratio of the distance $d$ in the forward direction $\Theta = 0$ to the distance in the backward direction $\Theta = 180^\circ$. This ratio is $(a - 1)/(a + 1)$. For example
if \( a = 1.5 \) for fixed \( d_0 \), the distance \( d \) in the backward direction is five times that in the forward direction. Figure 4 shows a line of constant \( d(\ell,j,I,k,i) \) for \( a = 1.5 \). We do not want to bias too much in the forward direction, for this would tend to wash out resonance loops. However, by suitably choosing the value of \( a \) we can be assured the computer will select the path 1,2,3,4a as the "shortest" in Fig. 2. We found the best value of \( a \) to be \( a \approx 1.5 \).

We also try weighting the function \( d(\ell,j,I,k,i) \) with a factor \((j + 1/2)\), however, this tends to force the computer to work hard at smoothing the high partial waves at the expense of the smoothness of the low partial waves.

In Figures 5-17 we compare the resultant minimum paths found by the following methods:

Path A--We exhibit CERN experimental solution found by Lovelace;\(^2\)

Path B--We use \( D^{\text{sum}} \) and \( d(\ell,j,I,k,i) = d_0(\ell,j,I,k,i) \);

Path C--We use \( D^{\text{Euc}} \) and \( d(\ell,j,I,k,i) = d_0(\ell,j,I,k,i) \);

Path D--We use \( D^{\text{sum}} \) and

\[
d(\ell,j,I,k,i) = [1/(1.5 + \cos \Theta)] d_0(\ell,j,I,k,i);
\]

Path E--We use \( D^{\text{Euc}} \) and

\[
d(\ell,j,I,k,i) = [1/(1.5 + \cos \Theta)] d_0(\ell,j,I,k,i);
\]

Path F--We use \( D^{\text{Euc}} \) and

\[
d(\ell,j,I,k,i) = [1/(1.5 + \cos \Theta)] d_0(\ell,j,I,k,i) \text{ and weight with a factor } (j + 1/2).
\]

We compare our five paths \((B,C,D,E,F)\) with path A, which is the path found by Lovelace using a sophisticated smoothing technique.\(^2\)
We have not bothered to show the G37, G39, F17, G17, and G19 waves, since they are not interesting at these energies. In Table II we list the total \( \chi^2 \)'s for each of our five paths, where the total number of degrees of freedom is 2069. To get the total \( \chi^2 \) for a path we add the \( \chi^2 \)'s for each solution along the path. We see that the two methods D and E, which use the smoothing function, give better total \( \chi^2 \). In addition paths D and E are indeed smoother than paths B and C, as can be seen by comparing Fig. 5C with Fig. 5E, or Fig. 7B with Fig. 7D.

In general our paths, although not as smooth, agree with Lovelace's path. There are, however, several differences between Lovelace's path and our paths that are worth noting. Firstly, we are better able to exhibit the complicated structure of the \( \text{S}1\text{l1} \) wave (Fig. 12). Secondly, we are unable to produce the same structure Lovelace has for the \( \text{P}1\text{l1} \). For reasons not understood all our paths have \( \text{P}1\text{l1} \) waves that are very messy above \( p_{\text{lab}} = 707 \) MeV/c (Fig. 13). Finally, there appears to be additional resonance-like structure in the following waves:

<table>
<thead>
<tr>
<th>Partial wave</th>
<th>Mass region</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>P31</td>
<td>1570-1680 MeV</td>
<td>6D</td>
</tr>
<tr>
<td>D35</td>
<td>1570-1680 MeV</td>
<td>9F</td>
</tr>
<tr>
<td>P13</td>
<td>1470-1680 MeV</td>
<td>14D</td>
</tr>
</tbody>
</table>

Whether this behavior actually corresponds to resonances or is just caused by inadequacies of our method is a question requiring further study. As mentioned earlier, the methods we use require as many
solutions with reasonable $\chi^2$ at each energy as can be found. Hence, the next step in improving our results would be to search for more solutions at each energy. Perhaps two or three times as many solutions can be found. It remains to be seen if the resonance-like structure in the above partial waves will then persist.

It is reassuring that a method based on very simple ideas of continuity and smoothness can select an energy-continued path that is reasonably well behaved.
ACKNOWLEDGMENTS

I am grateful to Professor Herbert Steiner, without whose encouragement this work would never have been accomplished, and to C. H. Johnson for introducing me to this subject. Also, I thank Professor Owen Chamberlain for the generous use of computer time.
FOOTNOTES AND REFERENCES

* This work was supported by the U.S. Atomic Energy Commission.


3. This double loop behavior is seen by many other groups. See, for example, P. Baryre, C. Bricman, A. V. Stirling, and G. Villet, Phys. Letters 18, 342 (1965).
Table I. Number of fits at each momenta.

<table>
<thead>
<tr>
<th>$P_{\text{lab}}(\text{MeV/c})$</th>
<th>Number of fits</th>
<th>Degrees of freedom</th>
<th>Best $\chi^2$</th>
<th>Worst $\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>385</td>
<td>1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2.</td>
<td>427</td>
<td>3</td>
<td>100</td>
<td>117.4</td>
</tr>
<tr>
<td>3.</td>
<td>490</td>
<td>3</td>
<td>90</td>
<td>86.0</td>
</tr>
<tr>
<td>4.</td>
<td>532</td>
<td>27</td>
<td>67</td>
<td>77.6</td>
</tr>
<tr>
<td>5.</td>
<td>614</td>
<td>30</td>
<td>60</td>
<td>64.5</td>
</tr>
<tr>
<td>6.</td>
<td>658</td>
<td>33</td>
<td>65</td>
<td>71.9</td>
</tr>
<tr>
<td>7.</td>
<td>675</td>
<td>37</td>
<td>77</td>
<td>68.5</td>
</tr>
<tr>
<td>8.</td>
<td>707</td>
<td>56</td>
<td>71</td>
<td>65.0</td>
</tr>
<tr>
<td>9.</td>
<td>726</td>
<td>48</td>
<td>55</td>
<td>53.8</td>
</tr>
<tr>
<td>10.</td>
<td>745</td>
<td>57</td>
<td>98</td>
<td>73.4</td>
</tr>
<tr>
<td>11.</td>
<td>777</td>
<td>30</td>
<td>57</td>
<td>74.4</td>
</tr>
<tr>
<td>12.</td>
<td>826</td>
<td>43</td>
<td>92</td>
<td>68.7</td>
</tr>
<tr>
<td>13.</td>
<td>875</td>
<td>41</td>
<td>114</td>
<td>86.5</td>
</tr>
<tr>
<td>14.</td>
<td>899</td>
<td>72</td>
<td>106</td>
<td>88.7</td>
</tr>
<tr>
<td>15.</td>
<td>925</td>
<td>60</td>
<td>74</td>
<td>46.6</td>
</tr>
<tr>
<td>16.</td>
<td>975</td>
<td>80</td>
<td>54</td>
<td>48.9</td>
</tr>
<tr>
<td>17.</td>
<td>1000</td>
<td>75</td>
<td>71</td>
<td>46.8</td>
</tr>
<tr>
<td>18.</td>
<td>1030</td>
<td>73</td>
<td>54</td>
<td>41.1</td>
</tr>
<tr>
<td>19.</td>
<td>1080</td>
<td>37</td>
<td>86</td>
<td>52.9</td>
</tr>
<tr>
<td>20.</td>
<td>1121</td>
<td>56</td>
<td>77</td>
<td>47.8</td>
</tr>
<tr>
<td>21.</td>
<td>1180</td>
<td>53</td>
<td>100</td>
<td>75.8</td>
</tr>
<tr>
<td>22.</td>
<td>1280</td>
<td>60</td>
<td>99</td>
<td>73.6</td>
</tr>
</tbody>
</table>

Table I continued next page
Table I (Continued).

<table>
<thead>
<tr>
<th>$p_{lab}$ (MeV/c)</th>
<th>Number of fits</th>
<th>Degrees of freedom</th>
<th>Best $\chi^2$</th>
<th>Worst $\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>23. 1360</td>
<td>80</td>
<td>104</td>
<td>79.3</td>
<td>196.5</td>
</tr>
<tr>
<td>24. 1440</td>
<td>69</td>
<td>94</td>
<td>99.7</td>
<td>179.0</td>
</tr>
<tr>
<td>25. 1579</td>
<td>59</td>
<td>86</td>
<td>86.3</td>
<td>152.3</td>
</tr>
<tr>
<td>26. 1700</td>
<td>60</td>
<td>118</td>
<td>79.6</td>
<td>179.6</td>
</tr>
</tbody>
</table>
Table II. List of total $\chi^2$ for our five paths, where the total number of data points is 2069.

<table>
<thead>
<tr>
<th>Path</th>
<th>Total $\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>2393</td>
</tr>
<tr>
<td>C</td>
<td>2417</td>
</tr>
<tr>
<td>D</td>
<td>2353</td>
</tr>
<tr>
<td>E</td>
<td>2315</td>
</tr>
<tr>
<td>F</td>
<td>2447</td>
</tr>
</tbody>
</table>
FIGURE CAPTIONS

Fig. 1. Argand diagram illustrating the function $d_0(\ell,j,I,k,i)$, which is just the geometric distance between the two points $k$ and $k-1$.

Fig. 2. Argand diagram illustrating two possible paths.

Fig. 3. Argand diagram illustrating the vectors $\vec{A}$ and $\vec{B}$ and the angle $\theta$ used in defining the function $d(\ell,j,I,k,i) = \frac{1}{1.5 + \cos \theta} d_0$.

Fig. 4. Shows a line of constant $d(\ell,j,I,k,i)$, where $d(\ell,j,I,k,i) = \frac{1}{1.5 + \cos \theta} d_0$ and $|\vec{B}| = d_0$.

Fig. 5. Argand diagram of the $S_{31}$ partial wave.

Fig. 6. Argand diagram of the $P_{31}$ partial wave.

Fig. 7. Argand diagram of the $P_{33}$ partial wave.

Fig. 8. Argand diagram of the $D_{33}$ partial wave.

Fig. 9. Argand diagram of the $D_{35}$ partial wave.

Fig. 10. Argand diagram of the $F_{35}$ partial wave.

Fig. 11. Argand diagram of the $F_{37}$ partial wave.

Fig. 12. Argand diagram of the $S_{11}$ partial wave.

Fig. 13. Argand diagram of the $P_{11}$ partial wave.

Fig. 14. Argand diagram of the $P_{13}$ partial wave.

Fig. 15. Argand diagram of the $D_{13}$ partial wave.

Fig. 16. Argand diagram of the $D_{15}$ partial wave.

Fig. 17. Argand diagram of the $F_{15}$ partial wave.
Fig. 1.
Fig. 5E,F

XBL711-2709
**Fig. 6A, B**

- **CERN Proton-Nucleon Experimental Fits.**
  - From Heidelberg, Natl.
  - Origin at Bottom Scale = 1.0

---

- **PLOTS TO FIT LEAST TOTAL OSET**

- **PLOTTING TO FIT LEAST TOTAL OSET**

---

- **Fig. 6A, B**

---

- **UCRL-20250**

---

- **XBL711-2704**

---
Fig. 7A,B
Fig. 8E,F

XBL71II-2700
Fig. 9A, B
Fig. 10A, B
Fig. 12A,B
Fig. 12C,D

UCRL-20250

Fig. 12C,D

XBL711-2690
Fig. 12E,F
Fig. 13A, B
Fig. 13E,F
Fig. 16A, B
This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.