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Introduction

Beams consisting of closely spaced bunches crossing at a small angle are the most promising solution. Bunched beams allow to reach high luminosity with a moderate total charge in the machine while the close bunch spacing keeps the peak event rate low. The non-zero crossing angle is required to avoid unwanted collisions in the vicinity of each IP where the beams remain close.

Outline of Procedure

The quantities of interest are the following:

1. Major Input Data (Performance goals):
   Luminosity $\mathcal{L}$
   Beam Energy $E$

2. Critical assumptions:
   $\Delta Q(0)$ tolerable beam-beam tune shift at each IP
   $\Delta Q(LR)$ tolerable long range beam-beam tune shift in the vicinity of each IP

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3. Crucial free (within some range) parameters:

   - $S_B$  bunch separation (bunch to bunch distance)
   - $\beta^*$  value of $\beta$-function at IP

4. Parameters describing beam and collision geometry:

   - $N_B$  # of particles per bunch
   - $\epsilon_n$  normalized rms emittance, $\epsilon_{nx} = \epsilon_{ny}$
   - $\sigma_z$  rms bunch length
   - $\sigma_y$  rms energy spread, $\sigma_y = \sigma_E/m_p$
   - $\alpha$  crossing angle

To derive the quantities (4) we will proceed along the following lines:

From an assumed value $\Delta Q(0)$ we will derive $\epsilon_n, N_B$ as functions of the dimensionless quantity $r = \alpha \sigma_z / 2 \alpha^*$. From an upper limit on $\Delta Q(LR)$ we will derive $\alpha(r)$, therefore obtaining a one to one correspondence between $r$ and $\sigma_z$. We will then use auxiliary considerations to narrow the choice of $\sigma_z$ (and therefore $r$) as well as $\sigma_y$.

Derivation of $\epsilon_N(r), N_B(r), \sigma_z(r)$

The following relations hold: (1)

$$\mathcal{L} = \frac{C}{S_B} \frac{1}{4\pi} \frac{\gamma}{\beta^*} \frac{N_B^2}{\epsilon_N} (1+r^2)^{-1/2}$$  \hspace{1cm} (1)

$$|\Delta Q_y(0)| = \frac{1}{4\pi} \frac{r_o N_B}{\epsilon_N} 2 \frac{(1+r^2)^{1/2} - 1}{r^2}$$  \hspace{1cm} (2)
\[ r = \frac{\alpha \sigma_x}{2 \sigma^*}. \]

\( \Delta Q_y \) as given by (2) is the linear tune shift for a particle centered in its bunch and \(|\Delta Q_y| \geq |\Delta Q_x|\) for crossing in the x-z plane. Expression (2) is valid for all \( r, 0 \leq r \leq \infty \). From (1) and (2) follow:

\[ N_B = \beta^* S_B \frac{r_o}{c \gamma} \frac{\phi}{|\Delta Q(0)|} \frac{g(r)}{f(r)} \]  
(3)

\[ \epsilon_N = \beta^* S_B \frac{r_o^2}{c \gamma} \frac{\phi}{|\Delta Q(0)|^2} \frac{q^2(r)}{4 \pi} \frac{f(r)}{f(r)} \]  
(4)

with \( f(r) = (1+r^2)^{-1/2} \), \( g(r) = 2 \frac{(1+r^2)^{1/2} - 1}{r^2} \)  
(5a,b)

\( \epsilon_N(r), N_B(r) \) are plotted in Figs. (1) and (2).

To calculate \( a(r) \) we consider the long range beam-beam interaction due to the close encounters at distances \( z = \pm n S_B/2 \) away from the IP. A detailed investigation\(^{(2)}\) has shown that:

1) For ratios \( \sigma_x/\sigma_y \) of "a few" and at distances \( r \geq 10 \text{ Max} (\sigma_x, \sigma_y) \) a simple \( 1/r \) dependence is a very good approximation for the fields created by a bi-gaussian charge distribution.

2) With typical insertion designs, with little betatron phase advance in the insertion quadrupoles, the resulting tune shift is, at least for antisymmetric configurations approximated to within \( \pm 10\% \) by
\[ \left| \Delta Q_{z,y}(LR) \right| = \frac{r_0 N_B}{4\pi} \frac{1}{\epsilon_N} \frac{8}{\eta^2} \frac{D}{s_B} \]

Where we have substituted a drift length \( D \) extending from the IP to the separating dipoles and where \( \eta \) is defined as:

\[ \eta = \frac{\alpha z}{\sigma(z)} \]

From this we can evaluate \( \alpha(r) \), \( \sigma(r) \):

We obtain:

\[ \alpha(r) = D^{1/2} \frac{r_0}{\gamma^2} \left( \frac{2c}{\pi} \frac{\mathcal{L}}{\left| \Delta Q(0) \right|} \right)^{1/2} \left( \frac{g(r)}{f(r)} \right)^{1/2} \]

\[ \sigma_z(r) = \sigma^* \left( \frac{s_B}{D} \right)^{1/2} 2r \left( \frac{\left| \Delta Q(LR) \right|}{8 \left| \Delta Q(0) \right|} g(r) \right)^{1/2} \]  

Fig. (2) shows \( \epsilon_n(r) \), \( N_B(r) \), \( \alpha(r) \), \( \sigma_z(r) \) in the appropriate normalization for \( \gamma = 2.13 \times 10^4 \), \( \mathcal{L} = 10^{33} \text{ cm}^{-2}\text{s}^{-1} \), \( |\Delta Q(0)| = 0.003 \), \( |\Delta Q(LR)| = 0.00025 \).

Carrying out these calculations we have implicitly assumed that a tolerable upper limit for \( \Delta Q(0) \) is independent of \( r \) and that we should keep \( |\Delta Q(LR)| \ll |\Delta Q(0)| \). In view of the fact that the nonlinear aspects of the beam-beam interaction are really what counts, this assumption will need closer inspection. The general structure of our argument would not be changed but the \( r \) dependence of the relations (5) through (8) might have to be modified.
Choice of $r$ and $\sigma_z(r)$

The choice of $\sigma_z$, and therefore $r$, is to some extent arbitrary. Low values are favored because they result in lower currents and larger emittances, thus reducing total synchrotron radiation power and the demands on injector brightness. Considerations of RF-technology and collective stability also enter. We will not discuss these but look at intra beam scattering (IBS). IBS growth rates depend critically on the six-dimensional phase space density. Fig. (3) shows growth rates for two different cases ($\beta^* = 2m$, $\beta^* = 1m$) as function of $r$ for different energy spreads $\sigma_y$. IBS essentially determines a lower limit for $\sigma_y$. Fig. (3) shows rapid increase of growth rates for very low values of $\sigma_z$, a minimum and then again a slow increase for large $\sigma_z$ (corresponding to small $\epsilon_n$).
References

1) Christoph Leemann, SSC Note-3.
2) Christoph Leemann, SSC Note-13.
Figure Captions

Fig. 1 Shown are $\varepsilon_N$, $N_B$ as well as the approximate synchrotron radiation power in a 6.5T ring. Values are calculated for $\mathcal{L} = 10^{33} \text{cm}^{-2}\text{s}$, $|\Delta Q(0)| = 0.003$, $\beta^* = 2 \text{m}$ and $S_B = 15 \text{m}$.

Fig. 2 Beam parameters $\varepsilon_N$, $N_B$, $\sigma$ and $\alpha$ normalized with the appropriate scaling factors. Assumed is $\mathcal{L} = 10^{33} \text{cm}^{-2}\text{s}$, $|\Delta Q(0)| = 0.003$, $|\Delta Q(LR)| = 2.5 \times 10^{-4}$.

Fig. 3 IBS growth rates calculated following Bjorken and Mtingwa's theory for $\beta^* = 2 \text{m}$ and $1 \text{m}$ and $S_B = 15 \text{m}$. Calculations are performed on the basis of the regular arcs only ($L_c = 160 \text{m}$, $\mu = 60^\circ$, $\phi = 0.71^\circ$).
\[ L = 10 \text{ cm} \quad \Delta Q_{\text{ub}} = -0.003, \quad \beta^* = 2m, \quad s_B = 15m \]

Scaling: \( N_{\text{eb}}, P_{\text{sync}} \ll \beta^* \)  
\( \epsilon_N, N_B \ll \beta^* s_B \)  

\[ P_{\text{sync}} \quad [\text{kW}] \]

\[ \epsilon_N \quad [\text{um}] \]

\[ N_B \quad [10^9] \]

\[ \frac{F}{L} = \frac{\alpha \sqrt{E}}{2c} \]
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