Title
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Diagnostic Tests Determining The Thermal Response Of A House

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ABSTRACT

Two different approaches to the determination of the thermal response of buildings are possible: deterministic models and methods based on equivalent thermal parameters (ETP's) of a building. While the former are computer applications of heat transfer theory, the latter consist of data oriented techniques that infer the ETP's of a particular building by multiple correlation of indoor temperature and weather. The ETP method is convenient to provide a rank ordering of different houses by their thermal performance and to assess the overall effects of retrofits on a house. Like deterministic methods, the ETP method can also predict accurate free-floating indoor temperatures and heating loads as a function of weather.

A convenient set of ETP's is established for a residential townhouse by means of a simple, single thermal mass model. Multiple step regressions of actual data on indoor temperature and weather yield estimates for the ETP's. The model tracks the measured data well. The regressed ETP's agree with what is expected from theoretical calculations and are consistent with the result from a different, constant-indoor temperature experiment.

INTRODUCTION

A large number of methods have been developed to predict the heating/cooling load or the indoor temperature of a building in response to weather and usage profiles [1-2]. Most of these methods involve computer oriented algorithms of heat transfer theory that balance all heating and cooling terms caused by equipment, appliances, outdoor weather and other factors. The time delays caused by heat conduction through walls greatly add to the complexity of these algorithms. While many of the resulting computer packages have been increasingly successful in predicting energy requirements for buildings of widely different construction and location, they also are inherently deterministic; once all input data concerning building construction, weather and operating schedules are fed into the computer, the predicted temperatures and/or heating or cooling loads are fixed. If this output does not agree with measured data to the user's satisfaction, there is no obvious way to correct the prediction.

A different approach, using equivalent thermal parameters (ETP's) of a building, has been proposed [3-4]: Instead of telling the computer how the building is built and asking it for the indoor temperature, one tells the computer the measured indoor temperature and asks it for the building parameters - parameters describing what the building is like. The same can be done using cooling and/or heating load as input instead of indoor temperature. A convenient set of ETP's is 1) equivalent thermal mass, 2) equivalent solar window area, 3) furnace field efficiency and 4-6) equivalent heat transfer constants (between indoor room air and outdoors, between indoors and a "temperature clamp", and between indoors and the solid house structure). The concept and the experimental determination of the furnace field efficiency is described elsewhere [5]. How the other ETP's are defined and measured, is the subject of this paper.
Identical houses exposed to identical weather have consistently shown considerable variation in space heating energy usage (up to a ratio of two to one), only in part related to differences in occupant behavior [6,7]. The experimental assessment of such variations and their patterns, possibly leading to insights into construction flaws, are exciting prospects for the ETP method.

Deterministic methods, in turn, by definition will not predict differences where there should be none, since the houses have identical floor plans.

A set of ETP's for a particular building can be used to estimate the same building's thermal performance in other weather situations; or, it can quantify the building's performance before and after retrofits. In many cases, the ETP's are best thought of in analogy to measured "miles per gallon" for a car. Once the actual data on heating/cooling load and indoor temperature of a building exist, it is comparatively easy to obtain the building's ETP's. This can be done without knowing anything else about the building. Specifically, no floor plans or other construction data are needed. To be able to catalogue the energy requirements and the thermal response of any house by a quite general set of four or five numbers should look attractive to policy makers, real estate agents and homeowners.

Deterministic methods, in turn, can tell you what any building does anywhere, not only the one on which you have load data; they are especially useful when the building exists only on the drawing board. The price for this generality is a large and complex program and considerable paper work on input sheets, listing everything from the composition of each wall layer to the efficiency of the heating system.

There are also hybrid approaches like estimating a set of ETP's from load data predicted by a deterministic computer package [8]. In this paper, in turn, we will compare the ETP's obtained from measured indoor temperature data with what is calculated by theoretical means.

The ETP method has been applied to experimental buildings [3]. This paper describes a similar approach, including the contribution by the sun and the internal heat sources, applied to an actual occupied townhouse. The free-floating indoor temperature, the solar flux and the electric power consumption are continuously monitored. The house's ETP's obtained from correlation of this data are discussed in view of their physical interpretation and compared to what is calculated using detailed floor plans and construction data. A similar experiment, with a constant, thermostatically controlled indoor temperature yielded a subset of the same ETP's and the field furnace efficiency [5].

The experiment described in this paper is part of the ongoing project on energy conservation carried out by the Center for Environmental Studies of Princeton University at Twin Rivers, New Jersey, a 3,000 dwelling, Planned Unit Development [6]. The experiment was carried out in a three-bedroom, two-story wood-frame townhouse, occupied by the author and contiguous to other identical units within the same block.

A SIMPLE INDOOR TEMPERATURE MODEL

The indoor temperature of a house is a function of heat gains by the heating plant, the appliances and the sun and of heat losses through the above ground house shell and the foundations. The thermal mass of the house smoothes out the effect of changes in such heat gains and heat losses on indoor temperature. Strictly speaking, each wall, floor and ceiling section should be treated as separate thermal masses. And each thermal mass is not constant; it depends on the rate of change in the thermal variables (temperatures, heat fluxes) at its boundaries.

We will take a bold shortcut by postulating the concept of a single, constant equivalent thermal mass for the house. The range of applicability of this concept is much wider than it would appear at first sight [3, 7]. It will work for most wood-frame or light masonry construction; it will work with heavier masonry walls, too, provided the time histories of the boundary temperatures and the heat gains are relatively smooth.

Equivalent thermal parameters (ETP's) like thermal mass and heat transfer constants can be determined both experimentally and theoretically. This paper presents an experimental method applied to data collected from a townhouse. The physical interpretation of the results will be discussed, with a cursory reference to the theoretical framework that is involved.

We now proceed to the establishment of the two master equations of our simple house model. Subsequently, we will determine the parameters of these equations by multiple correlation of experimental data. The first equation expresses a simple energy balance between all heat gains in the house, including the solar heat gain, and the heat transfers 1) to the "massive" house
structure, 2) to a "constant temperature clamp" and 3) to the outdoors. The second equation expresses the balance between the heat stored in the house structure and the heat transferred from room air to structure:

\[ Q = HS(T - T_s) + HC(T - TC) + H(T - TO) \]  

\[ C \frac{dT_s}{dt} = HS(T - T_s) \]  

Q is the sum of all heat gains within the house, including the sun [Watt];
T is the average indoor room air temperature [°C];
T_s is the temperature associated with the massive house structure [°C];
TC represents a "constant temperature clamp", a joint effect of neighbors and basement [°C];
TO is the outdoor temperature [°C];
H, HS are the equivalent heat transfer constants between room air and outdoors and between room air and the house structure, respectively [Watt/°C];
HC is the equivalent heat transfer constant between the room air and the temperature clamp [Watt/°C];
C is the equivalent thermal mass of the house [kWh/°C].

The equivalent electrical circuit is shown in Fig. 1, with temperatures represented by voltages, heat fluxes by currents, heat conductances by inverse resistances and the thermal mass by a capacitor.

The "constant temperature clamp" is represented by the (constant) temperature TC and results from the stabilizing action of the basement and the neighboring dwelling units. The basement temperature is held nearly constant by the bare basement masonry and the soil surrounding it. The temperatures of the neighbors, in turn, are held constant by action of their thermostats. The two effects were not modeled separately in Eqs. 1 mainly because the multiple correlations of the data do not warrant an additional parameter to be regressed. Such temperature clamps exist in most buildings, except possibly in a single zone, entirely above grade building with a well insulated floor.

The definition of the structure temperature, T_s, is somewhat fuzzy, since it changes across the profile of any wall. To avoid complications, we eliminate T_s by combining the two equations la and lb into a single expression:

\[ C(1 + \frac{HC}{HS})\dot{T} = C\frac{H}{HS}\dot{TO} + \frac{HC}{HS}\dot{TO} + \frac{Q}{HS} - (HHC)T + H\cdot TO + HC\cdot TC + Q \]  

where \( \dot{T} = \frac{dT}{dt}, \dot{TO} = \frac{dTO}{dt}, \) etc.

Equation 2 expresses the relation between easily measurable temperatures and heat gains only. The term in Q is the sum of all heat gains originating from: 1) four special electrical resistance heaters, introduced for experimental purposes, and all electric appliances, E; 2) the people working and living in the house, P; 3) the latent load of the humidifier and the plants, L; 4) the solar heat gain, A*S, where A is the equivalent solar window area [m²] and S is the solar flux impinging on the south walls [Watt/m²].

\[ Q = E + P + L + A*S \]  

* Defined by Jan Beyea, Center for Environmental Studies, Princeton University.
The equivalent solar window area, \( A \), is defined as the area of the 100% transparent, perfectly insulated opening in the south wall that would allow for the same degree of indoor solar heating as what is actually obtained. It gives the net effect of the sun shining through the windows, as well as heating the opaque outside walls and the roof [6], and is slightly dependent on the season (see Appendix). The solar term, \( A \cdot S \), in the only portion of \( Q \) that changes appreciably over the day, and thus the only significant contributor to the derivative, \( \frac{dQ}{dT} \). The other energy sources, \( E + P + L \), are approximately constant in time, along with the temperature of the "clamp," \( T_C \). From the general equation for the indoor temperature, Eq. 2, we can now derive the equation pertinent to this particular experiment, using Eq. 3:

\[
\frac{dT}{dt} = \left( \frac{H \cdot T_0 + A \cdot S}{H_S + H + H_C} \right) + \frac{H}{C^*} (T_0 - T) + \frac{H C}{C^*} (T_C - T) + \frac{A}{C^*} S + \frac{(E + P + L)}{C^*}
\]

where

\[
C^* = C(1 + \frac{H + HC}{H_S}).
\]

The term most crucial for the determination of the ETP's is the energy term, \( \frac{(E + P + L)}{C^*} \). An accurate determination of the quantity \( C^* \) (an "effective" thermal mass, to be distinguished from the equivalent thermal mass, \( C \)) is the key to a reliable derivation of \( H, HC \) and \( A \) from the three appropriate coefficients in Eq. 4. Along with \( C \), these are the most important of the ETP's. It is thus imperative to introduce a large, well determined heat source \( (E + P + L) \), as an energy "yardstick." The central gas furnace installed in the house is unsuited to this task because of its high power and its uncertain efficiency; it was thus shut off during the experiment. In its place, four portable electrical resistance heaters, equipped with fans, placed upstairs and downstairs, provided a constant, * well determined heat source of 5.65 kilowatt. The four heaters provided thus about 90% of the total quasi-constant heat gain \( (E + P + L) \).

As a simplifying assumption, no distinction between radiant and convective heat transfers is made. This and other simplifications are implemented to preserve the significance of the results obtained from fitting our model to experimental data. The number of free parameters of any such model is limited by the number of independent variables in the data. Compromises between the accuracy of physical modeling and the significance of the fitted parameters will always be required.

In the following section, we will describe the determination of the equivalent thermal parameters \( C, H, HC, HS \) and \( A \), by multiple correlation of experimental data.

### MULTIPLE REGRESSIONS OF THE DATA

The experiment presented in this paper was carried out during 4 days of last winter's natural gas crisis (January 1977). Figure 2 shows the most relevant of the collected data. The indoor temperature is obtained as an average of the readings of 12 thermistor probes distributed over the two living space floors. The solar flux was measured with a Lyntronix temperature-compensated pyranometer placed on the south-facing outside wall. The power drawn by the four heaters, all other electrical appliances and the experimental equipment was monitored twice daily. As the weather got warmer and overheating was imminent, one heater was turned off for the last 42 hours of the experiment. All regressions performed in this paper apply to the first 52 hours, while the remaining 42 hours serve as a check on the consistency of the model, after the appropriate adjustment is made for the decrease in electrical heat.

The equation used for the regressions is obtained from transforming the differential equation, Eq. 4, into a difference equation:

\[
T_{t+1} = a_T T_t + b_1 \cdot T_0 + b_2 \cdot A \cdot T_0 + c_1 S_{t} + c_2 \cdot A S_{t} + d
\]

\( t, t+1 \) are subscripts indicating the data sampling times (every hour, for example);

* A sinusoidal oscillation in time with a period between about 6 and 30 hours (but not 24 or 12 hours, to avoid correlation with outdoor temperature and sun) may be even more desirable. The necessary equipment, though, is more complicated and was not available at the time.
The relationships between the regression coefficients $a$, $b_1$, $b_2$, $c_1$, $c_2$ and $d$, and the equivalent thermal parameters $HC$, $H$, $A$, $C$ and $HS$, are derived by comparing Eqs. 4 and 5. We will see that the estimates of the coefficients $b_2$ and $c_2$ of the time differences of outdoor temperature, $\Delta T_{o}$, and solar flux, $\Delta S$, are less reliable than the others. Without using $b_2$ and $c_2$, we can derive the first three ETP's:

$$HC = \frac{(1-a-b_1)}{d_1}$$  \hspace{1cm} (6a)
$$H = \frac{b_1}{d_1}$$  \hspace{1cm} (6b)
$$A = \frac{c_1}{d_1}$$  \hspace{1cm} (6c)

where $d_1 = \frac{[d - (1-a-b_1)T_C]}{(E + P + L)}$.  \hspace{1cm} (6d)

Only the determination of the equivalent thermal mass, $C$, and the "structural" heat transfer constant, $HS$, depend on the coefficients $b_2$ and $c_2$:

$$C = \frac{\left[\frac{l+a}{2} - (1-a)c_2/c_1\right]}{\Delta t/d_1}$$  \hspace{1cm} (6e)
$$HS = \frac{\left[\frac{l+a}{2} b_1/b_2 - (1-a)\right]}{d_1}$$  \hspace{1cm} (6f)

$$HS = \frac{\left[\frac{l+a}{2} c_1/c_2 - (1-a)\right]}{d_1}$$  \hspace{1cm} (6g)

where $\Delta t$ is the time interval between successive data in the regression.

The redundancy in the determination of $HS$ is of little practical use because of the problems in the reliability of $b_2$ and $c_2$.

Table 1 shows the regression coefficients and the derived equivalent thermal parameters, along with the values of the constant variables, $E+P+L$ and $T_C$. The numbers in parentheses following the regression coefficients are the $t$-statistics of the estimates. That the R-squared is so high is a consequence of the strong autocorrelation in successive values of indoor temperature, $T_t$ and $T_{t+1}$.

**TRACKING ABILITY OF THE MODEL**

Figure 3 gives an idea of the tracking ability of this one-capacity (or first order) model. The measured indoor temperature is a magnification of the uppermost curve in Figure 2. The predicted indoor temperature was obtained by using the regression coefficients of Table 1, the measured time series of outdoor temperature and solar flux and the (measured) initial value for the indoor temperature, $T_0$. Using Eq. 5, the model prediction for $T_t$ is calculated, then "plugged back" into the r.h.s. of Eq. 5, obtaining $T_{t+1}$, and so on. The predicted values, $T_t'$ (rather than the measured values, $T_t$), are "recycled" into the r.h.s. of Eq. 5, to ensure that the tracking test is self consistent. After the first 52 hours of indoor temperature have been predicted in this fashion, the regression intercept, $d$, is adjusted using Eq. 6d, to compensate for the shutdown of one heater. Thus the last 42 hours in Fig. 3 amount to an extrapolation of the model. The indoor temperature is only temporarily underpredicted by the model, as stored heat is released and as a new heat transfer equilibrium is approached after the step-function "jolt" due to the heater shutdown. To give a better feeling for the stability of the model, we assume an arbitrary initial condition, $T_0 = 18^\circ C$, in Figure 4. The time the model requires to "forget" the wrong initial condition is related to the principal time constant of the house.

Inspection of the first-order linear differential equation of the model, Eq. 4, indicates that the principal time constant, $\tau$, is equal to

\[ \tau = \frac{1}{a} \]

The $t$-statistic is defined, here, as the ratio of an estimated regression coefficient and the standard error of the estimate. Generally, a coefficient can be called "significant" (or statistically different from zero) if $t > 2$. 

-5-
\[ \tau = \frac{C}{H+HC} = C\left( \frac{1}{HS} + \frac{1}{H+HC} \right) = 6.8 \text{ hours.} \] (7)

This is the time constant of the exponential functions, \( \exp(-t/\tau) \), appearing in the homogeneous solution of Eq. 4. Using the values listed in Table 1, \( \tau \) is computed to 6.8 hours and is indicated in Fig. 4. This time constant is a measure for the "natural sluggishness" of a house. Though a large \( C \) is a prerequisite for a large \( \tau \), as shown in Eq. 7, the popular idea of equating a heavy building with a large time constant does not necessarily follow.

PHYSICAL INTERPRETATION OF THE REGRESSED ETP's

In this section we will relate the regressed equivalent thermal parameters displayed in Table 1 to the physical properties of the house. The parameters easiest to interpret are the equivalent heat transfer constant per unit indoor-outdoor temperature difference, \( H \), and the equivalent solar window area, \( A \). We modeled the combined heat transfer by air infiltration through cracks and by conduction through walls, roof and windows, as a pure resistance (introducing no time delays). This is a valid assumption, considering the time scale of our data. Thus the calculation of \( H \) boils down to a mere static heat load calculation. The measurement of air infiltration (through automated measurement of the decay of SF\textsubscript{6} tracer gas), averaging 0.5 exchanges per hour during the experiment, eliminates the largest source of error usually present in static heat load calculations. The value of \( H \) computed in detail in the appendix, equals 211 W/°C, in excellent agreement with the 204 W/°C obtained from regression.

The equivalent solar window area, \( A \), is also derived in the appendix, using the measured optical transmission of the windows (74%) and the concept of sol-air temperature, to account for the solar heating of the opaque outside walls; the result is 6.04 m\(^2\). The agreement with 5.94 m\(^2\) from regression is better than the uncertainties introduced by some assumptions in the computation of \( A \) in the appendix, and should not be overemphasized.

The physical interpretation of \( HC \), the equivalent heat transfer constant between room air and the "temperature clamp", is less straightforward. It results from the combined action of the neighbors and the masonry/ground in which the basement is embedded. The greatest cause for uncertainty in estimating \( HC \) by theoretical means is that the magnitude of the thermal resistance between living space and basement cannot be well established. Apart from conduction through the floor, substantial convective heat transfer may occur through the open basement door and by means of leaks in the air ducts that run along the basement ceiling: the furnace fan was running continuously to ensure proper air mixing necessary for the simultaneous measurement of air infiltration. Based on earlier measurements, the joint effect of these duct leaks and the open basement door are estimated to correspond at most to 510 m\(^3\)/hr, or 50% of the fan capacity. This volume is assumed to flow from living space to basement, mix perfectly and return to the living space. The value for \( HC \) resulting from conduction alone (through the firewalls to the neighbors and through the floor to the basement) is 234 W/°C. By adding the extra air flow to and from the basement, \( HC \) is increased to 405 W/°C. The regressed value for \( HC \), shown in Table 1, is 330 W/°C, in reasonable agreement when we consider the uncertainty of our assumptions. We should point out, however, that we neglected the time delays introduced by the massive firewalls (20 cm cinder blocks lined by gypsumboard) in the previous computation of \( HC \) and that we assumed the basement to be a constant temperature clamp, which is only partially true. Exact calculations show, however, that the introduced error is smaller than the uncertainty in estimating the quantity of air flow exchanged between basement and living space [7].
Table 2 shows the comparison between regression estimates and theoretical calculations of the ETP's. It should be pointed out that this free floating temperature experiment is not the only way to experimentally determine the three ETP's we discussed so far: an experiment with a constant, thermostatically controlled temperature was also performed and provided ETP's consistent with what we found here [5].

The remaining two ETP's, the equivalent thermal mass, C, and the equivalent heat transfer constant between room air and house structure, HS, involve some knowledge of that portion of the solid house structure that effectively participates in the transient storage of heat. As one would expect, the regressed equivalent thermal mass, C = 3.22 kwh/°C, is smaller than the total static mass of the house, excluding the basement and the attic, of 8.27 kwh/°C. The theoretical estimate of C is obtained by matching the exact transfer function of the house, linking indoor temperature to total indoor heat gain, to the corresponding transfer function of the equivalent model we proposed in Eq. 1. The range of equivalent masses thus obtained is 21% lower than the regressed value. This discrepancy is likely caused by 1) assuming a homogeneous, isotropic structure for the firewall cinder blocks, instead of their actual air cavity structure (the total cinder block thermal mass makes up 42% of the thermal mass of the house); 2) neglecting the studs within the walls whose thermal mass would add 10% to the house thermal mass; 3) neglecting the mass of the furniture (about 2% of the house mass). All of these omissions and simplifications were necessary in order to remain within the framework of one-dimensional heat transfer through homogeneous wall layers.

Similar techniques of matching transfer functions were used to calculate the value range for the other remaining parameter, HS, and are detailed in [7]. The correspondence appears to be excellent. It can be shown that the equivalent heat transfer constant between room air and structure, HS, is essentially the sum of all air boundary layer conductances (in W/°C) at the inside surfaces of walls, floors and ceilings. In other words, the sum of all products of the areas of inside structural surfaces (walls, ceilings, floors, but not windows or other "light" structures) and the appropriate film coefficients yields a close estimate of HS [7].

The parameters HC and H could be similarly obtained by matching appropriate transfer functions. However, this procedure can be shown to be practically equivalent to the simpler, steady-state derivation we described previously [7]. The theoretical values of HC and H, obtained from matching transfer functions, differ only by a few percent from those listed in Table 2.

It can also be shown that the equivalent thermal parameters are somewhat dependent on the frequency distribution of the boundary conditions. The frequency dependence is the more dramatic, the larger the thermal mass involved in the heat transfers with which the ETP's are associated. This frequency dependence can be shown in an indirect way, by changing the time interval, \( \Delta t \), between successive data points in the time series used for the regression. Table 1 displayed the regression coefficients and the derived ETP's for a time interval \( \Delta t = 1 \) hour. Table 3 shows the ETP's obtained from regressions using time intervals, \( \Delta t \), ranging between 20 minutes and 3 hours. The longer the time interval, the more high-frequency components are "filtered out" from the data time series. The consistency in the results for H and HC is a confirmation of our assumption that they are simple resistors. Because the heat transfers associated with HC involve massive firewalls and the basement, one would expect this to be a less valid assumption. And, in fact, the equivalent conductance, HC, increases slightly as the data frequency spectrum is extended on the high side by going to shorter time.
intervals. The equivalent solar window area, \( A \), also remains almost constant, confirming that the time delays involved in solar heating are smaller than the regression time frame. The equivalent thermal mass, \( C \), increases slightly as we filter out high frequencies from the data. In the (theoretical) limit of extremely slow varying temperatures and solar flux it should approach the static thermal mass of the entire house.

One also expects a decrease in \( HS \) as one goes to larger time intervals, eliminating higher frequencies. This trend is reflected in Table 3, but far exceeds what one would expect, especially for the two "extreme" time intervals, 20 minutes and 3 hours. This is simply a consequence of the diminished statistical significance in the determination of the regression coefficient \( b_2 \) (and, to a lesser extend, \( c_2 \)) in Eq. 5. The two "central" time intervals, 1 hour and 2 hours, beside providing better estimates for \( HS \), also yield the best tracking of the model (in the sense of least RMS).

CONCLUSION

A set of equivalent thermal parameters (ETP's) for an occupied townhouse has been proposed and experimentally determined. The values obtained from a free-floating indoor temperature experiment are consistent with what one would expect from theoretical consideration. The set of ETP's gauges the combined effect of residents and construction characteristics on the overall thermal response of the house. The comparative accuracy with which the ETP's can be determined from experimental data suggests their potential use as meaningful indices in the ranking of the thermal performance of different, occupied houses and, possibly, of larger buildings as well. Another potential use, experimentally demonstrated in a different paper [5], is an assessment of the overall effects of retrofits on a house.

Finally, the good predictive characteristics of the model should allow accurate, hour-by-hour estimates of indoor temperature in different weather situations, for the same house for which the ETP's were experimentally determined. Current research on the functional relationships between construction characteristics and ETP's is being completed [7]. If sufficiently reliable, such functional relationships could be used for the development of much simpler and cheaper, though less accurate and less general, algorithms to predict heating/cooling loads and indoor temperature.

APPENDIX

Heat Load Calculations

The floor plans of the townhouse and the detailed breakdown of all contributions in the steady-state heat load calculation are given in [7], using heat conductances from [9]. A summary is given in Table 4.

Equivalent Solar Window Area \( A \)

This parameter is the result of a contribution from the transparent windows (about 80% of the total \( A \), in our townhouse with south facing double pane windows making up 17% of the total south wall surface) and a contribution from the opaque walls. Using the sol-air temperature concept, the equivalent solar window area \( A \) is defined as

\[
A = \varepsilon_s W + H'(a/h)
\]

where
- \( \varepsilon_s \) is the average net transmissivity of the window glass to solar radiation (0.74 measured in January),
- \( W \) is the net transparent glass area of the south-facing windows (6.39 m\(^2\))
- \( a \) is the absorptivity of the outside opaque walls to solar radiation;
- \( h \) is the outside film coefficient (\( a/h = 0.035 (\text{"C}m^2)/W \) from [9]);
- \( H' \) is the heat conduction per unit temperature difference through the opposite outside walls and the attic (37.2 W/("Cm"))

While the net glass area \( W \) can be determined quite easily, the net transmissivity \( \varepsilon_s \) of glass to solar radiation is a complicated function of solar altitude, time of day, glass properties, number of window panes and more. For this experiment \( \varepsilon_s \) was determined experimentally with the use of two solar flux meters placed inside and outside the window.
Table 1. Regression Coefficients and Derived ETP's

<table>
<thead>
<tr>
<th>Miscellaneous Info.</th>
<th>Regression Coefficients$^1$</th>
<th>ETP's</th>
</tr>
</thead>
<tbody>
<tr>
<td>E+P+L = 5.976 kW</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TC = 18.4 °C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δt = 1 hour</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

|                | a = 0.85964 (73.6)          | HC = 330 W/°C  |
|                | b_1 = 0.05368 (14.0)        | H = 204 W/°C   |
|                | b_2 = 0.03986 (1.5)         | A = 5.94 m^2   |
|                | c_1 = 1.5625 (24.1)         | C = 3.22 kWh/°C|
|                | c_2 = 0.9237 (8.0)          | HS = 4,229 W/°C$^2$ |
|                | d = 3.1862 R^2 - 0.997     | HS = 5,448 W/°C$^3$ |

(1) The variables in the regression of Eq. 5 have dimensions [°C] (temperatures) and [kW/m^2] (solar flux).
(2) Computed using Eq. 6f.
(3) Computed using Eq. 6g.

Table 2. Comparison of ETP's Obtained from Regression and from Theory

<table>
<thead>
<tr>
<th></th>
<th>Regression</th>
<th>Theory</th>
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<tbody>
<tr>
<td>HC [W/°C]</td>
<td>330 ± 32</td>
<td>234 - 405</td>
</tr>
<tr>
<td>H [W/°C]</td>
<td>204 ± 20</td>
<td>211</td>
</tr>
<tr>
<td>A [m^2]</td>
<td>5.94 ± 0.67</td>
<td>6.04</td>
</tr>
<tr>
<td>C [kWh/°C]</td>
<td>3.22 ± 0.32</td>
<td>2.5 - 2.6</td>
</tr>
<tr>
<td>HS [W/°C]</td>
<td>4,229 ± 2,000$^1$</td>
<td>4,200 - 4,4000</td>
</tr>
<tr>
<td></td>
<td>5,448 ± 800$^2$</td>
<td></td>
</tr>
</tbody>
</table>

(1) Using Eq. 6f.
(2) Using Eq. 6g.

Table 3. ETP's Obtained from Regressions Using Different Time Intervals Δt

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>20 min.</td>
<td>0.998</td>
<td>363</td>
<td>197</td>
<td>5.40</td>
<td>2.85</td>
<td>10,596$^1$</td>
</tr>
<tr>
<td>1 hr.</td>
<td>0.994</td>
<td>330</td>
<td>204</td>
<td>5.94</td>
<td>3.22</td>
<td>4,229</td>
</tr>
<tr>
<td>2 hrs.</td>
<td>0.988</td>
<td>306</td>
<td>208</td>
<td>6.15</td>
<td>3.31</td>
<td>4,439</td>
</tr>
<tr>
<td>3 hrs.</td>
<td>0.977</td>
<td>308</td>
<td>211</td>
<td>6.10</td>
<td>3.60</td>
<td>965</td>
</tr>
</tbody>
</table>

(1) Computed using Eq. 6f.
(2) Computed using Eq. 6g.
### Table 4. Heat Load Calculation for 3-Bedroom-Wood-Frame Townhouse

<table>
<thead>
<tr>
<th>Contribution from</th>
<th>(U) [W/(°Cm²)]</th>
<th>(A) [m²]</th>
<th>(UA) [W/°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outside walls</td>
<td>0.556</td>
<td>63.3</td>
<td>35.2</td>
</tr>
<tr>
<td>Front Door</td>
<td>3.07</td>
<td>1.9</td>
<td>5.7</td>
</tr>
<tr>
<td>Double Pane Windows¹</td>
<td>4.26</td>
<td>11.67</td>
<td>49.7</td>
</tr>
<tr>
<td>Single Pane Patio Door¹</td>
<td>6.07</td>
<td>5.58</td>
<td>33.9</td>
</tr>
<tr>
<td>Ceiling</td>
<td>0.488</td>
<td>70.9</td>
<td>34.6</td>
</tr>
<tr>
<td>Roof²</td>
<td>3.95</td>
<td>84.2</td>
<td>332.8</td>
</tr>
<tr>
<td>Attic: Ceiling &amp; Roof in Series</td>
<td>--</td>
<td>--</td>
<td>31.3</td>
</tr>
<tr>
<td>Air Infiltration</td>
<td>0.5 ex/hr</td>
<td>(V = 328) m³</td>
<td>55.1</td>
</tr>
<tr>
<td>Total Equivalent Heat Transfer Constant</td>
<td></td>
<td></td>
<td>(H = 210.9)</td>
</tr>
<tr>
<td>1. &amp; 2. Floor Firewalls</td>
<td>0.965</td>
<td>104.2</td>
<td>100.6</td>
</tr>
<tr>
<td>Basement Ceiling</td>
<td>1.98</td>
<td>67.4</td>
<td>133.4</td>
</tr>
<tr>
<td>Basement-Living Space Convection</td>
<td></td>
<td>510 m³/hr</td>
<td>(HC = 234.0) (lower value)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>170.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>404.9 (higher value)</td>
</tr>
</tbody>
</table>

(1) 80% glass area, metal sash.
(2) Includes 3 exchanges per hour attic ventilation (measured).
REFERENCES


9. ASHRAE Handbook of Fundamentals (1972 ed.).

Acknowledgement

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EQUIVALENT ELECTRICAL CIRCUIT OF THE HOUSE

FIG. 1

XBL 7711-10445
INDOOR AND OUTDOOR TEMPERATURE AND SOLAR FLUX

FIG. 2
MEASUREMENT AND PREDICTION OF INDOOR TEMPERATURE

FIG. 3

XBL 7711-10787
INDOOR TEMPERATURE FROM MEASUREMENT AND FROM MODEL, ASSUMING WRONG INITIAL CONDITION

FIG. 4
This report was done with support from the United States Energy Research and Development Administration. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the United States Energy Research and Development Administration.