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PION PRODUCTION
IN NEUTRON-PROTON COLLISIONS
AT BEVATRON ENERGIES

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PION PRODUCTION
IN NEUTRON-PROTON COLLISIONS
AT BEVATRON ENERGIES

Fred Nels Holmquist
(Thesis)

December 4, 1958
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PION PRODUCTION
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AT BEVATRON ENERGIES
Fred Nels Holmquist
Lawrence Radiation Laboratory
University of California
Berkeley, California
December 4, 1958

ABSTRACT

Neutrons from a target bombarded by 6.2-Bev protons produced 330 three-prong and 34 five-prong events in a hydrogen-filled cloud chamber. These events were analyzed in an IBM program of constrained solutions. Frequencies for nine different particle configurations are estimated. The energy spectrum of the beam neutrons is obtained. Histograms for center-of-mass (c.m.) momentum and angles as well as $Q$ values between emitted particles and their c.m. angles are shown for the $pp$, $pp^+$, $pp^-$, $pn^-$, $pp^+0$, $pp^-0$, and $pn^+-0$ particle configurations. In all of these particle configurations, the two nucleons tend to be emitted in opposite directions in the c.m. system, one going forward and the other going backward. Both the opposite-direction effect and forward-backward effect are greater at Bevatron energies than at Cosmotron energies and decrease as the number of pions produced is increased. In the $pn^+$ reaction, ideograms of $Q(p\pi^+)$, $Q(n\pi^-)$, and especially the sum of these distributions show a good fit to the $T = 3/2$ interaction cross section observed in $\pi^+p$ scattering, as weighted by the two-body phase-space factor. In the $pp^-$ reaction, the $Q(p\pi^-)$ distributions show evidence for the $T = 3/2$ resonance and the proposed $T = 1/2$ resonance observed in single $\pi^+$ photo-production. In the $pn^+$ reaction, the protons are peaked strongly backward, while in the $pn^-0$, and $pn^+-0$ the protons tend toward forward emission. In the $Q(\pi\pi)$ distributions, evidence is found for pion-pion interactions in isotopic spin state $T = 2$. A proposed phenomenological, directional-isobar model of multiple-pion production is found to be successful in its predictions. In this model it is
assumed that each nucleon during a grazing-type collision is excited to the $T = 3/2$ isobaric state or a higher $T = 1/2$ isobaric state by absorbing or emitting a single pion, and subsequently decays by single- or double-pion emission.
INTRODUCTION

This paper is a report on pion production in cloud chamber-observed neutron-proton collisions at Bevatron energies. Preliminary reports on this experiment, the p-p cloud chamber experiment at Bevatron energies, and the completed report on the related \( \pi^-p \) cloud chamber experiment have already been published.\(^1\)-\(^3\)

This cloud chamber experiment is similar in many respects to the two n-p cloud chamber experiments done at the Cosmotron. In the experiment of Wallenmeyer, hereinafter referred to as I, neutrons of maximum energy 1.5 Bev were used.\(^4\) Fowler, Shutt, Thorndike, and Whittemore, hereinafter referred to as II, used neutrons of maximum energy 2.2 Bev.\(^5\) No five-prong events were observed. The analysis of three-prong events gave data on the kinematics of single and double-pion production events.

The present experiment, using neutrons with a maximum energy of 6.2 Bev, serves as a sequel to I and II. The analysis of the three- and five-prong events yields information on events showing up to four pion production.\(^6\) Where statistics are adequate, the histograms for single and double pion production events are compared with those of I and II to show the effects of increasing the incident neutron energy.
OBJECTIVES OF THE EXPERIMENT

It was intended for this experiment to provide information on a number of specific questions that are of importance from a theoretical or experimental point of view. The following questions are of interest:

Multiplicity of Pion Production

The ambiguity in the classification of events increases rather rapidly with the increase in incident-neutron energy. It is to be expected that the available information on the multiplicity of three-prong events will be of less utility than at the lower Cosmotron energies. The ratio of double to single pion production observed in I and II were instrumental in ruling out the Fermi statistical model of pion production. But it may be difficult, on the basis of multiplicities alone, to rule out any of the several theories on pion production in nucleon-nucleon collisions that take account of the \( T = 3/2, J = 3/2 \) resonance.\(^7\)\(^-\)\(^12\) Nevertheless, approximate frequencies of the various particle configurations are useful in helping to evaluate the prospects of a given theory.

Angular Distribution of Particles

Evidence for forward-backward asymmetries in the direction of the emitted particles in the center-of-mass system (c.m.s.) was first noted in II, where it was observed that in the reaction \( np \rightarrow pn^- \), protons and \( \pi^+ \) show a tendency to be emitted backward while neutrons and \( \pi^- \) tend toward forward emission.\(^13\) In I these distributions are more isotropic. This forward-backward asymmetry may be expected to increase with increase in incident-neutron energy. Since this effect does not occur in p-p collisions, the present n-p experiment gives a unique opportunity to study the effect. Forward-backward asymmetries are of course at variance with any theory that assumes nuclear forces to be so strong that a state of thermodynamic equilibrium is attained during the collision process.
Momentum Distribution of Particles
Considerable theoretical work has been done on the momentum distributions of particles in the c.m. system, making this an important parameter for experimental study. 8,11,12

Angular Correlations Between Emitted Particles
The angular correlations between pairs of emitted particles in the c.m. s. can perhaps be most easily visualized from the point of view of the isobaric model of pion production. In this model a nucleon is excited to a more massive, isobaric state during the collision process. In its simplest form, the isobar decays by emitting a single pion and returns to the ground state of proton or neutron. In this case the mass of the isobar is given by \( M' = M_n + M_\pi + Q \). Here \( M_n \) and \( M_\pi \) are the rest masses of the nucleon and pion respectively, and \( Q \) is the total kinetic energy of the nucleon and pion in their c.m.s.

Consider an n-p collision in which both nucleons are excited to an isobaric state, each of which subsequently decays by emitting a pion. At low bombarding energies the two isobars move slowly apart in the c.m.s. of the event. A pion and nucleon that are decay products from an isobaric state move in directions opposite to one another in their own c.m. and at low bombarding energies also tend to move in opposite directions in the c.m.s. of the event. As the bombarding energy is increased, the velocity of the isobars increases, with the result that the decay products from a given isobar tend to move in the same direction in the c.m.s. of the event. Another effect is that particles associated with one isobar tend to move in opposite directions to particles associated with the other isobar.

Q-Value Distributions Between Pairs of Particles
Q-value distributions may give the best evidence for pion production through a particular excited state of a nucleon. At Bevatron energies a wide range of possible isobaric mass values may be investigated. Evidence for pion-pion interactions may also be looked for in the Q-value distributions of these particles.
Neutron-Beam Energy

Events that can be classified unambiguously as to particle configuration yield the incident-neutron energy for that event. These events should give some idea of the energies of neutrons emitted from the Bevatron target.
EXPERIMENTAL PROCEDURE

Cloud Chamber Operation

The observations were made with a diffusion cloud chamber filled with hydrogen at a pressure of 35 atmospheres with methyl alcohol as condensable vapor. The chamber was operated in a pulsed magnetic field of 15,300 gauss. The sensitive region of the chamber was approximately 12 inches in diam. and 2.3 inches deep. An automatic camera was mounted on the cloud chamber 30 inches above the sensitive volume. Stereoscopic pairs of photographs were taken through two 50-mm Leitz Summitar lenses. A third lens viewed a number register and an ammeter indicating the current in the magnet coils. Details of the cloud chamber and the magnet have been described elsewhere. 14, 15

Experimental Arrangement

A schematic drawing of the experimental arrangement is shown in Fig. 1. The neutrons were produced by bombarding an internal copper target with 6.2-Bev circulating protons in the Bevatron. The cloud chamber was placed 75 ft from the target along a line tangent to the proton beam. The beam at the chamber was collimated to 5/8 inch by 2.5 inch. A 19-inch paraffin filter was inserted into the beam 55 ft from the cloud chamber to reduce the number of low-energy neutrons; and a 2-inch lead filter, followed by a small sweeping electromagnet, was inserted 40 ft from the cloud chamber to remove the γ rays.

Scanning Procedure

About 8700 Bevatron pulses were photographed. The pictures were scanned for three-prong, five-prong, and seven-prong events. An n-p interaction results in an odd number of emitted charged particles. One-prong events were not counted. Because of gaps or "holes" in the sensitive region caused by local depletion of the methyl alcohol vapor, it is often difficult to distinguish one-prong events from other tracks in the chamber. Furthermore, if one-prong events had been counted, their interpretation would generally have been
Fig. 1. Schematic drawing of the experimental arrangement.
ambiguous. For example, if a positive pion could have been identified by density of ionization, it would still have been impossible to determine the number of neutral pions emitted. Accordingly, it was decided to concentrate on events with three or more prongs.

The cloud chamber pictures were all scanned at least three times. The over-all scanning efficiency was probably greater than 98%. A total of 523 events was counted consisting of 473 three-prong events, 48 five-prong events, and one seven-prong event.

Preliminary Analysis
An early, preliminary analysis of the five-prong events indicated that in at least 16 events four or more pions are produced, and that in at least three events transverse momentum balance is shown, indicating that three pions are produced. It was pointed out that because it is easier to establish the absence of momentum balance than the presence of momentum balance, the ratio 3:16 can only be considered within statistics as a lower limit of three-pion to four-or-more-pion production in the five-prong events. The seven-prong event was believed to be of charge configuration \( np \rightarrow pp^{+}+\ldots \), but six-pion production was not ruled out. Here, \(+\) indicates \( \pi^{+} \) and \(-\) \( \pi^{-} \).

Since the publication of this preliminary report, a program of constrained solutions has been developed, using an IBM 650 digital computer, which gives a considerably more penetrating analysis of the events of this experiment. It is this high-speed IBM program that has made this study possible.

Measurement of Events
The events were measured on a stereoscopic projector or space table which is essentially the reverse of the optical system of the camera. The projector permits the reconstruction in space of events that occurred within the chamber. After alignment of the two stereoscopic pictures is achieved, a track is measured by rotating the surface of the space table in a horizontal plane and tipping and displacing it up and down in such a manner as to make the two track images coalesce on the surface of the table. Fine adjustment is achieved by alternately viewing first one image and then the other in rapid succession and watching closely for a corresponding rapid movement of the track.
For each event the film number, the position of the origin, the number of prongs, and the magnet current were recorded. For each prong or track the following data were recorded:

1. The dip angle $\alpha$ between the track and its projection in the horizontal plane, and the estimated uncertainty $\Delta\alpha$. (Typically $\Delta\alpha \approx 1.0^\circ$.)

2. The radius of curvature $\rho$, measured with the space table tipped $\alpha$ degrees, and the estimated uncertainty $+\Delta\rho$, $-\Delta\rho$. The $+\Delta\rho$ was generally larger than $-\Delta\rho$, especially for short prongs with little visible curvature. Radii were measured with ruled Plexiglass templates. The uncertainty in $\rho$ is determined by the uncertainty in sagitta. For an average-quality track the sagitta could be measured to $\pm 0.05$ mm. Abnormally dense and diffuse tracks and short tracks terminating in "holes" in which local turbulence was evident had sagitta uncertainties two to five times as large.

3. The azimuthal angle $\beta$ between the horizontal direction of the track and the direction of the neutron beam, and the estimated uncertainty $\Delta\beta$. (Typically $\Delta\beta \approx 0.5^\circ$.)

4. The estimated ionization density, $dE/dx$, and its uncertainty.

5. The visible length of the track.

6. The height of the center of the track and the horizontal distance from the center of the track to the magnet centerline.

7. Identification of the track. Most positive tracks with momentum less than 0.9 Bev/c [$[dE/dx]_p > 1.5 I_{\text{min}}$] were identified as protons or positive pions. Particle identification was the most useful information in determining the configuration of an event.

In events used in this experiment, all negative tracks were assumed to be pions, and all positive tracks were consistent with pions or protons. When the sign of a short track could not be determined directly, it could often be inferred from the known signs of the other two tracks and conservation of charge. In three-prong events, nucleons have a maximum angle of emission of about $80^\circ$ with respect to the direction of the incident neutron. Pions can be emitted in all directions. Tracks at angles larger than the maximum nucleon angle were identified as pions.
Uncertainties in momentum and angle measurements also had to be taken into account. Varying a measured quantity within its range of experimental uncertainty may lead to a number of different classifications of the same event. Selecting only well-measured events might lead to a bias of the results, since the geometry of an event partially depends on its type. For this reason, only events that were hopelessly immeasurable were omitted. In the acceptable region of the cloud chamber, defined by specified x, y, z limits, only five events were omitted because the momenta were not measurable. A much larger number of events were omitted because their origins occurred in gaps in the sensitive layer. This omission was necessary because estimation of ionization density, which is of the greatest importance in this experiment, was made by comparing ionization of the tracks at the origin of the event, where conditions for ionization and lighting were similar. Another group of events were omitted because their origins were outside the specified x, y, z limits in the cloud chamber.

Strange-Particle Production

Two events were eliminated because a negative track seemed clearly to have the ionization density of a K⁻ meson. Also eliminated were three events that showed a backward positive track of too great an ionization for a π⁺, and which, because protons cannot go backwards, were assumed to be K⁺ mesons. Nine of the ten events with associated V-particles have been discussed previously.¹⁹ The analysis of additional wall-produced V⁰ decays has also been discussed.²⁰ These strange-particle events are not discussed in this paper. The undetected events with strange-particle production may be expected to cause a small background of error, which may be ignored in this experiment. No events showing antinucleon production were detected.

A small number of θ₂-on-p three-prong events were expected.²¹ An attempt was made to balance energy and momentum, assuming an incident θ₂ particle in several of the three-prong events in which a K particle was indicated. The attempt was unsuccessful, however, and no systematic effort was made to identify these events.
Direction of Neutron Beam

It was originally estimated that the direction of the neutron beam was known to within about ± 1°. A study of about a dozen high-energy electron-positron pairs (which serve as target pointers) showed that the incident neutron azimuthal angle was about \(1.0^\circ\) different from the X-axis of the cloud chamber. Accordingly, a \(1.0^\circ\) correction was made on \(\beta\) and a \(0.0^\circ\) change on \(\alpha\). It was then estimated that the probable error in the direction of an incoming neutron was \(\Delta \beta \pm 1.0^\circ\) and \(\Delta \alpha \pm 0.7^\circ\). In the analysis program, these angular uncertainties were added to the estimated measurement uncertainties of \(\Delta \alpha\) and \(\Delta \beta\).
ANALYSIS OF EVENTS

Introduction

The problem of analyzing an event is difficult for two reasons: First, only the direction of the incoming neutron is known. Second, the number of neutral particles produced in a reaction is unknown. These neutral particles carry off momentum $P_n$ and energy $E_n$. Information on the presence of neutral particles -- and, in some cases, their probable minimum number -- was obtained from the requirement that energy and momentum be conserved in the event. As stated before, all negative particles were assumed to be pions. Positive particles, when not identified by ionization, were called protons or pions, in a way consistent with nucleon conservation. A separate calculation was made for each possible interpretation of the positive particles.

Consider first, the simpler case in which only one neutral particle is emitted. In an inelastic event having $n = 1, 3, 5, \text{ or } 7$ charged outgoing particles whose momenta and angles have been measured, the mass $M_n$ of the neutral particle is calculated by

$$M_n = (E_n^2 - P_n^2)^{\frac{1}{2}},$$

where

$$E_n = E_0 + M_p - \sum_{i=1}^{n} E_i$$

$$P_n = P_0 - \sum_{i=1}^{n} P_i.$$  

Where $E_0$, $P_0$ are the total energy and momentum of the incident neutron, $M_p$ is the mass of the struck proton, and $E_i$, $P_i$ are the total energy and momentum of the $i$th outgoing track. The energy $E_n$ depends on the masses chosen for the charged outgoing particles. Once these masses are specified, however, there are only two unknowns in Equations (2) and (3), and the mass of the emitted neutral particle is determined.

In the more complicated case in which two neutral particles are emitted, it is possible to represent the two-particle system as if
it were a single particle with an effective mass given by

\[ M_{\text{eff}} = \left[ (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2 \right]^\frac{1}{2} \]  

(4)

The effective mass is a minimum when the two particles go off hand in hand with the same velocity, and is a maximum when the particles are emitted with equal and opposite momenta. The most probable effective mass may be computed by assuming a statistical distribution of the momenta and angles of the particles. In this experiment, in which the incoming neutron momentum is not known, we attempted to distinguish only between particle configurations in which one and more than one neutral particles are emitted. For example, we attempted to distinguish between the particle configurations \( \text{pn}^+ \) and \( \text{pn}^+0 \ldots \), where the latter group includes \( \text{pn}^+0, \text{pn}^+00, \) etc. Here the zero indicates \( \pi^0 \) and dots other possible neutral pions. In the \( \text{pn}^+ \) reaction, the mass of the emitted neutron is 940 Mev. In the \( \text{pn}^+0 \ldots \) group, the effective mass was taken to be 1400 Mev. Other effective masses used in this experiment were 493 and 2200 Mev for the \( 00 \ldots \) and \( n0 \ldots \) neutral configurations respectively. Not knowing the incident-neutron momentum was the greatest single deterrent to obtaining detailed information in this experiment. It is usually difficult to distinguish between \( pp0 \) and \( pp00 \ldots \) and often difficult to distinguish between \( \text{pn}^+ \) and \( \text{pn}^+0 \ldots \). In fact, these and other particle configurations can only be classified on a probability basis.

Table I lists the various particle configurations that could be distinguished, together with the number of pions produced and the mass or effective mass of the neutral particles emitted. In each event, each of the possible interpretations listed in Table I was tried in a separate calculation.
<table>
<thead>
<tr>
<th>Type of event</th>
<th>Particle configuration</th>
<th>No. of pions</th>
<th>Mass or effective mass of neutral particles (Mev)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-prong</td>
<td>pn (elastic)</td>
<td>0</td>
<td>940</td>
</tr>
<tr>
<td></td>
<td>pn0..</td>
<td>&gt; 1</td>
<td>1400</td>
</tr>
<tr>
<td></td>
<td>nn+..</td>
<td>&gt; 1</td>
<td>2200</td>
</tr>
<tr>
<td>3-prong</td>
<td>pp-</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>pn+-</td>
<td>2</td>
<td>940</td>
</tr>
<tr>
<td></td>
<td>pp-0</td>
<td>2</td>
<td>135</td>
</tr>
<tr>
<td></td>
<td>pn+-0..</td>
<td>&gt; 3</td>
<td>1400</td>
</tr>
<tr>
<td></td>
<td>pp-00..</td>
<td>&gt; 3</td>
<td>493</td>
</tr>
<tr>
<td></td>
<td>nn+++--</td>
<td>&gt; 3</td>
<td>2200</td>
</tr>
<tr>
<td>5-prong</td>
<td>pp++--</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>pn++--</td>
<td>4</td>
<td>940</td>
</tr>
<tr>
<td></td>
<td>pp++-0</td>
<td>4</td>
<td>135</td>
</tr>
<tr>
<td></td>
<td>pn++--0..</td>
<td>&gt; 5</td>
<td>1400</td>
</tr>
<tr>
<td></td>
<td>pp++--00..</td>
<td>&gt; 5</td>
<td>493</td>
</tr>
<tr>
<td></td>
<td>nn++++--</td>
<td>&gt; 5</td>
<td>2200</td>
</tr>
<tr>
<td>7-prong</td>
<td>pp+++--</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>pn++++--</td>
<td>6</td>
<td>940</td>
</tr>
<tr>
<td></td>
<td>pp++++--0</td>
<td>6</td>
<td>135</td>
</tr>
</tbody>
</table>
A typical three-prong event is shown schematically in Fig. 2. One of the positive prongs was identified as a $\pi^+$ and the other was not identified. The three possible interpretations are $pn^+$, $pn^+\pi^0$, and $nn^+\pi^0$.

When neither of the positive prongs were identified, eight different interpretations of the event were tried. This large number of possible choices made it probable that the event would be relegated to the ambiguous group.

\[ p_n = p_0 - p_1 - p_2 - p_3 \]
\[ E_n = E_0 + M_p - E_1 - E_2 - E_3 \]
\[ M_n = (E_n^2 - p_n^2)^{1/2} \]

$n + p \rightarrow pn\pi^+\pi^-$

$n + p \rightarrow pn\pi^+\pi^0 + (\text{possible } \pi^0, \text{s})$

$n + p \rightarrow nn\pi^+\pi^- + (\text{possible } \pi^0, \text{s})$

Fig. 2. Schematic drawing of a typical three-prong event.
Program of Constrained Solutions

To meet the two-fold problem of not knowing the momentum of the incoming neutron, $P_0$, or the effective mass of the emitted neutral particles, an IBM program of constrained solutions was used. The program was called "Cloudy." For each of the permitted mass values of the positive particles, the machine calculated $P_n$, $E_n$, $M_n$, and the derivatives of $M_n$ with respect to all the input parameters. A three-prong event had ten such parameters: $P_0$, $P_1$, $P_2$, $P_3$, $a_1$, $a_2$, $a_3$, $\beta_1$, $\beta_2$, $\beta_3$. In this unadjusted solution, the specified probable values of the parameters were used. This procedure requires an estimated value of $P_0$. The exact value of $P_0$ is given by

$$P_0 = \sum_{i=1}^{n} P_{ix} + P_{nx}, \tag{5}$$

where $P_{ix}$ and $P_{nx}$ are the forward components of the charged and neutral prongs respectively. As an approximation, we assume that $P_{nx}$ is a constant $K_n$, whose value depends on the particle configuration involved. We have

$$P_0 \approx \sum_{i=1}^{n} P_{ix} + K_n. \tag{6}$$

The unadjusted value of $M_n$ is seen to depend on the choice of $K_n$. The upper limit of $P_0$ is taken as 7.08 Bev/c (6.2 Bev), corresponding to the upper energy limit of neutrons from the target inside the Bevatron. The lower limit is taken to be 1.00 Bev/c (0.44 Bev), corresponding to a lower limit for reasonable pion production.

Starting with derivatives of $M_n$ with respect to the input parameters at the unadjusted mass value, and using an iterative procedure, the IBM machine adjusted each input parameter within its estimated error so as to bring the adjusted value of $M_n$ to a value corresponding to the appropriate mass or effective mass of the emitted neutral configuration. This was done in such a way that the fractional change, $\tau$, of the estimated error of each parameter was the same. The sign of the change was chosen to reduce the error in $M_n$. This adjustment $\tau$ is defined by $\tau = (\hat{u}_\tau - u_0)/\Delta u$ where $u$ is any of the
input parameters, \( \Delta u \) is the estimated error of \( u \) (in the direction of the adjustment), \( u_0 \) is the measured value, and \( u_r \) is the value used in the adjusted solution. The magnitude of \( \tau \) serves as a measure of the probability that the corresponding adjusted solution is indeed the correct one. The maximum and minimum values of \( M_n \), corresponding to \( \tau = \pm 1 \), were also calculated. In most events the main variation in \( M_n \) was due to \( P_0 \) and a few other input parameters (usually the momenta). Although all the measured quantities were adjusted, some had such small errors or small \( \partial M_n / \partial u \) that almost the same adjustment factor \( \tau \) would have been obtained if only a few input parameters had been adjusted. An event that was difficult to measure, and which had correspondingly large errors on the input parameters, often had a relatively small \( \tau \) adjustment on several of the possible particle-configuration interpretations. The probability that a given interpretation was indeed the correct one depended not only on the absolute value of \( \tau \), but also on the relative \( \tau \) values for the different particle configurations viewed in relation to one another.

It would have been desirable to make a plot of \( \tau \) vs \( P_0 (K_n) \) for each possible particle configuration. A configuration having a small adjustment factor over a considerable range of Bevatron energies would be more probable than another configuration having larger adjustment factors at one end of the neutron spectrum. A knowledge of the neutron spectrum would then permit many events to be classified as to particle configuration, with a good probability that the classification was the correct one.

Unfortunately, the cost of using the IBM 650 computer for such a detailed analysis program is prohibitive. However, the cost of using the IBM 704 computer for such a program would not be prohibitive. Such a program is now in preparation.

Because the Cloudy program was already pushing the limit of capabilities of the IBM 650 computer, it was only possible to choose different \( K_n \) values for nonbalanced particle configurations containing 0, 1, or 2 emitted neutrons. On the basis of a first study of a group of events in which \( K_n \) was taken to be equal to zero for all particle
configurations, \( K_n \) was then taken to be equal to 0.3, 1.6, and 3.0 Bev/c for three-prong events containing 0, 1, and 2 emitted neutrons, respectively. In order to maintain a homogeneous group of analyzed events, these values were not changed throughout the analysis. On the basis of a similar first study of the five-prong events, average values for \( K_n \) of 0.5 and 1.0 Bev/C were taken for 0 or 1 emitted neutron, respectively.

In attempting to determine which of the various possible charge configurations of an event is the correct one, the adjustment factors of the different configurations are of primary importance. These \( \tau \) values, together with the requirement that the energy of the incoming neutron be less than the Bevatron energy of 6.2 Bev, were often sufficient in themselves to make a classification decision. Other criteria used in classifying an event were unadjusted mass values, number of possible choices, and the relative size of \( +\Delta P_0 \) and \( -\Delta P_0 \). In a small number of events, an interpretation showing both nucleons emitted in the same hemisphere in the c.m.s. was used as evidence against that particular particle configuration. These criteria are discussed below. Values of \( \tau > 1 \) are excluded as possibilities; \( \tau > 0.6 \) can seldom be classified in an unambiguous manner. Other criteria being equal, \( \tau \) values < 0.1 are considered equally probable. Considering \( \tau \) values by themselves, a rough rule of thumb for identifying an event is that a well-identified particle configuration must have a value at least 0.4 less than its nearest competitor's, and that less-well-identified, merely probable events must be at least 0.2 less than the next best. This rule applies to three-prong events in which either a proton or a \( \pi^+ \) or both a proton and a \( \pi^+ \) are identified by ionization density. For example, in an event with three possible charge configurations having \( \tau \) values of 0.20, 0.45, and 0.80, the first configuration would be in the probable class. An event can be classified as either well identified, probable, improbable, ambiguous, or unclassifiable. If an event is classified as either well identified or probable, then it cannot also have any other classification. Three-prong events in which neither of the positive prongs are identified can seldom be classified as probable.
An unadjusted mass value close to the corresponding neutral mass value for a particle configuration was considered as evidence for that configuration, especially for particle configurations having a single emitted neutral particle. Proximity of an unadjusted mass value to the required value for that configuration did not necessarily mean that the corresponding adjustment parameter was small. Values of $\pm \Delta P_0$ were used especially in a number of borderline cases in which $+\Delta P_0$ was considerably larger or smaller than $-\Delta P_0$. To understand the effect of $\pm \Delta P_0$ on the adjustment parameters, consider a well-measured event in which $P_0$ is the only parameter that is not accurately known. Suppose $+\Delta P_0 = 1.00$ Bev/c, which makes $-\Delta P_0 = 5.08$ Bev/c. In adjusting to a neutral mass value larger than the unadjusted mass, it is characteristic for the momentum of the incoming neutron to be adjusted upward and for the corresponding $\tau$ value to be positive. In adjusting to a smaller mass value, the opposite is true. All charge configurations of a given event have the same $\pm P_0$ values. If the specified value for $K_n$ yields an unadjusted mass value that is less than the required neutral mass value, the amount of adjustment will be larger than if the specified value for $K_n$ yields an unadjusted mass that is greater than the required neutral mass by a similar amount. This is a direct result of the smallness of $+P_0$. For a given difference between the unadjusted and adjusted mass values, the positive adjustment factors among the different particle configurations would tend to be larger than the negative adjustment factors. Therefore, in considering events in which $+P_0$ is considerably larger or smaller than $-P_0$, and especially in considering events difficult to interpret, the sign of the adjustment factors was taken into account.

For the well-identified and probable events, a priori probabilities based on theoretical relative abundance of the different charged configurations were not used in the evaluation process. Thus a majority-type event such as pn+ is more likely to be correctly identified than a minority-type such as pp-0. In no cases were $Q$ values used to help identify the type of event.

At Bevatron energies, a very pronounced kinematical effect, common to all particle configurations, is for the two nucleons to be
in opposite hemispheres in the c.m.s. Another pronounced effect is for one nucleon to go forward in the c.m.s. and the other to go backward. Except for one minority group (pp-0, pp-00), such information was used as evidence against a particle configuration in only a small fraction of events and only in events difficult to interpret. For example, consider an event for which the IBM program of constrained solutions gives a good fit to both the pn+ and the pp-0 configuration. If in the former type the p and n go in approximately the same direction in the c.m.s., this information could be used as evidence against the pn+- type and therefore for the pp-0 type.

**Supplementary Program for Balanced Events**

In the balanced pp- and pp+- reactions, no neutral particles are emitted. In the program of constrained solutions, it is typical for the unadjusted mass $M_n$ to be small and negative and for both the limiting values of $M_n$, corresponding to $\tau = \pm 1$, to be large and negative. Unfortunately, the program is not amenable to adjustment to a zero-mass value. It was found advantageous to use a supplementary IBM program, designed for adjustment to zero mass. In this program, each of the measured angles and momenta has a different adjustment factor instead of the same adjustment factor as in the standard program of constrained solutions described above. An average value of these adjustment factors was used in evaluating the events.
MULTIPLICITY OF PION PRODUCTION

As stated previously, 473 three-prong events, 48 five-prong events, and one seven-prong event were counted when the film was scanned. One-prong events were not counted. Of these events, 330 three-prong and 34 five-prong events were accepted for analysis in this study. The great majority of the events not included were removed because the origins of the events occurred in holes where they could not be seen or because the origins were outside the specified x, y, z limits. A much smaller number of events were removed because they were hopelessly immeasurable, because the analyzing magnetic field was below a specified limit, or because they involved strange-particle production.

Three-Prong Events

Of the 330 three-prong events that were analyzed, 64 were well-identified as to particle configurations listed in Table I; 120 were classified as probable; 126 were ambiguous or improbable; and 20 were not obviously consistent with any particle configuration. The latter group contained 16 of the poor-quality events that were difficult to measure.

In each event it was possible to give with reasonable probability the number or minimum number of pions that could be produced in that event. The breakdown on the number of pions produced in three-prong events is given in Table II. It is assumed that the well-identified and probable events were correctly identified.

The data of Table II indicate that the average number of pions produced per three-prong event is greater than 2.0, and that the most probable number of pions produced in any given three-prong event is 2.

The 330 three-prong events may also be classified as to positive prongs identified by density of ionization. This breakdown gives 69 p+, 105 px-, 100 +x-, 14 ++-, and 42 xx-, where x denotes an unidentified positive prong.
Table II

<table>
<thead>
<tr>
<th>Number of three-prong events</th>
<th>Number of pions produced</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>≥ 1</td>
</tr>
<tr>
<td>20</td>
<td>≥ 1</td>
</tr>
<tr>
<td>116</td>
<td>= 2</td>
</tr>
<tr>
<td>88</td>
<td>&gt; 2</td>
</tr>
<tr>
<td>71</td>
<td>&gt; 3</td>
</tr>
</tbody>
</table>

In discussing the different particle configurations, we shall often denote the pp- reaction by B (for Balance), pp-0 by B₀, pp-00 by B₀₀, pn+ by A, pn+0 by A₀, and nn++ by C.

In Table III we have attempted to summarize the data on three-prong events that are pertinent in estimating the multiplicities of these events. Four main groups contain the bulk of available information. These are: (a) the well-identified and probable events; (b) the doubly ambiguous events; (c) the B₀B₀₀A₀ group; (d) the very ambiguous xx-group. The 37 events not included are not expected to change the overall picture as far as multiplicities are concerned. For example, the 20 events that were not obviously consistent with any particle reactions had particle identification of 6 p+, 9 px-, 4 px-, and 1 xx-. This distribution is not very different from that of the total three-prong group.

Except for the nn++ configuration, the well-identified and probable events are assumed to be correctly identified. Eleven of the 12 nn++ events are so interpreted because both positive pions are identified. When one considers that 183 three-prong events have at least one π⁺ identified, it is reasonable to assume, in at least a few borderline cases in which the second positive particle is really a proton (e.g. p = 1.5 lₘᵋₜₜ), that because of a "hole" in the sensitive layer it will look minimum and therefore be called a π⁺. To point out this effect, we have reduced the number 12 to 10. From the five-prong data of Table IV, the probable number of pp++ plus pp++-0 reactions is seen to be 13. By charge symmetry this number is also
Table III

Observed multiplicities of three-prong events

<table>
<thead>
<tr>
<th>Abbreviated notation</th>
<th>Particle configuration</th>
<th>Well-identified</th>
<th>Probable</th>
<th>Total</th>
<th>B</th>
<th>A</th>
<th>B₀</th>
<th>A₀</th>
<th>B₀₀</th>
<th>C</th>
</tr>
</thead>
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<td>Well-identified &amp; probable Group</td>
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<td></td>
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<td></td>
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<td>B</td>
<td>pp⁻</td>
<td>12</td>
<td>23</td>
<td>35</td>
<td>35</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>A</td>
<td>pn⁻</td>
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<td>6</td>
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<td></td>
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<tr>
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<td>pn⁻⁻⁻⁻</td>
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<td>34</td>
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<td>pp₀₀⁻⁻⁻⁻</td>
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<td>0</td>
<td>0</td>
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<tr>
<td>C</td>
<td>mn++⁻⁻⁻⁻</td>
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<tr>
<td>Doubly ambiguous Group</td>
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<td>8</td>
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<td>4</td>
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<td>12</td>
<td>9</td>
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<td>4+1</td>
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<tr>
<td>Very ambiguous xx- Group (counted at half value)</td>
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<tr>
<td>A₀</td>
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<td>4</td>
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<td>2</td>
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<td>132±23</td>
<td>18±8</td>
<td>67±16</td>
<td>7±5</td>
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</table>
expected for the \( nn^{++} \ldots \) or \( C \) configuration. Since we have ten of these events already, by identifying both positive pions we might expect to find at least another ten events in which both positive pions are not identified. More information on this point will be discussed later; for the present, we assume that the number of \( pp^{++} \) plus \( pp^{+-} \) events is small by statistical fluctuation, and that the number of \( nn^{++} \ldots \) is somewhat larger than 13, perhaps about 17.

The second group of Table III is made up of four subgroups that are primarily doubly ambiguous in the sense of the particle configurations listed in Table I. We know that the \( AC \) and \( A_0C \) subgroups must be largely \( A \) and \( A_0 \) respectively. The breakdown is shown in the right-hand columns of the table. The division of the \( AB \) and \( AA_0 \) subgroups was made according to previous expectations.

The \( B_0 \) and \( B_{00} \) events are generally mutually ambiguous. They also have a tendency to be ambiguous with \( A \) and to a lesser extent with \( A_0 \). The \( B_0B_{00}AA_0 \) group is a composite of a number of subgroups whose preferred components are shown in Table III. An attempt was made to reduce this ambiguity by accepting as \( B_0B_{00} \) reactions those events in which the directions of \( p \) and \( n \) in the \( pn^{+-} \) choice were less than 90° apart in the c.m.s. This procedure gave 12 events, probably of type \( B_0B_{00} \). These 12 included three of the five events that were probably \( B_0 \) or \( B_{00} \) by the regular criteria. In order to arrive at working numbers for the different configurations, we considered 8 of the 12 \( B_0B_{00} \) events to be \( B_0 \) and 4 \( B_{00} \). In the histograms, however, we shall include all 12 events with the \( B_0 \) group. This procedure will be substantiated later when it is shown that there is an expected correspondence effect between certain particles of \( pn^{+-} \) reaction and the particles of \( pp^{0} \) events selected in the above manner. The 14 \( B_0B_{00}AA_0 \) events with \( p \) and \( n \) in opposite hemispheres are taken to be 9 \( A \), 3 \( A_0 \), 1 \( B_0 \), and 1 \( B_{00} \).

Of the 42 \( xx \) events, 26 have four or more possible interpretations as to particle configuration. These interpretations were rated as good fits and poor fits. The good fits showed an abnormally large number of possible \( B_0 \) and \( B_{00} \) reactions. The six particle configurations
were then weighted in proportion to their number of good fits. Half of them were counted. To the $B_0$ reaction group, this added three events. This is not too much to add, considering that 15 of the 26 events could be fitted reasonably well to this reaction.

The total number of events counted for each of the six three-prong configurations is given at the bottom of Table III. These numbers will later be compared with the predicted theoretical values. Because of the numerous ambiguous events, the minority configurations tend to have larger upper limits than lower limits. The errors cannot be calculated in quantitative terms. Those stated in Table III are twice the statistical errors.

**Five-Prong Events**

As only one seven-prong event of configuration $pp++---$ was observed, five-pion production events must occur much less often than four-pion production events, and six-pion production events may be ignored. One probable $nn+++--$ event was distinguished by identifying all three positive pions. In the program of constrained solutions we first considered the five-pion production events to be on an equal basis with three- and four-pion production events. Of the 34 five-prong events accepted for analysis, five were well-identified, 12 were classified as probable, 14 were ambiguous or improbable, and three of the poorer-quality events were not obviously consistent with any of the particle configurations.

In Table IV the data pertinent to an estimation of the multiplicities of the five-prong events are summarized. In the first group, in which the numbers of well-identified and probable events are listed, five-pion production events are evaluated on an equal basis with three- and four-pion production events. A shorthand notation for the different reactions similar to that used for the three-prong events is shown in the table.

In the remaining three groups of Table IV, five-pion production is neglected. This assumption adds two more $pn++--$ events, which were ambiguous with $nn+++--$, to the eight events of this configuration from the first group. In the histograms for the $pn++--$ reaction, these two events are classed as probable.
Table IV

<table>
<thead>
<tr>
<th>Abbreviated Notation</th>
<th>Particle configuration</th>
<th>Well-identified configuration</th>
<th>Probable</th>
<th>Well-identified &amp; probable</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E'$</td>
<td>pp++--</td>
<td></td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>$A'$</td>
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<td>1</td>
</tr>
<tr>
<td>$C'$</td>
<td>nn++--</td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$A'_0$</td>
<td>pn++--0</td>
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<td>0</td>
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</tr>
<tr>
<td>$B'_{00}$</td>
<td>pp++-0</td>
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<td>0</td>
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<td><strong>Probable group</strong></td>
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<td>$A'A'B'$</td>
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<td><strong>Improbable-fit group</strong></td>
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*(counted at half value)* $\frac{1}{2} \times 5 = 2\frac{1}{2}$

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<tr>
<th>Events counted</th>
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<th>$A'_{i}$</th>
<th>$B'_{i}$</th>
<th>$C'_{i}$</th>
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<table>
<thead>
<tr>
<th>Events observed</th>
<th>Events counted</th>
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<td>$E'$</td>
<td>6</td>
</tr>
<tr>
<td>$A'$</td>
<td>8</td>
</tr>
<tr>
<td>$B'$</td>
<td>1</td>
</tr>
<tr>
<td>$C'$</td>
<td>1</td>
</tr>
<tr>
<td>$A'_0$</td>
<td>0</td>
</tr>
<tr>
<td>$B'_{00}$</td>
<td>0</td>
</tr>
<tr>
<td>$A'$</td>
<td>2</td>
</tr>
<tr>
<td>$B'_{0}$</td>
<td>2</td>
</tr>
<tr>
<td>$A'A'B'_{0}$</td>
<td>1</td>
</tr>
<tr>
<td>$A'B'_{0}$</td>
<td>1</td>
</tr>
<tr>
<td>$A'A'$</td>
<td>1</td>
</tr>
<tr>
<td>$A'A'B'$</td>
<td>1</td>
</tr>
<tr>
<td>$A'B'$</td>
<td>1</td>
</tr>
<tr>
<td>$B'B'$</td>
<td>1</td>
</tr>
<tr>
<td>$A'$</td>
<td>$2\frac{1}{2}$</td>
</tr>
<tr>
<td>$A'A'$</td>
<td>1</td>
</tr>
<tr>
<td>$B'$</td>
<td>1</td>
</tr>
</tbody>
</table>

5-pion production on equal basis with 3- and 4-pion production.

5-pion production neglected.
The six ambiguous events listed in Table IV are divided into $B^1$, $A'$, and $B_0'$ in proportion to their respective components.

In the improbable-fit group, two of the events are taken from the three poor-quality events that did not obviously fit any reaction but whose particle identification of $++--x$ and $p++--$ requires the $A'$ classification. The other three events show poor fits to the reactions indicated. Events in this last group are counted at half value.

The errors estimated for the totals of the different five-prong events include the statistical errors.

**Comparison of Experimental Multiplicities with Theory**

These experimental multiplicities may be compared with those predicted by the pion-production model of Belinki and Nikishov,\(^9\) and also with those predicted by the similar model of Yamamoto.\(^10\) Multiplicities calculated using the former model were in very good agreement with the observed frequencies of experiment II. For one, two, three, or four pions produced, these models include the multiplicity ratios used in the Fermi statistical theory. For single- or double-pion production they also include the multiplicity ratios of the Peaslee model of pion production in which one or both of the nucleons is excited to a $T = 3/2$ state, which subsequently decays with emission of a pion.

In the model of Belinki and Nikishov, the $T = 3/2$, $J = 3/2$ isobaric state is included in the statistical theory of multiple formation of particles. They consider the strongly interacting pion-nucleon system as a single particle, and assume that during a nucleon-nucleon collision particles with a mass of $1.32 M_0$ can be produced, where $M_0$ is the mass of a nucleon. They also assume that the probability of the formation of such particles is determined by the statistical weight. In two-pion production, for example, the states that originate during the collision of nucleons are $NN\pi\pi$, $NN'\pi$, and $NN'$, where $N$ and $N'$ denote a nucleon and the isobaric state of the nucleon respectively. The weightings of the three groups vary with the energy of the incident nucleon and are computed on the basis of the statistical theory.
In Table V, the charge distributions for particle configurations with m pions are shown for the three cases in which 0, 1, or both of the nucleons are excited to the T = 3/2 isobaric state. Equal weights of the T = 0 and 1 are assumed for the initial state of the nucleons. For the case in which neither nucleon is excited to the isobaric state, the fractional probabilities are those used in the Fermi statistical theory. The fractional probabilities for single-pion production with one excited nucleon and double-pion production with both nucleons excited are those used in the Peaslee model.

The fractional probabilities listed in Table V and the calculated frequencies at 5.3 Bev are taken from a more recent paper by Maksimenko and Nikishov in which the multiplicities predicted by the Belenkii and Nikishov model are calculated at this energy. Numbers at 5.3 Bev are normalized to the total number of 306 three-prong and five-prong events used in this experiment.

The frequencies at 3.0 Bev are calculated on the basis of a similar model of Yamamoto, in which the mass of the excited nucleon is taken to be 1.17 M0. Since four-pion production is neglected at this energy, the numbers at 3.0 Bev are normalized to the sum of the observed pp-, pp-0, pn+, and pp++ events.

The median energy for all well-identified and probable events in this experiment is 3.8 Bev. We may expect better agreement at 3.0 Bev for one-, two-, and three-pion production than at 5.3 Bev. A legitimate comparison with either of these models, of course, requires calculated frequencies integrated over the incident-neutron energy spectrum. We may, however, point out a few observations. The one-, two-, and three-pion production events are roughly consistent with the calculated values at 3.0 Bev, and the four-pion production events are consistent with the 5.3 Bev calculated frequencies. It appears, however, that the calculated frequencies integrated over the neutron energy spectrum might not be consistent with the multiplicities observed in this experiment. For example, the calculated ratio of pn++-/pp+-- is 0.72 at 5.3 Bev and decreases at lower bombarding energies. The observed ratio is $15^{+6}_{-5} / 9^{+5}_{-4}$, making agreement between theory and experiment difficult.
Table V

Charge distributions in n-p collisions for 0, 1, and 2 excited T = 3/2 nucleons, assuming equal weights of T = 0, 1; predicted multiplicities of the M' = 1.32 M0 Belinki-Nikishov model at 5.3 Bev; predicted multiplicities of the M' = 1.17 M0 Yamamoto model at 3.0 Bev; and multiplicities observed in this experiment at 3.8 ± 2.4 Bev. Five pion production is ignored.

<table>
<thead>
<tr>
<th>No. pions</th>
<th>Particle configuration</th>
<th>Fractional probabilities</th>
<th>Multiplicities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>NNmπ</td>
<td>NN'(m-1)π</td>
</tr>
<tr>
<td>0</td>
<td>pn</td>
<td>1.000</td>
<td>0.167</td>
</tr>
<tr>
<td>1</td>
<td>pp-</td>
<td>0.278</td>
<td>0.167</td>
</tr>
<tr>
<td></td>
<td>pn0</td>
<td>0.444</td>
<td>0.666</td>
</tr>
<tr>
<td></td>
<td>nn+</td>
<td>0.278</td>
<td>0.167</td>
</tr>
<tr>
<td>2</td>
<td>pp-0</td>
<td>0.109</td>
<td>0.137</td>
</tr>
<tr>
<td></td>
<td>pn++</td>
<td>0.460</td>
<td>0.563</td>
</tr>
<tr>
<td></td>
<td>pn00</td>
<td>0.156</td>
<td>0.163</td>
</tr>
<tr>
<td></td>
<td>nn+0</td>
<td>0.159</td>
<td>0.137</td>
</tr>
<tr>
<td>3</td>
<td>pp+-</td>
<td>0.138</td>
<td>0.124</td>
</tr>
<tr>
<td></td>
<td>pp-00</td>
<td>0.100</td>
<td>0.087</td>
</tr>
<tr>
<td></td>
<td>pn+-</td>
<td>0.462</td>
<td>0.508</td>
</tr>
<tr>
<td></td>
<td>nn++</td>
<td>0.138</td>
<td>0.124</td>
</tr>
<tr>
<td></td>
<td>pn000</td>
<td>0.062</td>
<td>0.070</td>
</tr>
<tr>
<td></td>
<td>nn+00</td>
<td>0.100</td>
<td>0.087</td>
</tr>
<tr>
<td>4</td>
<td>pp-+-</td>
<td>0.179</td>
<td>0.163</td>
</tr>
<tr>
<td></td>
<td>pn+++-</td>
<td>0.209</td>
<td>0.229</td>
</tr>
<tr>
<td></td>
<td>pp-000</td>
<td>0.048</td>
<td>0.043</td>
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<tr>
<td></td>
<td>pn-000</td>
<td>0.316</td>
<td>0.338</td>
</tr>
<tr>
<td></td>
<td>nn+-0</td>
<td>0.179</td>
<td>0.163</td>
</tr>
<tr>
<td></td>
<td>pn0000</td>
<td>0.021</td>
<td>0.021</td>
</tr>
<tr>
<td></td>
<td>nn+0000</td>
<td>0.048</td>
<td>0.043</td>
</tr>
<tr>
<td></td>
<td>pn+-0 plus pn-00</td>
<td>120.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>nn+- plus nn+-0</td>
<td>29.9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>pp-00 plus pp-000</td>
<td>28.4</td>
<td></td>
</tr>
<tr>
<td>3 total</td>
<td></td>
<td>260.9</td>
<td></td>
</tr>
<tr>
<td>5 total</td>
<td></td>
<td>45.1</td>
<td></td>
</tr>
</tbody>
</table>

*aDiffraction scattering is not included.
The assumption of all the statistical theories that the possible final states reach statistical equilibrium during the collision process requires that the angular distributions of the emitted particles have forward-backward symmetry in the c.m.s. Maenchen has pointed out that in $\pi^-p$ interactions at 5 Bev, some of the distributions are quite asymmetric, indicating that complete statistical equilibrium was not attained. 3 Forward-backward asymmetries were also noted in II. It will be seen that in our experiment a number of the distributions show strong forward-backward asymmetries. It seems that good agreement between statistical theory and experiment should not be expected at Bevatron energies.
ENERGY SPECTRUM OF INCIDENT NEUTRONS

Events which can be classified in an unambiguous manner yield the incident-neutron energy for that event. The momentum distributions of the neutrons producing the various observed particle configurations are shown in Fig. 3. All the well-identified and probable events of this experiment are included except those for the \( nn^+-- \) configuration, which are not as reliable as the others. (See the section on the \( nn^+-- \) group in the next chapter.) In the neutron momentum distribution for the \( pp^- \) reaction, seven of the events classed as probable were difficult to distinguish from the \( pn^+ \) reaction. (See the section on the \( pp^- \) reaction.) These events are indicated in Fig. 3.

The median momenta for one-, two-, three-, four-, and five-pion production are approximately 4.4, 4.65, 4.9, 5.1, and 6.8 Bev/c. The median momenta for all the events is 4.7 Bev/c.

The proximity of the median momenta for one-, two-, three-, and four-pion production permitted us to make a reasonable estimate of the peak energy of the beam neutrons. On the basis of statistical theory, the cross section for single-pion production in the energy range of the Bevatron neutron spectrum decreases with increasing incident neutron energy, while the cross sections for three- and four-pion production increase.\(^{10}\) Also, experiment has shown that three- and four-pion production increases in this energy range. If it is assumed that the neutron-proton inelastic cross section is fairly constant in this energy range, as it is for proton-proton inelastic scattering (26 mb), then it also may be assumed that single-pion production decreases at Bevatron energies.\(^{25}\) The peak energy of the beam neutrons is therefore taken to be greater than the 4.4 Bev/c median energy for single-pion production and less than the 4.9 Bev/c value for triple-pion production. The peak of the incident-neutron-beam momentum spectrum is estimated to be at 4.7 Bev/c in coincidence with the median momenta for all the events of Fig. 3. If the 7 \( pp^- \) events that were difficult to distinguish from the \( pn^+ \) reaction had been excluded, the median energy of the \( pp^- \) events would have been
Fig. 3. Momentum distributions of neutrons producing the observed particle configurations.
4.8 Bev. The fact that triple- and quadruple- pion production increases with increase in incident neutron energy is consistent with the same peak energy of the beam neutrons.

The corresponding neutron kinetic energy of 3.8 Bev is in good agreement with the value obtained by Barrett using nuclear emulsions as a detector. In Barrett's experiment, absorption cross sections for Bevatron-produced neutrons were measured for Pb, Al, Cu, and nuclear emulsion. His analysis of the stars indicates that the energy of the incident neutrons is about 4 Bev.

In experiment I, one-third of the total three-prong events observed were produced by neutrons with energies of less than 1 Bev. Practically all of these were of pp- configuration. In the present experiment, only one poorly identified pp- event with incident neutron energy less than 2.0 Bev/c (1.3 Bev) was observed. It was concluded that the scarcity of pp- events below 2.0 Bev/c is due to the low intensity of the neutron beam in this portion of the energy spectrum.

In I, double production came into prominence at about 1 Bev. This is consistent with the results of the cloud chamber experiment of Batson, Culwick, and Riddiford which indicated that in 530 interactions of 950-Mev protons with deuterons, there were only three certain cases of the pn+-configuration. In our experiment, no double-pion production events with incident-neutron momentum less than 2.0 Bev/c were observed. This is additional evidence that the intensity of the neutron beam below 1.3 Bev is small. The upper limit of the incident-neutron energy spectrum extends to the 6.2-Bev limit set by the energy of the circulating protons in the Bevatron. The data of this experiment suggest a smooth neutron-energy spectrum peaked at 3.8 Bev with maximum energy limits of ± 2.4 Bev; i.e., $E_n = 3.8 \pm 2.4$ Bev.
KINEMATICS OF PARTICLE CONFIGURATIONS IN THE C. M. SYSTEM

Some General Features Common to All Configurations

One very pronounced effect common to all types of particle configurations is for the two nucleons to be emitted in directions opposite to one another in the c.m.s. There is good evidence that this effect decreases with increase in the number of pions produced. There is also evidence that this effect is greater at Bevatron energies than at Cosmotron energies. These three effects may be seen in the histograms of \( \cos \theta_{ab}^* \) for those particle configurations which do not have more than one neutral particle emitted. \( \theta_{ab}^* \) is the angle between particles \( a \) and \( b \) in the c.m.'s., and in the case under consideration \( a \) and \( b \) are the two nucleons involved. When \( \cos \theta_{ab}^* \) is +1, particles \( a \) and \( b \) are emitted in the same direction; when \( \cos \theta_{ab}^* \) is -1, they are emitted in opposite directions.

The decrease in the opposite direction effect between nucleons with increase in the number of pions produced may be seen by comparing the \( \cos \theta_{ab}^* \) histograms for configurations pp- (Fig. 15b), pn+ (Fig. 6c), pp++- (Fig. 24b), and pn+++ (Fig. 27b) for one-, two-, three-, and four-pion production, respectively. The distribution for the pp-0 configuration (Fig. 15b) is consistent with the pn+ distribution.

The increase in the opposite direction effect between nucleons at Bevatron energies over those at Cosmotron energies may be seen for the pn+- reaction in Fig. 6 for I, II, and this experiment. The data of II are plotted against \( \theta_{ab}^* \) and \( \Delta N/\Delta \cos \theta_{ab}^* \). The resulting histogram is essentially equivalent to the \( \cos \theta_{ab}^* \) vs. the number of particles per unit interval of \( \cos \theta_{ab}^* \) used in this experiment and in I.

Another very pronounced effect common to all particle configurations is for the angular distributions of the nucleons to be peaked in the forward and backward directions. This effect is strongest in the pp- configuration, in which the angular distributions of the protons are in sharp forward and backward spikes; the effect decreases as the
number of pions produced increases. These angular distributions are
given by \( \cos \theta^*_a \). \( \theta^*_a \) is the angle in the c.m.s. between the line of
flight of the incident neutron and the direction of the emitted particle.
In the case under discussion, \( a \) is for the nucleons. \( \cos \theta^*_a = +1 \) is the
forward direction, and cosine \( \theta^*_a = -1 \) is the backward direction. The
decrease of the forward and backward nucleon peaks with an increase
in the number of pions produced may be seen in the \( \cos \theta^*_a \) histograms
for particle configuration pp-. with single-pion production (Fig. 17c),
for configurations pn+- (Fig. 5c) and pp-0 (Fig. 14b) with double-pion
production, for configurations pp+-+ (Fig. 23b) and pn+-0.. (Fig. 21b)
with triple- or quadruple-pion production, and for configuration pn++--
with quadruple-pion production (Fig. 26b).

The forward-backward peaking of the angular distributions of
the nucleons is considerably greater at Bevatron energies than at Cos-
motron energies. This energy effect is seen in the histograms for
the pn+- configuration in Fig. 5 for I, II, and this experiment. It is
seen that the forward or backward peaking increases from an approxi-
mately isotropic distribution in I to a sharp forward or backward
spike at Bevatron energies. This energy effect is also seen in the
histograms for the pp- configuration in Fig. 17 for I, II, and this experi-
ment. In this single-pion case, the forward and backward peaks ob-
served in I decrease somewhat in II and increase to sharp spikes at
Bevatron energies.

Another feature common to all particle configurations is that
the nucleons usually carry off most of the momentum. An approximate
empirical rule at Bevatron energies is that in any given particle con-
figuration the median c.m.s. nucleon momentum is roughly 2,3 times
the median pion momentum. The pions carry off an increasing share
of the momentum as the number of pions produced is increased. Hist-
ograms of \( P^*_a \), the momentum of particle \( a \) in the c.m.s., are shown
for the following particle configurations: pp-(Fig. 16c), pn+- (Fig.
4c), pp-0 (Fig. 14a), pp+-+ (Fig. 23a), pn+-0.. (Fig. 21a), and pn++--
(Fig. 26a). The median c.m.s. nucleon momentum is seen to be
greatest in single-pion production events and to decrease as the number
of pions produced is increased. Approximate values of the median c.m.s. nucleon momentum for single-, double-, triple-, and quadruple-pion production are 0.95, 0.75, 0.65, and 0.5 Bev/c, respectively.

The momentum of particles in the c.m.s. appears to be a more basic physical quantity than the kinetic energy of particles in the c.m.s. Table VI shows the median c.m.s. momenta and kinetic energies of the nucleons and pions in the pp- and pn+- reactions. The large increase in the ratio of nucleon kinetic energy to pion kinetic energy with increase in incident neutron energy hardly seems compatible with a statistical model of pion production.

Table VI

Median c.m.s. momenta and kinetic energy of nucleons and pions in the pp- and pn+- reactions.

<table>
<thead>
<tr>
<th></th>
<th>Experiment I</th>
<th>Experiment II</th>
<th>This experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>E&lt;sub&gt;n&lt;/sub&gt; &lt; 1.5 Bev</td>
<td>E&lt;sub&gt;n&lt;/sub&gt; &lt; 2.2 Bev</td>
<td>E&lt;sub&gt;n&lt;/sub&gt; &lt; 6.2 Bev</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P* K.E.*</td>
<td>P* K.E.</td>
<td>P* K.E.</td>
</tr>
<tr>
<td></td>
<td>Bev/c Bev</td>
<td>Bev/c Bev</td>
<td>Bev/c Bev</td>
</tr>
<tr>
<td>PP-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nucleons</td>
<td>0.33 0.055</td>
<td>0.50 0.13</td>
<td>0.95 0.40</td>
</tr>
<tr>
<td>Pions</td>
<td>0.24 0.14</td>
<td>0.33 0.22</td>
<td>0.38 0.27</td>
</tr>
<tr>
<td>Ratio</td>
<td>1.4 0.4</td>
<td>1.5 0.6</td>
<td>2.5 1.5</td>
</tr>
<tr>
<td>pn+-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nucleons</td>
<td>0.28 0.042</td>
<td>0.43 0.094</td>
<td>0.73 0.25</td>
</tr>
<tr>
<td>Pions</td>
<td>0.16 0.072</td>
<td>0.22 0.12</td>
<td>0.32 0.21</td>
</tr>
<tr>
<td>Ratio</td>
<td>1.8 0.6</td>
<td>2.0 0.8</td>
<td>2.3 1.2</td>
</tr>
</tbody>
</table>
These general features of nucleon behavior in the c.m.s. may be explained in part by a consideration of conservation of momentum. The opposite-direction effect decreases as the number of pions produced is increased. Since in any given event the nucleons usually carry off most of the momentum, conservation of momentum tends to make the nucleons go in opposite directions. The effect is diminished as the pions carry off a greater share of the momentum, as the number of pions emitted increases.

If this conservation-of-momentum argument were extended, it might be rationalized that the increase in the opposite-direction effect at Bevatron energies implies that the nucleons carry off a correspondingly larger share of the momentum at these energies. This effect is indeed observed in particle configurations pp- and pn+. In the $P_a^*$ histograms for the pp- configuration, the ratios of the median nucleon momentum to the median pion momentum are 1.4, 1.5, and 2.5, for experiments I, II, and this experiment, respectively. Similarly, in the histograms for pn+, the ratios are 1.8, 2.0, and 2.3 for the respective experiments.

The opposite direction effect, however, is closely associated with the forward-backward peaking of the nucleons. Forces that would cause the forward-backward peaking to increase at Bevatron energies would surely also cause the opposite-direction effect to increase. Further, except for the pp- reaction of this experiment, the opposite-direction effect appears to be more pronounced than the forward-backward peaking effect. This may result from the augmentation of the opposite-direction effect of conservation of momentum by the forces causing the forward-backward peaking.

The forward-backward peaking of the nucleons in the c.m.s. may be explained by a consideration of conservation of angular momentum. Fermi has pointed out that it is much more probable for two particles to collide off center of each other than directly on center, and that therefore a particle system will generally be left with a considerable amount of angular momentum after a collision. In conserving angular momentum the emitted particles would tend toward forward and backward directions.
If we assume that the nucleons undergo a grazing type of collision and continue approximately in their respective forward or backward directions, the opposite-direction effect and the forward-backward peaking effect are self-explanatory. In I, Wallenmeyer refers to Brookhaven experiments which show an apparent tendency for the colliding particles to continue after collision in approximately their incident directions. The $\pi^- p$ experiment at Bevatron energies and our experiment also show this effect. This assumption also makes possible a simple phenomenological model explaining some of the observed forward or backward directions of the different particles in the c.m.s.

The pn$^+$ Reaction in the C. M. System

The pn$^+$ reaction is perhaps the most interesting of the three-prong events from both the experimental and the theoretical points of view. Experimentally, it is observed to be the majority-type reaction with sufficient statistics in both the well-identified and probable groups to give significant meaning to the histograms of $P_a^*$, $\cos \theta_a^*$, $Q_{ab}$, and $\cos \theta_{ab}^*$. Also, charge symmetry between p$^+$ and n$^-$ and between p$^-$ and n$^+$ demands symmetry in the $Q_{ab}$ and $\cos \theta_{ab}^*$ histograms of these particles. The degree of symmetry serves as a measure of the reliability of the data. From a theoretical point of view, the pn$^+$ reaction is of special interest in establishing the importance of the isotopic spin state $T = 3/2$ in pion production in nucleon-nucleon collisions.

Histograms of $P_a^*$ for pn$^+$ events in all three experiments are shown in Fig. 4. The shaded areas are for the 42 well-identified events. Nonshaded areas are for the 55 probable pn$^+$ events. Of the 42 well-identified events all have protons identified by density of ionization, their momentum being less than 0.9 Bev/c, and only three have positive pions not identified by density of ionization. In the 55 probable pn$^+$ events, however, 19 have protons not identified by ionization density, and 23 have positive pions not identified by ionization density. All events have at least one positive track identified. Thus, the well-identified group of events is characterized by the identification of both the proton and positive pion, and the probable group is largely characterized by the identification of only the proton or the positive pion.
Fig. 4. Momentum distributions in c.m.s. of particles emitted in the $p\pi^+\pi^-$ reaction in experiments I and II and this experiment.
evaluating the observed distributions in the histograms and the differences between the well-identified and probable distributions, the biased but unavoidable conditions that went into their selection should be kept in mind. When both positive tracks were identified, the chief problem was to distinguish between the pn+- and the pn+-0 particle configurations. When only one positive track was identified, other possible particle configurations were considered, and the events tended to be more ambiguous. The probable events are more ambiguous than the well-identified events and contain a larger proportion of misidentified events. However, the restrictions on momentum of the positive particles in the laboratory system are different from the restrictions of the well-identified events, resulting in a different bias in the distributions. Therefore, in evaluating the histograms, the well-identified group of events and the sum of well-identified and probable events should be considered separately.

In the well-identified events, the momentum distribution of the protons resembles that of the neutrons, and the median nucleon momentum is about 0.77 Bev/c. The median nucleon momentum for all the pn+- events is about 0.73 Bev/c. The momentum distribution of the positive pions in the well-identified events is similar to that of the negative pions; the median momentum is about 0.28 Bev/c. In the probable events, the momentum distribution of the pions is somewhat greater, resulting in a total average pion momentum in the pn+- events of about 0.32 Bev/c.

The $P^*_a$ histograms for the pn+- events of experiments I and II, are shown in Figs. 4a and 4b, respectively. In I, the incident-nucleon energy $E_n$ is less than 1.5 Bev/c, with a peak neutron-beam energy of about 1 Bev. In II, $E_n$ is less than 2.2 Bev/c, with a mean energy of about 1.73 Bev/c. Comparison of the three sets of histograms shows that the general shape of both the nucleon and pion $P^*_a$ distributions is maintained as the incident nucleon energy is increased from the Cosmotron energies to the Bevatron energies.

Histograms of $\cos \theta^*_a$ for the three experiments are shown in Fig. 5. $\cos \theta^*_a$ and $\cos \theta^*_ab$ histogram areas of I, II, and this experiment have been normalized to facilitate comparison. In the well-identified
Fig. 5. Angular distributions in c.m.s. of particles emitted in the \pm \pi^+\pi^- \text{ reaction in experiments I and II and this experiment.} Areas under histograms normalized.
events of this experiment, the protons are observed to be peaked strongly backward, with 29 of the 42 events having \( \cos \theta^p \) less than -0.9. The neutrons are peaked strongly forward, with 31 of the 42 events having \( \cos \theta^n \) greater than +0.9. The positive pions of the well-identified group are predominantly backward; 31 are in the backward hemisphere and 11 in the forward hemisphere. The negative pions are predominately forward; 30 are in the forward hemisphere and 12 in the backward. Within statistical uncertainty, the \( \pi^+ \) and \( \pi^- \) distribution of the well-identified events can be called antisymmetrical. The neutron and proton distributions are also antisymmetrical. This follows from the fact that the nucleons carry off most of the momentum, and conservation of momentum tends to make their angular distributions antisymmetrical. The close antisymmetry between the proton and neutron distributions and also between the positive and negative pion distributions gives a certain degree of reliability to the well-identified \( pn^+ \) events.

The well-identified events, however, can be expected to have proton and positive-pion distributions which are biased in the backward direction. This is a direct result of the fact that they are almost wholly made up of events in which both the proton and positive pion were identified by density of ionization. These relatively low-momentum particles in the laboratory system will tend to be backwards in the c.m.s. Less biased distributions of \( \cos \theta^p \) and \( \cos \theta^n \) should be found in the sum of the well-identified and probable events, if the number of misidentified events in the probable class is not too large.

The distributions of \( \cos \theta^p \) and \( \cos \theta^n \) for all the \( pn^+ \) events are again antisymmetrical within the statistical uncertainty. These distributions are somewhat less peaked than the well-identified events by themselves and have a straggling of events in the forward direction for the proton and in the backward direction for the neutron. The negative-pion distribution is also somewhat less peaked; 68 events have negative pions in the forward hemisphere and 29 events have negative pions in the backward hemisphere. The positive pions, however, have only 56 in the backward hemisphere and 41 in the forward hemisphere. In the forward hemisphere, 23 events did not have the positive pion
identified, as compared with only three such events in the backward hemisphere. Since the rather large forward group in the positive-pion distribution does not have a similarly large counterpart in the backward component of the negative-pion distribution, it might be concluded either that some of the events having $\pi^+$ in the forward hemisphere are mis-identified or that pn+- events with negative pions in the backward hemisphere tend to be ambiguous with other particle configurations.

These $\cos \theta^*_a$ distributions may be compared with those of experiments I and II, shown in Fig. 5. It was first observed in II that protons in the pn+- reaction had a rather strong tendency toward emission in the backward direction and that neutrons had a tendency toward emission in the forward direction. It was pointed out in I that the more isotropic distributions of that experiment were probably a result of the lower energy of the incident neutrons. The trend toward forward peaking of the neutrons and backward peaking of the protons, established in I and II, is seen to be accentuated at Bevatron energies. In a similar, but less pronounced manner, the forward and backward peaking of the pions is observed to increase from the approximately isotropic distributions of I to the more sharply defined peaks at Bevatron energies. The strong forward peaking of the negative pions, observed in II and in this experiment, is not antisymmetrical with the flatter positive-pion distributions of II and the total histogram of this experiment, both of which show a weaker forward peak as well as the stronger backward peak. The well-identified pn+- events of II (not shown separately in Fig. 5b) and of our experiment show stronger backward peaking and little or no forward peaking. It would seem that the rather similar positive-pion distributions and negative-pion distributions of II and this experiment and the mediocre antisymmetry between positive- and negative-pion distributions might result from similar difficulties in identifying the pn+- reaction in the two experiments.

In the pn+- reaction, charge symmetry requires that the distribution of correlation angle, $\theta^*_{ab}$, between particles a and b be symmetrical for the pairs $p\pi^+$ and $n\pi^-$ and also for $p\pi^-$ and $n\pi^+$. Charge symmetry also requires that the distribution of Q values between these pairs be symmetrical. The shape of the Q-value distributions, of
course, depends on the nuclear forces involved. The term nuclear forces is here used in its most general sense.

Since the nucleons carry off most of the momentum in the $p\pi^+$-reaction, conservation of momentum tends to make the proton and neutron go in opposite directions and thus to be antisymmetric. Charge symmetry then requires that the $\pi^+$ and $\pi^-$ distributions be antisymmetric. Also, since the proton and neutron show strong forward-backward asymmetries, it is to be expected that the $\pi^+$ and $\pi^-$ distributions will also show forward-backward asymmetries. It is to be noted, however, that the charge symmetry of nuclear forces cannot require the proton in the $p\pi^+$-reaction to go backwards in the c.m.s. or the $\pi^+$ to go in the same direction as the proton. Information on the nuclear forces themselves will be necessary to explain the forward-backward asymmetries.

Histograms of $\cos \theta_{ab}$ for the different pairs of particles of the $p\pi^+$-reaction are shown in Fig. 6 for all three experiments. The angular correlation between protons and neutrons shows a very pronounced opposite-direction peaking. The $p\pi^-$ and $n\pi^+$ pairs both show definite peaking in the opposite direction. This is especially noticeable in the 42 well-identified events, of which in each of these groups there are 33 in the opposite hemispheres. The $p\pi^-$ distribution is more in the opposite direction than the $n\pi^+$ distribution in the total histogram of this experiment. Comparison with corresponding distributions of I and II shows that this latter effect is also true for those experiments. There is a definite trend in the three experiments for the opposite direction effect of the $p\pi^-$ and $n\pi^+$ distributions to increase with energy of the incident neutrons.

The pronounced tendency of the $p\pi^+$ and $n\pi^-$ distributions of the well-identified events to be predominantly in the same direction reflects the bias condition arising from the fact that both the proton and positive pion were almost always identified by density of ionization in this group. Comparison of $p\pi^+$ and $n\pi^-$ distributions with corresponding distributions of experiments I and II shows a definite increase in the proportion of events in the same-direction hemisphere with increase in incident-neutron energy. The predominantly opposite-direction
Fig. 6. Angular correlation distributions in c.m.s. for each pair of particles emitted in the pnπ⁺π⁻ reaction in experiments I and II and this experiment. Areas under histograms normalized.
effect of these distributions in I has, at Bevatron energies, shifted
toward a flatter distribution or same-direction effect. A similarity
in the distributions of this experiment and those of II is to be noted. A
greater opposite-direction effect of the pπ+ distribution over that of the
nπ- distribution is observed in both experiments.

The fact that several of the distributions of this experiment and
of II are similar in their dissimilarities between charge-symmetric
pairs of particles and their lack of complete antisymmetry between
charge-symmetric particles should not be interpreted as substantial
evidence against the charge symmetry of nuclear forces. Yukawa has
suggested that certain difficulties in meson theory might indicate a
breakdown in the rigorous framework of special relativity. On the
strength of this suggestion and the fact that similar features in the angu­
lar distributions were observed for the pn+- reaction of both II and our
experiment, the data may be interpreted as suggesting the validity of
only an approximate but not rigorously correct special relativity. In
an n-p collision producing a pn+- particle configuration, the neutron is
observed to go forward in the laboratory system with a median velocity
of 0.92 the velocity of light, and the proton with a median velocity of
0.53 the velocity of light. If we assume that the proton and π+ and also
the neutron and π- decay from isobaric states of the nucleon having a
mean life in the rest system of about 0.5 X 10^-23 seconds as indicated
by the width of the (3/2, 3/2) resonance, and that the average values of
β of the excited nucleons in the laboratory system are 0.53 and 0.92,
with time dilation factors γ = 1.18 and 2.68, then the excited nucleon
with charge -1 will decay into a neutron and π- after traveling a dis­
tance of 3.8 X 10^-13 cm, and the excited nucleon with charge +2 will
decay into a proton and π+ after traveling a distance of 0.94 X 10^-13 cm.
Suppose that in the case of the nucleon with charge -1, the decay occurs
outside the range of nuclear forces (~1.4 X 10^-13 cm) so that decay
products do not interact with the other excited nucleon or its decay
products, and suppose that in the case of the nucleon with charge +2,
the excited nucleon decays into a proton and π+ inside the range of
nuclear forces. This supposition is in contradiction to special relati­
vity, if charge symmetry of nuclear forces is assumed. Interaction of
the proton and $\pi^+$ with the other excited nucleon would distort the $Q(p\pi^+)$ distribution, and attractive forces between the $\pi^+$ and the other excited nucleon would cause the $\pi^+$ to go in a more forward direction and to go more in the same direction as the neutron. The $Q$ value of the high-velocity nucleon, however, would not be distorted, and the $\pi^-$ would tend to maintain its preferential forward direction. These effects are indicated in the data.

It was observed in I, and to a lesser extent in II, that protons and $\pi^+$ and also neutrons and $\pi^-$ tend to be emitted more frequently in the opposite directions than protons and $\pi^-$ or neutrons and $\pi^+$. At Bevatron energies this effect has reversed itself, so that the $p$ and $\pi^-$ and $n$ and $\pi^+$ are now in more opposite directions.

The angular-correlation distribution between the $\pi^+$ and $\pi^-$ shown in Fig. 6c shows that the pions have a tendency toward emission in opposite directions. The stronger opposite-direction effect of the well-identified events probably results from the previously mentioned bias condition of this group. The pions carry off less than half the momentum, and their angular correlation is only weakly dependent on conservation of momentum. Comparison of the $\pi^+\pi^-$ distribution of this experiment with those of I and II seems to indicate that the opposite direction effect decreases with increase in incident-neutron energy. The observed ratios of opposite-hemisphere events to same-hemisphere events for $\pi^+\pi^-$ pairs are 2.5, 1.7, and 1.5, for I, II, and this experiment, respectively. Such an effect may be consistent with an increase in attractive pion-pion force, with increase in relative energy of the two pions.

The $Q$-value distributions for the $pn+\text{-reaction}$ are shown for all experiments in Fig. 7. It has already been mentioned that within the statistical uncertainties, charge symmetry of nuclear forces requires the distributions of $Q(p\pi^+)$ to be similar to $Q(n\pi^-)$, and requires $Q(n\pi^+)$ to be similar to $Q(p\pi^-)$. The $Q(p\pi^+)$ and $Q(n\pi^-)$ distributions are seen to be strongly peaked in the neighborhood of 0.15 Bev with a sharp drop-off at 0.3 Bev. The well-identified events show a peak between 0.1 and 0.15 Bev in both the $Q(p\pi^+)$ and the $Q(n\pi^-)$ distributions. The $Q(p\pi^+)$ distribution exhibits a sharp dip between 0.15 and 0.20 Bev.
Fig. 7. $Q$-value distributions for each pair of particles emitted in the $p^n\pi^+\pi^-$ reaction in experiments I and II and this experiment.
This gives a doubly peaked appearance to the distribution, the primary peak being at about 0.12 Bev and a secondary peak being at about 0.25 Bev. In spite of this dip in the $Q(p\pi^+)$ distribution, the well-identified events show considerable symmetry between the $Q(p\pi^+)$ and $Q(n\pi^-)$ groups. The probable events dominate the higher-energy portions of the distributions. These higher-energy $Q$ values are characterized by having only the proton or the $\pi^+$ identified by ionization. The direction and momentum of the emitted neutron were determined by energy-momentum balance. The $Q$ value between two particles is an invariant quantity, depending only on the rest mass and momentum of the particles and the angle between them. The $Q(p\pi^+)$ distribution can thus be expected to be a more accurate representation of the true picture than the $Q(n\pi^-)$ distribution, which makes use of the derived momentum and direction of the neutron. However, the fact that there is a greater number of high-energy $Q$ values in the $Q(p\pi^+)$ distribution than in the $Q(n\pi^-)$ distribution probably arises out of bias resulting from the selection criteria used in identifying the events. The $Q$ values used in these histograms are the adjusted values from the IBM program of constrained solutions, and are not obtained directly from the measured angles and momenta of the particles. The adjusted and nonadjusted values are generally quite similar. The former, however, should on the average be more accurate than the latter, especially for $Q$ values involving short tracks with correspondingly large probable errors.

The shape of the $Q(p\pi^-)$ distribution is roughly similar to $Q(n\pi^+)$ distribution. The general shape of these two histograms is different from those of the $Q(p\pi^+)$ and $Q(n\pi^-)$ distributions; the $Q(p\pi^-)$ and $Q(n\pi^+)$ distributions are less peaked in the low energy portion of the $Q$-value spectrum and have a more gradual decline beyond the maximum at about 0.15 Bev.

The well-identified events of the $Q(pn)$ distribution are seen to have a higher median energy than the probable events. Since the incident-neutron energy is approximately the same for the well-identified and probable events, it appears that the momentum exchange between nucleons during the collision process is less in the well-identified than in the probable events.
The \(Q(\pi^+\pi^-)\) distribution is similar to the \(Q(p\pi^-)\) and \(Q(n\pi^+)\) distributions, with a maximum in the vicinity of 0.15 Bev and a gradual decline with increasing energy. There is an important difference, however, which is best seen in the histogram for the well-identified events. The peaks between 0.05 to 0.10 Bev and also between 0.20 and 0.25 Bev are also seen in other \(Q(\pi\pi)\) distributions of other particle configurations. Both of these peaks may have theoretical significance. Evidence for pion-pion interactions will be discussed in a later chapter.

**Isobar Model of Pion Production**

It has been stated that the \(p\pi^+\) reaction is of special interest in establishing the importance of the isotopic spin state \(T = 3/2\) in pion production in nucleon-nucleon collisions. The assumption that the isotopic spin state \(T = 3/2\) is dominant in the excited-nucleon model of pion production follows from the \((3/2, 3/2)\) resonance (isotopic spin \(= 3/2\), angular momentum \(= 3/2\)) observed in the \(\pi^+p\) total cross section. The simple phenomenological arguments presented in this paper depend on the isotopic spin and do not explicitly involve the angular momentum. The shape of the \(\pi^+p\) total cross section, however, is of fundamental importance to the phenomenological model. If the cross section is plotted against the total kinetic energy of the pion and nucleon in their own c.m.s., the peak of the \((3/2, 3/2)\) resonance is at approximately 140 Mev. The energy width at half maximum is about 100 Mev. The smaller secondary peak occurs at about 840 Mev. The width of the resonance peak implies the existence of an isobaric state of the nucleon of very short lifetime \((0.5 \times 10^{-23} \text{ seconds})\). The mass of this isobar, \(M'\), which is a variable, is given by

\[
M' = M_p + M_\pi + (K.E._p^* + K.E._\pi^*) = M_p + M_\pi + Q(p\pi),
\]

where \(M_p\) and \(M_\pi\) are the rest masses of the proton and pion, \(K.E._p^*\) and \(K.E._\pi^*\) are the kinetic energies of the proton and pion in the rest of the two particles, and \(Q(p\pi)\) is the sum of these kinetic energies. \(Q(p\pi)\) is simply the "mass" that must be added to the rest masses of the nucleon and pion to give the rest mass of the isobar. The mass of the \(T = 3/2, J = 3/2\) isobar at the peak of the resonance curve is approximately 0.94 Bev + 0.14 Bev + 0.14 Bev = 1.22 Bev.
In nucleon-nucleon collisions one may assume after Peaslee\(^7\) that single- or double-pion production occurs when one or both of the nucleons is excited to the \(T = 3/2\) isotopic spin state and subsequently decays to the ground state by pion emission. In the collision of a neutron and proton, the total isotopic spin may be either 0 or 1. The \(z\) component of isotopic spin, \(T^z\), is 0. Assuming that \(T\) and \(T^z\) are conserved, that pion production proceeds only through the \(T = 3/2\) state, and, for simplicity, that the \(T = 0\) and \(T = 1\) states are equally probable, \(^{31}\) the charge ratios for double production are given by:

\[
n + p \rightarrow \frac{7}{10} n\pi^+, \frac{1}{30} p\pi^+, \frac{1}{30} n\pi^-, \frac{2}{30} p\pi^-, \frac{1}{30} n\pi^0, \frac{2}{30} p\pi^0 + \frac{2}{30} n\pi^0, \frac{4}{30} n\pi^0, \frac{1}{30} p\pi^0.
\]  

(8)

Here commas separate pion-nucleon pairs decaying from the same \(T = 3/2\) isobar. Equation (8) gives ratios for the charge configurations \(p\pi^+, pp-0, nn+0\), and \(pn00\) of 11, 1, 1, and 2, respectively. (See Table V.)

In the terms involving the \(pn+\) reaction of Equation (8), one can see that the positive pion is associated with the proton and the negative pion with the neutron in 21 cases out of 22. If the proton and \(\pi^+\) and also the neutron and \(\pi^-\) as a rule decay from an \((T = 3/2, J = 3/2)\) isobar, the distribution of \(Q\) values between these pion-nucleon pairs might be expected to resemble the total \(\pi^+p\)-interaction cross section, when the latter distribution is plotted against the total kinetic energy of the proton and pion in the rest system of these particles. This phenomenological relationship between the probability for isobar formation and the observed \(\pi^+p\) total-interaction cross section was originally made by Yuan and Lindenbaum. \(^{32}\) More recently, Lindenbaum and Sternheimer have developed an isobaric nucleon model for pion production in nucleon-nucleon collisions which is primarily based on this premise. \(^{11}\)

There are other pertinent implications of Eq. (8). The strong association of \(p\pi^+\) and \(n\pi^-\) requires that the angular correlations between these pairs be essentially different from those of \(p\pi^-\) and \(n\pi^+\). At low incident-neutron energies, where the velocity of
separation of the isobars is small, the $p\pi^+$ and $n\pi^-$ angular-correlation distributions tend to be in opposite directions. As the energy of the incident neutron increases, the velocity of separation of the isobars also increases. The result is a decrease in the opposite direction of $p\pi^+$ and $n\pi^-$ pairs and an increase in the opposite direction of $p\pi^-$ and $n\pi^+$ pairs. These effects are observed in the $\cos \theta_{ab}$ histograms for the pn+ reaction in Fig. 6.

Equation (8), however, does not predict the forward-backward asymmetries of nucleons in the c.m.s. In particular, for the pn+ reaction, this equation cannot explain why the protons go backward in the c.m.s. and the neutrons go forward. It may with good assurance be assumed that the nucleons or nucleon cores undergo a grazing-type collision and continue in their respective forward or backward direction. But the calculation of Eq. (8) was made on the basis of conservation of the charge +1 of the neutron-proton system, the implication being that the emitted nucleons have no memory of their precollision charge states. Equation (8) further implies that in the formation of a pn+ particle configuration, the nucleon with charge +1 going backwards in the c.m.s. before collision would with equal probability have charge 0 or 1 after collision. If it is assumed that the charge state of a nucleon is changed by the emission and absorption of pions, the complete randomization of the charge of a nucleon during collision would suggest that a large number of virtual pions are exchanged during the collision process. This supposition would be consistent with strong coupling between nucleon and pion in which many virtual pions are expected to be closely associated with the nucleon. However, the proton and nucleon tend to maintain their respective charge states in the pn+ reaction, which may be consistent with intermediate coupling.

In spite of this difficulty, we may accept the prediction of Eq. (8) that $p\pi^+$ and $n\pi^-$ pairs are quite generally associated decay products. Then, since the proton goes backward and the neutron goes forward, we might expect that, with increasing incident-neutron energy, the $\pi^+$ would tend to go increasingly backward with the proton, and would expect the $\pi^-$ to go increasingly forward with the neutron. This conclusion follows from the assumption that the effect of the Q-value disintegration energy,
which tends to make the $\pi^+$ and $p$ go in opposite directions, does not increase as fast as the effect of the velocity of the isobars in the c.m.s., which effect tends to make the $\pi^+$ and $p$ go in the same direction. Comparison of the $\cos \theta^*_a$ histograms for I, II, and this experiment, shown in Fig. 5 confirms these expectations between I, II, and the well-identified events of this experiment for both $\pi^+$ and $\pi^-$ distributions, and also between I, II, and the probable events of the $\pi^-$ distribution. It does not hold between the lower-energy experiments and the probable $\pi^+$ group of this experiment. This breakdown, however, is consistent with the large proportion of probable pn$^+$ events in this experiment which have large $Q(p\pi^+)$ values.

The increase in the opposite-direction effect between nucleons with increase in incident-neutron energy, as well as its decrease with increase in the number of pions produced, may also be understood on the basis of the excited-nucleon model of pion production. After collision, the isobars move in opposite directions in the c.m.s. Assuming again that the average $Q$ value does not increase rapidly with increase in incident-neutron energy, the average deflection of the nucleon from the path of its parent isobar decreases as the velocity of the isobar in the c.m.s. increases. The decrease in the opposite-direction effect of nucleons with increase in the number of pions produced follows from the expectation that an isobar emitting two pions will be deflected from its original path more than if it emitted one pion. The similar effects in the forward-backward peaking of the nucleons can be explained by similar reasoning: if it is assumed that the isobars continue in approximately their respective forward and backward directions after collision.

All of the angular distributions and angular correlations observed in the pn$^+$ reaction of I, II, and this experiment may be qualitatively in agreement with an isobaric nucleon model of pion production in which each nucleon is excited to an intermediate state which subsequently decays by emission of a pion. However, the $Q$-value distributions between nucleon-pion pairs in experiments I and II were not so suggestive of an isobaric model of pion production. Since the mass of an isobar is simply the sum of the rest masses of the nucleon and pion
plus the Q value of the nucleon-pion decay products, it might be expected that the Q-value distributions would give the most direct kind of evidence for isobaric pion production, or at least would show the importance of a given resonant state. The Q-value distributions of I (See Fig. 7a) were too low in energy to show any peaking in the vicinity of 0.14 Bev/c. In experiment II, however, where incident-neutron energies were sufficiently high to show the \( T = 3/2, J = 3/2 \) resonance, the Q-value distributions in the \( pn^+ \) reaction (See Fig. 7b) did not clearly show the resonance in the \( Q(p\pi^+) \) and \( Q(n\pi^-) \) distributions where they would be expected if the \( (3/2, 3/2) \) resonance dominated, and the \( Q(p\pi^+) \) and \( Q(n\pi^-) \) distributions were not dissimilar from the \( Q(p\pi^-) \) and \( Q(n\pi^-) \) distributions. They were not inconsistent with a \( (3/2, 3/2) \) resonance. We have already pointed out that both the \( Q(p\pi^+) \) and \( Q(n\pi^-) \) histograms of this experiment, shown in Fig. 7c, show large peaks in the vicinity of 0.15 Bev with a predominance of events in the low-energy portion of the spectrum less than 0.3 Bev. This is confirmatory evidence that the \( (3/2, 3/2) \) resonance is of primary importance in nucleon-nucleon pion production, at Bevatron energies, and probably also at lower energies.

The \( Q(p\pi^-) \) histogram for the well-identified events seems to be roughly consistent with the total interaction cross section of negative pions on protons. The \( Q(p\pi^-) \) and \( Q(n\pi^+) \) histograms for all \( pn^+ \) events have fewer events in the low-energy portion of the spectrum and have a more gradual tapering off at increased Q values.

The Q-value data by themselves apparently do not rule out a final-state interaction model in which pions produced in the collision process are forced into the observed Q-value distributions. But the Q-value data, the angular-distribution data, and the angular-correlation data all appear to be consistent with an isobaric-nucleon model in which each nucleon is excited to an intermediate state which subsequently decays by emission of a pion.

The fact that the characteristic \( (3/2, 3/2) \) peaking of the \( Q(p\pi^+) \) and \( Q(n\pi^-) \) distributions was not observed at Cosmotron energies and was observed at Bevatron energies might be explained on the basis of a greater separation between the excited nucleons before they decay in...
the higher-energy experiment. A crude calculation indicated that the two excited nucleons could generally be outside the range of nuclear forces at Bevatron energies and inside at Cosmotron energies.

By the addition of the $Q(p\pi^+)$ distribution to the $Q(n\pi^-)$, the statistics are improved, and bias conditions resulting from the selection criteria should be reduced. (See Fig. 8a) In order to provide a comparison, the $Q(p\pi^-)$ histogram was added to the $Q(n\pi^+)$ histogram. (See Fig. 8b.) These composite distributions probably give the most reliable $Q$-value distributions. Since all events that could possibly be measured were included in the above-mentioned histograms, it was apparent that some of the more poorly determined $Q$ values would do more harm than good. Therefore, a selection of the better events was made, the main criteria for selection being the requirement that the total length of the three tracks be greater than 18 cm, the maximum counted length of any one track being 8 cm. (A number of well-measured events were removed by this selection.) The principal effect of this selection was to remove events with a track less than 2 cm long. Pronounced turbulence was also the basis for removal in a small number of cases, and a few events were removed in which estimation of density of ionization was especially difficult. Histograms of these better events are shown for $Q(p\pi^+)$, $Q(n\pi^-)$, and their sum in Fig. 9.

The principal drawback to the $Q$-value histograms is that a $Q$ value involving a particle with a barely measurable track with large probable errors is given equal weight, in its position on the histogram, with a $Q$ value involving particles with long well-measured tracks. Also, the sensitivity of $Q$ values to errors in the angle and momentum measurements depends on the magnitude of the angle and the momentum. The significance of an observed peak in a $Q$-value distribution might turn out to be very small if the probable errors on the $Q$ values are considered. In order to help circumvent these difficulties, weighted histograms or ideograms were made of the $Q(p\pi^+)$, $Q(n\pi^-)$, $Q(p\pi^-)$, and $Q(n\pi^+)$ events. These weighted histograms can be expected to bring out the detailed structure of the $Q$-value distributions in a more penetrating manner than the histograms, because they contain the
Fig. 8. $Q$-value distributions of $Q(p\pi^+)+Q(n\pi^-)$ and $Q(p\pi^-)+Q(p\pi^+)$ in the $pn\pi^+\pi^-$ reaction.
Fig. 9. $Q$-value distributions of $Q(p\pi^+)$, $Q(n\pi^-)$, and $Q(p\pi^+) + Q(n\pi^-)$ for the better events in the $pn\pi^+\pi^-$ reaction.
additional information of errors on the individual Q values. In the weighted histograms, each Q value contributes a rectangle of unit area, the base of which is $2\Delta Q$ or twice the uncertainty of the Q value. The rectangle is centered on the calculated value of Q. $\Delta Q$ values were taken at their maximum positive value. In the case of $Q(n\pi^-)$ or $Q(n\pi^+)$, the uncertainty of the neutron momentum was taken to be $\pm 10\%$ of its calculated momentum, and the uncertainty in the two angles specifying neutron direction were taken to be $\pm 1.0$ degree.

Weighted histograms of $Q(p\pi^+)$, $Q(n\pi^-)$, and their sum are shown for the total events in Fig. 10. The weighted comparative histograms of $Q(p\pi^-)$, $Q(n\pi^+)$, and their sum are shown in Fig. 11. Also, weighted histograms are shown for the better events described above, for the $Q(p\pi^+)$ and $Q(n\pi^-)$ distributions and their sum in Fig. 12.

The dashed curve shown in the weighted histograms for $Q(p\pi^+)$, $Q(n\pi^-)$, and their sum is the total $\pi^+p$ interaction cross section, $\sigma(\pi^+p)$, plotted against the total c.m.s. kinetic energy, as weighted by the two-body phase-space factor. $\sigma(\pi^+p)$ is equivalent to the cross section for isotopic spin $T = 3/2$, i.e., $\sigma(\pi^+p) = \sigma 3/2$. Following the procedure of Lindenbaum and Sternheimer, $^{11}$ we take the probability of forming two isobars $N'_1$ and $N'_2$ with masses $M'_1$ to $M'_1 + dM'_1$ and $M'_2$ to $M'_2 + dM'_2$, respectively, to be

$$d^2 P(M'_1, M'_2) = F \sigma(M'_1) \sigma(M'_2) dM'_1 dM'_2,$$  \hspace{1cm} (9)

where $\sigma(M'_1)$ and $\sigma(M'_2)$ are the $\pi^+p$ total-interaction cross sections for masses $M'_1$ and $M'_2$, respectively. $F$ is the two-body phase-space factor given by

$$F = \frac{P}{E_1 E_2},$$  \hspace{1cm} (10)

where $P$ is the momentum of either isobar in the c.m.s. $E_1$ and $E_2$ are the total c.m.s. energies of isobars 1 and 2, respectively, and $E = E_1 + E_2$ is the total energy in the c.m.s. of initial nucleons.

Units are such that $c = 1$. Q values are related to the isobar masses by $Q_1 = M'_1 - M_\pi - M_0$ and $Q_2 = M'_2 - M_\pi - M_0$ where $M_\pi$ and $M_0$ are
Fig. 10. Ideograms of $Q(p\pi^+)$, $Q(n\pi^-)$, and $Q(p\pi^+) + Q(n\pi^-)$ for particles emitted in the $pnp^+\pi^-$ reaction. $Q_{\text{min}} = 30$ Mev. The dashed curve is the total $\pi^+p$ interaction cross section as weighted by the two-body phase-space factor and plotted against the total c.m.s. kinetic energy.
Fig. 11. Ideograms of $Q(p\pi^-)$, $Q(n\pi^+)$, $Q(p\pi^-) + Q(n\pi^+)$ for particles emitted in the $pn\pi^+\pi^-$ reaction. $\Delta Q_{\text{min}} = 30$ Mev.
Fig. 12. Ideograms of $Q(p\pi^+)$, $Q(n\pi^-)$ and $Q(p\pi^+)+Q(n\pi^-)$ for particles emitted in the better events of the $pnn\pi^+\pi^-$ reaction. $\Delta Q_{\text{min}} = 30$ Mev. The dashed curve is the total $\pi^+p$ cross section as weighted by the two-body phase-space factor and plotted against the total c.m.s. kinetic energy.
The rest masses of the pion and nucleon, respectively. The effect of the phase-space factor is to give greater weight to the lower-mass isobars which have a larger momentum for a given c.m.s. energy. The probability of having an isobaric mass between \( M'_{1} \) and \( M'_{1} + dM'_{1} \) was found by summing over all values of \( M'_{2} \) commensurate with \( E \). The distribution of \( E \) values is obtained from the incident-neutron momentum spectrum for the pn+- reaction shown in Fig. 3.

The area under the dashed curve has been normalized to the area under the weighted histogram for all the pn+- events. The weighted histograms were smoothed by taking the minimum value of \( \Delta Q = 30 \text{ Mev} \). This is a legitimate procedure, since the width of the \((3/2, 3/2)\) resonance that we are seeking is about 100 Mev at half maximum. Putting no restriction on the minimum \( \Delta Q \) value would cause those \( Q \) values that involve long well-measured tracks to appear as sharp peaks and would make comparison with the weighted \( \sigma(\pi^+ p) \) curve difficult.

Fig. 10a and b shows that the \( Q(p\pi^+) \) and \( Q(n\pi^-) \) distributions are good fits to the total \( \pi^+ p \)-interaction cross section. Fig. 11a and b shows that the \( Q(p\pi^-) \) and \( Q(n\pi^+) \) distributions do not fit the \( \sigma 3/2 \) curve. The sum of \( Q(p\pi^+) \) plus \( Q(n\pi^-) \) (Fig. 10c) has the advantage of better statistics with less expected bias. The good fit of this weighted histogram with the \( \sigma 3/2 \) curve and its contrast with the weighted histogram of \( Q(p\pi^-) \) plus \( Q(n\pi^+) \) (Fig. 11c) is offered as convincing evidence that pions are often produced via the \( T = 3/2, J = 3/2 \) resonance observed in the \( \pi^+ p \) total-interaction cross section. Even better fits with the \( \sigma 3/2 \) curve are observed in the weighted histograms of the better events shown in Fig. 12 for \( Q(p\pi^+) \), \( Q(n\pi^-) \), and their sum.

The secondary peak at about 0.26 Bev observed in the \( Q(p\pi^+) \) distribution, however, deserves special attention. Although the \( Q(n\pi^-) \) distribution is expected to be less representative of the \( T = 3/2 \) cross section than the \( Q(p\pi^+) \) distribution, it actually is a somewhat better fit. From another point of view, however, the \( Q(n\pi^-) \) distribution can be thought of as a washed-out \( Q(p\pi^+) \) type of distribution, the flat portion of the \( Q(n\pi^-) \) histogram in the vicinity of 0.25 Bev of Fig. 7c being the vestige that remains of the secondary peak observed in the \( Q(p\pi^+) \)
distribution. The existence of the secondary peak in these distributions would be of particular interest, because just such a peak is predicted in the $\pi^+p$ cross section by the intermediate-coupling theory of Tomonaga. This theory and the significance of the secondary peak will be discussed later.

Another difficulty with the fit between the $\sigma 3/2$ curve and the $Q(p\pi^+)$ and $Q(n\pi^-)$ distributions is that these $Q$-value distributions are somewhat large on the low side of the resonance. Part of this excess is due to a natural levelling off of the distributions as a result of measurement errors. In the case of the weighted histograms, the events with large $\Delta Q$ tend to flatten out the $Q$-value distributions and thus to build up all low-cross-section portions of the curve. An important part of the low-energy excess may be due to the inclusion of $pn+\rightarrow$ events with the $pn+$ events. Since the large majority of events with low $Q(p\pi^+)$ values had both positive tracks identified by density of ionization, the $pn+\rightarrow$ particle configuration is the most probable contaminant. This speculation is consistent with the observed $Q(p\pi^+)$ distribution of the $pn+\rightarrow$ events. (See Fig. 22a). The distribution of $Q(p\pi^-)$ may also be weighted toward the low-energy end by an influx of type $pn+\rightarrow$ events.

A final representation of the $Q$ values is shown in Fig. 13a in which $Q(p\pi^+)$ is plotted against $Q(n\pi^-)$ for the $pn+$ events. The probability of having a given $Q(p\pi^+)$ and $Q(n\pi^-)$ value is calculated for the isobaric nucleon model of pion production by Eqs. (9) and (10). In order to compare the observed and calculated density distributions, the $Q$ space of Fig. 13a is divided into six regions with $Q(p\pi^+)$ and $Q(n\pi^-)$ boundaries at 0.3 Bev and a 45° dividing line. The number of observed and calculated events in each region is shown in Fig. 13b. As previously stated, the fact that there is a greater number of events with large $Q(p\pi^+)$ than with large $Q(n\pi^-)$ is probably due to selection-criteria bias for the former or against the latter or both. The average number of events in the two long regions defined by $Q(p\pi^+)>0.3$, $Q(n\pi^-)<0.3$, and $Q(p\pi^+)<0.3$, $Q(n\pi^-)>0.3$ is 23, which is consistent with the calculated value of 13. Also the presence of the six events in the region where $Q(p\pi^+)$ and $Q(n\pi^-)$
Fig. 13. (a) Scatter diagram of \(Q(p\pi^+)\) and \(Q(n\pi^-)\) for the reaction \(pnn\pi^+\pi^-\). At the top of the scatter diagram the \(Q(n\pi^-)\) histogram is given, while at the right side the \(Q(p\pi^+)\) histogram is plotted. The observed number of events in each designated region of the \(Q\) space is shown. (b) Calculated numbers of events in the different regions of the \(Q\) space.
are both $> 0.3$ is consistent with the calculated number of one event in this region, in view of the fact that misidentified events, poor quality events, and pn+- events with the $p$ and $\pi^-$ decaying from the $T = 3/2$ isobaric state may tend to have points scattered over the whole portion of the diagram.

The pp-0 Reaction in the C. M. System

The difficulties in sorting out the pp-0 reactions have already been discussed in the chapter on multiplicities. Only six probable pp-0 events were resolved by the standard technique of comparing adjustment factors in the IBM program of constrained solutions. A larger group of 12 events, which were primarily ambiguous between the pp-0 and pn+- reactions, were classed as probable pp-0 reactions, on the basis that they were very likely not pn+- reactions, since the proton and neutron in pn+- reactions were less than $90^\circ$ apart in the c.m.s. A contamination by pp-00 events is expected in this larger group. This larger group of 12 events is shown by crosshatched areas in the histograms of $P_a^\pi$, $\cos \theta_a^\pi$, $Q_{ab}$, and $\cos \theta_{ab}^\pi$ in Figs. 14a and 15a, and 15b, respectively. The protons have been classed as forward or backward ($p_f$ or $p_b$) according to their forward or backward direction in the c.m.s. (See $\cos \theta_{p}^\pi$ of Fig. 14b.) The model of pion production presented in the last chapter of this paper requires, under certain conditions, a correspondence between the particles of the pn+- and pp-0 reactions in which $p$, $n$, $\pi^+$, and $\pi^-$ correspond to $p_b$, $p_f$, $\pi^0$, and $\pi^-$, respectively. Therefore, in order to provide for easy comparison, the corresponding histograms of the pp-0 reaction are arranged in the same order as those of the pn+- reaction. It is seen that, within the statistical uncertainties, the pp-0 distributions are consistent with the corresponding pn+- distributions. In the $\cos \theta_a^\pi$ histograms for the pp-0 reaction (Fig. 14b), the $\pi^-$ tends toward forward emission and the $\pi^0$ tends toward backward emission. However, this effect is due to the 12 crosshatched events, which suggests that this group is more reliable than the six events sorted out by the standard method. In the $\cos \theta_{ab}^\pi$ histograms (Fig. 15b), the $p_f\pi^0$ and $p_b\pi^-$ tend toward emission in opposite directions, and the $p_f\pi^-$ and $p_b\pi^0$ tend toward emission in the same direction, but with a flatter
Fig. 14. Momentum distributions and angular distributions in c.m.s. of particles emitted in the $pp\pi^-\pi^0$ reaction.
Fig. 15. $Q$-value distributions and c.m.s. angular correlation distributions for each pair of particles emitted in the $pp\pi^-\pi^0$ reaction.
distribution, just as in the corresponding pn+- distributions. The \( \pi^- \pi^0 \) tend toward opposite directions. Again, these effects are due to the 12 crosshatched events. In the \( Q_{ab} \) histograms (Fig. 15 a), the \( p_f \pi^- \) and \( p_b \pi^0 \) distributions are similar and have a majority of \( Q \) values less than 0.3 Bev. The \( p_f \pi^0 \) and \( p_b \pi^- \) are similar but have a more dispersed distribution, just as do their corresponding pairs in the pn+- reaction.

The pp- Reaction in the C.M. System

In all of the identified pp- reactions, one of the protons was identified by density of ionization. The requirement of transverse momentum balance was a primary consideration in sorting out these events, but this alone was not a sufficient reason for classification as pp-. A good indication that an event might be balanced was for the unadjusted mass \( M_n \) to be small and negative, and for both the limiting values of \( M_n \), corresponding to \( \tau = \pm 1 \), to be large and negative. However, in evaluating the probability that an event was of configuration pp-, the supplementary IBM program for balanced events was primarily relied upon.

Twelve events identified as pp- were classed as well-identified, and 23 events identified as pp- were classed as probable. A difficulty was encountered in separating some of the pp- and pn+- events. The difficulty was due in large part to the fact that the balanced events were evaluated by means of the supplementary program, while the pn+- reaction was evaluated by means of the regular program of constrained solutions. A direct comparison of the adjustment factors of the two IBM programs was difficult to interpret. Five events showed good fits to both the pp- and pn+- configurations. Seven events which were difficult to distinguish from the pn+- reaction but which apparently showed considerably better fits to the pp- events were classified as probable pp- events. These seven events are indicated in the histograms.

Histograms of \( P^*_a \) for the pp- events are shown in Fig. 16 for all three experiments. Well-identified events of this experiment are shaded. Protons are classed as forward or backward (\( p_f \) or \( p_b \)) according to their positions in the forward or backward peak of Fig. 16c.
Fig. 16. Momentum distributions in c.m.s. of particles emitted in the ppπ⁻ reactions in experiments I and II and this experiment.
The median momentum of the pions and protons in I, II, and this experiment have already been discussed (See Table VI). In all three experiments, $P_{\pi^-}$ is peaked between 0.2 and 0.4 Bev/c.

Histograms of $\cos \theta^*$ are shown in Fig. 17. The very strong forward-backward peaking of the nucleons has already been discussed. A principal feature of interest is the apparent tendency of the $\pi^-$ to go increasingly forward as the incident-neutron energy is increased. The ratio of the number of pions in the forward hemisphere to the number in the backward hemisphere in I, II, and this experiment is $52/42 = 1.24$, $17/10 = 1.7$, and $23/12 = 1.9$, respectively. It may be argued that $pn^+$ events that are mistakenly called $pp^-$ events would tend to show a forward peaking of the $\pi^-$. This results from the fact that the $\pi^-$ is peaked forward in the $pn^+$ reaction. The same argument may also be applied to the $pp^-$ events mistakenly called $pp^-$. Of the seven probable $pp^-$ events that were difficult to distinguish from the $pn^+$ particle configuration in this experiment, five negative pions were forward and two backward, and in the five events that were good fits to both $pp^-$ and $pn^+$, all five negative pions were forward in the $pp^-$ interpretation. The tendency toward forward $\pi^-$ emission in both the well-identified and probable $pp^-$ events of this experiment and in I and II indicate that this effect is real. However, the statistics are too poor, and the chances of misidentified $pp^-$ events too great, to draw a definite conclusion. On the basis of the isobar model, it is assumed that the tendency of the $\pi^-$ to be emitted in a forward direction would result from its usually being a decay product of the forward-moving isobar; the neutron apparently undergoes a grazing type of collision and converts into a proton and a $\pi^-$. 

Histograms of $\cos \theta^*_{ab}$ for the $pp^-$ reaction are shown in Fig. 18b. The very strong peaking of the protons in the opposite direction has already been noted. Other principal features to be noted are the strong opposite-direction peaking of the $p_b \pi^-$ and the somewhat-less-strong same-direction peaking of the $p_f \pi^-$. On the basis of the isobar model, it appears that these distributions are to be expected if the $\pi^-$ is usually a decay product of the forward-moving isobar, and if the effect of the velocity of separation of the isobar and the nucleon in the
Fig. 17. Angular distributions in c.m.s. of particles emitted in the ppπ⁻ reactions in experiments I and II and this experiment. Areas under histograms normalized.
Fig. 18. Q-value and c.m.s. angular-correlation distributions for each pair of particles emitted in the pπ⁻ reaction.
c.m.s. is sufficiently large to overcome the Q-value effect which tends to make the pion-nucleon decay products go in opposite directions.

Histograms of $Q_{ab}$ for the pp-reaction are shown in Fig. 18a. The large group of $Q(p_f \pi^-)$ events that have Q values of less than 0.25 Bev is only roughly consistent with the (3/2, 3/2) resonance with a peak at 0.14 Bev. There is also a rather isolated group of $Q(p_b \pi^-)$ events peaking sharply between 0.40 and 0.45 Bev. The $Q(p_b \pi^-)$ distribution also has a group of low-energy Q values which are consistent with the (3/2, 3/2) resonance; this group, however, is hardly distinct from the larger group with Q values less than 0.45 Bev. The small isolated group of events at 0.7 Bev is perhaps also worth noting.

The isobar model requires that the $\pi^-$ be associated in decay with either the forward or the backward proton. At Bevatron energies, the $\pi^-$ may be associated with the correct proton by noting whether the direction of the $\pi^-$ is in the forward hemisphere, $\pi^-_f$, or in the backward hemisphere, $\pi^-_b$. Then the $\pi^-_f$ is associated with $p_f$ and the $\pi^-_b$ with $p_b$. That this is a reasonable procedure may be seen from the pn+- reaction, in which the negative pions were peaked forward with the neutrons and the positive pions tended toward backward emission with the protons. For single-pion production, there should be a greater tendency for the $\pi^-$ to go in the forward or--if its associated proton goes in the backward direction--in the backward direction, provided that the Q values are not too large. This is also indicated by the greater forward-backward peaking of the nucleons in the pp-reaction, compared with the pn+- reaction.

A scatter diagram of $Q(p_f \pi^-)$ versus $Q(p_b \pi^-)$ is shown in Fig. 19. In the histogram bordering the $Q(p_f \pi^-)$ axis, events in which the $\pi^-$ is backward are indicated. The $Q(p_b \pi^-_b)$ distribution shows the (3/2, 3/2) peaking at 0.14 Bev. The $Q(p_f \pi^-_f)$ distribution still shows only rough consistency with the (3/2, 3/2) resonance in the group of events with less than 0.25 Bev. By the addition of $Q(p_f \pi^-_f)$ to $Q(p_b \pi^-_b)$, the statistics are improved, and there may be a better Q-value distribution, showing the masses of the isobars that produce the pp-reaction. This histogram is shown in Fig. 20 b. The histogram of
Fig. 19. Scatter diagram of $Q$ values obtained for $p_{f}\pi^-$ and $p_{b}\pi^-$ in the $pp\pi^-$ reaction. The shaded areas in the top and side histograms represent the $p_{f}\pi^-$ and $p_{b}\pi^-$ components, respectively.
all $Q(p\pi^-)$ values is shown in Fig. 20c. The $(3/2, 3/2)$ resonance is in evidence in Fig. 20b and to a lesser extent in Fig. 20c. A small secondary peak with maximum between 0.40 and 0.45 Bev is also evident in both Figs. 20c and b.

It is of interest to compare the distributions of Figs. 20c and b with the excitation function recently published by Dixon and Walker\textsuperscript{34} for single-$\pi^+$ photoproduction at 90°. This excitation function is shown in Fig. 20a, where comparison is made of $\gamma p$ and $\pi^- p$ systems at the same energy in the c.m.s. The energy scale is reduced to the total kinetic energy of the proton and $\pi^-$ in the c.m.s. The observed peak at 700 Mev of the incident photon energy in the laboratory system is reduced on this scale to 402 Mev. The resonance peak at 300 Mev of the incident photon energy in the laboratory system is reduced on this scale to 123 Mev, which is 17 Mev below the 140 Mev value that is taken as the peak of the $(3/2, 3/2)$ resonance. If the 402 Mev value is adjusted upward by 17 Mev to bring it in line with the $\pi^+ p$ cross-section data, the adjusted value of the secondary peak is 419 Mev. The secondary peak observed in Figs. 20c and b is consistent with this value or the unadjusted 402 Mev value. Considering the small number of pp- events, the good fit of the c.m. differential cross section at 90° for single-$\pi^+$ photoproduction with the histogram of $Q(p\pi^-)$ plus $Q(p\pi^-)$ must be largely fortuitous. Nevertheless, a secondary peak between 0.40 and 0.45 Bev is in evidence.

In the reaction $n + p \rightarrow p p$, we essentially have $n \rightarrow p^-$. In the reaction $\gamma + p \rightarrow n \pi^+$, we have $p \rightarrow n \pi^+$. The two reactions are, in a sense, charge-symmetric relations. From this point of view, it is perhaps not unreasonable that the two distributions described above are similar.

It has been pointed out that if the resonant behavior of the second peak is to be assigned a definite state of isotopic spin, then the fact that the peak is more pronounced in $\pi^+$ photoproduction than in $\pi^- p$ photoproduction indicates that the assigned state of isotopic spin should be $T = 1/2$, as suggested by Wilson.\textsuperscript{34, 35}

It is of interest to note that nucleon excitation energy corresponding to the "adjusted" value of the secondary peak at 419 Mev,
Fig. 20. Comparison of Q-value distributions of the $p\pi^-$ pairs emitted in the $pp\pi^-$ reaction with the c.m. differential cross section at 90° for photoproduction of single positive pions from hydrogen. [Curve (a) after Dixon and Walker.34]
is 559 Mev, or just twice the nucleon excitation energy of 280 Mev for
the peak of the (3/2, 3/2) resonance. The next resonance might then
be expected at three times the 280 Mev value, which gives a Q value
of 700 Mev. The small isolated peak at 700 Mev observed in the
Q(p, \pi^-) might then have some significance.

Pn+0+0+ Configuration in the C. M. System

The pn+0+ group includes the pn+0, pn+00, and perhaps
a few events of the pn+00 particle configuration. This group had a
tendency to be ambiguous with the pn+ and nn+0+ configurations.
Of the 67 ± 16 events that were estimated to belong to the pn+0+ group, only two events were well-identified, one of which was with
the aid of an electron-positron pair; and 32 were classed as probable.
Both of the well-identified events and five of the probable events had
both positive prongs identified by density of ionization, 26 of the prob­
able events had only the \pi^+ identified; and one had neither prong iden­
tified. The apparent requirement that the \pi^+ be identified in order to
have a pn+0+ type is unreasonable, and some bias in the histogram
distributions can be expected because these 31 of 32 events had a \pi^+
identified.

Histograms for P_a^* and \cos \theta_a^* are shown in Fig. 21. The
median momentum of the p, \pi^+, and \pi^- are 0.80, 0.23, and 0.30,
respectively. Greater confidence can be had in the \pi^- distribution
than in the \pi^+, because of the above-mentioned bias. The ratio of
median proton momentum to pion momentum is 2.6. The distribu­
tions of \cos \theta_a^* for the \pi^+ and \pi^- are essentially flat, indicating iso­
tropic emission of pions in the c.m.s. Some bias against forward
\pi^+ emission was expected as a result of the above-mentioned effect.
The most interesting feature is the tendency of the proton to go in the
forward direction. Twenty-three protons are emitted in the forward
hemisphere and 11 in the backward hemisphere. This is just the re­
verse of the proton direction in the pn+- reaction where, as a rule,
the protons went in the backward direction. This tendency toward
forward peaking is of special importance to the isobar model of pion
production proposed below and deserves further consideration at this
time. Twenty-three events were primarily ambiguous between
Fig. 21. Momentum distributions and angular distributions in the c.m.s. of particles emitted in the pπ⁺π⁻π₀ configuration.
(See Table III). If these are assumed to be pn+-0.. and nn++-.., then 18 of the protons are forward and five backward. Charge symmetry and the small number of pp+- events assures us that most of these are not nn++-.., and therefore are probably pn+-0.. events. Seventeen events were quite ambiguous between pn+-0.. and pn+-.

If these are called pn+-0.., then ten protons are forward and seven backward. Neither of the above ambiguous groups would then be expected to greatly change the observed forward-to-backward ratio of 23:11. It is of course possible that a number of events that were classed as pn+- are actually pn+-0.. This possibility arises out of the lack of knowledge about the incoming-neutron momentum and the use of an effective mass to represent the n0.. combination. Such an eventuality would increase the number of protons in the backward direction.

Histograms of $Q_{ab}$ and $\cos \theta_{ab}^*$ are shown in Fig. 22. In order to point up certain peaking effects, the better events of Fig. 22a (as defined above) are shaded with diagonal lines. The $p\pi^+$, $p\pi^-$, and $\pi^+\pi^-$ combinations all show a tendency toward emission in opposite directions with a ratio of opposite hemisphere to same hemisphere of about 2 to 1. The most interesting feature in the $Q$-value distributions is the sharp peaking of the $p\pi^+$ distribution between 0.10 and 0.15 Bev, indicating that the $p$ and $\pi^+$ may often be the disintegration products of the $(3/2, 3/2)$ isobar, or at least that the $(3/2, 3/2)$ resonance is important in the pn+-0.. reaction. The distribution of events in this peak, however, is on the low side of the $\pi^+p$ resonance. The small peak between 0.40 and 0.50 Bev coincides approximately with the secondary peak discussed in the section on the pp- reaction, suggesting that the second resonance level is a mixture of isotopic spin states $T = 3/2$ and $T = 1/2$. However, four of the seven events in this interval are of the poorer type, which indicates that the probable errors on these $Q$ values may be large. In the $Q(\pi^+\pi^-)$ distribution of the better events, the peaks between 0.05 and 0.10 and between 0.20 and 0.25 should be noted.
Fig. 22. $Q$-value and c.m.s. angular-correlation distributions for each pair of particles emitted in the $p\pi^+\pi^-\pi^0$ configuration.
The pp+-- Reaction in the C.M. System

The pp+-- reactions were sorted out by means of the same techniques used for the pp- reaction. Of the 35 acceptable five-prong events, one was well-identified as pp+--, and six were classified as probable pp+-- events. Histograms of $P_a^*$, $\cos \theta_a^*$, $Q_{ab}$, and $\cos \theta_{ab}$ are shown in Figs. 23 and 24. The forward-backward peaking and opposite-direction tendency of the protons has already been mentioned. Protons are distinguished by being more forward or backward ($p_f$ or $p_b$). Since the statistics are very poor, we shall point out only the most noteworthy features. In the $\cos \theta_a^*$ histograms, the $\pi^-\pi^+$ is seen to tend forward emission. This tendency is reflected in the $\cos \theta_{ab}^*$ distribution for $p_b\pi^-$ pairs, in which 11 pairs are in the opposite hemisphere and three pairs are in the same hemisphere. Nine of the 14 pairs of the $Q(p_b\pi^-)$ distribution have values of between 0.3 and 0.5 Bev, indicating that $p_b\pi^-$ may often be associated with the proposed second resonance whose peak is at about 0.4 Bev. The $p_f\pi^-$ distribution tends toward smaller $Q$ values and exhibits a peak between 0.1 and 0.2 Bev, indicating that this pair may often be associated with the $(3/2, 3/2)$ resonance.

The nn++-- Configuration in the C.M. System

Except for the $Q_{ab}$ distributions, histograms for the 12 nn++-- events in the c.m.s. are not shown. The nucleons carry off most of the momentum. Since the direction and momentum of both the nucleons was unknown in this case, the use of an effective mass for the emitted neutral particles does not permit a reliable estimate of the directions and momenta of the visible particles in the c.m.s. The $Q$ values, however, are invariant and are therefore shown in Fig. 25. The peaks in the vicinity of 0.05 to 0.10 Bev and 0.20 and 0.25 Bev for both $Q(\pi^+\pi^-)$ and $Q(\pi^+\pi^-)$ should be noted. It was pointed out in the chapter on multiplicities that in view of the principle of charge symmetry, too many nn++-- events were apparently observed by "identifying" both of the positive pions, which automatically put them in this group. Part of the excess may be explained by the effect of the numerous small gaps in the sensitive layer of the cloud chamber in
Fig. 23. Momentum and angular distributions in c.m.s. of particles emitted in the $pp\pi^+\pi^-\pi^-$ reaction.
Fig. 24. Q-value and c.m.s. angular-correlation distributions for each pair of particles emitted in the pp\(\pi^+\pi^-\pi^-\) reaction.
Fig. 25. $Q$-value distributions of $Q(\pi^+\pi^+)$ and $Q(\pi^+\pi^-)$ in the nn$\pi^+\pi^-\pi^-$ configuration.
which the methanol vapor was largely depleted. These gaps made it inevitable that a few proton tracks with ionization density of perhaps 1.5 minimum would appear as minimum tracks and thus would be identified as pions. Another part of the apparent excess may be explained by the apparently strong tendency of the $\pi^-$ to be emitted in the forward direction in the pp+- reaction. Since the $\pi^-$ distribution is forward in this reaction, then, in view of the principle of charge symmetry, the $\pi^+$ will be backward in the nn+- reaction. We have already pointed out that positive particles going backward in the c.m.s. tend to be identified in the laboratory system. Thus a majority of the nn+- events might be expected to be identified by identifying the two positive pions.

The pn+- Reaction in the C. M. System

Of the ten events used in the histograms for the pn+- reaction, four were well-identified, four were probable, and two were ambiguous with nn+++. The probable pn+- reactions showed a tendency to be ambiguous with pn++0 and nn+++. When we consider the improbability of five-pion production, the probable events may be considered more in the well-identified category. In the histograms, the two ambiguous events are classified as probable events.

Histograms of $P_a^*$ are shown in Fig. 26a. It is seen that the momentum distribution of the proton is similar to that of the neutron and that the distribution of the $\pi^+$ is similar to the $\pi^-$. The median momenta of the nucleons and pions are about 0.44 and 0.21 Bev/c, respectively, giving a ratio of nucleon to pion momentum of 2.1.

Histograms of $\cos \theta_a^*$ are shown in Fig. 26b. The most striking feature is the forward peaking of the protons and the backward peaking of the neutrons. Nine of the ten protons are in the forward hemisphere, and all ten neutrons are in the backward hemisphere. If we assume a grazing type of collision in which the nucleons continue after collision in approximately the same forward or backward direction they had before collision, then it follows that the neutron and proton have changed roles, the one being transformed into the other. The $\cos \theta_a^*$ histograms for $\pi^+$ and $\pi^-$ are similar and are consistent with a flat distribution.
Fig. 26. Momentum and angular distributions in c.m.s. of particles emitted in the pnπ⁺π⁺π⁻ reaction.
Histograms of $\cos \theta^*_{ab}$ are shown in Fig. 27b. The neutron and proton show a strong tendency toward emission in the opposite direction. The charge-symmetric pairs $\pi^+ \pi^+$ and $\pi^- \pi^-$ both show a tendency to go in the opposite direction. The evidence thus far suggests that the proton goes forward with a $\pi^+$ and a $\pi^-$, and the neutron goes backward with a $\pi^+$ and $\pi^-$. The roughly flat distribution of $\pi^+ \pi^-$ is consistent with this picture. So also are the charge-symmetric pairs $p\pi^+$ and $n\pi^-$, which are both consistent with a flat distribution. The charge-symmetric pairs $p\pi^-$ and $n\pi^+$ are similar in shape, but both show a tendency toward emission in the opposite direction. These distributions may also be consistent with this simple picture.

Histograms of $Q_{ab}$ are shown in Fig. 27a. The charge-symmetric pairs appear to be similar within the statistical uncertainties. The $p\pi^+$ distribution shows more consistency with the $(3/2, 3/2)$ resonance than the other three pion-nucleon pairs. The $p\pi^+$ distribution is expected to be a more accurate distribution than the $n\pi^-$, because of the calculated direction and momentum of the neutron. In the $Q(\pi^+ \pi^-)$ distribution the peaking between 0.05 and 0.10 Bev and between 0.20 and 0.25 Bev is to be noted. These peaks may also be seen in the sum of $Q(\pi^+ \pi^+)$ plus $Q(\pi^- \pi^-)$ distributions.
Fig. 27. $Q$-value and c.m.s. angular-correlation distributions for each pair of particles emitted in the $pn\pi^+\pi^+\pi^-\pi^-$ reaction.
EVIDENCE FOR PION-PION INTERACTIONS

The pion-pion wave functions may be analyzed into eigenstates of total isotopic spin \( T = 0, 1, \) and 2. The \( \pi^+ \pi^+ \) and \( \pi^- \pi^- \) wave functions are in state \( T = 2 \). The \( \pi^+ \pi^- \) wave function may be in states \( T = 0, 1, \) or 2. In Fig. 28b the \( Q(++) \) and \( Q(--) \) values of particle configurations nn++-- , pp+-, and pn++-- are shown added together. In Fig. 28a the \( Q(+-) \) values are shown added together. In this case, the particle configurations included were pn+, pn-0.., nn+-.., pp+-, and pn++--. The \( Q(+-) \) distribution shows two large peaks. One is centered between 0.05 and 0.10 Bev, and the other between 0.20 and 0.25 Bev. These two peaks are largely due to the well-identified events. The \( Q(++) \) plus \( Q(--\) distribution of Fig. 28b also shows peaks between 0.05 and 0.10 and between 0.20 and 0.25 Bev. The fewer number of events in this distribution gives these peaks less statistical significance. The fact that this \( Q(++) \) plus \( Q(--\) distribution must be in isotopic spin state \( T = 2 \) indicates that the peaks of the \( Q(+-) \) distribution are at least partly due to resonances in the \( T = 2 \) state.

Both of these peaks may have a theoretical basis. The position of the higher-energy peak coincides with the mass of a K meson minus two pion masses. This value of 0.213 Bev, shown in Fig. 28a by an arrow, is simply the energy released in the theta decay mode of the K meson. The arrow at 0.213 Bev is seen to bisect the higher-energy peak. This peak seems to indicate a strong interaction between pions in the isotopic spin state \( T = 2 \).

The lower-energy peak may be associated with the pion-pion resonance predicted by Mitra and Saxena. They solved an integral equation for pion-pion scattering for the cases of isotopic spin \( T = 0, 2, \) and assumed the pions to be in s states with respect to each other. It was found that the interaction in the state \( T = 0 \) is too strongly attractive to give a resonance. For the case \( T = 2 \), however, a resonance was obtained at a momentum, in the c.m.s. of the two pions, of about 93 Mev/c for each pion, taking \( G^2/4 = 15.5. \) This is equivalent to each of the pions having 28-Mev kinetic energy, which gives a \( Q \) value of 0.056 Bev. An arrow at 0.056 Bev is shown in Fig. 28a.
Fig. 28. Summation of $Q(\pi^+\pi^-) + Q(\pi^-\pi^-)$ and $Q(\pi^+\pi^-)$ distributions from particle configurations $pn\pi^+\pi^-$, $pnn^+\pi^-\pi^-$, $nnn^+\pi^+\pi^-$, $pp\pi^+\pi^-\pi^-$, and $pnn^+\pi^+\pi^-\pi^-$. 

(a) $Q(\pi^+\pi^-)$

(b) $Q(\pi^+\pi^+) + Q(\pi^-\pi^-)$
The observed low-energy peak in the $Q(\pi^+\pi^-)$ and the $Q(\pi^-\pi^-)$ plus $Q(\pi^+\pi^+)$ distribution is apparently consistent with this predicted resonance. In a more recent paper, Mitra considered the role of pion-pion interaction in $\tau$-meson decay. The results of this study give supporting evidence for the $T = 2$ resonance.
THE DIRECTIONAL-ISOBAR MODEL FOR MULTIPLE-PION PRODUCTION

The purpose of this section is to compare the predictions of an isobar model with the facts observed in this experiment and experiments I and II. As will be seen, many of the experimental results agree sufficiently closely with this model to make this phenomenological approach significant.

General Features of the Directional-Isobar Model

In this phenomenological model the following assumptions are made:

1. The nucleons continue after collision in approximately the same direction that they were going before collision.

2. During the collision process a single pion is absorbed or emitted by each nucleon. This intermediate process may be thought of as the scattering of a pion out of one nucleon and into the other. The three possible combinations of nucleon-pion pairs are then \((p^+, n^-)\), \((p^0, n^0)\), and \((p^-, n^+)\). The proton-pion pairs are backwards in the c.m.s., and the neutron-pion pairs are forward in the c.m.s. The nucleon-pion pairs form isobaric states of the nucleon having an isotopic spin of \(T = 3/2\) or \(T = 1/2\), which subsequently decay to the ground state via pion emission.

3. The three combinations of nucleon-pion pairs given above occur in proportion to their \(T = 1/2\) and \(T = 3/2\) components and to the effective pion-nucleon scattering cross sections associated with the \(T = 1/2\) and \(T = 3/2\) states. In the case in which the \(T = 3/2\) and \(T = 1/2\) states are equally effective, the three combinations of nucleon-pion pairs should be equally probable. In this case, there are two pions of each charge state among the three combinations. The condition is consistent with the hypothesis of charge independence.

The \(T = 3/2\) isobar is associated with the \((3/2, 3/2)\) resonance, and decays by emission of a single pion. The \(T = 1/2\) isobar is assumed to have a considerably greater excitation energy (perhaps the \(T = 1/2\) resonance observed in photoproduction), and to decay either by single-pion emission to the ground state or by double-pion emission via the
$T = 3/2$ state. Five-pion production events are ignored. No interference effects are assumed between the two isotopic spin states, either in the formation or decay of the isobars.

Fig. 29 shows the three combinations of pion-nucleon pairs, together with their forward or backward direction in the c.m.s. and the probability of their being in the $T = 3/2$ and $T = 1/2$ states. The notation $(T, T_z)$ is used to indicate the total isotopic spin and its $z$ component. $\sigma_{3/2}$ and $\sigma_{1/2}$ are the effective pion-nucleon scattering cross sections for the $T = 3/2$ and $T = 1/2$ isotopic spin states, respectively.

\[
\begin{align*}
\text{Backward} & \quad \sigma_{3/2}\left(\frac{3}{2}, \frac{3}{2}\right) \quad \leftarrow p^+ \quad \rightarrow \quad \text{Forward} \\
& \quad n^- \rightarrow \sigma_{3/2}\left(\frac{3}{2}, -\frac{3}{2}\right) \\
\frac{1}{3} \sigma_{1/2}\left(\frac{1}{2}, \frac{1}{2}\right) + \frac{2}{3} \sigma_{3/2}\left(\frac{3}{2}, \frac{1}{2}\right) & \quad \leftarrow p^0 \\
& \quad n^0 \rightarrow \frac{2}{3} \sigma_{3/2}\left(\frac{3}{2}, -\frac{1}{2}\right) + \frac{1}{3} \sigma_{1/2}\left(\frac{1}{2}, -\frac{1}{2}\right) \\
\frac{2}{3} \sigma_{1/2}\left(\frac{1}{2}, -\frac{1}{2}\right) + \frac{1}{3} \sigma_{3/2}\left(\frac{3}{2}, \frac{1}{2}\right) & \quad \leftarrow p^- \\
& \quad n^+ \rightarrow \frac{1}{3} \sigma_{3/2}\left(\frac{3}{2}, \frac{1}{2}\right) + \frac{2}{3} \sigma_{1/2}\left(\frac{1}{2}, \frac{1}{2}\right)
\end{align*}
\]

Fig. 29. Diagram showing forward and backward directions and weightings of $T = 1/2$ and $T = 3/2$ components assumed for the directional-isobar model of multiple-pion production.
Excitation Modes of the Directional Isobar Model

From the diagram of Fig. 29, it is evident that in this model during a nucleon-nucleon collision, there are three modes of excitation of the nucleons: (a) both nucleons are excited to the $T = 3/2$ state, (b) both nucleons are excited to the $T = 1/2$ state, (c) one nucleon is excited to the $T = 3/2$ state and the other to the $T = 1/2$ state.

Both Nucleons Excited to the $T = 3/2$ State

This case is limited to double-pion production. The probability that both nucleons will be excited to the $T = 3/2$ state is seen from the diagram of Fig. 29 to be:

$$ P_{3/2}^2 = K \sigma_{3/2}^2 \left[ \left( \frac{3}{2}, \frac{3}{2} \right) \left( \frac{3}{2}, \frac{3}{2} \right) + \frac{4}{9} \left( \frac{3}{2}, \frac{1}{2} \right) \left( \frac{3}{2}, \frac{1}{2} \right) + \frac{1}{9} \left( \frac{3}{2}, \frac{1}{2} \right) \left( \frac{3}{2}, \frac{1}{2} \right) \right] $$ (11)

where arrows $\rightarrow$ and $\leftarrow$ indicate backward and forward directions, respectively, and $K$ is a constant of proportionality. Reducing the wave functions to their component pion-nucleon pairs, we have

$$ P_{3/2}^2 = \frac{K \sigma_{3/2}^2}{81} \left[ \begin{array}{c}
\text{p}^{+}\text{n}^{+}, \text{n}^{-}\text{p}^{-} \\
\text{n}^{+}\text{p}^{-}, \text{p}^{+}\text{n}^{-} + 8 \text{p}^{0}, \text{p}^{-}\text{p}^{0} + 8 \text{n}^{+}, \text{n}^{+}\text{p}^{0} + 16 \text{p}^{0}, \text{n}^{0}\text{n}^{0} \\
+1 \text{p}^{-}\text{n}^{+} + 2 \text{p}^{-}\text{p}^{0} + 2 \text{n}^{0}, \text{n}^{+}\text{n}^{0} + 4 \text{n}^{0}\text{p}^{0}, \text{p}^{0}\text{p}^{0}
\end{array} \right] $$ (12)

The particle configurations $\text{p}^{+}\text{n}^{-}$, $\text{p}^{-}\text{n}^{0}$, $\text{n}^{+}\text{n}^{0}$, and $\text{p}^{0}\text{n}^{0}$ occur as 86, 10, 10, and 20. These numbers may be compared with those for the Peaslee model with equal parts of $T = 0$ and $T = 1$, which gives 110, 10, 10, and 20. In the Peaslee model for the $\text{p}^{+}\text{n}^{-}$ reaction, 21 out of 22 protons and positive pions decay from the same isobar. In our model, 81/86 or 20.7 out of 22 protons and positive pions decay from the same isobar. From Eq. (12) it is seen that 82 out of 86 protons are in the backward direction.

The experimental results show strong peaking of protons in the backward direction, with 85 out of 97 protons in the backward hemisphere. The $Q_{ab}$ and $\cos \theta_{ab}^*$ distributions of the $\text{p}^{+}\text{n}^{-}$ reaction are consistent with both models.
The ratio of $\frac{pn^+}{pp^-} = 8.6$ may be compared with the experimental ratios of this experiment and experiments I and II. In this experiment, the number of $pn^+$ and $pp^-$ events is $132 \pm 23$ and $18 \pm 8$, respectively. The ratio of $pn^+$ to $pp^-$ is 7.3, which is in good agreement with the 8.6 value. In I the numbers were $39 \pm 9$ and $8 \pm 4$, which is consistent with the 8.6 value. In II the ratio of $pn^+$ to $pp^-$ was $3.2 \pm 0.7 / 1 \pm 0.35$, which is not in probable agreement with the 8.6 value. However, when the contributions to the $pn^+$ reaction from isobaric nucleons decaying from the $T = \frac{1}{2}$ state are included, it will be seen that the ratios of $\frac{pn^+}{pp^-}$ in experiments I and II are consistent with our model.

Equation (12) states that negative pions generally decay with the forward proton in the $pp^-$ reaction, while they go with the forward neutron in the $pn^+$ reaction. A comparison of the histograms of $P_a^*$, $\cos \theta_a^*$, $Q_{ab}^*$, and $\cos \theta_{ab}^*$ for the two reactions gives a result consistent with Eq. 12 when the corresponding particles are $p_b^*$, $p_f^*$, $-, 0$ and $p, n, -, +$, respectively.

Both Nucleons Excited to the $T = 1/2$ State

The probability that both nucleons will be excited to the $T = 1/2$ state is seen from the diagram of Fig. 29 to be

$$P_{1/2}^2 = K \Omega_{1/2}^2 \left[ \frac{4}{9} \left( \frac{1}{2}, -\frac{1}{2} \right) \left( \frac{1}{2}, \frac{1}{2} \right) + \frac{1}{9} \left( \frac{1}{2}, \frac{1}{2} \right) \left( \frac{1}{2}, -\frac{1}{2} \right) \right].$$  \hspace{1cm} (13)

We assume that the $T = 1/2$ level is sufficiently high that two pions may be emitted in the decay of the isobar to the ground state. The $T = 1/2$ isobar may decay via the $T = 3/2$ state with probability $s$, in which case two pions are emitted, or directly to the ground state with probability $1-s$. Figure 30 shows the energy-level diagram for the ground states of $p$ or $n$, the $T = 3/2$ level with the four charge states $2, 1, 0,$ and $-1$, and the proposed $T = 1/2$ level with charge states $1$ and $0$. The probabilities for decay via the different charge states are simply calculated from tables of Clebsch-Gordan coefficients. The decay scheme of the terms of Eq. (13) is evident from Fig. 30.
The relative frequencies for the different particle configurations, for the case in which both nucleons are excited to the $T = 1/2$ isobaric state, are shown in the right-hand column of Table VII. Quantities in this column are to be multiplied by $\sigma_{1/2}^2$, to obtain the relative frequencies for the different particle configurations. The common factor of $K/81$ has been dropped from this table.
Table VII

Relative frequencies of particle configurations, calculated by the use of the directional-isobar model of multiple-pion production, for the three modes of nucleon excitation: both nucleons excited to the $T = 3/2$ state; one nucleon excited to the $T = 3/2$ state and the other to the $T = 1/2$ state; and both nucleons excited to the $T = 1/2$ state.

<table>
<thead>
<tr>
<th>No. of pions</th>
<th>No. of prongs</th>
<th>Particle configuration</th>
<th>Both $T = 3/2$</th>
<th>One $T = 1/2$, one $T = 3/2$</th>
<th>Both $T = 1/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>pp-0</td>
<td>$10 \sigma_{3/2}^2$</td>
<td>$20(1-s)\sigma_{1/2} \sigma_{3/2}$</td>
<td>$10(1-s)^2\sigma_{1/2}^2$</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>pn-+</td>
<td>86</td>
<td>16</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>pn00</td>
<td>20</td>
<td>16</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>nn+0</td>
<td>10</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total</td>
<td>126</td>
<td>72</td>
<td>45</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>pp++-</td>
<td>$20/3 s \sigma_{1/2} \sigma_{3/2}$</td>
<td>$50/3 s (1-s) \sigma_{1/2}^2$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>pp-00</td>
<td>$24/3$</td>
<td>$30/3$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>pn++-</td>
<td>$96/3$</td>
<td>$90/3$</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>nn++-</td>
<td>$20/3$</td>
<td>$50/3$</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>pn000</td>
<td>$32/3$</td>
<td>$20/3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total</td>
<td>$24/3$</td>
<td>$30/3$</td>
<td>90</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>pp++-0</td>
<td>$50/9 s^2 \sigma_{1/2}^2$</td>
<td>$525/9$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>pn++</td>
<td>$125/9$</td>
<td>$20/9$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>pp-00</td>
<td>$20/9$</td>
<td>$120/9$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>pn+00</td>
<td>$50/9$</td>
<td>$20/9$</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>pn0000</td>
<td>$20/9$</td>
<td>$20/9$</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>nn+000</td>
<td>$20/9$</td>
<td>$45$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total</td>
<td>$50/9$</td>
<td>$20/9$</td>
<td>45</td>
</tr>
</tbody>
</table>
The pn++-- reaction in the directional-isobar model. Equation (14) gives the number of pn++-- events in this model. The eight components of this reaction represent the different modes of decay of the two T = 1/2 isobars:

\[
N(pn++--) = K_1 \sigma_{1/2}^2 \frac{S^2}{9} \left[ 81 \pi^+ (n\pi^-)_{3/2} \pi^- (p\pi^+)_{3/2} + 9 \pi^- (n\pi^+)_{3/2} \pi^- (p\pi^+)_{3/2} + 9 \pi^+ (n\pi^-)_{3/2} \pi^+ (p\pi^-)_{3/2} + 1 \pi^- (n\pi^+)_{3/2} \pi^+ (p\pi^-)_{3/2} \right]
\]

\[
\frac{1}{4} \text{ (similar terms in opposite directions)}
\]

(14)

where \(K_1\) is a constant. The unbracketed \(\pi\) is the result of the decay of the T = 1/2 isobar to the T = 3/2 level. Four protons in the forward direction for each proton in the backward direction are predicted. Figure 26b shows that 9 out of 10 protons were forward and all 10 neutrons were backward. In each component of Equation (14) there is a \(\pi^+\) and a \(\pi^-\) in both the forward and the backward directions. The \(\cos \theta^a\) and \(\cos \theta^b\) histograms show this. However, the experimental fact that the proton and \(\pi^-\) and also the neutron and \(\pi^+\) tend to be in opposite directions is still a problem. A larger energy change in the transition between the T = 1/2 and the T = 3/2 states would produce this tendency.

One Nucleon Excited to the T = 3/2 State and the Other to the T = 1/2 State.

The probability that one nucleon will be in the T = 3/2 excited state and the other will be in the T = 1/2 excited state is seen from the diagram of Fig. 29 to be

\[
P_{1/2} P_{3/2} = K_0 \sigma_{1/2} \sigma_{3/2} \frac{2}{9} \left[ \left( \frac{1}{2}, \frac{1}{2} \right) \left( \frac{3}{2}, \frac{1}{2} \right) + \left( \frac{3}{2}, \frac{1}{2} \right) \left( \frac{1}{2}, \frac{1}{2} \right) \right]
\]

\[
+ \left( \frac{3}{2}, \frac{1}{2} \right) \left( \frac{1}{2}, \frac{1}{2} \right) + \left( \frac{1}{2}, \frac{1}{2} \right) \left( \frac{3}{2}, \frac{1}{2} \right) \right].
\]

(15)
From the symmetry of the terms it is evident that any particle in a given reaction will have forward-backward symmetry. Both double- and triple-pion production may occur. Probabilities for the different particle configurations may be worked out from the diagram of Fig. 30 and are shown in the "One T = 1/2, one T = 3/2" column of Table VII.

The \( p^{+}n^{+}0 \) configuration. This group includes the \( p^{+}n^{+}0 \) and the \( p^{+}n^{+}00 \) reactions. From Table VII it is seen that the number of events in this group is given by

\[
N(p^{+}n^{+}0...) = K_1 \left[ 32 s \sigma_{1/2} \sigma_{3/2} + 30 s (1-s) \sigma_{1/2} \sigma_{1/2} + \frac{120}{9} s^2 \sigma_{1/2} \sigma_{1/2} \right].
\]

(16)

The first term of Eq. (16) has as many protons in the forward direction as in the backward. In the second term, which represents one of the \( T = 1/2 \) isobars decaying directly to the ground state and the other decaying via the \( T = 3/2 \) state, \( 8/15 \) of the protons are forward. In the third term of Eq. (16), which represents the \( p^{+}n^{+}00 \) configuration, \( 29/40 \) of the protons are forward. The sharp peak of the \( Q(p\pi^+) \) distribution in the vicinity of the \((3/2, 3/2)\) resonance suggests that the proton and \( \pi^+ \) often decay from a common \( T = 3/2 \) isobar in the \( p^{+}n^{+}00 \) configuration. In the first, second, and third terms of Eq. (16), the proton and \( \pi^+ \) decay from a common \( T = 3/2 \) isobar in \( 3/8, 1/3, \) and \( 13/32 \) of the cases respectively. In each case this \( T = 3/2 \) isobar is in turn a decay product of the \( T = 1/2 \) isobar. The relative weighting of the terms is not important. This fraction of about \( 1/3 \) seems small in relation to the sharp peak of the distribution. Final-state interactions, however, may serve to bring more of the \( Q(p\pi^+) \) values into the vicinity of the \((3/2, 3/2)\) resonance.

The \( pp^{+-} \) reaction. Since there are only one well identified and six probable \( pp^{+-} \) events, little more than indications of consistency between theory and experiment may be expected. The number of \( pp^{+-} \) events is seen from Table VII to be:

\[
N(pp^{+-}) = K_1 \left[ \frac{20}{3} s \sigma_{1/2} \sigma_{3/2} + \frac{50}{3} s (1-s) \sigma_{1/2} \sigma_{1/2} \right].
\]

(17)
In the first term all the particles have forward-backward symmetry. In the second term the principal component is

\[ K_1 \ 12 \ 2 (\pi^- p)_{1/2} \ p_{(1-s)} (\pi^- p)_{1/2} + (\pi^- p)_{3/2} \]

The \((\pi^- p)_{1/2}\) part in the backward direction has a large disintegrating energy or \(Q\) value between its \(\pi^-\) and its backward proton. The large \(Q\) value should give the \(\pi^-\) an approximately isotropic distribution and make the \(\pi^-\) and backward proton go in opposite directions. However, the \((\pi^- p)_{3/2}\) part in the forward direction has a considerably smaller \(Q\) value between its \(\pi^-\) and its \((\pi^- p)_{3/2}\) isobar, which will give the \(\pi^-\) a more forward distribution. Experimentally, the \(\pi^-\) is observed to have a tendency towards forward emission, and the \(p_b \pi^-\) pair is observed to have a tendency toward emission in the opposite direction. Also, the \(p_b \pi^-\) pair has 9 out of 14 \(Q\) values between 0.3 and 0.5 Bev, which is an indication that a backward moving \(T = 1/2\) isobar often decays directly to the ground state. The seven events in the histograms thus favor the predominance of the second term of Eq. (17). This condition results from the initial excitation of both nucleons to the \(T = 1/2\) state.

Other terms in the \(pn^+\)- and \(pp^0\)-reactions. In the discussion of the two-pion reactions above, only that case was considered in which both the nucleons were excited to the \(T = 3/2\) state. Table VII shows the two other terms for each of these reactions. For the \(pn^-\) reaction, the number of events is given by

\[ N(pn^-) = K_1 86 \sigma_{3/2}^2 + K_1 (1-s) \sigma_{1/2} \sigma_{3/2} \left\{ 4(\pi^+ n)_{3/2} (\pi^- p)_{1/2} + 4(\pi^- p)_{3/2} (\pi^+ n)_{1/2} \right\} + \text{(similar terms in opposite directions)} \]

\[ + K_1 (1-s)^2 \sigma_{1/2} \left[ 16(\pi^- p)_{1/2} (\pi^+ n)_{1/2} + 4(\pi^+ n)_{1/2} (\pi^- p)_{1/2} \right] . \]

(18)
The first term was discussed previously. The large coefficient of 86 virtually insures its predominance over the second and third terms. In the second and third terms, the n and \( \pi^+ \) decay from a common isobar, and the p and \( \pi^- \) decay from a common isobar, whereas in the first term it is predominantly the p and \( \pi^+ \) and the n and \( \pi^- \) that decay from common isobars. Particles in the second term have forward-backward symmetry. In the third term, four out of five protons are in the backward direction, the predominant direction -- 82 of 86 -- of the protons in the first term.

The number of pp\(-\)0 events is given by

\[
N(pp-0) = K_1 \sigma^{3/2} + K_1 (1-s) \sigma^{1/2} \left[ \sigma^{3/2} \right. \\
+ \left. (\text{similar terms in opposite directions}) \right] \\
+ K_1 (1-s)^2 \sigma^{1/2} \left[ \sigma^{3/2} \right. \\
+ \left. (\text{similar terms in opposite directions}) \right]
\]

The first term was discussed previously. The second and third terms are relatively more important here than they were in the pn\(^+\)\(-\) reaction. In the third term, four out of five neutral pions and the forward proton decay from a common isobar. The opposite effect occurs in the first term, in which four out of five neutral pions and the backward proton decay from a common isobar. However, in the third term, the isobars decay from the higher \( T = 1/2 \) level, and the pions should have a more isotropic distribution. The previously noted correspondence between the particles p, n, \( \pi^- \), and \( \pi^+ \) and \( p_b \), \( p_f \), \( \pi^- \), and \( \pi^0 \), respectively, suggests that the first term is dominant at Bevatron energies.
Multicities for Double-, Triple-, and Quadruple-Pion Production

The relative frequencies for the different particle configurations, given in Table VII, are dependent on two parameters: One is the fractional probability, \(s\), that the \(T = 1/2\) isobar will decay via the \(T = 3/2\) state, emitting two pions, and the other is the ratio, \(r\), which is \(\sigma_{1/2}/\sigma_{3/2}\). Consider first what value the ratio, \(r\), may be expected to have at Bevatron energies. The directional-isobar model implies that each nucleon scatters a pion out of the other, or, more accurately perhaps, that the two nucleons mutually scatter a pion. If we ignore internal momentum of pions in the nucleon, then the neutron moving forward in the laboratory system with velocity \(\beta c\) has a relative velocity, with respect to a pion in the stationary proton, of just \(\beta c\). In the rest frame of reference of the neutron, the pion in the proton is moving backward with velocity \(\beta c\). This velocity determines the energy in the laboratory system that a pion would have in ordinary pion-nucleon scattering. The median energy of neutrons producing three- and five-prong events is 3.8 Bev, giving a \(\beta\) of 0.98. A pion with this \(\beta\) has a kinetic energy of about 600 Mev. This is near the minimum of the \(\pi^+ p\) total-interaction cross section where \(r\) is about 3. The median incident-neutron energy for the four-pion production events is 5.2 Bev, which gives a pion energy of about 800 Mev. This is near the maximum of the \(T = 1/2\) total-interaction cross section. Furthermore, all ten \(pn++--\) events are grouped in the energy region where \(r\) is greater than 2. At a pion energy of about 430 Mev the two cross sections are equal, and below this energy \(\sigma_{3/2}\) rapidly becomes greater than \(\sigma_{1/2}\). The 430-Mev value corresponds to an incident-nucleon energy of about 2.8 Bev. It is of interest to note that in the 2.75-Bev proton-proton experiment of Block et al, the cross section for triple-pion production was about one-third as large as the cross section for double-pion production. In II, the \(n-p\) experiment, with maximum incident-neutron energy of 2.2 Bev, not a single five-prong event was observed although 185 three-prong events were recorded. The \(\beta\) of the incident neutron can of course give only an indication of the effective cross-section values. The decrease in the velocity of the nucleons during the collision process reduces the relative \(\beta\) between nucleon and pion.
An accurate estimate would require that the integrated effect of the velocity on the cross section values be considered. The decrease in velocity would be expected to increase the relative importance of the $T = 3/2$ cross section. Interference effects between the $T = 1/2$ and $T = 3/2$ states could be important. The internal motions of the pions in the nucleon should be taken into account. For the present, however, only an effective value of $r$ is considered in this phenomenological model.

The multiplicities for the double-, triple-, and quadruple-pion-production events are calculated from the entries of Table VII. In Table VIII, multiplicities are shown for $s = 1/2$, $r = 3/2$ and for $s = 1$, $r = 1$. The calculated total number of three- and five-prong events has been normalized to 269, the observed total number of three- and five-prong events, (excluding the pp- configuration) used in this experiment. The observed experimental values are shown in the extreme right-hand column. Five pion-production events are ignored. Table VIII shows that the frequencies predicted by the directional-isobar model are not in conflict with the observed frequencies of the different particle configurations. Moreover, if $s = 1$, $r = 1$ is used, comparison of the predicted frequencies reveals a remarkably good fit. An even better fit could be made by properly adjusting $s$ and $r$. However, the calculated frequencies are approximate; for instance, no effort is made to take into consideration the large energy range of the incident neutrons. Such an adjustment would disguise the approximate nature of the calculation.

The directions of the nucleons predicted by the directional-isobar model using $s = 1$, $r = 1$ are in good agreement with the experimentally observed nucleon directions. The nucleon directions for the $pn^+$- and $pn^+$-- reactions have already been discussed. For the $pn^+0_-$ group, 1/2 of the predicted 45 $pn^+0_-$ events have protons in the forward direction, and 29/40 of the 19 predicted $pn^+0_-$ events have protons in the forward direction. Thus in the 34 events used in the histograms of the $pn^+0_-$ group, 19 protons are expected to be in the forward direction and 15 in the backward direction. This compares favorably with the observed distribution, in which 23 protons are in
Table VIII

Number of events predicted by the directional-isobar model of multiple-pion production using $s = 1/2$, $r = 3/2$ and $s = 1$, $r = 1$; and observed numbers of events. ($r = \sigma_{1/2}/\sigma_{3/2}$)

<table>
<thead>
<tr>
<th>No. of protons</th>
<th>No. of prongs</th>
<th>Particle configuration</th>
<th>$s=1/2$, $r=3/2$</th>
<th>$s=1$, $r=1$</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Both $T=3/2$</td>
<td>One $T=3/2$</td>
<td>Both $T=1/2$</td>
<td>One $T=1/2$</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>11</td>
<td>6</td>
<td>34</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>26</td>
<td>9</td>
<td>120</td>
<td>122</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>6</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>9</td>
<td>14</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>3</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>3</td>
<td>8</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>1</td>
<td>14</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>1</td>
<td>14</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>26</td>
<td>64</td>
<td>64</td>
<td>67±16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>28</td>
<td>37</td>
<td>37</td>
<td>28±6</td>
</tr>
</tbody>
</table>

\(a\)Estimated from 3- and 5-prong data.

\(b\)Excluding pp-
the forward hemisphere and 11 are in the backward hemisphere (see Table IX). The correspondence between the particles p, n, \( \pi^+ \), and \( \pi^- \) and \( p_b, p_f, \pi^0 \), and \( \pi^- \) described above is also in accordance with the predictions of this model, using \( s = 1, r = 1 \). For the \( pp^- \) configuration, however, there is no component resulting from the excitation of both nucleons to the \( T = 1/2 \) state, although this is the mode of excitation favored by experiment. However, since only seven \( pp^- \) events are represented in the histograms, six of which are probable, the discrepancy is not serious. The apparently large number of observed \( nn^- \) events compared to the number of \( pp^- \) events is partly explained by the use of \( s = 1, r = 1 \) in this model, since it thereby predicts almost as many \( nn^- \) events as \( nn^- \) events.

Table IX

Comparison of observed and calculated forward and backward distribution of protons in particle configurations in which a proton and a neutron are emitted, calculated on the basis of the directional-isobar model, using \( s = 1, r = 1 \). The calculated numbers of events having protons in the forward-backward directions are normalized to the observed numbers.

<table>
<thead>
<tr>
<th>Particle Configuration</th>
<th>Observed</th>
<th></th>
<th>Calculated</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Backward</td>
<td>Forward</td>
<td>Backward</td>
<td>Forward</td>
</tr>
<tr>
<td>( pn^- )</td>
<td>85</td>
<td>12</td>
<td>92</td>
<td>5</td>
</tr>
<tr>
<td>( pn^0 )</td>
<td>11</td>
<td>23</td>
<td>15</td>
<td>19</td>
</tr>
<tr>
<td>( pn^- )</td>
<td>1</td>
<td>9</td>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

The predictions of the directional-isobar model using \( s = 1/2, r = 3/2 \) are included for comparative purposes and to show the relative importance of the three components of the different particle configurations. Although the predicted numbers of events for this group do not compare as well with experimental results as the predictions of the model using \( s = 1, r = 1 \), nevertheless, they do not give an impossible fit.
The agreement of the predicted directions of nucleons in the pn+- and pn+-0.. reactions with the experimentally observed directions is not bad. The predicted 10 to 6 ratio of "Both T = 1/2" to "One T = 3/2 and one T = 1/2" in the pp+-- reaction suggests a fair agreement with the experimentally favored mode of excitation, "Both T = 1/2." Also, the predicted ratio of three-prong events (excluding pp-) to the total five-prong events --241 to 28 -- agrees with the experimentally observed ratio of 241 to 28. However, since there are two adjustable parameters in the present model, this ratio is not a sensitive one. The poor fit of the number of events in the pp-0 reaction, and the presence of modes of excitation other than "Both T = 3/2" in this reaction, are the chief discrepancies presented by the s = 1/2, r = 3/2 group. Only the mode of excitation in which both nucleons are excited to the T = 3/2 state can account for the observed correspondence between the particles of this pp-0 configuration and the pn+- configuration. In the predictions of the model using s = 1/2, r = 3/2, only 11/34 of the pp-0 events are in the "Both T = 3/2" mode of excitation. The ratio pn+-/pp-0 = 3.5, predicted by using s = 1/2, r = 3/2, is not in good agreement with the most probable ratio observed in this experiment, which is about 7.

This 3.5 ratio is in better agreement with the most probable observed ratios of pn+-/pp-0 events in experiments I and II, which were 4.9±1.1/1±0.5 and 3.2±0.7/1±0.35, respectively. If s equals 0, all isobars in the T = 1/2 state decay directly to the ground state and triple- and quadruple-pion production are eliminated. If we suppose that the T = 1/2 state is important at Cosmotron energies, such as through the effect of Fermi momentum of the pions in the nucleon, and if we consequently choose r = 1, then the pn+-/pp-0 ratio would be 3.1, which is in good agreement with II. In this case, three out of four pp-0 events and 36 out of 86 pn+- events would not have both nucleons excited to the T = 3/2 state. This would wash out the correspondence between the particles p, n, π⁺, and π⁻ and p⁺, p⁻, π⁺, and π⁻, respectively, and might obscure any (3/2, 3/2) peaking in the Q(pπ⁺) and Q(nπ⁻) distributions of the pn+- reaction. Better
agreement with experiment I is obtained by letting \( r = 0.4 \), in which case the \( \text{pn}+/\text{pp}-0 \) ratio is 4.9. This ratio may not be unreasonable in view of the lower bombardment energies of experiment I.

**Single-Pion Production and Elastic Scattering**

It is natural to try to extend the directional-isobar model for multiple-pion production to include single-pion production and elastic scattering. If, after absorption or emission of the single pion, one of the nucleons is in the ground state of \( T = 1/2 \), the reaction would be limited to single- or double-pion production. If both nucleons are in the ground state, elastic scattering would result. By defining a cross section, \( \sigma_0 \), for the mode in which one nucleon remains in the ground state, we may calculate the relative frequencies of the different reactions for elastic scattering, single-pion production, and the contributions of this mode to double-pion production, as well as the forward or backward direction of the particles, just as we did before. Table X shows the relative frequencies of the possible reactions in terms of \( \sigma_0 \), \( \sigma_{1/2} \), \( \sigma_{3/2} \), and \( s \). In the mode in which one nucleon is excited to the \( T = 3/2 \) isobaric state, the charge ratios are the same as those given by Peaslee. However, in the mode in which one nucleon is excited to the \( T = 1/2 \) isobaric state and the other remains in the ground state (with \( T = 1/2 \)), the particles are going in the wrong direction. In the \( \text{pn}+/- \) reaction, all the protons are going forward, and in the \( \text{pp}- \) reaction, all the negative pions and the backward-moving proton decay from a common \( T = 1/2 \) isobar. Inclusion of the \( \sigma_0 \sigma_{1/2} \) terms negates agreement with observed experimental results. If these terms are forbidden, however, several calculated quantities fit the observed data. If we take \( r = \sigma_{1/2}/\sigma_{3/2} = 1 \) as before, and adopt the 3.8-Bev \( p-p \) elastic and inelastic cross sections of 12 mb and 26 mb, respectively, for \( n-p \) collisions, it follows that \( \sigma_0/\sigma_{3/2} \) is 2.0. If \( s = 1 \) as before, and the proportionality constant \( K_1 = 1.415 \) for the \( s = 1, r = 1 \) group, there are 34 pp-events. This agrees with the estimated number of 37 ±12 events. On these assumptions, in half of the events the \( \pi^- \) and forward protons and the \( \pi^- \) and backward protons are decay products of a \( T = 3/2 \) isobar, and there should be equal numbers.
Table X

Contributions to the relative frequencies of particle configurations of Table VII, assuming a hypothetical cross section, $\sigma_0$, for a nucleon to remain in the ground state after absorption or emission of a pion during the collision process.

<table>
<thead>
<tr>
<th>No. of pions</th>
<th>No. of prongs</th>
<th>Particle config.</th>
<th>Both ground state</th>
<th>$1 T = 3/2$</th>
<th>$1 T = 1/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>pn</td>
<td>$45 \sigma_0^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>pp-</td>
<td>$12 \sigma_0 \sigma_{3/2}$</td>
<td>$30(1-s)\sigma_0 \sigma_{1/2}$</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>pn0</td>
<td>48</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>nn+</td>
<td>12</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>pp0</td>
<td>$10 s \sigma_0 \sigma_{1/2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>pn++</td>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>pn00</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>nn+0</td>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
of them. Actually, more pions are emitted in the forward direction. In this experiment 23 negative pions were emitted in the forward hemisphere and 12 negative pions were emitted in the backward hemisphere. As pointed out above, at Bevatron energies the forward or backward direction of the \( \pi^- \) in the c.m.s. should give a fair indication of its isobaric association with the forward or backward proton. In the group of 12 events that had the \( \pi^- \) in the backward hemisphere, 9 of the \( Q(p_b^b \pi^-_b) \) values were in the \( (3/2, 3/2) \) resonance. Also a similar number of the events with forward pion emission had \( Q(p_f^f \pi^-_f) \) values that were consistent with the \( (3/2, 3/2) \) resonance. Therefore, a majority of the \( pp^- \) events may be consistent with the directional-isobar model of pion production.

Of the elastic-scattering events, four out of five should show charge-exchange scattering in which the neutron goes backward in the c.m.s. An increase in charge-exchange scattering is observed at the onset of forward-backward asymmetry in neutron-proton scattering at energies just above 250 Mev; about twice as many neutrons are scattered backwards as forwards. Since in our model, the frequencies of elastic and single-pion production are related by the common factor \( \sigma_0 \), it is reasonable that the energy range in which the onset of greater charge exchange-scattering occurs is also that in which single-pion production commences. This model may be consistent with a portion of the elastic events which show a preferential forward scattering of protons and also with a portion of the \( pp^- \) events which show forward-backward symmetry of the \( \pi^- \).
INTERMEDIATE COUPLING OF NUCLEAR FORCES
IN MESON THEORY

This directional-isobar model of pion production may throw some light on the validity of intermediate- and strong-coupling theories. Two published articles on intermediate-coupling theory initiated the study which led to this isobar model. It was noted that in the first article, by Maki, Soto, and Tomonaga (MST), a small bump at about 250 Mev on a calculated \( \pi^+ p \) cross-section curve corresponded in location to that of the observed secondary peak (following the (3/2, 3/2) resonance) in the \( Q(p^{\pi^+}) \) distribution of the pn\(^+ \) reaction experiment. In the second article, Komatsuzawa, Munakato, and Hasegawa extended the work of MST to include double-pion production in \( \pi^- p \) collisions by a cascade process in which the isobar decays from the second excited level of the nucleon via the first, \( T = 3/2 \), level. The charge states of the first and second excited levels were such as to suggest the charge-exchange effect between the pn\(^+ \) and pn\(^{++} \) reactions, wherein the protons were predominantly backward in the former and consequently forward in the latter.

An accurate assessment of the significance of the secondary peak in the \( Q(p^{\pi^+}) \) distribution is difficult for several reasons. The well-identified and probable events and also the better and poorer events have rather different distributions. This shows on the histograms as a double peaking. In the histograms the double peaking arises primarily from the sharp dip in the interval between 0.15 and 0.20 Bev. If we assume that the \( Q(p^{\pi^+}) \) distribution should be a smooth curve resembling the \( T = 3/2 \) total interaction cross section, then in the total histogram, which includes well-identified and probable events (Fig. 7c), 16 or more events would be expected in the interval between 0.15 and 0.20 Mev. However, only four events are observed in this interval. In the well-identified group (Fig. 7c), about 11 events would be expected in this interval, while only two are observed. The double peaking effect is more pronounced in the better events than in the poorer events with short tracks. This effect primarily results from those events in which both positive particles
were identified, but it was also evident in the group (now shown separately) in which the proton was not identified. (The group in which the $\pi^+$ was not identified had a broad distribution of $Q(p\pi^+)$ values, centered at about 0.8 Bev, with only three values of less than 0.30 Bev, and is therefore not pertinent to this discussion.) The $Q(p\pi^+)$ ideograms, which include the additional information of measurement errors (See Figs. 10a or 12a), also clearly show the secondary peak.

The intermediate-coupling theory based on the original work of Tomonaga enables one to picture the physical meaning of nuclear coupling. A distinction is made between the inner or bound meson configurations and the outer or unbound ones. The various states of the inner configuration correspond to various isobaric states of the nucleon. Takeda points out that weak coupling corresponds to a single pion in the bound states, and that strong coupling corresponds to many pions in the bound states. Intermediate coupling is between these extremes. In this case, probability amplitudes with greater than four mesons are neglected. The outer states have a continuous spectrum. In the pion-nucleon scattering, for instance, the incident pion is in the outer unbound state, and the scattering process is pictured as the absorption and re-emission of the incident pion into and out of the bound configuration.

Another kind of scattering is to be expected since the wave function of the unbound pion is distorted by the effect of the orthogonality condition of the unbound state on the bound states. In their article on the scattering problem in intermediate-coupling theory, MST call this "potential scattering." It is this potential scattering that gives rise to the small bump superimposed on the ordinary resonance scattering curve. In this article, when coupling constant $V^2$ is taken to be 1 and when cutoff energy is taken arbitrarily to be four pion masses, then a sharp maximum is observed to occur at excitation energy 1.95 pion masses, and the potential scattering peak or small bump is observed to occur at about 2.8 pion masses. This curve calculated by MST is shown in Fig. 31. These peaks correspond to $Q$ values of 133 and about 250 Mev, which compare well with the main and secondary peaks observed in the $Q(p\pi^+)$ histogram of Fig. 7c.
Fig. 31. $\pi^+ p$ scattering cross section calculated by the use of intermediate coupling theory with coupling constants $V^2 = 1$ and cutoff energy equal to 4 pion masses. (After Maki, Soto, and Tomonaga. 40)
The shape of the calculated curve of Fig. 31 resembles the histogram of \( Q(p\pi^+) \) plus \( Q(n\pi^-) \) shown in Fig. 9c and also resembles the corresponding ideogram shown in Fig. 12c. The \( \pi^+p \) scattering cross section is calculated by the use of a charged longitudinal meson field and a number of other simplifying assumptions such as a fixed nucleon, only one pion in the unbound configuration, and only one level excited. The isobaric energy levels are computed in this article. The charge states of the first three in order of increasing energy are \((2, -1), (1, 0), \) and \((2, -1)\). The first excited level corresponds to \( p^+ \) and \( n^- \), and the second may be formed by \( p^- \) or \( n^+ \).

Komatsuzawa, Munakato, and Hasegawa extended this work to include double-pion production for the case of \( \pi^-p \) collisions. \( ^{41} \) If the second excited level decays directly to the ground state, ordinary scattering results. If the energy of the incoming pion is large enough, double-pion production occurs when the isobar at the second excited level decays via the first excited level to the ground state by first emitting a \( \pi^+ \) and then a \( \pi^- \). The reaction in this case is \( \pi^-p \rightarrow \pi^+\pi^-n \). In the case of \( \pi^+p \) collisions, only the first excited level need be involved, and the reaction is simply \( \pi^+p \rightarrow \pi^+p \). It follows from these reactions that in \( n-p \) collisions the proton in the \( pn^{+-} \) reaction might be expected to be predominantly forward, once it is known that the proton in the \( pn^{+-} \) reaction is predominantly backward.

Harlow and Jacobson used a version of the Tomonaga method and a symmetric pseudoscalar meson theory with a fixed extended source of the Yukawa shape to calculate the energies of excited states of nucleons. \( ^{44} \) For a coupling stronger than a critical strength which varies with the state and source size, isobars are found for isotopic spin \( 1/2 \) and \( 3/2 \) and angular momentum \( 1/2 \) and \( 3/2 \). The \((3/2, 3/2)\) state always lies lowest in energy. The second excited level is occupied by a degenerate \((3/2, 1/2)\) and \((1/2, 3/2)\) pair. Beyond these, a second \((3/2, 3/2)\) isobar occupies the next-highest level above this degenerate level, over most of the range of coupling strength.

Hasegawa has considered the two-nucleon system, using the Tomonaga intermediate coupling method and a charged scalar meson
field. He finds that there are two contributions of the unbound meson field to the interaction potential of two nucleons: One changes slightly the coefficient of the $e^{\mu r}$ potential; the other introduces the new $e^{-2\mu r}$ potential or potentials of a higher order. Such potentials might throw some light on the meaning of peaks observed in the various $Q$-value distributions.

The secondary peak in the $Q(\pi \pi^+)$ distribution at about 250 Mev and the presence of the $T = 1/2$ isobaric state in the second excited level were considered as possible evidence for the intermediate coupling. But a degenerate second excited level shared by $T = 1/2$ and $T = 3/2$ isobaric states is apparently in conflict with this directional-isobar model. If the second excited level is assumed to be a $T = 3/2$ isobaric state, the predicted forward or backward directions of particles in the c.m.s. would not agree with the experimentally observed results: The protons in both the $\pi n+\pi^+\pi^-$ reaction and the $\pi n+\pi^-$ reaction would be predominantly backward.
STRONG COUPLING

In the old strong-coupling theory, Pauli and Dancoff showed that the symmetric pseudoscalar theory predicts states for the nucleon for which the angular momentum and isotopic spin are equal. The first excited state has $T = J = 3/2$, and the second excited state has $T = J = 5/2$.

In a recent Physical Review letter, Landovitz and Margolis investigated the possibility that strong-coupling theory may predict other low-lying states. In the strong coupling limit, they find a new quantum number $y$, corresponding to an operator $\mathcal{Y}$, where $\mathcal{Y}^2 = (J + T)^2$. (The Pauli-Dancoff states have $y = 0$.) New states are predicted for $y = 1$. A spectrum for these states up to 1 Bev is shown. Only states having $T < 5/2$ are included. The $(3/2, 3/2)$ state having $y = 0$ is still the lowest excited level. Under the assumed value of the coupling constant and the source size and shape, the excitation energy of the $(3/2, 3/2)$ level is 300 Mev. The next-higher level is a $(1/2, 1/2)$ state, with $y = 1$ at 600 Mev, followed by $(3/2, 1/2)$ and $(1/2, 3/2)$ states, with $y = 1$ at 750 Mev, and a second $(3/2, 3/2)$ state with $y = 1$ at 900 Mev. Except for the extra $(1/2, 1/2)$ level above the $(3/2, 3/2)$ state, the order and identity of these excited states predicted by the strong-coupling theory are the same as those predicted by Harlow and Jacobsohn using an intermediate-coupling theory. It was pointed out above that the phenomenological, directional-isobar model proposed in this paper is consistent with a $(1/2, 1/2)$ state above the $(3/2, 3/2)$ level, and is apparently in conflict with a degenerate $(3/2, 1/2)$ and $(1/2, 3/2)$ state above the $(3/2, 3/2)$ level. If intermediate-coupling theory definitely predicts that the second excited level is a degenerate $T = 1/2$, $T = 3/2$ state, then the fact that the directional isobar model is apparently correct in its assumption that $T = 1/2$ is the second excited level might be interpreted as evidence for the validity of strong-coupling theory and against the validity of intermediate-coupling theory.

Both the strong-coupling theory and the intermediate-coupling theory predict that the lowest excited nucleon level has isotopic spin
and angular momentum of $3/2$. Evidence for or against these theories may be expected from the observation of kinematic effects which are not under the dominant influence of the $(3/2, 3/2)$ resonance.
CONCLUSIONS

The proposed phenomenological, directional-isobar model of multiple-pion production is found to be successful in its predictions. In this model, the nucleons are assumed to undergo a grazing type of collision and to continue in approximately the same forward or backward direction they had before collision. During the collision process, each nucleon is excited to the $T = 3/2$ isobaric state or a higher $T = 1/2$ isobaric state by absorbing or emitting a single pion. The $T = 3/2$ isobar decays by emission of a single pion. The $T = 1/2$ isobar may decay via the $T = 3/2$ level, emitting two pions, or may decay directly to the ground state.

The experimental evidence shows that in all of the particle configurations, the two nucleons tend to be emitted in opposite directions in the c.m.s. one going forward and the other going backward. Both this opposite-direction effect and this forward-backward effect are greater at Bevatron energies than at Cosmotron energies. These effects decrease as the number of pions produced is increased.

By the use of $s = 1$, $r = 1$ at Bevatron energies, the directional-isobar model is found to agree with observed multiplicities, with observed forward and backward distributions of protons and neutrons, and with the observed correspondence between the particles of the $pp$-0 and $pp$+- reactions. In the $pp$+- reaction, this model appears to be qualitatively in agreement with the $Q$-value distributions, with angular distributions, and with the angular-correlation distributions. By the use of $s = 0$ at Cosmotron energies, this model is consistent with the observed changes in these distributions between the n-p experiments at Cosmotron energies and at Bevatron energies. At Cosmotron energies, it is also consistent with the $pn$+- / $pp$-0 ratio.

In the $pn$+- reaction, weighted histograms or ideograms of $Q(p\pi^+)$ and $Q(n\pi^-)$, and especially the sum of these distributions, show a good fit to the total $T = 3/2$ interaction cross section, as weighted by the two-body phase-space factor. An even better fit is shown when only the better events are used. On the other hand, $Q(p\pi^-)$ and $Q(n\pi^+)$ weighted histograms do not resemble the total
T = 3/2 interaction cross section. These Q-value distributions, together with the angular distributions and angular-correlation distributions of this experiment and the angular distributions and angular correlation distributions of I and II, give conclusive experimental evidence, in agreement with the directional isobar model, that the (3/2, 3/2) resonance is of primary importance in double-pion production in n-p collisions at both Bevatron and Cosmotron energies. The good fit of the Q(pπ⁺) and the Q(nπ⁻) distributions with the T = 3/2 total interaction cross section, as weighted by the two-body phase-space factor, and the agreement of the observed and calculated density of points in the Q space shows that the isobars decay without large interference effects and gives a degree of quantitative agreement with this directional-isobar model.

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REFERENCES AND NOTES


6. The single seven-prong event is described in reference 16.


13. We adhere to the standard notation of +, 0, and - for \( \pi^+ \), \( \pi^0 \), and \( \pi^- \), respectively.


17. The analysis of the seven-prong event with this IBM program showed that it is with good probability a pp++-- event.
21. Myron L. Good, Lawrence Radiation Laboratory (private communication).
22. Howard S. White; Lawrence Radiation Laboratory (private communication).
29. The present use of the term antisymmetrical was defined in II to mean that the value of the first distribution at angle $\theta_0$ is equal to that of the second at $(180^\circ - \theta_0)$.
31. The assumption of equal parts of $T = 0$ and $T = 1$ is only approximate. Lindenbaum and Sternheimer deduced for experiment II the ratio $(\sigma_T = 0) / (\sigma_T = 1) = 17.3 \text{ mb} / 9.4 \text{ mb} = 1.84$.
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