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Publication Date
2008-09-19
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* Work supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098
The Standing Wave FEL/TBA: Realistic Cavity Geometry and Energy Extraction*

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Abstract
A set of parameters for standing wave free electron laser two beam accelerators (SWFEL/TBA) is evaluated for realistic cavity geometry taking into account beam-break-up and the sensitivity of output power to imperfections. Also given is a power extraction system using cavity coupled wave guides.

I. INTRODUCTION

For the next generation linear colliders, a high gradient acceleration structure is necessary. As a possible source of energy for such colliders, a SWFEL/TBA has been proposed[1,2]. A schematic diagram of a FEL/TBA is shown in Figure 1.

There have been some parameter studies of SWFEL/TBA parameters. Preliminary optimizations were made for the constancy of output energy and phase with respect to de-tuned energy for short pulse bunches[3]. The sensitivity has been reduced further by utilizing drift tubes between the cavities[4].

In this paper, we optimize the parameters with longer pulses (for better efficiency), with longer cavities (for BBU considerations[5]) than previously considered, and with realistic engineering constraints. Induction linacs have leakage current the order of 100 A and need much higher current than this level to be efficient. Switching time for such a high current takes about 10 nsec and thus beam pulse lengths significantly longer than this are needed for a good efficiency. An output energy of about 10 J/m is desired. Reacceleration can also not be too large, since about 30 cm long ferrite material is needed for 0.25 MeV reacceleration. With these constraints transverse and longitudinal beam dynamics are considered and lead to a set of parameters for SWFEL/TBA.

II. BASIC MODEL

Within a cavity the particles and fields of a SWFEL can be examined by the conventional wiggle-averaged FEL equations[6]. Using the subscripts s for signal waves and w for wiggler quantities, and representing the vector potential by the usual normalized quantity

\[
\mathbf{a} = \frac{\mathbf{A}}{mc^2/e},
\]

the equations for particle phase, \( \theta_j \), the \( j \)-th particle energy, \( \gamma_j \), and the field amplitude, \( a_s \), with phase, \( \phi \), are given by the following.

\[
\frac{d\theta_j}{dz} = s \left[ k_s + k_w + \frac{\omega_s}{c} - \frac{\omega_w}{2c\gamma_j^2} \right] + \frac{a_s^2}{2} - 2D_x a_w a_s \cos(\theta_j + \phi),
\]

\[
\frac{d\gamma_j}{dz} = -D_x \frac{\omega_s}{c} a_w a_s \sin(\theta_j + \phi),
\]

\[
\frac{d(a_s e^{i\phi})}{ds} = \frac{i\mathbf{a}_w^2}{a_w} \left[ \frac{2\gamma_j^2}{R} \frac{1}{\gamma_j} \left( e^{-i\phi} \right) \right].
\]

Here, \( \gamma_r \) is the resonant beam energy, \( I \) is the average beam current, \( z \) is the axial coordinate, \( s \) is the distance from the leading bunch, and the jitter term \( D_x = l^2/2 \). The angle brackets are averages over particles of a bunch. The brackets are redundant since in our analysis we make the approximation that only one macro particle per bunch. The shunt impedance is then given by

\[
R = \frac{4\pi}{VL} \left| \int_{L/2}^{+L/2} \frac{e^{-i\phi}}{\gamma_j} \right|^2,
\]

where \( \bar{v} \) is the particle velocity, \( v_z \) is the \( z \) component of the velocity, \( \omega \) is the FEL mode angular frequency, \( V \) is the volume of the cavity, and \( L \) is the length of the cavity.

Once a particle leaves a cavity, the FEL field bounces back to the other end of the cavity as a standing wave. Thus, the field should be updated accordingly.

* The work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U. S. Department of Energy under Contract No. DE-AC03-76SF00098.

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The bunches in a pulse are then considered as blocks of 2N. Since the particles do not interact much with the reflected waves, the first 2N bunches behave identically where \( N = L/c \). The next 2N bunches see the reflected field of the previous 2N bunches. Thus we need to examine only one bunch per every 2N bunches. Utilizing the drift tubes between cavities, and by adjusting the beam energy for each 2N particles, it is possible to make each 2N particles behave identically through each cavity, thus yielding a stable high power energy for a long device as described in Reference [4].

The output energy is then proportional to the square of \( R/Q \) and the total charge of the pulse \( I L_p \).

\[
 W_{out} \propto (I L_p)^2 \left( \frac{R}{Q} \right)^2. \tag{3}
\]

The proportionality constant not shown in the above equation, depends on the geometry of the cavity.

**III. GEOMETRY EFFECT**

The geometry of cavities and irises affect the field equation, the third equation of Eq.(1), through \( R/Q \). Although the quantity \( R/Q \) depends on the beam energy and wiggler fields, the geometry dependence affects the FEL performance through this quantity only. Thus, we can evaluate FEL performance for various geometry by simply evaluating this quantity and using the usual FEL equations.

Now it is easy to understand the geometry effect since the geometry term affects only the field equation, and at the same time the averaged beam current affects only the field equation. Therefore, the geometry effect can be easily compensated by adjusting current, keeping \( I R/Q \) constant. Thus sensitivity of output power to various errors is as in an ideal cavity. The output energy may be different. This, however, is not a problem since it is necessary to have constant output energy but the magnitude of the energy is not so important.

**IV. NUMERICAL RESULTS**

Eq.(1) is advanced by the fourth order Runge-Katta method. Assuming no slippage between particles and fields, the field equations are evolved in \( z \) (replacing \( s \) by \( z \)). The initial field phase was set to zero and the initial particle phase is given as \( \pi/3 \).

A typical evolution of FEL variables, in the absence of errors, are illustrated in Figure 2. In the absence of any errors, the dynamics of each 2N block is the same for the cavities. Thus only the first 3 cavities are shown. The particle lose energy (a) as it traverses a cavity while the field amplitude increases (b). The lost energy of the particle is replenished before entering the next cavity, thus repeating the same motion again. In (c) particle phase, field phase, and the ponderomotive phase are shown.

For stable FEL performance, it is important to keep the ponderomotive phase \( \psi = \theta + \phi \) invariant. In principle we do not have to consider phase change of \( \theta \) and \( \phi \) independently. However, best results are obtained by minimizing the variation of the particle phase within a cavity, by making the nonsinusoidal component of the particle phase equation in Eq.(1) small. For 17.1 GHz, \( k_w = 0.16 \) is good.

A set of parameters is summarized in Table 1, taking into account BBU, beam sensitivity and engineering.
constraints. For the parameters shown in Table 1, sensitivity of output energy and field phase to energy de-tune and local field errors are shown in Figure 3. It shows 6% fluctuation in output energy (a) and 0.7 radian phase fluctuation (b) with 1% energy de-tune. With 0.5% energy de-tune, the fluctuations are half of those 1% de-tune case. Similar results are obtained with half the cavity length but twice as many cavities. Imperfection in cavities are introduced as field errors in each cavity. When the input power was increased to 80 kW/m, the output energy fluctuation remained the same and the phase fluctuation was reduced to half. With 4% random variations of the field errors we obtained 4% energy fluctuation (c) and 0.0025 radian phase fluctuations (d). The reason for the small phase fluctuations is that the particle phase was adjusted with local field errors. With 2% field errors, the fluctuations are reduced by half.

Table 1. Parameters of a SWFEL for 17.1 GHz

<table>
<thead>
<tr>
<th>wiggler</th>
<th>parameter $a_w = 1.41$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B field = 0.9 kG</td>
</tr>
<tr>
<td></td>
<td>wiggler period $\lambda_w = 39$ cm; ($k_w = 0.1598$)</td>
</tr>
<tr>
<td></td>
<td>device length = 100 m</td>
</tr>
<tr>
<td>cavity length</td>
<td>86 cm</td>
</tr>
<tr>
<td>beam</td>
<td>energy $\gamma = 33.1$</td>
</tr>
<tr>
<td></td>
<td>average current $I = 1$ kA</td>
</tr>
<tr>
<td></td>
<td>pulse length = 100 ns</td>
</tr>
<tr>
<td>input power</td>
<td>8 kW/m</td>
</tr>
<tr>
<td>output energy</td>
<td>21.2 J/m</td>
</tr>
</tbody>
</table>

V. RF POWER EXTRACTION

The RF energy is extracted from both sides of the rectangular cavity, to rectangular wave guides coupled by circular holes. Assuming that the operating mode of the SWFEL is TE01p, then the wave guide modes are excited by TE10p mode, and propagate to both ends. Let $a$ be the width, $b$ is the height, $L$ is the length of the cavity, then $b$ is the width of the guide. (The height of the cavity shares the width of the wave guides.)

$$p_{10} = E_0 \sin \left( \frac{\pi y}{b} \right) \sin \left( \frac{p \pi z}{L} \right) \hat{z}; \text{ in cavity,}$$

$$p_{10} = D_0 \sin \left( \frac{\pi y}{b} \right) e^{i k z} \hat{z}; \text{ in waveguides,}$$

where $E_0$ and $D_0$ are amplitude of the modes and $k_z = p \pi / L$. The holes are located at the maximum magnitude of the operating mode. If the coupling hole radius $r$ is small compared to the $z$-wavelength of the cavity mode, each hole behaves like an electric dipole source to the wave guide. Then following Reference 7, we can estimate the output power per port as follows.

$$P_{out} = \frac{1}{2} \left[ \frac{\omega}{2} E_{10} \mp nP \right]^{2};$$

$$P = \frac{2}{3} \varepsilon_0 r^3 E_0 e^{-\alpha t} \hat{z}.$$

The exponent $\alpha$ is the attenuation of the dipole due to the thickness $t$ of the cavity / wave-guide wall, given by $\alpha = (\varepsilon_0 / r^2 - k^2)^{1/2}$, with $k$ being the wave-number of the operating FEL mode in free space. It is assumed that all the $n$-holes are in phase.

Noting that the amplitude of the wave guide mode $D_0 = \sqrt{Z_1} \sqrt{2 / bc}$ with $Z_1 = k_z / (\omega \varepsilon_0)$ we can determine the radius of holes to extract the energy in the cavity within $\tau = 100$ nsec. Since the energy in a cavity is $U = \varepsilon_0 (a b L / 8) E_0^2$, and $P_{out} = U / 4 \tau$ where the factor 4 in the denominator came from 4 ports, we obtain

$$P_{out} = \frac{n^2 k^2 \varepsilon_0}{bc Z_0} \left( 1 - \left( \frac{\pi}{kb} \right)^2 \right)^{1/2} e^{-2 \alpha t} E_0^2 = \frac{\varepsilon_0 a b L E_0^2}{32 \tau}.$$ (6)

The above equation gives a relation between geometry of the cavity and the extraction time. Note that the output power is proportional to $\rho^2$, apart from an $r$ dependence in $\alpha$.

For 17.1 GHz, $a=10$ m, $b=3$ cm, $c=1.5$ cm, $t=0.5$ cm, with $n=60$ we find that the radius of the hole to be 0.47 cm. For 11.4 GHz, if we keep all other parameters fixed, the radius is 0.37 cm.

VI. CONCLUSION

We propose a set of parameters for a SWFEL/TBA in Table 1. The parameters are chosen taking into account engineering constraints, transverse beam displacements, and beam sensitivity. Also presented is a calculation which shows that the cavity energy can be extracted adequately through cavity coupled wave guides.

VI. REFERENCES