Evidence for $B^0 \to \rho^0 \rho^0$ Decays and Implications for the Cabibbo-Kobayashi-Maskawa Angle $\alpha$

We search for the decays $B^0 \rightarrow \rho^0 \rho^0$, $B^0 \rightarrow \rho^0 f_0(980)$, and $B^0 \rightarrow f_0(980) f_0(980)$ in a sample of about $384 \times 10^6 Y(4S) \rightarrow BB$ decays collected with the BABAR detector at the PEP-II asymmetric-energy
$e^+e^-$ collider at Stanford Linear Accelerator Center. We find evidence for $B^0 \rightarrow \rho^0\rho^0$ with 3.5σ significance and measure the branching fraction $B = (1.07 \pm 0.33 \pm 0.19) \times 10^{-6}$ and longitudinal polarization fraction $f_L = 0.87 \pm 0.13 \pm 0.04$, where the first uncertainty is statistical, and the second is systematic. The uncertainty on the Cabibbo-Kobayashi-Maskawa quark-mixing matrix unitarity angle $\alpha$ due to penguin contributions in $B \rightarrow \rho\rho$ decays is 18° at the 1σ level. We also set upper limits on the $B^0 \rightarrow \rho^0f_0(980)$ and $B^0 \rightarrow f_0(980)f_0(980)$ decay rates.

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Measurements of CP-violating asymmetries in the $B^0\bar{B}^0$ system test the flavor structure of the standard model by over-constraining the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [1]. The time-dependent CP asymmetry in the decays of $B^0$ or $\bar{B}^0$ to a CP eigenstate dominated by the tree-level amplitude $b \rightarrow u\bar{u}d$ measures $\sin2\alpha_{CKM}$, where $\alpha_{eff}$ differs from the CKM unitarity triangle angle $\alpha = \arg(-V_{ud}^*V_{ut}\nu_u V_{ut}^*V_{ud})$ by a quantity $\Delta\alpha$ accounting for the contributions from loop (penguin) amplitudes. The value of $\Delta\alpha$ can be extracted from an analysis of the branching fractions of the $B$ decays into the full set of isospin-related channels [2].

Branching fractions and time-dependent CP asymmetries in $B \rightarrow \pi\pi$, $\rho\rho$, and $\rho\rho$ have already provided information on $\alpha$. Since the tree contribution to the $B^0 \rightarrow \rho^0\rho^0$ decay is color suppressed, the decay rate is sensitive to the penguin amplitude. The $B^0 \rightarrow \rho^0\rho^0$ decay has a much smaller branching fraction than $B^0 \rightarrow \rho^0\rho^0$ and $B^+ \rightarrow \rho^+\rho^0$ channels [4–9], and therefore a stringent limit on $\Delta\alpha$ can be set [2,7,10]. This makes the $\rho\rho$ system particularly effective for measuring $\alpha$.

In $B \rightarrow \rho\rho$ decays the final state is a superposition of CP-odd and CP-even states. An isospin-triangle relation [2] holds for each of the three helicity amplitudes, which can be separated through an angular analysis. The helicity angles $\theta_1$ and $\theta_2$ are defined as the angles between the direction of $\pi^+$ and the direction of the $B$ in the rest system of each of the $\rho^0$ candidates. The resulting angular distribution $d^2\Gamma/(d\cos\theta_1d\cos\theta_2)$ is

$$0 \left\{ \frac{1}{4} [1 - f_L] \sin^2\theta_1 \sin^2\theta_2 + f_L \cos^2\theta_1 \cos^2\theta_2 \right\},$$

(1)

where $f_L = |A_0|^2/(|A_1|^2)$ is the longitudinal polarization fraction and $A_{\lambda = -1,0,+1}$ are the helicity amplitudes.

In this Letter we present the first evidence for the $B^0 \rightarrow \rho^0\rho^0$ decay, the measurement of the longitudinal polarization fraction in this decay, and updated constraints on the penguin contribution to the measurement of the unitarity angle $\alpha$.

These results are based on data collected with the BABAR detector [11] at the PEP-II asymmetric-energy $e^+e^-$ collider [12]. A sample of 383.6 ± 4.2 million $B\bar{B}$ pairs was recorded at the Y(4S) resonance with the center-of-mass (c.m.) energy $\sqrt{s} = 10.58$ GeV. Charged-particle momenta and trajectories are measured in a tracking system consisting of a five-layer double-sided silicon vertex tracker and a 40-layer drift chamber, both within a 1.5-T solenoidal magnetic field. Charged-particle identification is provided by measurements of the energy loss in the tracking devices and by a ring-imaging Cherenkov detector.

We select $B \rightarrow M_1M_2 \rightarrow (\pi^+\pi^-)(\pi^+\pi^-)$ candidates, with $M_{1/2}$ standing for $\rho^0$ or $f_0^0$, from neutral combinations of four charged tracks that are consistent with originating from a single vertex near the $e^+e^-$ interaction point. We veto tracks that are positively identified as kaons or electrons. The identification of signal $B$ candidates is based on several kinematic variables. The beam-energy-substituted mass, $m_{ES} = (s/2 + p_\pi \cdot p_B^0/E_B^2 - p_B^0)^{1/2}$, where the initial $e^+e^-$ four-momentum $(E_i, p_i)$ and the $B$ momentum $p_B$ are defined in the laboratory frame, is centered near the $B$ mass with a resolution of 2.6 MeV for signal candidates. The difference $\Delta E = E_B^m - \sqrt{s}/2$ between the reconstructed $B$ energy in the c.m. frame and its known value $\sqrt{s}/2$ has a maximum near zero with a resolution of 20 MeV for signal events. Four other kinematic variables describe two possible $\pi^+\pi^-$ pairs: invariant masses $m_1, m_2$ and helicity angles $\theta_1, \theta_2$.

The selection requirements for signal candidates are the following: $5.245 < m_{ES} < 5.290$ GeV, $|\Delta E| < 85$ MeV, $550 < m_{1/2} < 1050$ MeV, and $|\cos\theta_{1/2}| < 0.98$. The last requirement removes a region corresponding to low-momentum pions with low and more uncertain reconstruction efficiency. In addition, we veto the copious decays $B^0 \rightarrow D^{(*)}\pi^+ \rightarrow (h^+\pi^-\pi^-)\pi^+$, where $h^+$ refers to a pion or kaon, by requiring the invariant mass of the three-particle combination to differ from the $D$-meson mass by more than 13.2 MeV, or 40 MeV if one of the particles is consistent with a kaon hypothesis.

We reject the dominant $e^+e^- \rightarrow q\bar{q}(q = u, d, s, c)$ (continuum) background by requiring $|\cos\theta_T| < 0.8$, where $\theta_T$ is the angle between the $B$-candidate thrust axis and that of the remaining tracks and neutral clusters in the event, calculated in the c.m. frame. We further suppress continuum background using a neural network discriminant $\mathcal{E}$, which combines a number of topological variables calculated in the c.m. frame. Among those are the polar angles of the $B$ momentum vector and the $B$-candidate thrust axis with respect to the beam axis. Other discriminating variables include the two Legendre moments $L_0$ and $L_2$ of the energy flow around the $B$-candidate thrust axis [13] and the sum of the transverse momenta of all particles in the rest of the event, calculated with respect to the $B$ direction.

After application of all selection criteria, $N_{\text{cand}} = 64843$ events are retained. On average, each selected event has...
1.05 signal candidates, while in Monte Carlo [14] samples of longitudinally and transversely polarized $B^0 \to \rho^0 \rho^0$ decays we find 1.15 and 1.03 candidates, respectively. When more than one candidate is present in the same event, the candidate having the best $\chi^2$ consistency with a single four-pion vertex is selected. Simulation shows that 18% of longitudinally and 4% of transversely polarized $B^0 \to \rho^0 \rho^0$ events are misreconstructed with one or more tracks not originating from the $B^0 \to \rho^0 \rho^0$ decay. These are mostly due to combinatorial background from low-momentum tracks from the other $B$ meson in the event.

Further background separation is achieved by the use of multivariate $B$-flavor-tagging algorithms trained to identify primary leptons, kaons, soft pions, and high-momentum charged particles from the other $B$ [15]. The discrimination power arises from the difference between the tagging efficiencies for signal and background in seven tagging categories ($c_{\text{tag}} = 1 \ldots 7$).

We use an unbinned extended maximum likelihood fit to extract the $B^0 \to \rho^0 \rho^0$ event yield and fraction of longitudinal polarization $f_L$. We also fit for the event yields of $B^0 \to \rho^0 f_0$ and $B^0 \to f_0 f_0$ decays, as well as of several background categories. The likelihood function is

$$L = \exp \left(-\sum_k n_k \prod_{i=1}^{N_{\text{var}}} \left( \sum_j n_{ij} P_j(\tilde{x}_i) \right) \right), \tag{2}$$

where $n_j$ is the unconstrained number of events for each event type $j$ ($B^0 \to \rho^0 \rho^0$, $B^0 \to \rho^0 f_0(980)$, $B^0 \to f_0(980) f_0(980)$, three background components from $B$ decays, and continuum), and $P_j(\tilde{x}_i)$ is the probability density function (PDF) of the variables $\tilde{x}_i = \{m_{\text{ES}}, \Delta E, \mathcal{E}, m_1, m_2, \cos \theta_1, \cos \theta_2, c_{\text{tag}}\}$ for the $i$th event.

We use simulated events to parameterize the background contributions from $B$ decays. The charmless modes are grouped into two classes with similar kinematic and topological properties: $B^0 \to a_1^+ \pi^+$ and a combination of other charmless modes, including $B^0 \to \rho^0 K^0$, $B^+ \to \rho^+ \rho^0$, $B \to \rho \pi$, and $B^0 \to \rho^0 \rho^-$. One additional class accounts for the remaining neutral and charged $B$ decays to charm modes. We ignore any other four-pion final states whose contributions are expected to be small in our invariant mass window.

Since the statistical correlations among the variables are found to be small, we take each $P_j$ as the product of the PDFs for the separate variables. Exceptions are the kinematic correlation between the two helicity angles in signal, and mass-helicity correlations in other $B$-decay classes and misreconstructed signal.

We use double-Gaussian functions to parameterize the $m_{\text{ES}}$ and $\Delta E$ PDFs for signal, and a relativistic Breit-Wigner functions for the resonance masses of $\rho^0$ and $f_0(980)$ [16]. The angular distribution at production for $B^0 \to \rho^0 \rho^0$, $B^0 \to \rho^0 f_0$, and $B^0 \to f_0 f_0$ modes [expressed as a function of the longitudinal polarization in Eq. (1) for $B^0 \to \rho^0 \rho^0$] is multiplied by a detector acceptance function $G(\cos \theta_1, \cos \theta_2)$, determined from Monte Carlo simulations. The distributions of misreconstructed signal events are parameterized with empirical shapes in a way similar to that used for $B$ background discussed below. The neural network discriminant $\mathcal{E}$ is described by three asymmetric Gaussian functions with different parameters for signal and background distributions.

The PDFs for nonsignal $B$ decay modes are generally modeled with empirical analytical distributions. Several variables have distributions identical to those for signal, such as $m_{\text{ES}}$ when all four tracks come from the same $B$, or $\pi^+ \pi^- \pi^- \pi^-$ invariant mass $m_{1,2}$ when both tracks come from a $\rho^0$ meson. Also for some of the modes the two $\pi^+ \pi^-$ pairs can have different mass and helicity distributions, e.g., when only one of the two combinations comes from a genuine $\rho^0$ or $f_0$ meson, or when one of the two pairs contains a high-momentum pion (as in $B \to a_1 \pi$). In such cases, we use a four-variable correlated mass-helicity PDF.

The signal and $B$-background PDF parameters are extracted from simulation. The Monte Carlo parameters for $m_{\text{ES}}, \Delta E$, and $\mathcal{E}$ PDFs are adjusted by comparing data and simulation in control channels with similar kinematics and topology, such as $B^0 \to D^- \pi^+$ with $D^- \to K^+ \pi^- \pi^-$. The continuum background PDF parameters are left free

| TABLE I. Summary of results: event yields ($n$); fraction of longitudinal polarization ($f_L$); selection efficiency (Eff) corresponding to measured polarization; branching fraction ($B_{\text{sig}}$); branching fraction upper limit (UL) at 90% CL; and significance including systematic uncertainties. The systematic errors are quoted last. We also show the background event yields for $a_1 \pi$, $q\bar{q}$, charmless, and other $B\bar{B}$ components (statistical uncertainties only). |
|---|---|
| $n(B^0 \to \rho^0 \rho^0)$ | 100 ± 32 ± 17 |
| $f_L$ | 0.87 ± 0.13 ± 0.04 |
| Eff (%) | 24.2 ± 1.0 |
| $B_{\text{sig}} \times 10^{-6}$ | 1.07 ± 0.33 ± 0.19 |
| Significance, stat only ($\alpha$) | 3.7 |
| Significance, syst included ($\alpha$) | 3.5 |
| $n(B^0 \to \rho^0 f_0)$ | 20 ± 21^{+7}_{-10}$ |
| Eff (%) | 26.1 ± 1.0 |
| $B_{\text{sig}} \times B(f_0 \to \pi^+ \pi^-) \times 10^{-6}$ | 0.19 ± 0.21^{+0.07}_{-0.10} |
| UL $\times B(f_0 \to \pi^- \pi^-) \times 10^{-6}$ | 0.53 |
| $n(B^0 \to f_0 f_0)$ | $-3 \pm 9 \pm 5$ |
| Eff (%) | 28.6 ± 1.1 |
| $B_{\text{sig}} \times B(f_0 \to \pi^+ \pi^-) \times 10^{-6}$ | $-0.03 \pm 0.08 \pm 0.04$ |
| UL $\times B(f_0 \to \pi^- \pi^-) \times 10^{-6}$ | 0.16 |
| $n(B^0 \to a_1^+ \pi^+)$ | 81 ± 25 |
| $n(\text{charmless})$ | $-17^{+10}_{-96}$ |
| $n(\bar{B}B)$ | 3198 ± 224 |
| $n(q\bar{q})$ | 61469 ± 311 |
PDF parameters are varied by their respective uncertainties to derive the corresponding systematic errors ($\pm 10\%$, $\pm 4$ events for $\rho^0\rho^0$, $\rho^0 f_0$, and $f_0 f_0$ respectively, and 0.03 for $f_{0\ell}$). We also assign a systematic error of 2 events for $\rho^0\rho^0$, 3 events for $\rho^0 f_0$, and 1 event for $f_0 f_0$ (0.01 for $f_{0\ell}$) to account for a possible fit bias, evaluated with Monte Carlo experiments. The above systematic uncertainties do not scale with event yield and are included in the calculation of the significance of the result.

We estimate the systematic uncertainty due to the interference between the $B^0 \to \rho^0 \rho^0$ and $a_1^\pm \pi^\mp$ decays using simulated samples in which the decay amplitudes for $B^0 \to \rho^0 \rho^0$ are generated according to this measurement and those for $B^0 \to a_1^\pm \pi^\mp$ correspond to a branching fraction of $(33.2 \pm 4.8) \times 10^{-6}$ [17]. Their amplitudes are modeled with a Breit-Wigner function for all $\rho \to \pi\pi$ and $a_1 \to \rho\pi$ combinations and their relative phase is assumed to be constant across the phase space. The strong phases and CP content of the interfering state $a_1^\pm \pi^\mp$ are varied between zero and a maximum value using uniform prior distributions. We take the rms variation of the average signal yield (14 events for the $\rho^0\rho^0$ yield, or 0.03 for $f_{0\ell}$) as a systematic uncertainty.

Uncertainties in the reconstruction efficiency arise from track finding (2%), particle identification (2%), and other selection requirements, such as vertex probability (2%), track multiplicity (1%), and thrust angle (1%).

To constrain the penguin contributions to $B \to \rho\rho$ decays, we perform an isospin analysis, by minimizing a $\chi^2$ term that includes the measured quantities expressed as the lengths of the sides of the isospin triangles. We use the measured branching fractions and fractions of longitudinal polarization of the $B^+ \to \rho^+ \rho^0$ [6] and $B^0 \to \rho^+ \rho^-$ [7] decays, the CP-violating parameters $S_2^{\pm \pm}$ and $C_2^{\pm \pm}$ determined from the time evolution of the longitudinally polarized $B^0 \to \rho^+ \rho^-\$ decay [8], and the branching fraction and polarization of $B^0 \to \rho^0\rho^0$ from this analysis. We assume uncertainties to be Gaussian and neglect $I = 1$ isospin

![Figure 1](color online). Projections of the multidimensional fit onto (a) $m_{ES}$, (b) $\Delta E$, (c) dipion invariant mass ($m_1$ is shown, distribution of $m_2$ is similar), and (d) cosine of the helicity angle ($\cos\theta_1$ is shown), after a requirement on the signal-to-background probability ratio with the plotted variable excluded. This requirement enhances the fraction of signal events in the sample. The data points are overlaid by the solid black line, which corresponds to the full PDF projection. The individual $B^0 \to \rho^0\rho^0$ PDF component is also shown with a solid red line. The sum of all other PDFs (including $B^0 \to \rho^0 f_0$ and $B^0 \to f_0 f_0$ components) is shown as the dashed blue line. The $D$-meson veto causes the acceptance dip seen in (d).

in the fit. Finally, the discrete $B$-flavor tagging PDFs for signal modes are obtained in dedicated fits to events with identified exclusive $B$ decays. The tagging PDFs for inclusive $B$ backgrounds are determined by Monte Carlo simulations and their systematic uncertainties are studied in data.

Table I shows the results of the fit. The $B^0 \to \rho^0\rho^0$ decay is observed with a significance of 3.5$\sigma$, as determined by the quantity $\sqrt{-2\log(L_0/L_{\text{max}})}$, where $L_{\text{max}}$ is the maximum likelihood value, and $L_0$ is the likelihood for a fit with the signal contribution set to zero. It corresponds to a probability of background fluctuation to the observed signal yield of $2 \times 10^{-4}$, including systematic uncertainties, which are assumed to be Gaussian distributed. We do not observe significant event yields for $B^0 \to \rho^0 f_0(980)$ and $B^0 \to f_0(980)f_0(980)$ decays. Background yields are found to be consistent with expectations. In Fig. 1 we show the projections of the fit results onto $m_{ES}$, $\Delta E$, $m_1$, and $\cos\theta_1$ variables.

Dominant systematic uncertainties in the fit originate from statistical errors in the PDF parameterizations, due to the limited number of events in the control samples. The

![Figure 2](color online). $\Delta \chi^2$ as a function of $\Delta \alpha$ obtained from the isospin analysis discussed in the text. The dashed lines at $\Delta \chi^2 = 1$ and $\Delta \chi^2 = 2.7$ are taken for the 1$\sigma$ (68%) and 1.64$\sigma$ (90%) interval estimates.
contributions, electroweak loop amplitudes, nonresonant and isospin-breaking effects.

With the $B^0 \to \rho^0 \rho^0$ measurement we obtain a 68% (90%) CL limit on $|\Delta \alpha| = |\alpha - \alpha_{\text{eff}}| < 18^\circ$ ($<20^\circ$). Figure 2 shows $\Delta \chi^2$ as a function of $\Delta \alpha$. The central value of $\alpha$ obtained from the isospin analysis is the same as $\alpha_{\text{eff}}$, which is constrained by the relation $\sin(2\alpha_{\text{eff}}) = S^+_{L}/(1 - C^+_L - 2)^{1/2}$ and is measured with the $B^0 \to \rho^+ \rho^-$ decay [8].

The error due to the penguin contribution becomes the dominant uncertainty in the measurement of $\alpha$ using $B \to \rho \rho$ decays. However, once the sample of $B^0 \to \rho^0 \rho^0$ decays becomes more significant, time-dependent angular analysis will allow us to measure the CP parameters $S^0_L$ and $C^0_L$, analogous to $S^+_L$ and $C^+_L$, resolving ambiguities inherent to isospin-triangle orientations.

In summary, we find evidence for $B^0 \to \rho^0 \rho^0$ decay with 3.5$\sigma$ significance. We measure the $B^0 \to \rho^0 \rho^0$ branching fraction of $(1.07 \pm 0.33 \pm 0.19) \times 10^{-6}$ and determine the longitudinal polarization fraction for these decays of $f_L = 0.87 \pm 0.13 \pm 0.04$. The measurement of this branching fraction combined with that for $B^+ \to \rho^0 \rho^+$ and $B^0 \to \rho^+ \rho^-$ decays provides a constraint on the penguin uncertainty in the determination of the CKM unitarity angle $\alpha$. These results supersede our previous measurements [4]. We find no significant evidence for the decays $B^0 \to \rho^0 f_0$ and $B^0 \to f_0 f_0$.

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[3] Charge conjugate $B$ decay modes are implied in this Letter.