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A mini-landscape of exact MSSM spectra in heterotic orbifolds

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Abstract

We explore a “fertile patch” of the heterotic landscape based on a $\mathbb{Z}_6$-II orbifold with SO(10) and E$_6$ local GUT structures. We search for models allowing for the exact MSSM spectrum. Our result is that of order 100 out of a total $3 \times 10^4$ inequivalent models satisfy this requirement.

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1. Introduction

Although there are only a few consistent 10D string theories, there is a huge number of 4D string compactifications [1,2]. This leads to the picture that string theory has a vast landscape of vacua [3]. The (supersymmetric) standard model (SM) corresponds to one or more possible vacua which \textit{a priori} might not be better than others. To obtain predictions from string theory one can employ the following strategy: First seek vacua that are consistent with observations and then study their properties. Optimistically, one might hope to identify certain features common to all realistic vacua, which would lead to predictions. Even if this is not the case, one might still be able to assign probabilities to certain features, allowing one to exclude certain patches of the landscape on a statistical basis. However, realistic vacua are very rare. For instance, in the context of orientifolds of Gepner models, the fraction of models with the chiral matter content of the standard model is about $10^{-14}$ [4,5]. The probability of getting something close to the MSSM in the context of intersecting D-branes in an orientifold background is $10^{-9}$ [6,7], even if one allows for chiral exotics.

In this study, we show that certain patches of the heterotic landscape are more “fertile” in the sense that the analogous probabilities are at the percent level.

We base our model scan on the heterotic E$_8 \times$ E$_8$ string [8,9] compactified on an orbifold [10–16]. Our study is motivated by recent work on an orbifold GUT interpretation of heterotic string models [17–19]. We focus on the $\mathbb{Z}_6$-II orbifold, which is described in detail in [17,19,20]. The search strategy is based on the concept of “local GUTs” [20–23] which inherits certain features of standard grand unification [24–26]. Local GUTs are specific to certain points in the compact space, while the 4D gauge symmetry is that of the SM. If matter fields are localized at such points, they form a complete GUT representation. This applies, in particular, to a 16-plet of a local SO(10), which comprises one generation of the SM matter plus a right-handed neutrino [26,27],

$$16 = (\mathbf{3}, \mathbf{2})_{1/6} + (\overline{\mathbf{3}}, \mathbf{1})_{-2/3} + (\bar{\mathbf{3}}, \mathbf{1})_{1/3} + (\mathbf{1}, \mathbf{2})_{-1/2} + (\mathbf{1}, \mathbf{1})_{1} + (\mathbf{1}, \mathbf{1})_{0},$$

where representations with respect to SU(3)$_C \times$ SU(2)$_L$ are shown in parentheses and the subscript denotes hypercharge. On the other hand, bulk fields are partially projected out and form incomplete GUT multiplets. This offers an intuitive explanation for the

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observed multiplet structure of the SM [20–23]. This framework is consistent with MSSM gauge coupling unification as long as the SM gauge group is embedded in a simple local GUT $G_{\text{local}} \supseteq \text{SU}(5)$, which leads to the standard hypercharge normalization.

We find that the above search strategy, as opposed to a random scan, is successful and a considerable fraction of the models with SO(10) and $E_6$ local GUT structures pass our criteria. Out of about $3 \times 10^4$ inequivalent models which involve 2 Wilson lines, $O(100)$ are phenomenologically attractive and can serve as an ultraviolet completion of the MSSM.

2. MSSM search strategy: Local GUTs

It is well known that with a suitable choice of Wilson lines it is not difficult to obtain the SM gauge group up to $U(1)$ factors. The real challenge is to get the correct matter spectrum and the GUT hypercharge normalization. To this end, we base our strategy on the concept of local GUTs. An orbifold model is defined by the orbifold twist, the torus lattice and the gauge embedding of the orbifold action, i.e. the gauge shift $V$ and the Wilson lines $W_n$. We consider only the gauge shifts $V$ which allow for a local SO(10) or $E_6$ structure. That is, $V$ are such that the left-moving momenta $p$ (we use the standard notation, for details see e.g. [18–20]) satisfying

$$p \cdot V = 0 \mod 1, \quad p^2 = 2$$

are roots of SO(10) or $E_6$ (up to extra group factors). Furthermore, the massless states of the first twisted sector $T_1$ are required to contain 16-plets of SO(10) at the fixed points with SO(10) symmetry or 27-plets of $E_6$ at the fixed points with $E_6$ symmetry.

Since these massless states from $T_1$ are automatically invariant under the orbifold action, they all survive in 4D and appear as complete GUT multiplets. In the case of SO(10), that gives one complete SM generation, while in the case of $E_6$ we have $27 = 16 + 10 + 1$ under SO(10). It is thus necessary to decouple all (or part) of the 10-plets from the low energy theory.

The Wilson lines are chosen such that the standard model gauge group is embedded into the local GUT as

$$G_{\text{SM}} \subset \text{SU}(5) \subset \text{SO}(10) \text{ or } E_6,$$

such that the hypercharge is that of standard GUTs and thus consistent with gauge coupling unification. The spectrum has certain features of traditional 4D GUTs, e.g. localized matter fields form complete GUT representations, yet there are important differences. In particular, interactions generally break GUT relations since different local GUTs are supported at different fixed points. Also, gauge coupling unification is due to the fact that the 10D (not 4D) theory is described by a single coupling.

Our model search is carried out in the $\mathbb{Z}_6$-II orbifold compactification of the heterotic $E_8 \times E_8$ string, which is described in detail in [19,20]. In this construction, there are 2 gauge shifts leading to a local SO(10) GUT [28],

$$V^{\text{SO}(10),1} = \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0, 0, 0, 0, 0 \right), \quad \left( \frac{1}{3}, 0, 0, 0, 0, 0, 0, 0, 0 \right),$$

and 2 shifts leading to a local $E_6$ GUT,

$$V^{\text{E}_6,1} = \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0, 0, 0, 0, 0 \right), \quad \left( 0, 0, 0, 0, 0, 0, 0, 0, 0 \right),$$

$$V^{\text{E}_6,2} = \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0, 0, 0, 0, 0 \right), \quad \left( \frac{1}{6}, \frac{1}{6}, 0, 0, 0, 0, 0, 0, 0 \right).$$

We will focus on these shifts and scan over possible Wilson lines to get the SM gauge group. The $\mathbb{Z}_6$-II orbifold allows for up to two Wilson lines of order 2 and for one Wilson line of order 3 (cf. [19,20,29]).

The next question is how to get 3 matter generations. The simplest possibility is to use 3 equivalent fixed points with 16-plets [21] which appear in models with 2 Wilson lines of order 2. If the extra states are vectorlike and can be given large masses, the low energy spectrum will contain 3 matter families. However, this strategy fails since all such models contain chiral exotic states [20]. In the case of $E_6$, it does not work either since one cannot obtain $G_{\text{SM}} \subset \text{SU}(5) \subset \text{SO}(10) \subset E_6$ with 2 Wilson lines of order 2.

The next-to-simplest possibility is to use 2 equivalent fixed points which give rise to 2 matter generations. The third generation would then have to come from other twisted or untwisted sectors. The appearance of the third family can be linked to the SM anomaly cancellation. Indeed, the untwisted sector contains part of a 16-plet. Then the simplest options consistent with the SM anomaly cancellation are that the remaining matter either completes the 16-plet or provides vector-like partners of the untwisted sector. In more complicated cases, additional 16- or 16-cones can appear. The localized 16- and 27-plets are true GUT multiplets, whereas the third or “bulk” generation only has the SM quantum numbers of an additional 16-plet. We find that the above strategy is successful and one often gets net 3 families. The other massless states are often vector-like with respect to the SM gauge group and can be given large masses consistent with string selection rules.

In our MSSM search, we focus on models of this type (although we include all models with 2 Wilson lines in the statistics). These are realized when 1 Wilson line of order 3 and 1 Wilson line of order 2 are present. We require that the spectrum contain 3 matter families plus vector-like states. Furthermore, we discard models in which the SU(5) hypercharge is anomalous. Although a non-anomalous hypercharge could be defined, typically it would not have the GUT normalization and thus would not be consistent with gauge unification.
Let us now present our results for models with the SO(10) local structure. For each of the SO(10) shifts of Eq. (4), we follow the steps:

1. Generate Wilson lines $W_3$ and $W_2$.
2. Identify “inequivalent” models.
3. Select models with $G_{\text{SM}} \subset SU(5) \subset SO(10)$ (or $E_6$).
4. Select models with three net $(3, 2)$.
5. Select models with non-anomalous $U(1)_Y \subset SU(5)$.

Our results are presented in Table 1. The models with the chiral MSSM matter content are listed in [30].

In the analysis of [34] looking at non-supersymmetric heterotic string vacua, about 10% of the models scanned contained the SM gauge group. Our result (step 3) is comparable.
where \( s_a, s_b, \ldots \) are SM singlets. Some singlets are required to get large (close to \( M_{\text{Pl}} \)) VEVs in order to cancel the Fayet–Iliopoulos (FI) term of an anomalous U(1). The supersymmetric field configurations are quite complicated and generally there are vacua in which all or most of the SM singlets get large VEVs. This breaks many of the gauge group factors, such that the low energy gauge group can be \( G_{\text{SM}} \) up to a hidden sector,

\[
G_{\text{SM}} \times G_{\text{hidden}},
\]

where the SM matter is neutral under \( G_{\text{hidden}} \). Furthermore, if the relevant Yukawa couplings are allowed by string selection rules, this makes the vector-like matter heavy; thus it decouples from the low energy theory. We note that there are in general several pairs of Higgs doublets with a matrix of \( \mu \)-like mass terms, for which we require only one small eigenvalue.\(^2\)

Clearly, one cannot switch on the singlet VEVs at will. Instead one has to ensure that they are consistent with supersymmetry. Supersymmetry requires vanishing of the \( F \)- and \( D \)-terms. The number of the \( F \)-term equations equals the number of complex fields \( s_a \), therefore there are in general non-trivial singlet configurations with vanishing \( F \)-potential. The \( D \)-terms can be made zero by complexified gauge transformations [42] if each field enters a gauge invariant monomial [43]. Thus, to ensure that the decoupling of exotics is consistent with supersymmetry, one has to show that all SM singlets appearing in the mass matrices for the exotics enter gauge invariant monomials involving only SM singlets and carrying anomalous charge. In this Letter, we assume that the relevant singlets develop large supersymmetric VEVs.

In the process of decoupling, the vector-like states can mix with the localized \( 16 \)- and \( 27 \)-plets (if it is allowed by the SM quantum numbers) such that the physical states at low energies are neither localized nor “true” GUT multiplets. Nevertheless, it is clear that whatever the mixing, in the end exactly 3 SM families will be left, if the mass matrices have maximal rank.

To show that the decoupling of exotics is consistent with string selection rules is a technically involved and time consuming issue. In order to simplify the task and to reduce the number of models, we first impose an additional condition. We require that the models possess a renormalizable top-Yukawa coupling as motivated by phenomenology. Then we consider only superpotential couplings up to order 8.

First, we require a renormalizable \( O(1) \) Yukawa coupling \( (3, 2)_{1/6}(\bar{3}, 1)_{-2/3}(1, 2)_{1/2} \), i.e. one of the following types

\[
UUU, \quad UTT, \quad TTT,
\]

where \( U \) and \( T \) denote generic untwisted and twisted fields, respectively. The \( UUU \) coupling is given by the gauge coupling, \( UTT \) is a local coupling and thus is unsuppressed, while the \( TTT \) coupling is significant only when the twisted fields are localized at the same fixed point. We discard models in which the above couplings are absent or suppressed.

In the next step \( \circ \), we select models in which the mass matrices for the exotics (cf. Eq. (7)) have a maximal rank such that no exotic states appear at low energies. Here, we consider only superpotential couplings up to order 8 and for this analysis we assume that all relevant singlets can obtain supersymmetric vevs.\(^3\) We find that a significant fraction of our models passes requirements \( \triangleright \) and \( \circ \) (see Table 2 and for further details [30]). In particular, we identify 93 models that can serve as an ultraviolet completion of the MSSM in string theory.

To verify whether an MSSM candidate is consistent with phenomenology requires addressing several questions. The most important issues include

- realistic flavour structures,
- absence of fast proton decay,
- hierarchically small supersymmetry breaking.

\(^2\) To get a pair of massless Higgs doublets usually requires fine-tuning in the VEVs of the SM singlets such that the mass matrix for the \((1, 2)_{-1/2}, (1, 2)_{1/2}\) states gets a zero eigenvalue. This is the notorious supersymmetric \( \mu \)-problem. The fine-tuning can be ameliorated if the vacuum respects certain (approximate) symmetries [40,41].

\(^3\) We also address to some extent the question of \( D \)-flatness. In many of the models, we find that all SM singlets enter gauge invariant monomials. A full analysis of this issue is deferred to a subsequent publication.
The standard SU(5) hypercharge generator is given by
\[ U(UU)_\mu \] interaction reduces to \[ \mu \] depends on the form of the matrix of that, due to the absence of the \[ E_8 \] lattice vector to simplify computations. The Wilson lines are chosen as
\[ W_2 = (\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}), \]
\[ W_3 = (-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}); \]
(10)
(11)
The standard SU(5) hypercharge generator is given by
\[ t_y = (0, 0, 0, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}) \]
(12)
The gauge group after compactification is
\[ G_{SM} \times SO(8) \times SU(2) \times U(1)^7, \]
(13)
while the massless spectrum is given in Table 3.

Renormalizable Yukawa couplings involving the SM fields are shown in Table 4. The top Yukawa coupling comes from the \( UUU \) interaction \( q_i \ell_j \bar{u}_k \), which allows us to identify the right-handed top, the up-type Higgs doublet and the quark doublet of the third generation. (Here we denote the leptons and Higgses collectively by \( \ell_i, \bar{\ell}_i \).) Other renormalizable interactions \( q_i \ell_j \bar{d}_k \) and \( \bar{\ell}_j \ell_k \bar{\ell}_k \) can produce the down-type quark and lepton masses as well as lepton number violating interactions. What happens precisely depends on the form of the matrix of \( \mu \)-like mass terms for the vector-like states and, thus, on the vacuum configuration. We note that, due to the absence of the \( \bar{u}_i \bar{d}_j \bar{d}_k \) operator, the proton is stable at this level.

The model has three generations of SM matter plus vector-like exotics. Once the SM singlets \( s_i \) get VEVs, the gauge group reduces to
\[ G_{SM} \times G_{hidden}, \]
(14)
where \( G_{hidden} = SO(8) \times SU(2) \). At the same time, the vector-like states get large masses. We have checked that the rank of all the mass matrices is maximal, such that the exotics do decouple (assuming all singlets acquire supersymmetric vevs). Below we present most of them. An entry \( s^n \) indicates that the coupling appears first when \( n \) singlets are involved. Each entry usually contains

A model that passes all of our criteria ③–⑧ and comes very close to the supersymmetric standard model has been presented in [20,22]. In our scan, we obtain many comparable models. In what follows, we substantiate this statement by studying a specific example, leaving a complete survey for future work.

Example

The model is based on the gauge shift
\[ V^{SO(10)} = \left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0, 0 \right) \left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right), \]
(10)
many terms and involves different singlets as well as coupling strengths

\[
\mathcal{M}_{d\bar{d}} = \begin{pmatrix}
  s^6 & s^6 & s^3 & s^6 & s^6 & s^1 & s^1 \\
  s^6 & s^6 & s^3 & s^6 & s^6 & s^1 & s^1 \\
  s^3 & 0 & 0 & s^3 & 0 & s^6 & s^6 \\
  s^6 & s^3 & 0 & s^6 & s^3 & s^6 & s^6 
\end{pmatrix}, \quad \mathcal{M}_{\ell\ell} = \begin{pmatrix}
  s^3 & s^1 & s^1 & s^1 & s^1 & s^1 & s^1 \\
  s^1 & s^3 & s^3 & s^3 & s^3 & s^3 & s^3 \\
  s^1 & s^3 & s^3 & s^3 & s^3 & s^3 & s^3 \\
  s^1 & s^3 & s^3 & s^3 & s^3 & s^3 & s^3 \\
  s^1 & s^3 & s^3 & s^3 & s^3 & s^3 & s^3 \\
  s^1 & s^3 & s^3 & s^3 & s^3 & s^3 & s^3 \\
  s^1 & s^3 & s^3 & s^3 & s^3 & s^3 & s^3 
\end{pmatrix},
\]

\[
\mathcal{M}_{m'm'} = \begin{pmatrix}
  s^1 & s^1 \\
  s^1 & s^1 
\end{pmatrix}, \quad \mathcal{M}_{\nu\nu} = \begin{pmatrix}
  s^5 & s^5 & s^5 & s^5 \\
  s^5 & s^5 & s^5 & s^5 \\
  s^5 & s^5 & s^5 & s^5 \\
  s^5 & s^5 & s^5 & s^5 
\end{pmatrix}, \quad \mathcal{M}_{mm} = \begin{pmatrix}
  0 & s^5 & s^6 & s^6 \\
  s^5 & 0 & s^6 & s^6 \\
  s^6 & s^6 & 0 & s^5 \\
  s^6 & s^6 & s^5 & 0 
\end{pmatrix}.
\]

Similarly, the mass matrices for \(s_i^\pm\) and \(\tilde{s}_i^\pm\) have a maximal rank. The \(d\bar{d}\) mass matrix is \(4 \times 7\) such that there are 3 massless \(\tilde{d}\) states. The \(\ell\ell\) mass matrix is \(8 \times 5\), so there are 3 lepton doublets. By choosing a special vacuum configuration one can reduce the rank of the \(\ell\ell\) mass matrix to 4 such that there is a pair of massless Higgs doublets. (This is just the supersymmetric “\(\mu\)-problem”). Thus we end up with the exact MSSM spectrum.

We have checked that the required vacuum configuration is \(D\)-flat. That is, one can assign large VEVs to the singlets without inducing the \(D\)-terms. Since the number of the \(F\)-term equations equals the number of the field variables, there are generally non-trivial solutions to \(F = 0\). Then, using complexified gauge transformations, one can make the \(F\)- and \(D\)-terms vanish simultaneously. Such supersymmetric vacua would correspond to isolated solutions in field space. Although we expect such solutions to exist, their explicit form remains undetermined and will be studied elsewhere.

Finally, the model allows us to define a suitable \(B-L\) generator which leads to the standard charges for the SM matter,

\[
t_{B-L} = \left(1, 1, 0, 0, 0, -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}\right) \left(2x - \frac{1}{2}, \frac{1}{2}, 0, x, x, 0, 0, 0\right),
\]

with arbitrary \(x\). An interesting feature is that the spectrum contains a pair of fields which have \(B-L\) charges \(\pm 2\). If \(B-L\) gauge symmetry is broken by VEVs of these fields, the matter parity (or family reflection symmetry \([44,45]\)) \((-1)^{3(B-L)}\) is conserved and proton decay is suppressed.

5. Conclusion

We have analyzed the heterotic \(E_8 \times E_8\) string compactified on a \(\mathbb{Z}_6\)-II orbifold, allowing for up to two discrete Wilson lines. Employing a search strategy based on the concept of local GUTs, we have obtained about \(3 \times 10^4\) inequivalent models. Almost 1% of these models have the gauge group and the chiral matter content of the MSSM. This result shows that orbifold compactifications of the heterotic string considered here correspond to a particularly fertile region in the landscape and the probability of getting something close to the MSSM is significantly higher than that in other constructions.

The most important outcome of our scan is the construction of \(O(100)\) models consistent with the MSSM gauge group and matter content, amended by a hidden sector. A detailed phenomenological analysis of these models is in progress.

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References