Title
ANTIPAIRING EFFECTS IN THE COLLECTIVE NUCLEAR MOTION AND THEIR RELEVANCE TO THE SADDLE TO SCISSION DESCENT IN FISSION

Permalink
https://escholarship.org/uc/item/5151346t

Author
Moretto, L.G.

Publication Date
1975-03-01
ANTIPAIRING EFFECTS IN THE COLLECTIVE NUCLEAR MOTION AND THEIR RELEVANCE TO THE SADDLE TO SCISSION DESCENT IN FISSION

L. G. Moretto

March 1975

Prepared for the U. S. Energy Research and Development Administration under Contract W-7405-ENG-48

For Reference

Not to be taken from this room
ANTIPAIRING EFFECTS IN THE COLLECTIVE NUCLEAR MOTION AND THEIR RELEVANCE TO THE SADDLE TO SCISSION DESCENT IN FISSION*

L. G. Moretto†

Department of Chemistry and Lawrence Berkeley Laboratory University of California Berkeley, California 94720

ABSTRACT

The dependence of the pairing correlation upon collective velocity is studied in terms of the classical equations of motion. A decrease in pairing with increasing collective velocity similar to the Coriolis anti-pairing effect, is predicted. The consequences for nuclear viscosity and for the saddle-to-scission descent are discussed.

* Work done under the auspices of the U. S. Energy Research and Development Administration.
† Sloan Fellow 1974-76.
The description of collective nuclear motion in terms of the intrinsic degrees of freedom still remains one of the most intriguing open problems in nuclear physics. An essential aspect of this problem is the extent to which the collective and intrinsic modes are coupled, which, in classical terms, is represented by the form and the magnitude of the viscosity tensor. The descent from the saddle to the scission point in the fission of heavy nuclei, as well as the interaction of heavy ions with nuclei, potentially contains information regarding this matter. Liquid drop calculations, performed by Nix, on the saddle-to-scission descent assuming an irrotational non-viscous flow, gives reasonably good agreement with experimental kinetic energies and implies a large amount of pre-scission kinetic energy (~ 30 to 40 MeV). Further liquid-drop calculations containing viscosity seem to indicate that indeed the motion is not very viscous.

Despite these arguments against large viscosity, serious doubts can still be cast on these calculations. The outset of a viscous dynamical evolution depends not only upon the magnitude but also and perhaps essentially upon the form of the viscosity tensor. In fact, one can postulate a viscosity tensor with large components and of a form such that it leads to a scission configuration close to that of two touching spheres. This would generate an apparent paradox: as the viscosity increases, the scission configuration approaches the shape of two touching spheres, and the fission fragment kinetic energy increases.
While there is no evidence for such a form of the viscosity tensor, the example shows that the case of large viscosity can still be argued.

Some recent calculations by Boneh and Fraenkel\textsuperscript{3} based upon the shell model without pairing, indicate that the coupling between collective and intrinsic degrees of freedom in the descent from saddle to scission is very large and that large dissipative forces are at work. The standard criticism to calculations of this kind is the fact that pairing is not accounted for. It is generally believed that pairing, by dramatically increasing the quasi-particle energies, and in particular by making the energy separation between the zero and two quasi-particle states very large, would substantially prevent the creation of quasi-particle excitations.

The calculations performed so far with the inclusion of pairing, determine the value of the gap parameter $\Delta$ in the standard way by requiring the expectation value of the Hamiltonian to be stationary (a minimum) with respect to small variations in $\Delta$. While such a condition is valid for a stationary state, it is not reasonable in the case of dynamical motion where, from a semiclassical standpoint, the Action and not the Hamiltonian is expected to be stationary. Therefore the new gap equation necessary to define $\Delta$ must reflect not the stationary properties of the ground state but the dynamical state of the system, as it is moving along one or more collective coordinates. Here we shall use the Least Action principle instead of the Hamiltonian principle because it does not contain the time explicitly and directly defines a classical trajectory in the collective coordinate space.

For simplicity let us consider a simple collective coordinate $q$. 
The classical Action integral takes the form:

\[ S = \int_{q_1}^{q_2} \left[ 2M(q, \Delta) \left( E - V(q, \Delta) \right) \right]^{\frac{1}{2}} dq = \int_{q_1}^{q_2} I(\Delta, q) dq \]  

(1)

where \( M(q, \Delta) \) is the inertia; \( E \) is the total energy of the system; \( V(q, \Delta) \) is the potential energy and \( q_1, q_2 \) are two points of the trajectory. The gap parameter \( \Delta(q) \) is to be determined in such a way that the integral (1) be minimized.

In order to define the problem, one must specify the form of \( V(q, \Delta) \) and of \( M(q, \Delta) \). The easier quantity to specify is \( V(q, \Delta) \). If one employs the uniform model, one obtains in second order:

\[ V(q, \Delta) = V(q, \Delta_0) + g(\Delta - \Delta_0)^2 \]  

(2)

where \( V(q, \Delta_0) \) is the potential energy for a value of \( \Delta = \Delta_0 \) corresponding to the stationary solution of the gap equation, and \( g \) is the doubly degenerate single particle level density.

Much more difficult is the evaluation of the inertia \( M(q, \Delta) \). The cranking model suggests that the inertia is inversely proportional to the square of \( \Delta \):

\[ M(q, \Delta) = \frac{K(q)}{\Delta(q)^2} \]  

(3)

If this is indeed the case, or, more generally, if \( \frac{\partial M}{\partial \Delta} \) < 0, then a most remarkable conclusion follows. The variational principle applied to (1) gives

\[ \delta S = 0 \]

Since the integrand (1) does not contain \( d\Delta/dq \), the Euler equation,
resulting from the variational principle becomes an algebraic equation:

\[ \frac{\partial I}{\partial \Delta} = 0. \quad (4) \]

Equation (4) is an algebraic equation which requires that the integrand be minimized with respect to \( \Delta \) at each point in the integration interval. The result is

\[ \frac{\Delta}{\Delta_0} = 1 - \frac{E - V_0(q, \Delta_0)}{g\Delta_0^2} \quad (5) \]

where \( \Delta_0 \) is the solution of the usual gap equation, and \( g \) is the doubly degenerate single particle level density. The above equation is only valid for values of \( \Delta \) sufficiently close to \( \Delta_0 \). However, one can appreciate the main qualitative conclusion even from this very crude equation. As one tries to inject kinetic energy into the collective degree of freedom, the system reacts in such a way as to decrease the pairing correlation.* This phenomenon, which is analogous to the Coriolis anti-pairing effect, depends only upon the sign of \( \partial M/\partial \Delta \) and it is a very fundamental property of the classical equations of motion.

If one calls the actual kinetic energy \( E_{\text{kin}} = E - V(\Delta, q) \) and the expected kinetic energy \( E_{\text{kin}}^0 = E - V_0(\Delta_0, q) \) one obtains for the ratio of the two quantities:

\[ \frac{E_{\text{kin}}}{E_{\text{kin}}^0} = 1 - \frac{E_{\text{kin}}^0}{g\Delta_0^2} = \frac{\Delta}{\Delta_0} \quad (6) \]

This ratio, as one can see, decreases continuously with increasing \( E_{\text{kin}}^0 \). The actual kinetic energy \( E_{\text{kin}} \) first increases with \( E_{\text{kin}}^0 \) and then

* Note: It is worth noticing that the opposite effect occurs during the process of barrier penetration where an increase rather than a decrease in \( \Delta \) is predicted.
decreases. The maximum occurs at:

\[
\frac{dE_{\text{kin}}^0}{dE_{\text{kin}}} = 1 - \frac{2E_{\text{kin}}^0}{g \Delta_0^2} = 0
\]  

or at

\[
E_{\text{kin}}^0 = \frac{1}{2} g \Delta_0^2
\]

where \( E_{\text{kin}} \) assumes the value:

\[
E_{\text{kin}} = \frac{1}{2} E_{\text{kin}}^0
\]

and

\[
\Delta = \frac{1}{2} \Delta_0.
\]

For larger values of \( E_{\text{kin}}^0 \) the true kinetic energy decreases. Again, one must be very cautious in using these results for values of \( \Delta \) much different from \( \Delta_0 \), both because of the quadratic expansion (2) and because of the poor knowledge of the dependence of \( M(\Delta, q) \) upon \( \Delta \).

In the above treatment one has assumed that \( \Delta \) acts as a parameter which adjusts adiabatically without having any kinetic energy term associated with it. One may improve the picture by considering \( \Delta \) to be a dynamical variable. In this case, \( M(\Delta, q) \) becomes a 2\times2 matrix and the Action becomes:

\[
S = \int_{q_1}^{q_2} \left[ 2(E - V(\Delta, q)) \left\{ M_{qq} + 2M_{q\Delta} \Delta_q + M_{\Delta\Delta} \Delta_q^2 \right\} \right]^{\frac{1}{2}} dq
\]  

where \( \Delta_q \) is the first derivative of \( \Delta \) with respect to \( q \). The Euler equation now becomes a differential equation. Because, at least in the uniform model,
\[ M_{qq} \gg 2 M_{q\Delta} \Delta_q \quad \text{and} \quad M_{qq} \gg M_{\Delta\Delta} \Delta_q^2 \]}

The previously developed formulas still hold qualitatively. The overall qualitative result of this exercise is the following. A collective mode cannot bear much more kinetic energy than an amount of the order of the condensation energy \( \frac{1}{2} g \Delta_0^2 \), without dramatically decreasing the pairing correlation.

In the descent from saddle to scission, as the system starts increasing its velocity, pairing should quickly decrease to the point where quasiparticle excitations become very likely due to their decreased energy. The creation of quasiparticles should further decrease pairing through "blocking", thus generating a catastrophic breakdown of the pairing correlation.

In conclusion it appears that:

i) A dynamical treatment of the pairing correlation must be used when dealing with nuclear collective motion;

ii) The reaction of the pairing correlation to the collective velocity depends critically upon sign and magnitude of \( \frac{\partial M}{\partial \Delta} \).

iii) A negative \( \frac{\partial M}{\partial \Delta} \) implies a decrease in pairing as the collective velocity increases, possibly leading to a catastrophic breakdown of the pairing correlation;
iv) Any attempt to determine the degree of viscosity in the descent from saddle to scission should be preceded by a careful evaluation of $M_{\Delta q}(\Delta, q)$.

ACKNOWLEDGMENT

It is a pleasure to acknowledge many fruitful discussions with the Drs. Z. Fraenkel, R. Babinet and J. R. Nix.
REFERENCES


LEGAL NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.