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INVESTMENT AND COORDINATION
IN OLIGOPOLISTIC INDUSTRIES

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Key words: Investment, oligopoly, game theory.

Abstract

Investment by firms in 24 chemical product industries is examined to determine whether firms invest preemptively to achieve persistent increases in market share or whether there is evidence of maintain-market-share behavior. The data indicate that investment reduces the probability of capacity expansion by rival firms, but the effect is temporary. Large firms tend to display maintain-market-share behavior, while smaller firms tend to invest simultaneously with rivals. The role of preemptive investment is limited to that of permitting a firm to invest with a lower probability of redundant investment by rivals. Preemption does not allow a persistent increase in market share, but instead acts as a means by which firms may coordinate capacity investment to help avoid episodes of industry over-capacity.

JEL Classification: 611, 616, 631
Investment and Coordination in Oligopolistic Industries

by

Richard Gilbert*
and
Marvin Lieberman**

October 1986

1. Introduction

Competition with irreversible investment poses a problem of coordination, as errors in expectations about rival actions can lead to redundant capacity. Dynamic models of oligopoly identify strategies by which firms that recognize their interdependence can coordinate their actions. This paper examines investment behavior in twenty-four chemical product industries and compares the behavior to the predictions of some common models of oligopoly.

The first model is Cournot-Nash in capital investment. This model, described in Gilbert-Harris [1984], determines equilibria in investment plans when firms can commit to a time path of new investment. The equilibria have the property that firms take turns making new investments and redundant investment is avoided. With identical firms there is an investment round-robin: no firm builds plant \( K+1 \) until all other firms have built plant \( K \).

The Cournot-Nash model has the characteristic that, in equilibrium, firms maintain approximately constant market shares. However, the model is restrictive in its assumptions about investment behavior. If firms have

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similar technologies, their capacities can differ by no more than one plant in a Cournot-Nash equilibrium. A weaker hypothesis is that firms succeed in coordinating their investment behavior by relying on some mechanism that provides for approximately stationary market shares. Examples of the maintain market share model (MMS) include Osborne [1976] and Spence [1978a, b]. Dynamic models in which constant market shares are sustained as an equilibrium consequence of threat strategies are described by Friedman [1978] and in a stochastic context by Brock and Scheinkman [1981] and Green and Porter [1984], which build on work by Stigler [1964]. Stationary market shares also emerge from the "kinked-demand" model (proposed by Sweezy [1939] and recast in game-theoretic form by Anderson [1984]) when firms have approximately similar technologies and demand. ²

Another means by which firms may regulate investment is preemption. Preemption can be identified with strategic investment to exploit a first-mover advantage or with a race to appropriate a profitable investment opportunity. Examples of the former include the entry-deterring investment in Dixit [1980] and the growth-deterring investment in Spence [1977] and Fudenberg-Tirole [1985]. In either case, successful preemption acts to deter or delay investment by rival firms, and in so doing successful preemption mitigates redundant (although not necessarily excess) capacity.

This paper develops a logit model of industry capacity investment to test these various hypotheses of firm investment activity. The probability that a firm expands capacity at each moment of time is taken to be a linear function of observable characteristics of the firm and the industry in which it operates. The hypotheses of maintain market share and preemption are formulated as restrictions of the general model to those explanatory variables that are expected to influence investment under the maintained
hypothesis. The results show that firm investment behavior is sensitive to industry capacity utilization and that new plant investment can be successful in deterring expansions by rival firms. The data also show a tendency for large-share firms to invest in a way that tends to maintain their market shares. The restriction of behavior to a hypothesis that firms follow a strict rule of MMS is rejected in favor of a more general model that allows for preemptive investment. Not surprisingly, the Cournot-Nash model of industry investment, which is a restricted version of the MMS model, is also rejected.

The hypothesis that firms can preempt rival investment independently of the history of market shares also can be rejected in favor of the more general model in which firms adjust to counteract changes in their market shares. The empirical results support the conclusion that preemptive investment can be effective; however, its role is limited to producing some order in the sequencing of investment and does not extend to persistent deterrence of rival investment. This sequencing role of preemptive investment is important to the attainment of efficiency when new plant investment exhibits economies of scale since it reduces the probability of redundant investment with corresponding excess capacity.

The data also reveal an asymmetry in the response of firms to rival investment. The larger firms in an industry tend to invest in opposition to their rivals, while smaller firms tend to follow the investment activity of others. This behavior of "jumping on the bandwagon" by smaller firms can be an equilibrium outcome. Investment activity is a signal about rival firms' expectations of market trends. Equilibria exist where smaller firms may be able to profit from the information that is provided by the
investment activity of larger firms, and larger firms may not be able to profitably follow the investments of smaller firms.

The next section describes the data sample and the variables used in the model of industry investment. Section 3 describes the anticipated results under the maintained hypotheses of Cournot-Nash investment, maintain-market share, and preemption. Section 4 discusses the results of the model estimations and the last section offers concluding remarks.

2. Data Sample and Explanatory Variables

The data sample covers the 24 chemical products listed in Table 1. There are approximately 20 years of coverage for each product. All are homogeneous, undifferentiated chemicals or related products with well-defined production capacities. Output was often consumed captively in firms' downstream operations, but for all products at least 25 percent of industry output was sold through arms-length channels. All products in the sample demonstrated positive net output growth from the earliest year of coverage through at least 1975. Thus, the sample represents products with growing demand, although in a few cases output declined after 1975.

The products in the sample are also characterized by intermediate levels of seller concentration. The sample, which was selected from a larger dataset, includes products for which the number of producers was three or greater, but less than twenty. This screening was performed to limit the sample to industries with oligopoly market structures, and to reduce the computational load.3
### Table 1

**Products Included in Data Sample**

<table>
<thead>
<tr>
<th>Product Name</th>
<th>Coverage Period</th>
<th>Number of Firms</th>
<th>Number of Observations</th>
<th>Number of Expansions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acrylic Fibers</td>
<td>1953-82</td>
<td>3</td>
<td>146</td>
<td>36</td>
</tr>
<tr>
<td>Acrylonitrile</td>
<td>1959-82</td>
<td>4</td>
<td>114</td>
<td>32</td>
</tr>
<tr>
<td>Aluminum</td>
<td>1956-82</td>
<td>4</td>
<td>159</td>
<td>30</td>
</tr>
<tr>
<td>Aniline</td>
<td>1961-82</td>
<td>4</td>
<td>64</td>
<td>10</td>
</tr>
<tr>
<td>Bisphenol A</td>
<td>1959-82</td>
<td>3</td>
<td>89</td>
<td>17</td>
</tr>
<tr>
<td>Caprolactam</td>
<td>1962-82</td>
<td>3</td>
<td>34</td>
<td>10</td>
</tr>
<tr>
<td>Ethylene Glycol</td>
<td>1960-82</td>
<td>9</td>
<td>161</td>
<td>24</td>
</tr>
<tr>
<td>Formaldehyde</td>
<td>1962-82</td>
<td>14</td>
<td>278</td>
<td>59</td>
</tr>
<tr>
<td>Isopropyl Alcohol</td>
<td>1964-82</td>
<td>3</td>
<td>60</td>
<td>12</td>
</tr>
<tr>
<td>Maleic Anhydride</td>
<td>1959-82</td>
<td>3</td>
<td>102</td>
<td>12</td>
</tr>
<tr>
<td>Methanol</td>
<td>1957-82</td>
<td>8</td>
<td>234</td>
<td>33</td>
</tr>
<tr>
<td>Pentaerythritol</td>
<td>1952-82</td>
<td>4</td>
<td>123</td>
<td>12</td>
</tr>
<tr>
<td>Phenol</td>
<td>1959-82</td>
<td>8</td>
<td>221</td>
<td>39</td>
</tr>
<tr>
<td>Phtalic Anhydride</td>
<td>1955-82</td>
<td>8</td>
<td>223</td>
<td>43</td>
</tr>
<tr>
<td>Polyethylene-LD</td>
<td>1958-82</td>
<td>8</td>
<td>204</td>
<td>56</td>
</tr>
<tr>
<td>Polyethylene-HD</td>
<td>1958-82</td>
<td>6</td>
<td>219</td>
<td>76</td>
</tr>
<tr>
<td>Sodium Chlorate</td>
<td>1957-82</td>
<td>3</td>
<td>126</td>
<td>19</td>
</tr>
<tr>
<td>Sodium Hydrosulfite</td>
<td>1964-82</td>
<td>3</td>
<td>41</td>
<td>10</td>
</tr>
<tr>
<td>Sorbitol</td>
<td>1965-82</td>
<td>3</td>
<td>46</td>
<td>7</td>
</tr>
<tr>
<td>Styrene</td>
<td>1958-82</td>
<td>8</td>
<td>169</td>
<td>32</td>
</tr>
<tr>
<td>Titanium Dioxide</td>
<td>1964-82</td>
<td>5</td>
<td>99</td>
<td>25</td>
</tr>
<tr>
<td>1,1,1-Trichloroethane</td>
<td>1966-82</td>
<td>3</td>
<td>40</td>
<td>6</td>
</tr>
<tr>
<td>Vinyl Acetate</td>
<td>1960-82</td>
<td>5</td>
<td>116</td>
<td>28</td>
</tr>
<tr>
<td>Vinyl Chloride</td>
<td>1962-82</td>
<td>9</td>
<td>170</td>
<td>26</td>
</tr>
</tbody>
</table>

Total 3238  654
Computation of Variables

The basic data consist of production capacities for each product by firm and year, and total industry output for each product by year. These data are denoted as follows:

\[ K_{i,j,t} = \text{total capacity of firm } i \text{ to produce product } j \text{ on January 1 of year } t \; ; \]

\[ Q_{j,t} = \text{total industry output of product } j \text{ during year } t \; . \]

The capacity data represent capacity "stocks" observed on January 1st of each year, while the output data represent "flows" over the course of each year. Both capacity and output are measured in physical units (e.g., pounds or gallons). The data were analyzed at the firm level; there is one observation per firm for each product and year in which the firm was a producer. The sample includes a total of 3238 observations on 75 firms.

Dependent Variable

Investment behavior was estimated using a logit model of the probability that a firm builds a new plant. The binary dependent variable was set equal to 1 for all observations where firm \( i \) increased its net production capacity for product \( j \) by more than 5% during the observation year, i.e.,

\[
(1) \quad y_{i,j,t} = 1 \text{ if } \frac{K_{i,j,t+1} - K_{i,j,t}}{K_{i,j,t}} > .05 .
\]

Our choice of a dichotomous measure for investment stems from the fact that, with economies of scale in new capacity, the ratio in (1) takes on extremely large values for small, growing firms. Defining investment as a dichotomous variable avoids this capacity scaling problem. Table 1 shows that the fraction of observations for which \( y_{i,j,t} = 1 \), or the mean probability of expansion, is about 20 percent.

In the chemical industry, firms can expand capacity in three ways:
(1) by constructing a new "greenfield" plant, (2) by adding additional processing units at an existing plant site, or (3) by expanding existing processing units incrementally, e.g., by eliminating bottlenecks in the process flow. Incremental expansions often stem from learning-based improvements achieved at negligible investment cost. Our choice of a 5% threshold on expansion size, although arbitrary, is designed to screen out incremental expansions of this sort.\(^7\)

An expansion is identified with the calendar year in which a capacity addition was completed. Typically, a gestation period of up to two years is required for completion of a new plant, although incremental expansion of an existing plant usually can be implemented more quickly. Completed expansions thus reflect the influence of firm and industry conditions prevailing one to two years prior to the observed expansion date.

Some of the independent variables used in this study are based on first-differences over the two-year plant gestation period. As a result, for any given firm, observations begin at least two years after the firm entered the industry. This excludes the initial capacity investment by new entrants, and hence the dependent variable registers expansions by incumbent firms only.\(^8\)
Independent Variables

Capacity utilization. \( CU_{j,t} \) represents the average rate of capacity utilization over the prior two-year period, i.e.,

\[
(2) \quad CU_{j,t} = \frac{Q_{j,t-1}}{\sum_i (K_{i,j,t} + K_{i,j,t-1})} + \frac{Q_{j,t-2}}{\sum_i (K_{i,j,t-1} + K_{i,j,t-2})}.
\]

Output growth rate. The historical growth rate of output for product \( j \) over the prior four year period was defined as

\[
(3) \quad GROW_{j,t} = \left( \frac{Q_{j,t}}{Q_{j,t-4}} \right)^{\frac{1}{4}} - 1.
\]

Firm's capacity share. Firm \( i \)'s share of total industry capacity to produce product \( j \) at the start of year \( t \) is:

\[
(4) \quad SHARE_{i,j,t} = \frac{K_{i,j,t}}{\sum_i K_{i,j,t}}.
\]

Change in capacity share. The change in capacity share of firm \( i \) from the start of year \( t - 2 \) to the start of observation year \( t \) was defined as:

\[
(5) \quad DELSHR_{i,j,t} = \frac{SHARE_{i,j,t}}{SHARE_{i,j,t-2}}.
\]

A two-year period over which to measure the change in capacity shares was selected because it corresponds to the lag associated with construction of new plants. A similar construction lead time was observed by Mayer (1960).

"Bandwagon" effect. It is not possible with the available data to capture all of the determinants of firms' investment expectations. For example, firms may anticipate an increase in the rate of growth of demand and this will lead to an increase in investment by all (optimistic) firms. A technological breakthrough may open new investment opportunities and lead
to a surge of new capacity. These possibilities are not captured by the explanatory variables in the capacity expansion model and would be recorded as unexplained influences on investment behavior.

Define the "bandwagon" variable:

\[
BAND_{i,j,t} = \frac{\sum_{m=i}^{m=i} (K_{m+1, j, t} - K_{m, j, t})}{\sum_{m=i}^{m=i} K_{m, j, t}}
\]

BAND is the percentage by which all producers other than firm \( i \) collectively increased their capacity during the observation year. A positive logit coefficient for BAND implies that the firm's expansions tended to be correlated with expansions of other firms, after controlling for the influence of historical growth and capacity utilization.

The BAND variable is inversely related to the change in firm \( i \)'s capacity share during the observation year that results from the actions of competing firms. It differs from the DELSHR variable because DELSHR refers to changes in capacity share that result from own as well as rival investment and is measured over the preceding two years. The correlation between BAND and DELSHR is very small (\( r = .03 \)).

A positive BAND coefficient implies that two or more firms expanded together in the same year. To the extent that each firm is knowledgeable of the investment plans of its rivals, a positive BAND coefficient suggests that firms have a tendency to invest simultaneously in lieu of staggering their investments over time; i.e., they tend to "hop on the bandwagon."

Public announcements of investment plans are one source of information about rival firms' investment intentions. The trade literature contains frequent announcements of planned investments. To assess the reliability of these announcements, we analyzed capacity expansion announcements in the trade literature from the start of sample coverage through 1973 for four
products in the sample (maleic anhydride, methanol, pentarythritol and vinyl acetate). A total of 92 expansions were announced. Three-fourths of these were carried through to completion. Of the 69 completions, 11 were delayed by more than one year beyond the completion date originally announced, and 12 involved significantly less capacity than what had originally been announced. This pattern was similar across all four products. These observations from the trade literature suggest that capacity expansion announcements can be taken seriously and that firms are likely to be informed about the current investment decisions of their competitors.

**Interaction terms.** The major asymmetry across firms producing each product relates to their market shares. To detect possible differences in expansion behavior that vary systematically with market share, multiplicative interaction terms were computed between the capacity share of each firm (SHARE) and the level of each of the dependent variables (excluding SHARE itself).

3. **Expected Results Under the Maintained Hypotheses**

   **H.1 Cournot-Nash and Round-Robin Investment**

   The Cournot-Nash model with identical firms predicts an investment round-robin, with no firm building its $K+1$ plant until every other firm has at least $K$ plants.

   The assumptions that firms take rival expansion plans as given and are identical with regard to technology and product characteristics are very
restrictive, and it would not be surprising if the "round-robin" investment behavior predicted by the Cournot-Nash model is rarely observed in our sample. Nonetheless, this hypothesis of firm investment can be tested empirically.

Although the round-robin theory assumes that firms are symmetric, this may be generalized to allow for different capacity increments by different firms. Also, firms that are clearly members of a competitive fringe may be excluded from the round robin. With these modifications, consider an industry with \( I \) competitors. Let \( d_{jt} \) be an indicator variable that takes on a value of 1 if firm \( j \) invests at date \( t \), \(-1/(I-1)\) if a rival firm invests at \( t \), and zero otherwise. Define the variable

\[
D_{jt} = \sum_{\tau=0}^{t} d_{jt}
\]

and let the start date \((\tau=0)\) correspond to a point in time at which all firms have made equal numbers of investments (with possibly unequal market shares). If firms invest in a round-robin, then \( D_{jt} \) will be bounded above by \( +1 \) and below by \(-1 \). (If firms have unequal numbers of investments at the start date, but follow a round robin, the bounds on \( D_{jt} \) will differ. However, the difference between the upper and lower bounds of \( D_{jt} \) still will be at most two.) An examination of the frequency distribution of \( D_{jt} \) thus allows a quantitative assessment of the presence of an investment round robin. Any observations of \( D_{jt} \) separated by more than two units cannot belong to a proper investment round-robin. Of course, even if firms coordinate an investment round-robin, mistakes in actions and/or observations may occur. This can be incorporated by allow-
ing a confidence level for the fraction of observations of \( D_{jt} \) that must lie outside a bound of two before the investment round-robin hypothesis can be rejected.

H.2 Maintain Market Share

In the static MMS equilibrium models of Osborne [1976] and Spence [1978a,b] firms invest lock-step so there is no variation in market shares. In a more realistic competitive environment, market shares will fluctuate with random changes in factors that determine demand and supply even if firms are investing with the objective of maintaining their market shares. Firms following a MMS strategy could be expected to invest if (and only if) they detect reductions in their market shares that they consider substantial relative to historical random variations. This is consistent with Porter's [1983] interpretation of the oligopoly game in Green and Porter [1984] when market share rather than price is the basis for monitoring oligopoly behavior. It is also consistent with Brock and Scheinkman [1981]. Although these models provide for episodes of price wars, the symmetry of firm behavior implies that market shares will remain relatively stable whether firms are in the cooperative or aggressive phases of their equilibrium strategies.

The models in Green and Porter [1984] and Brock and Scheinkman [1981] presume a stationary environment with perfectly divisible production. For the industries in the data sample, market and technological circumstances were subject to rapid change, and new investment exhibited economies of scale. These factors complicate the attainment of equilibrium strategies that support cooperative behavior. Furthermore, indivisibilities in new plant investment make detection of cheating more difficult because market shares must fluctuate over time. Nonetheless, strategies of the type described in the literature on repeated games are not irrelevant to these
industries and we proceed under the hypothesis that an investment-coordinating strategy involving MMS has been attained.

In an MMS equilibrium, the average rate of investment for each firm over time should be proportional to the product of the growth rate and the market share. Capacity utilization rates per se should have no effect on firm investment behavior. Firms should respond negatively to DELSHR, the measure of the change in capacity share. They should respond positively to BAND. If rival firms announce capacity expansion programs, corresponding to a high value for BAND, firm i must respond with a capacity expansion of its own in order to maintain its share of industry capacity.

H.3 Preemption

Preemption acts to deter or delay investment by rival firms. Successful preemptive investment should allow an increase in market share that lasts for some time and should not stimulate investment activity by competing firms. Unlike the maintain-market-share model, a reduction in a firm's market share should not be an inducement to invest when firms engage in effective preemption strategies. The preemption model does not impose a necessary pattern on firm's market shares. Hence the DELSHR variable (and its interaction with SHARE) should not have any explanatory value in the preemption model.

Industry capacity utilization data can be used to test whether a preemption strategy will work. When a firm invests, total capacity goes up and if there is no change in price, capacity utilization goes down. If industry capacity utilization has any effect on investment, it must be
positive if preemption can be effective. Otherwise, investment would only serve to stimulate investment by other firms. Thus a positive correlation between capacity utilization and the probability of investment is a necessary condition for effective preemption.

Similarly, if all firms other than firm $i$ announce a capacity expansion, the information should cause firm $i$ to consider a delay in its own capacity expansion decisions. This implies that the BAND variable should be negatively correlated with investment, although the correlation could be reversed if firms invest simultaneously for reasons that are not captured by other explanatory variables (for example, a process innovation could lower costs and trigger new investment by all firms in the industry).

These relationships between the probability of investment and changes in capacity share and industry capacity utilization should hold whether firms engage in growth-deterring investment as in Spence [1977], Fudenberg-Tirole [1983] or Dixit [1980] or whether they engage in a sequence of investments as in Gilbert-Harris [1984]. In both situations the preempting firm fills an investment niche and its capacity serves to deter investment by rival firms. In the sequencing model of Gilbert-Harris [1984] the deterrence is only temporary. Nonetheless, if such a strategy can be successful for even a short period of time, neither an increase in a firm's market share nor an increase in capacity utilization should stimulate investment by other firms.

4. Results and Discussion

The major results are summarized in Table 2. The dependent variable is binary, equal to one for firm $i$ if the firm expanded by more than 5% in year $t$, and zero otherwise. The first column lists the independent
Table 2  
Logit Analysis of Expansion Probability†
\[ Y_{it} = 1 \text{ if firm } i \text{ expanded by more than } 5\% \text{ in year } t \]

<table>
<thead>
<tr>
<th>X (mean value)</th>
<th>Equation (1)</th>
<th>Equation (2)</th>
<th>MMS (3)</th>
<th>Preemption (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant (1.00)</td>
<td>-0.74** (-11.5)</td>
<td>-0.58** (-5.8)</td>
<td>-0.23** (-10.3)</td>
<td>-0.73** (-12.2)</td>
</tr>
<tr>
<td>SHARE (.137)</td>
<td>0.23** (3.8)</td>
<td>-0.66 (-1.2)</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>CU (.818)</td>
<td>0.58** (8.2)</td>
<td>0.23* (2.1)</td>
<td>--</td>
<td>0.52** (7.2)</td>
</tr>
<tr>
<td>CU x SHARE (.111)</td>
<td>--</td>
<td>2.38** (3.9)</td>
<td>--</td>
<td>0.38** (4.3)</td>
</tr>
<tr>
<td>GROW (.088)</td>
<td>0.28** (3.7)</td>
<td>0.21 (1.8)</td>
<td>--</td>
<td>0.27* (2.5)</td>
</tr>
<tr>
<td>GROW x SHARE (.012)</td>
<td>--</td>
<td>0.76 (1.7)</td>
<td>0.72* (2.4)</td>
<td>0.10 (0.3)</td>
</tr>
<tr>
<td>DELSHR (1.020)</td>
<td>-0.02 (-1.1)</td>
<td>0.09** (2.8)</td>
<td>-0.03 (-1.5)</td>
<td>--</td>
</tr>
<tr>
<td>DELSHR x SHARE (.144)</td>
<td>--</td>
<td>-1.03** (-4.0)</td>
<td>0.08 (1.2)</td>
<td>--</td>
</tr>
<tr>
<td>BAND (.067)</td>
<td>0.10* (2.1)</td>
<td>0.28** (3.4)</td>
<td>0.37** (5.0)</td>
<td>0.24** (3.0)</td>
</tr>
<tr>
<td>BAND x SHARE (.011)</td>
<td>--</td>
<td>-1.02** (-2.6)</td>
<td>-0.72* (-2.2)</td>
<td>-0.78* (-2.3)</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>-1565.4</td>
<td>-1546.0</td>
<td>-1604.5</td>
<td>-1561.6</td>
</tr>
<tr>
<td>No. of obs.</td>
<td>3238</td>
<td>3238</td>
<td>3238</td>
<td>3238</td>
</tr>
</tbody>
</table>

†Numbers in parentheses are asymptotic t-statistics.

**Significant at the .01 level, two-tailed test.

*Significant at the .05 level, two-tailed test.
variables and their values at the sample mean. The next column reports parameter estimates and t-statistics for a linear logit model that includes all of the independent variables but omits interaction terms. The next column reports the parameter estimates and t-statistics for the full model including interactions with the SHARE variable to account for possible effects of market share. The next two columns report parameter estimates and t-statistics for restricted models corresponding to the maintain-market share and preemption hypotheses. In each case the parameter estimates are the partial derivatives of the logit probability of expansion with respect to the independent variable, calculated at the sample mean. For example, in equation (1), a firm in an industry with a growth rate of 9.8% per year (1% above the sample mean of 8.8% per year) had, _cet. par._, an expansion probability that exceeded the probability at the mean by 0.28% per year.

A comparison of models (1) and (2) shows that, across the sample, the larger firms in an industry tend to behave differently from the smaller firms. On average the DELSHR variable has a (weak) negative coefficient, suggesting that there is a slight tendency for firms to invest in a way that compensates for past changes in market share. However, a different picture emerges when interaction effects with market share (SHARE) are included. The coefficient for the DELSHR variable turns positive (and highly significant) and the interaction variable has a negative (and highly significant) coefficient. The interaction terms are negligible for firms with very small market shares. Thus the data show that for small firms, investment activity is positively correlated with recent changes in market share (increases in market share are followed by more investment, decreases in market share are followed by less), while larger firms in an industry tend to invest in opposition to recent changes in their market share.
The model in column (3) is a restriction of model 2 intended to include only the variables that should have explanatory power in a MMS model of industry investment behavior. Excluded are the variables SHARE, CU and CU x SHARE. SHARE and GROW are omitted from this specification because in a MMS equilibrium each firm's investment should be proportional to the product of its market share and the industry growth rate. Hence the variable GROW x SHARE should capture the effects of market growth of industry investment in a MMS equilibrium. Capacity utilization (CU) and the interacted variable CU x SHARE are omitted because capacity utilization should not be a determinant of growth in a MMS equilibrium.

The parameter estimates in model 3 do not confirm the MMS hypothesis. The DELSHR variables are only weakly significant and their signs are reversed compared to model 2. Model 3 is easily rejected in favor of model 2, based on a likelihood ratio test.

The weak support for the MMS strategy suggests that the "round-robin" investment cycle of the Cournot-Nash model (of which MMS is a special case) should also be rejected. To examine this further, the round-robin statistic $D_{jt}$ was constructed for the firms in the sample according to the definition given in section 3. Only 24% of the observations on $D_{jt}$ were within one unit of its mean value, which is inconsistent with the round-robin hypothesis for firms with identical costs for new plants. This result is not unexpected given the strong symmetry implied by the investment round-robin and the variations in the amount and timing of capital investment by the sample firms.

The last column shows the parameter estimates and t-statistics for a restricted model intended to reflect the elements of preemptive investment behavior. The important variables are CU and BAND. Industry capacity
This is consistent with a view that small firms either fail or mature into larger firms and larger firms tend to invest in a way that reflects MMS behavior. The combined effects of the DELSHR and the DELSHR x SHARE variables cancel at a market share of about 9%. For firms with more than about 9% of the market, the probability of investment is negatively correlated with recent changes in market share.

A similar result occurs with the BAND variable. The BAND variable is positive at the 5% confidence level in equation 1, which excludes the share interaction term. BAND remains positive (and significant) when interaction effects are included, but the BAND variable interacted with SHARE is significantly negative. Again, with respect to BAND, large share firms appear to act differently from small share firms.

The coefficient of the capacity utilization variable is positive and significant at the 1% level in model 1, indicating a positive correlation between industry capacity utilization and the probability that a firm expands, cet. par. Model 2 shows that a positive correlation with industry capacity utilization holds for both large and small firms; moreover, the correlation increases for firms with larger market shares. For example, cet. par. if industry capacity utilization rises by 1% above its mean value of 81.8%, the probability of expansion goes up by about 0.47% for a firm with a 10% capacity share and by about 0.94% for a firm with a 30% capacity share.

The GROW variable has a positive and significant correlation with expansion in model 1. Unlike the other explanatory variables, interacting GROW with capacity share in model 2 does not show a statistically significant difference in the behavior of small and large firms with respect to market growth.
utilization (CU) has a highly significant positive effect on investment for firms of all sizes. As discussed above, this result is necessary for a preemptive strategy to be successful. The effect of the BAND variable on investment is positive for small firms and negative for firms with a market share greater than about 30%.

These results are consistent with the hypothesis that firms can successfully preempt at least the larger firms in an industry. Smaller firms have a tendency to invest when other firms announce a capacity expansion decision (the BAND coefficient is positive for smaller firms). Larger firms do not tend to follow the investment decisions of their rivals and their own investments are sensitive to industry capacity utilization. Thus the parameter estimates support the feasibility of using capacity expansions to preempt investment by (at least) larger firms.

The tendency of smaller firms to invest simultaneously with their rivals suggests that smaller firms are relying on rival investment as a signal of market opportunities. However, this begs the question of why smaller firms should follow investment by others, while larger firms tend to invest against the tide. Appendix A provides a derivation of a signalling model in which there exist multiple equilibria corresponding to market expectations which have the property that some firms will tend to emulate the investment activity of their competitors. In particular, if larger firms expect that their smaller rivals will follow their investment activities, then there exist equilibria in which this strategy is individually rational. Equilibria also exist in which larger firms follow investments by smaller firms. The fact that this behavior is not observed may depend on other factors that lead to market expectations in which larger firms are considered barometers for future market conditions.
The preemption model in equation 4 omits the DELSHR terms relating to MMS behavior. These terms appear significant in equation 2, and a likelihood ratio test (at the .01 level) confirms that the preemption model must be rejected in favor of the more general model, equation 2.

The partial correlations in the general model suggest elements of MMS behavior and at least the potential for preemptive investment. The DELSHR coefficient for large-share firms is consistent with MMS, while the coefficients for the capacity utilization and bandwagon variables provide a basis for investment to have a temporary deterring effect on the probability of expansion by rival firms. But in order to assess the predicted impact of an investment on the behavior of rival firms, it is necessary to trace the consequences resulting from the effects of all the explanatory variables. We do this below for the general model given by equation 2.

The coefficients in Table 1 are defined as

\[ \hat{a}_k = \frac{\partial P_{i,\tau}}{\partial x_{i,k,\tau}} \]

where \( P_{i,\tau} \) is the probability that firm \( i \) expands in year \( \tau \) and \( x_{i,k,\tau} \) is the \( k \)th explanatory variable for year \( \tau \). (The values reported in Table 2 are evaluated at the sample means.)

If firm \( j \) expands in year \( \tau \), this is first recorded in the data as an increase in \( j \)'s capacity in year \( \tau+1 \). But the expansion also increases \( j \)'s capacity in years \( \tau+2, \ldots, \tau+n \) where \( n \) is the life of the plant. The effect of \( j \) expanding in year \( \tau \) on firm \( i \)'s probability of expansion in year \( \tau \) is
\[ \frac{dP_{i,t}}{dK_{j,t+1}} = \sum_{\ell=1}^{n} \frac{3P_{i,t}}{3K_{j,t+2}} \frac{dK_{j,t+1}}{dK_{j,t+1}} \]

Since we will be concerned about the effects only over a few years, we can ignore depreciation and take \( \frac{dK_{j,t+1}}{dK_{j,t+1}} = 1 \). Then, applying the chain rule, using (8), and writing the result in logarithmic form gives

\[ \frac{d}{d \ln K_{j,t+1}} \frac{d \ln P_{i,t}}{d \ln K_{j,t+1}} = \frac{1}{P_{i,t}} \left[ \frac{\sum_{\ell=1}^{n} \sum_{k=1}^{K} a_{k} x_{i,k,t}}{a_{1} x_{i,1,t}} \frac{\delta \ln x_{i,k,t}}{\delta \ln K_{j,t+1}} \right] \]

Appendix B lists the derivatives of the explanatory variables with respect to the capacity of firm \( j \). This information permits estimates of the effects of an expansion by firm \( j \) on the probabilities that rival firms will invest.

Figures 1 and 2 illustrate the results of these estimates for a simulation of an industry with five firms. Initially, each firm has the same capacity share. The figures contrast two cases, defined by the actions of the first firm. In the "no preemption" case, firm 1 does not invest. Its share at date \( t \) is 20 percent, the same as the share of each of its rivals. In the "preemption" case firm 1 (and only firm 1) invests at date \( t \), raising its capacity share from 20 to 40 percent, while the share of each rival drops to 15 percent at date \( t \). Figure 1 shows the probability that any of the rival firms invests by the indicated date. Figure 2 shows the probability that three or more rival firms invest by the indicated date.

The probability of rival investment is close to one by \( t+10 \) in all of the simulations. This is expected because the probability is cumulative, and with growing demand and hence increasing capacity utilization, the probability that a rival firm will invest is increasing. 13
Figure 1
Probability of Rival Expansion
Market Shares (20,20 (4))→(40,15 (4))

Figure 2
Probability 3 or 4 Rivals Expand
Market Shares (20,20 (4))→(40,15 (4))
1, the probability that at least one of the first firm's rivals will invest is high at every date. Preemptive investment by firm 1 has only a modest depressing effect on the probability that at least one rival expands (it even increases the probability in the first year, which is consistent with the "bandwagon" effect for small firms). The conclusion to be drawn from figure 1 is that capital investment is not effective in preempting investment by all rivals and thereby allowing a firm to achieve a persistent increase in its capacity share. Investment-deterring preemption of the type described by Dixit [1980], Spence [1977], or Fudenberg-Tirole [1985] does not appear to play a significant role in this industry. Simulations of other industry configurations using the econometric estimates also support this conclusion.

Figure 2 tells a different story. The estimates here are of the probability that three or four of the rival firms will expand by some date. Expansions by only one or two firms (or none) are not counted. The probability that three or more firms expand within a few years from date \( t \) is rather low. But if they do expand, the reduction in industry capacity utilization would have adverse impacts on all of the competitors.

Firm 1's expansion at date \( t \) has a significant impact on the probability that three or more rival firms expand. From the years \( t+2 \) through \( t+6 \), preemptive investment by firm 1 cuts the probability that three or more rivals will expand approximately in half. This does not guarantee the absence of excess capacity, because the market must contend with the capacity that firm 1 contributed at date \( t \). Indeed, expected capacity utilization could be lower in the preemption case. What figure 2 does show is that expansion by firm 1 significantly lowers the conditional probability that (many) other firms will invest. Although excess capacity may result, firm 1 is assured that the scales will tip in its favor. Firm
has extra capacity, and the probability that (too many) other firms will invest is reduced.

Figure 2 reveals that preemptive investment has a coordinating function in that it allows firms to sequence their investments. When firm 1 invests at date t, it significantly reduces the probability that three or more firms will follow. This result would not emerge if firms invested randomly, without regard to the actions of their rivals. Note that this preemptive behavior is effective only for a limited period. Preemptive investment allows the industry to increase the probability of an efficient investment program in which lumpy investments are sequenced over time. While the preemption shown in figure 2 helps avoid inefficient simultaneous expansions, there is no evidence that preemption allows a persistent increase in market share.

The evidence of "short-term preemption" behavior is consistent with the model of sequential preemption in Gilbert and Harris (1984).14 It is also in accord with the interpretation of preemption as enforced turn-taking by Smith (1981). Furthermore, it is not inconsistent with a tendency for firms to maintain their market shares over an extended time period. Indeed, if an industry is to succeed in achieving a non-cooperative equilibrium that sustains supra-competitive earnings, some mechanism to govern the sequencing of capacity investment in the short term is essential.

Other Interaction Effects

The theoretical and empirical analysis above presumes that the major asymmetries in firms' investment behavior are based on market share. Nevertheless, it is possible that other asymmetries are important, and share may simply proxy for other factors. Hence, we tested a number of
additional variables to determine whether share in fact represented the important asymmetry in investment behavior. Each of these variables was added to the full model in equation 2. Virtually all of these additional variables proved insignificant.

The other interaction variables tested included:

(a) Level of producer concentration. It is plausible that the investment asymmetries attributed to market share may actually result from differences in industry concentration. Products for which firms have large shares also tend to have high concentration levels. To differentiate the two hypotheses, the Herfindahl index of producer concentration was computed for each product and observation year. A second set of interaction terms (comparable to the share interactions reported in Table 2) was defined using the Herfindahl index as the interaction variable. These concentration interactions were then included along with the SHARE interactions in equation 2. When this test was performed, none of the Herfindahl interaction terms proved statistically significant at the .05 level. However, all of the SHARE interaction terms remained statistically significant. This confirms the importance of individual firm shares rather than industry concentration levels in influencing ologopolistic investment behavior.

(b) Size of firm. The observed asymmetries could also be related to overall firm size rather than capacity shares. To test this possibility, data on each firm's total book value of assets was obtained from the COMPSTAT files. The logarithm of firm's assets was then interacted with CU, DELSHR, and BAND. These
asset interaction terms failed to prove statistically significant; however, the SHARE interactions remained significant even when the assets interactions were included.

(c) **Major industry group of firm.** Most firms in the sample are either (1) large diversified chemical companies (SIC 280) (2) smaller chemical companies (SIC 282-289), or (3) petroleum companies (SIC 291). Conceivably, these groups might differ in their investment behavior. To test this hypothesis, dummy variables were defined for each of these three groups of firms. The dummies were then interacted with CU, DELSHR, and BAND, and tested in the LOGIT model. The results showed no significant difference in expansion behavior across groups.

(d) **Corporate financial liquidity.** In the absence of perfect capital markets, firms' investment decisions may be constrained by the limited availability of internally generated funds. Less profitable firms may undertake fewer expansion projects. Moreover, all firms may be less inclined to initiate investments during years of low corporate earnings, and relatively more likely to expand during high profit periods. We tested for such liquidity effects using yearly COMPUSTAT data on firms' after tax return on invested capital.

The following liquidity measures were tested as independent variables in the LOGIT equations: (1) actual level of firms' after tax (percent) return during the observation year, (2) mean level of firms' after tax return across all observation years, and (3) deviation of return from this mean level (tested for the observation year and years t - 1 and t - 2, to account for ges-
tation lags in investment). A statistically significant positive relation (.05 level) was found between the deviation from the mean level in year $t - 2$ and expansion in year $t$. Otherwise, none of these liquidity variables was found to significantly influence the probability of expansion. The liquidity variables were also interacted with GU, DELSHR, and BAND, but these interactions failed to prove statistically significant.

5. **Summary and Conclusions**

Both exogenous and market uncertainty contribute to the risk of capital investment. The former stems from the unpredictability of events that determine the size and profitability of markets, while the latter is caused by the actions of rival firms. Market concentration should facilitate behavioral mechanisms to reduce the risks associated with the timing of investment in new plant. Two candidates for market coordination are the maintain-market-share model of oligopoly and the preemption model. In the MMS model, coordination is achieved by firms' commitments to matching the investments of their rivals. In the preemption model, investment by one firm deters simultaneous investment by others.

Given the intricacies of dynamic games, it is presumptuous to suppose that actual market conditions should correspond to a particular view of investment behavior. The data examined in this study reveal elements of both preemptive behavior and a tendency to maintain market shares. The results suggest that firms can employ preemptive investment as a means to sequence new additions to industry capacity. The tendency to maintain market shares implies that such preemptive behavior would not be effective in deterring rival investment over the longer term. Thus the main role of
preemptive activity would be to coordinate new investment and promote efficiency by avoiding excess capacity from redundant investment.

Both the potential for preemption and the tendency for MMS behavior are more pronounced for large-share firms, a result which is consistent with the view that these are behavioral responses to reduce the risks associated with the timing of investment in new plant. The costs associated with these risks are more likely to be borne by large-share firms or firms in more concentrated markets, as small-share firms may more easily profit by deviating from a coordinated investment sequence. Thus these investment data suggest that firm size is associated with investment behavior that can moderate fluctuations in total industry capacity and thereby mitigate an important source of market uncertainty.
Footnotes

1. Richardson (1960) articulates the problems that investment coordination with sunk costs pose for efficient production in competitive as well as oligopolistic industries.

2. Constant market shares would not be expected if firms are price-takers because the lumpiness of capacity additions results in different marginal costs among firms with otherwise similar technologies. Lumpiness also may upset a kinked-demand equilibrium.

3. The data sample also excludes chemicals with production processes involving significant joint products and chemicals for which production capacity can be switched easily from one product to another in response to shifts in market demand. A detailed description of the larger data set from which the sample was selected is available from the authors.

4. The capacity data are from annual issues of the Directory of Chemical Producers, published by SRI International. The output data are from U.S. International Trade Commission and Census Bureau publications. Additional data on firm size and financial liquidity are from Compustat.

5. The 5% threshold is arbitrary, but virtually identical results were obtained with different threshold values.
6. Experimentation using a Tobit model with the ratio in (1) as the dependent variable produced results qualitatively very similar to those reported in Table 2.

7. Of the total number of expansions in the sample, 13% fall below the 5% threshold. The Logit results are almost identical if the threshold is set at zero or at other small values.

8. The behavior of incumbent firms in response to entry is examined in Lieberman [1986, forthcoming].

9. BAND is also only weakly correlated with lagged values of DELSHR.

10. The full model (equation 2) with interaction terms also was estimated with separate constant terms for each product and year in order to identify factors that might be specific to particular industries or time periods. The inclusion of these dummy variables changed only the sign of the non-interacted growth variable, with no loss of significance for the other variables. This suggests that the qualitative conclusions from the statistical analysis do not reflect circumstances unique to particular industries or points in time.

11. If \( \hat{\beta} \) is the vector of maximum likelihood estimates, the partial derivative of the logit probability is

\[
(\exp [X'\hat{\beta}]) (1 + \exp [X'\hat{\beta}])^{-2} \beta_i
\]

12. In what follows, "large" and "small" refer to market share, not absolute size.
13. Growing demand does not rule out the possibility of preemption by investing in anticipation of market growth, but it does complicate the task.

14. See also Fudenberg and Tirole [1985] for a discussion of preemptive behavior.
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Appendix A

To see how the bandwagon behavior might come about, consider two firms with initial capacities \( K_1^0 \) and \( K_2^0 \) \((K_1^0 \geq K_2^0)\). Inverse demand is linear in total output (equal to total capacity, \( K \)) and a random demand parameter, \( \tilde{X}: P(K, \tilde{X}) = a - bK + \tilde{X} \). Firm i's prior information about \( \tilde{X} \) is summarized by a single observation \( \tilde{\theta}_i \), drawn from an independent normal population with unknown mean and known variance \( \sigma^2 \), which we take equal to unity.

Each firm chooses to expand by \( k \) or not at all. For firm i the value of an investment depends not only on \( \tilde{\theta}_i \) (its best estimate of the mean of \( \tilde{X} \)), but also on the probability \( p_j \) that its rival expands. Firm i should invest if \( \tilde{\theta}_i \) exceeds a critical value \( \hat{\theta}_i \). It is easy to see that if the investment decision hinges on the incremental profit from the new plant, the critical value \( \hat{\theta}_i \) depends on \( p_j \) with \( \frac{\partial \hat{\theta}_i(p_j)}{\partial p_j} > 0 \) (firm i requires a better demand signal to invest if competitive investment is more likely). Furthermore, if the incremental costs of investment are such that \((K_1^0, K_2^0)\) were equilibrium values given initial expectations, then \( \hat{\theta}_1(p) = \hat{\theta}_2(p) \). If \( K_1^0 > K_2^0 \) and both firms have the same technology for new investment, then \( \hat{\theta}_1(p) \geq \hat{\theta}_2(p) \) (because firm 1 has higher inframarginal losses to contend with). We will proceed under the assumption that \( \hat{\theta}_1(p) = \hat{\theta}_2(p) \).

Suppose that firm 1 has invested in a new plant. Its competitor has two pieces of information: its own signal \( \tilde{\theta}_j \) and the information that \( \tilde{\theta}_1 \geq \hat{\theta}_1 \). The likelihood of these two independent observations, given that the true (unknown) mean of \( \tilde{X} \) is \( \mu \), is
\begin{equation}
\begin{split}
(A.1) \quad p(\theta_j, \theta_i \geq \hat{\theta}_j | u, \sigma^2 = 1) = \left[ \frac{1}{2\pi} e^{-\frac{1}{2}(\theta_j - u)^2} \right] [1 - \phi(\hat{\theta}_j - u)]
\end{split}
\end{equation}

where \( \phi(\cdot) \) is the cumulative of the standard normal probability density function, which we write as \( \phi(\cdot) \).

The value of \( u \) that maximizes the likelihood of these observations is the solution to

\begin{equation}
(A.2) \quad u_j^* = \theta_j + \frac{\phi(\hat{\theta}_j - u_j^*)}{1 - \phi(\hat{\theta}_j - u_j^*)}
\end{equation}

or

\begin{equation}
(A.3) \quad u_j^* = \theta_j + h(\hat{\theta}_j - u_j^*)
\end{equation}

where \( h(\cdot) \) is the conditional probability density function for the standard normal density. Since \( h(\cdot) \) is monotone increasing and

\begin{equation}
\frac{d u_j^*}{d \theta_j} = \frac{h'(\hat{\theta}_j - u_j^*)}{1 + h'(\hat{\theta}_j - u_j^*)}
\end{equation}

it follows that firm \( j \)'s best estimate of the expected value of \( \bar{X} \) conditional on observation of investment by firm \( i \) is an increasing function of \( \hat{\theta}_j \).

Suppose firm 1 believes that \( p_2 > p_1 \) (firm 2 is more likely to follow the investment of firm 1 than vice-versa), and this belief is common knowledge.

With \( p_2 > p_1 \), firm 1's critical value for investment exceeds firm 2's: \( \hat{\theta}_1(p_2) > \hat{\theta}_2(p_1) \). Since firm i's maximum likelihood estimate \( \hat{\theta}_1 \) of the mean of \( \bar{X} \) is an increasing function of \( \hat{\theta}_j \), it follows that

\begin{equation}
E_{\theta_2} [u_j^* | \theta_2, \text{ firm 1 invests}] > E_{\theta_1} [u_j^* | \hat{\theta}_1, \text{ firm 2 invests}]
\end{equation}

where "firm 1 invests" is equivalent to the information that \( \theta_i \geq \hat{\theta}_i(p_j) \).

We will show that equilibria exist in which these expectations are self-fulfilling. If firm 1 believes that \( p_2 > p_1 \), firm 2 will be more likely to follow firm 1.
We have the following result. If firm 1 thinks that \( p_2 > p_1 \), it will invest only when it receives a very optimistic signal about future demand. When firm 1 invests, it is a signal to firm 2 that future demand will be high. The same information transmission occurs when firm 2 invests. But because \( p_2 > p_1 \), the critical value for firm 2 is lower and therefore its investment does not require as much optimism as an investment by firm 1.

These expectations are self-fulfilling. If firm 1 thinks \( p_2 > p_1 \), firm 2 should take advantage of this belief and it will be more likely that firm 2 will follow firm 1 than the opposite. Of course the opposite would occur if initial beliefs were such that \( p_1 > p_2 \); then firm 1 would be more likely to follow firm 2.

The bandwagon effect can vary systematically with market share if there were a mechanism to establish initial beliefs that depend on market shares. There are several ways this might happen. The data suggest that larger firms tend to make new investments that are larger than the new investments of smaller firms. This would raise the critical value \( \hat{\theta}_i(p_j) \) for larger firms and cause an asymmetry in beliefs. Also, if the size differences of firms are the result of historical events and not justified by the incremental costs of new investments, then larger firms would have higher inframarginal losses from new investment. This would imply a higher critical value for the demand signal and again result in asymmetric investment probabilities. Furthermore, larger firms can exploit scale economies in the acquisition and use of information, and therefore small firms may correctly view the actions of large firms as the result of superior information (see, e.g., Eckard [1982]).
Appendix B

Table B.1 lists the derivatives

\[ \sum_{i=1}^{n} \frac{3 \ln x_{i,k,t}}{3 \ln K_{j,t+1}} . \]

normalized to the capacity share of firm \( j \) in year \( t+1 \). The factor \( g \) is the rate of growth of industry capacity, which is assumed equal to the mean growth rate of output. The derivatives for years \( t+s, s \geq 3 \), are the same as for year \( t+3 \) except that the factor \( (1+g) \) in the BAND and BAND \( \times \) SHARE terms is raised to the power \( s-1 \).
Table B.1

\[
[\text{SHARE}_{j,t+1}]^{-1} \sum_{i=1}^{n} \frac{\partial \ln x_{i,k,t}}{\partial \ln \text{SHARE}_{j,t+1}}
\]

<table>
<thead>
<tr>
<th>YEAR: ( t = )</th>
<th>( t )</th>
<th>( t+1 )</th>
<th>( t+2 )</th>
<th>( t+3 )</th>
</tr>
</thead>
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<tr>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SHARE</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>CU</td>
<td>0</td>
<td>(-\frac{1}{2} \left( \frac{1+g}{2+g} \right))</td>
<td>(-\frac{1}{2} \left( \frac{2+g}{2+g} \right))</td>
<td>(-\frac{1}{1+g} )</td>
</tr>
<tr>
<td>CU \times SHARE</td>
<td>0</td>
<td>(-\frac{1}{2} \left( \frac{5+3g}{2+g} \right))</td>
<td>(-\frac{1}{2} \left( \frac{7+3g}{2+g} \right))</td>
<td>(-\frac{2+g}{1+g} )</td>
</tr>
<tr>
<td>GROW</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>GROW \times SHARE</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>DELSHR</td>
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<td>0</td>
</tr>
<tr>
<td>DELSHR \times SHARE</td>
<td>0</td>
<td>-2</td>
<td>-2</td>
<td>-1</td>
</tr>
<tr>
<td>BAND</td>
<td>\frac{1+g}{g}</td>
<td>A_{t+1}</td>
<td>A_{t+2}</td>
<td>A_{t+3}</td>
</tr>
<tr>
<td>BAND \times SHARE</td>
<td>\frac{1+g}{g}</td>
<td>B_{t+1}</td>
<td>B_{t+2}</td>
<td>B_{t+3}</td>
</tr>
</tbody>
</table>

where

\[
A_{t+2} = \left[ (1+g)^{t-1} - \text{SHARE}_{i,t+1} \right]^{-1}
\]

\[
B_{t+2} = \left[ \frac{1 + (1+g)^{t-1} - \text{SHARE}_{i,t+1}}{(1+g)^{t-1} - \text{SHARE}_{i,t+1}} \right]^{-1}
\]