To See or Not to See: The Visual Component of Complex Mental Arithmetic

Wendy Ann Deslauriers (wadeslau@connect.carleton.ca)
Centre for Applied Cognitive Research, Institute of Cognitive Science, Carleton University, Ottawa, ON K1S 5B6 Canada

Gene P. Ouellette (gouellette@mta.ca)
Psychology Department, Mount Allison University, Sackville, NB E4L 1C7 Canada

Martin Barnes (mbarnes@connect.carleton.ca)
Institute of Cognitive Science, Carleton University, Ottawa, ON K1S 5B6 Canada

Jo-Anne LeFevre (jlefevre@connect.carleton.ca)
Centre for Applied Cognitive Research, Institute of Cognitive Science, Department of Psychology, Carleton University, Ottawa, ON K1S 5B6 Canada

Abstract

Does visual processing influence mental arithmetic? The relationship between high and low visual similarity and horizontal versus vertical presentation was examined in three-digit addition problems with two carries (ex. 518 + 294). Adults (20 female, 20 male) solved vertically presented problems faster than horizontally presented problems and solved problems with low visual similarity faster than those with high visual similarity. Similar main effects were found for accuracy, but these were qualified by a significant interaction between presentation format and visual similarity, as the advantageous effects of vertical presentation were negated by high visual similarity. These results suggest that, in contrast to the findings of Noel et al. (2001), there is a role for visual processes in mental addition.

Keywords: mathematics, numerical cognition, arithmetic, visual processing, working memory, presentation format, education

Introduction

Psychologists have assumed for some time that complex mental arithmetic relies upon an individual’s working memory. Intuitively, mental arithmetic appears to require working memory to encode the problem, hold interim results and carries, and for computational processes and response retrieval (DeStefano & LeFevre, 2004; Hitch, 1978, 2000; Heathcote, 1994; Logie, Gilhooly, & Wynn, 1994; Noël, Desert, Aubrun, & Seron, 2001; Seitz & Schumann-Hengsteler, 2000, 2002; Trbovich & LeFevre, 2002). Despite this intuitive appeal, and the importance of arithmetic in daily life, only recently has empirical research begun to focus on the role of working memory in complex mental arithmetic. The present study adds to this relatively new body of literature, in examining the specific role of a visual working memory component in multi-digit addition.

To date, research on working memory in mental arithmetic has referenced Baddeley’s (1986) multi-component model; in particular, research has focused on delineating the roles of the phonological loop, visual-spatial sketchpad, and central executive. For consistency, the present study will also be discussed with respect to these components of Baddeley’s model. Adopting this framework is not meant to dismiss the relevance of alternate accounts of working memory. For a comprehensive discussion and comparison of working memory models, the reader is directed to Miyake and Shah (2004). We begin with a brief overview of research that has explored the role of Baddeley’s specific working memory components in complex mental arithmetic.

The majority of research into working memory involvement in mental arithmetic has relied upon dual-task interference paradigms. For instance, Logie et al. (1994) investigated the effects of various concurrent tasks on adult’s addition of two-digit numbers. The most pronounced effect was found for a concurrent task that required the generation of random letters, a task commonly assumed to tap into the central executive (LeFevre, DeStefano, Coleman, & Shanahan, 2005). Lesser, yet significant, interference was also evident when the original addition equation was presented visually for concurrent tasks of articulatory suppression and presentation of irrelevant pictures. These results suggest involvement of both the phonological loop (as reflected by influence of articulatory suppression) and the visual-spatial sketchpad (as reflected by influence of irrelevant pictures), when addition problems are presented visually.

In a series of studies, Heathcote (1994) evaluated the roles of both phonological and visual components of working memory in mental arithmetic. Importantly, by manipulating the presence of carries in 3-digit addition problems, Heathcote found that interference from presentation of irrelevant visual stimuli was restricted to equations involving carries. Conversely, articulatory suppression interfered with performance regardless of the presence of carries. Interestingly, Heathcote also manipulated the visual similarity of the digits involved in the addition problems and found no clear effects on response time. Heathcote thus concluded that the visual-spatial sketchpad is important for the retention of carries only, whereas the phonological loop is needed for subvocal rehearsal.

Together, the experiments of Logie et al. (1994) and Heathcote (1994) seem to establish a major role of the
central executive and phonological loop in mental arithmetic (see also Imbo, Vandierendonck, & Vergauwe, 2007), and a minor involvement of the visual-spatial sketchpad. It must be noted, however, that these studies have been criticized on methodological grounds (see Noël et al., 2001), including small sample sizes. Further, other research has challenged the conclusion that mental arithmetic relies upon a visual component of working memory at all. Specifically, Noël et al. (2001) followed the procedure of Heathcote (1994), and investigated the effects of manipulating the visual similarity between digits used in complex addition equations. In contrast to Heathcote, Noël et al. did not find an effect of visual similarity on either the response time or accuracy of adding two 3-digit numbers, regardless of the presence or number of carries. Conversely, the phonological similarity of the digits affected both speed and accuracy. Noël et al. (2001) thus concluded that the visual-spatial sketchpad of working memory is not involved in mental addition.

In interpreting the results of Noël et al. (2001), attention must be directed towards presentation format. That is, Noël et al. presented all equations one addend at a time, and thus the entire equation was never present simultaneously. In addition, the presentation duration was only 1500 ms per operand. This presentation format may force participants into relying more on a phonological rehearsal strategy (DeStefano & LeFevre, 2004; Hitch, 2000) and less on visual memory, than may be the case in typical arithmetic solving where the entire equation is visible.

In summary, previous research has suggested major roles of the central executive and phonological loop in the performance of mental calculations. The role of the visual-spatial sketchpad, however, is not clear. The present study thus attempts to elucidate the role of a visual component of working memory in mental arithmetic. In particular, the present study incorporates the paradigms used by Heathcote (1994) and Noël et al. (2001), in contrasting complex addition equations with high versus low visual similarity, but with an additional condition of presentation format: half of the equations were presented in horizontal format and half in vertical format with one addend under the other (see Table 1). As such, the present study provides a more thorough evaluation of the involvement of a visual working memory component in mental arithmetic, with special consideration given to presentation format.

Previous research on mathematical computations has indeed suggested an important influence of presentation format and related involvement of visual processes. Trbovich and LeFevre (2002) for instance, showed that performance on simple addition problems was both faster and more accurate when the equations were presented vertically as opposed to horizontally (see also Heathcote, 1994). Trbovich and LeFevre further argued that horizontal presentation of mathematical equations increases the phonological demands of the task, as a secondary task with a high phonological load interfered with performance on horizontal equations more so than it did with vertically presented equations. Conversely, a visual-spatially loaded task negatively influenced performance on vertically presented equations more so than for equations presented horizontally. The visual load associated with visually similar equations may therefore be presumed to interfere only with the processing of vertically presented equations.

Hypotheses

Based on previous research, it was hypothesized that participants would solve problems presented vertically faster and more accurately than problems presented horizontally. It was also reasoned that, due to a visual memory component of mental arithmetic, participants would solve problems with low visual similarity faster and more accurately than those with high visual similarity. Further, it was hypothesized that vertical presentation would maximize visual processing demands, thus resulting in increased involvement of visual working memory and an interaction between visual similarity and presentation format. That is, the benefits of a vertical format would be negated by additional visual processing demands associated with a higher visual similarity between the digits.

Method

Participants

A total of 40 participants (20 female, 20 male) voluntarily took part in this study. Participants were undergraduate students enrolled in Psychology courses at a large university in a Canadian urban centre. Participants ranged in age from 18 to 34 years, with a median of 21.5 years.

Tests and Measures

Participants solved 20 experimental trials involving three-digit by three-digit addition problems with two carries, that were presented on a laptop computer. Problems were presented either horizontally or vertically and were also subdivided according to high or low visual similarity, calculated according to a 7-point scale developed by Campbell and Clark (1988). Participants also solved multi-digit addition problems from one subtest of the Kit of Factor-Reference Cognitive Tests (French, Ekstrom, & Price, 1963) as a measure of addition fluency. The Visual Patterns Test (Della Sala, Gray, Baddeley, & Wilson, 1997) was also administered to all participants as a measure of visual working memory.

Addition Equations

The equations used by Noël et al. (2001) to examine the influence of visual similarity on mental arithmetic were obtained. In order to simplify the investigation, only problems with two carries were used for this study.

Equations were classified as high or low in visual similarity by Noël et al. (2001), according to a 7-point scale (1 = low visual similarity; 7 = high visual similarity) developed by Campbell and Clark (1988). For example, 1 and 8 have a visual similarity index of 1.00, whereas 3 and 8
have a visual similarity index of 6.00. Identical digits were
assumed to be very highly visually similar and assigned an
index of 7.00. Although Campbell and Clark found that the
visual similarities between digits varied according to the
order of presentation, these differences were assumed to not
affect this stimuli, which included six digits presented
simultaneously, and the similarity matrix was, therefore,
adjusted to use the means between symmetrical digit
pairings. For example, 2 to 9 has a visual similarity index of
2.88, whereas 9 to 2 has a visual similarity index of 3.25;
for consistency then, both 2 to 9 and 9 to 2 were given the
mean value of 3.065. The visual similarity index for each
arithmetic problem was determined by calculating the mean
of the visual similarities between every pair of digits in the
problem. For example, the visual similarity index of 13 + 28
is 2.66, the mean of the similarity index of 1 to 3, 1 to 2, 1
to 8, 3 to 2, 3 to 8, and 2 to 8. Calculating the visual
similarity indices in this manner was also consistent with
Noël et al. (2001). Table 1 illustrates examples of three-digit
problems with both low and high visual similarity indexes.

<table>
<thead>
<tr>
<th>Visual Similarity</th>
<th>Presentation Format</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Horizontal</td>
</tr>
<tr>
<td>High</td>
<td>439 + 386</td>
</tr>
<tr>
<td>Low</td>
<td>518 + 294</td>
</tr>
</tbody>
</table>

Because Noël et al. (2001) found that phonological
similarity influenced accuracy rates and response times, it
was important that phonological similarity be controlled in
the present study. Given that Noël et al. used French-
speaking participants, the original phonological balance was
assumed to have changed. Phonological similarity was thus
calculated, in English, using a method originally developed
by Lhermitte & Desrouesné (1974). The phonological-
similarity index was obtained by counting the number of
identical phonemes within each pair of numbers and then
dividing by the total number of phonemes. The higher the
phonological-similarity index, the more acoustically similar
the pair of numbers is assumed to be.

Within each visual similarity classification (high vs. low)
the problems were randomly assigned to one of two sets,
balanced for phonological similarity. Thus, the problems
were divided into four conditions: horizontal presentation and
high visual similarity, horizontal presentation and low visual
similarity, vertical presentation and high visual
similarity, vertical presentation and low visual similarity.
Each condition contained five equations. In order to obtain
phonological and visual similarity balance within each
condition, four of Noël et al’s (2001) original stimuli were
not used in this study. Across participants, assignment of
sets of problems was counterbalanced with presentation
format. To ensure that the high and low visual similarity
equations did not differ in terms of phonological similarity
in English, an ANOVA was run contrasting all four
experimental conditions with respect to phonological
similarity. No significant effects were found $F(3,15) < 1,
p = .982$, thus confirming that phonological similarity did
not differ across conditions.

The 20 trials for each of the two protocols were then
arranged in pseudo-random order according to two
constraints. Experimental trials were preceded by two
practice trials. Consecutive problems did not end with the
same number and there were never more than two
consecutive problems from within the same group. To
control for possible practice effects, two additional testing
protocols were created by reversing the order of
presentation for both of the existing testing sequences. Thus
in all, four different testing protocols were used, carefully
counterbalancing presentation format and controlling for
practice effects.

**Addition Fluency** The addition subtest of the Kit of Factor-
Reference Cognitive Tests (French et al., 1963) was
administered to determine participants’ addition fluency.
Sixty multi-digit addition problems with three addends were
presented on each page. Before beginning the test,
participants were given ten practice problems and
encouraged to use them to practice for speed.

**Visual Span** The Visual Patterns Test (Della Sala et al.,
1997) is a standardized test that measures visual working
memory with patterns that cannot be verbally nor spatially
encoded. The patterns are checkerboard grids on which half
of the squares are black and half are white. The stimuli
include three pattern trials for each grid size and the grids
progress in size from a 2 x 2 matrix (with two black
squares) to a 5 x 6 matrix (with 15 black squares) by adding
two more squares at each level.

**Procedure** The addition fluency measure was administered first.
Participants were then shown the 3-digit by 3-digit
arithmetic problems, presented on a computer screen and
asked to compute a solution and respond orally. Following
completion of the 20 trials, the visual pattern test was
administered.

**Addition Equations** Stimuli were presented on a laptop
computer with a liquid crystal display. All stimuli were
presented in white in the centre of a black screen and were
preceded by a fixation point to indicate the coming trial.
Both addends were presented simultaneously and remained
on the screen until participants responded. Experimenters
clicked the mouse to record response times (ms) when the
participant completed their answer. A second mouse click
triggered the next trial. Participants were asked to respond
with a complete numerical answer from left-to-right since all but one participant in a pilot study had chosen left-to-right responses when offered a directional choice. The accuracy of each response was evaluated and recorded by the experimenter on a clipboard as correct or incorrect.

**Addition Fluency** Participants were given two minutes to complete each page of addition problems. They were instructed to work as rapidly as they could without sacrificing accuracy and to answer problems by first working across the first row of ten problems and then moving to the next row. Total problems correct across both pages was used as a measure of addition fluency.

**Visual Span** Participants were shown each pattern, on a stimulus card, for three seconds, and then asked to respond by sketching the pattern on a blank grid of the same shape as the stimulus. There was no response time limit, and participants received feedback on their success after every trial. Patterns were considered correctly recalled if all squares were marked in the correct positions. A stop condition was reached when a participant was unsuccessful with all three trials at a given level. The mean of the complexity of the last three patterns recalled correctly was used as each participant’s raw visual span.

**Results**

**Response Time Data**

Response-time data were analyzed in a 2 (visual similarity: high, low) x 2 (presentation format: horizontal, vertical) repeated measures ANOVA. As hypothesized, there was a significant main effect of presentation format $F(1,39) = 7.29$, $MSE = 9,147,204$, $p < .01$. Participants responded more rapidly to vertical stimuli than to horizontal stimuli (12,640 ms vs. 13,930 ms). There was also a significant main effect of visual similarity $F(1,39) = 21.26$, $MSE = 10,651,890$, $p < .001$. Participants responded more slowly to problems classified as high in visual similarity than to those low in visual similarity (14,475 ms vs. 12,095 ms).

There was no significant interaction in response times between presentation format and visual similarity $F(1,39) < 1$, $MSE = 7,225,987$, $p = .516$.

<table>
<thead>
<tr>
<th>Response Time (ms)</th>
<th>Vertical</th>
<th>Horizontal</th>
</tr>
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<tbody>
<tr>
<td>Low</td>
<td>11,589</td>
<td>12,601</td>
</tr>
<tr>
<td>High</td>
<td>13,689</td>
<td>15,260</td>
</tr>
<tr>
<td>$\text{(SD)}$</td>
<td>(829)</td>
<td>(1,289)</td>
</tr>
<tr>
<td>$\text{(SD)}$</td>
<td>(1,297)</td>
<td>(1,276)</td>
</tr>
</tbody>
</table>

Table 2: Mean response times and accuracy by presentation format and visual similarity.

**Accuracy Rates**

The proportion of correct trials were analyzed with a 2 (visual similarity: low, high) x 2 (presentation format: horizontal, vertical) repeated measures ANOVA. Congruent with the hypotheses, participants responded more accurately to vertical stimuli than to horizontal stimuli (.813 vs. .762), $F(1,39) = 4.89$, $MSE = .02$, $p < .05$. There was no significant main effect of visual similarity $F(1,39) = .836$, $MSE = .02$, $p = .366$. However, presentation format and visual similarity interacted, $F(1,39) = 4.47$, $MSE = .12$, $p < .05$. Figure 1 shows accuracy rates separated by condition. The advantage for vertical over horizontal format occurred only when the problems were low in visual similarity. The orientation advantage disappeared completely when the problems were high in visual similarity.

**Addition Fluency & Visual Span**

Table 3 shows the correlations of addition fluency and visual span to all measures along with the appropriate descriptive statistics. Note that addition fluency was moderately correlated with response time for all equation conditions, while the visual span was correlated only with response times from vertically presented equations. Response times were negatively associated with accuracy rate.

**Discussion**

The current study examined the influences of visual similarity and presentation format on response time and accuracy for complex mental addition. Congruent with the original hypotheses, a vertical presentation format was associated with reduced response times and increased accuracy, relative to horizontal presentation. There was also an apparent advantage in response times for equations with low visual similarity (relative to equations with high visual similarity). Also in accord with the original hypotheses,
there was a significant interaction between presentation format and visual similarity for participant accuracy. These results are suggestive of an important role of visual factors in complex mental arithmetic. Item analyses, treating the chosen equations as a random variable, confirmed the generalizability of these findings.

The present results are interpreted as support for an important role of visual working memory in mental arithmetic. This interpretation is in part based upon Trbovich and LeFevre (2002), who demonstrated that vertically presented mathematical equations are solved faster than horizontally presented equations and are subject to interference from a secondary task that taps visual working memory. It is interesting to note that in the present findings, vertical presentation resulted in faster response times, regardless of visual similarity. Thus, vertical arrangement of digits facilitated the visual processing demands, presumably by lining up the appropriate addends. This facilitative effect on response times suggests that visual working memory involvement is beneficial in completing mental addition. Response times were also slower overall for equations with higher visual similarity, indicating that a visual memory component is active regardless of the presentation format. That is, problems with higher visual similarity required longer response times in both presentation formats, suggesting that the solving of complex addition problems is subject to influence from the visual characteristics of the digits involved.

Performance accuracy in the present findings also supports an important role of visual working memory in mental addition. The main effect of presentation format again indicates a facilitative effect of vertically lining up the addends of an equation; further, the hypothesized interaction demonstrates that the advantage associated with such a vertical presentation format can in fact be negated by the increased visual processing demands brought about by higher visual similarity among the digits. It is proposed that high visual similarity amongst digits increases the visual processing and memory load, thus negating the beneficial effects of a vertical presentation format.

This importance attributed to visual processing and working memory in mental addition, is in accordance with Logie et al. (1994), yet in contrast to the conclusions of Noël et al. (2001) and Heathcote (1994). The apparent discrepancies between studies can be reconciled given the manipulation of presentation format in the present research design. That is, by manipulating presentation format, the importance of visual similarity was revealed. In addition, it is important to note that the methodology incorporated here involved the entire equation being presented simultaneously, which may further encourage reliance on visual working memory.

The finding that vertical presentation of equations results in faster and more accurate responses warrants further discussion. It may be that vertical presentation is easier because it assists solvers in lining up interim results and carries visually, directly above or below the corresponding digits of the problem. This explanation would also account for why vertical presentation may involve visual working memory more so than does horizontal presentation. In fact, it may well be the increased involvement of visual working memory that makes vertical equations apparently easier to solve, effectively distributing the working memory demands across the two slave systems. In contrast, horizontal or sequential presentation creates a heavy phonological load that, for these complex problems, quickly overwhelms the capacity of the phonological loop and stresses the central executive. On this view, the vertical format used in mathematics textbooks and associated with the ‘standard’ solution algorithm (adding the columns right-to-left) may be designed to minimize processing demands on the cognitive system by sharing the load across the working memory complex.

Although the present research provides important information about the role of visual-spatial working memory in mental calculation, it must be acknowledged that the present study included only a small number of equations per condition (five). Therefore it was important to reanalyze the data treating the equations as random variables as a test of generalizability over the “population” of all possible equations. Despite the loss of statistical power associated with this type of analyses, the overall pattern of results was replicated. It is also important to note that the present design did counterbalance the high and low visual similarity equations across presentation formats. Thus all equations were presented in both formats.

The pattern of correlations among measures reported here also warrant further attention. In particular, the addition subtest of the Kit of Factor-Reference Cognitive Tests (French, et al., 1963) was significantly correlated with
response time for all conditions, thus confirming the relevance of this test as a measure of addition fluency. Interestingly, the visual span from the Visual Patterns Test (Della Sala, et al., 1997) was significantly correlated only for response times from vertical presentations, again highlighting the substantial visual memory involvement in solving such presented equations.

Although not directly addressed in the present study, it is also important to consider whether visual working memory is important for particular elements or processes involved in solving equations. That is, it is suggested above that visual working memory may be relevant for holding interim results and carries. In this respect, the present study did not specifically examine the effect of varying the number or complexity of carries. As suggested by the results of Heathcote (1984), however, there may be an interaction between the number of carries and visual similarity. This is an area for future research to consider in more depth.

Noël et al. used Belgian participants and this replication was conducted with Canadian undergraduates. Educational standards and approaches vary between these environments and there may be cross-cultural differences in the use of visual-spatial working memory during arithmetic. Future research should examine whether varying presentation format elucidates a similar effect of visual similarity among Belgian participants.

Finally, it is of interest to note that the present study used stimuli previously developed by Noël et al. (2001) rather than examining the entire population of visual similarities. Future research should consider comparing stimuli from various points within the population distribution for visual similarity in order to investigate at which point a problem’s visual similarity begins to interfere with processing.

In summary, the present study extended previous research on the role of visual processing in mental arithmetic. By manipulating both visual similarity and presentation format within a carefully planned experimental design, the present study uncovered a potentially important role of visual working memory in solving mental addition. These results suggest that, to better understand mental arithmetic, it is important to consider both presentation format and visual similarity amongst digits.

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