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Pion Condensation and the Evolution of Neutron Stars*

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Abstract  Pion condensation induces an anisotropy in the pressure of dense matter. As a consequence dense neutron stars, if in this phase, will have a quadrupole deformation and, if rotating, will radiate gravitational waves. The resultant damping of the period is compared with pulsar data. A combination of gravitational radiation and dipole electromagnetic radiation, by which pulsars are detected, can account for the observed dispersion of pulsar periods and their derivatives.

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A new property of the pion condensate state of matter was noticed recently, namely, that the pressure in the medium is anisotropic. This is due to the broken spatial symmetry, which is the characteristic of the pion condensate. Apparently it was previously assumed that the pressure could be calculated from \( P = \rho \frac{\partial^2 \mathcal{A}(f)}{\partial \rho^2} \) where \( f \) and \( \rho \) are the free energy and number densities, respectively. This is true for an isotropic medium, but that is just what the pion condensed state is not. The existence of the anisotropy can be established by studying the stress-energy tensor.

The purpose of this note is two-fold. First, a simple model, the chiral \( \sigma \)-model, is employed, because of the ease of mathematical analysis, to prove that the anisotropy in pressure is finite at the equilibrium value of the pion condensate wave number. Second, an exploration of the consequences of this anisotropy for the properties and evolution of neutron stars is begun. Their interior densities are believed to be of the order of ten times normal nuclear density, which is above the critical density for condensation expected by most authors. Consequently, the pion condensed state is likely to be the ground state of the dense interior. The anisotropy in pressure of this state will tend to deform the star. This tendency will be resisted by gravity so that a stable deformation will be established. Since it is the repulsion of the nuclear force which prevents gravitational collapse, we can anticipate that the deformation could be appreciable, since it is induced in this case by the nuclear forces.

Using a chiral \( \sigma \)-model the longitudinal and transverse pressures are estimated. The equation for hydrostatic equilibrium is used to estimate the resulting deformation, which is prolate because the longitudinal pressure is greater than the transverse. Such a star rotating
about any axis, not its symmetry axis, will emit gravity waves. The power radiated in this way will damp the rotational motion. The consequent rate of change of the period is calculated and compared with the known pulsar data.

The chiral \( \sigma \)-lagrangian \( \mathcal{L}_\sigma \) (ref. 2) is supplemented by the lagrangian for the vector meson \( \omega_\mu \), which plays the important role of introducing a repulsion at high density.

\[
\mathcal{L} = \mathcal{L}_\sigma + \mathcal{L}_\omega = \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \tau \partial^\mu \tau) - \frac{1}{4} \lambda (\sigma^2 + \tau^2 - \nu^2)^2 + \frac{f}{\pi} \sigma \nonumber \\
+ \mathcal{N} [i \sigma - g (\sigma + i \gamma_5 \tau) \nu N - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} \sigma \omega_{\mu} \omega^{\mu} - g \omega \nu_{\nu} \psi N \psi N].
\]

For purpose of illustration, a neutral condensate in pure neutron matter is considered. Following Dautry and Nyman, make the ansatz for the scalar and pion fields,

\[
\sigma = \sigma_0 \cos k_0 x , \quad \pi_0 = \sigma_0 \sin k_0 x , \quad \pi_\pm = 0.
\]

The normal state corresponds to \( k_0 = 0 \) and the condensed state to \( k_0 \neq 0 \). The Dirac equation that follows from \( \mathcal{L} \) is obviously dependent on the oscillating \( \sigma \) and \( \pi \) fields, but this space variation can be removed by transforming to a new Dirac field defined by \( \psi = \exp (-i \frac{1}{2} \tau^3 \gamma_5 k_0 x) \psi_N \), with \( k_0 = 0 \). The Dirac equation, with nucleon mass \( m = g \sigma_0 \), becomes

\[
(i \sigma - g \nu \mu - m = \frac{1}{2} \gamma_5 \tau^3) \psi = 0.
\]

The spin degeneracy of the usual Dirac equation is broken in the condensed state, and the particle eigenvalues are split into two branches

\[
E_\pm = Q_0 = \left( Q^2 + m^2 + \frac{1}{4} k^2 \pm \Delta^2 \right)^{1/2},
\]

\[
\Delta^2 = k \sqrt{Q_3^2 + m^2}, \quad Q_\mu = p_\mu - g \omega_{\mu}.
\]
The ground state is built by filling these eigenvalues up to the Fermi surface. It can be seen that there is an axial symmetry in momentum space about the pion momentum, \( k = (0, 0, k) \). This distortion of the Fermi surface, produced by the interaction with the pions, is partly responsible for the anisotropic pressure. When \( E_\pm \) is expanded in powers of \( 1/m \), the asymmetry occurs at one order beyond the non-relativistic limit, namely \( 1/m^2 \). Therefore, it is at this order that the difference in longitudinal and transverse pressure will first appear. At this order, the Fermi surface of the lower eigenvalues \( E_- \) is a prolate spheroid and of the upper eigenvalue (if occupied), an oblate spheroid.

The ground state expectation of the stress-energy tensor, \( \mathcal{J}_{\mu\nu} \), corresponding to \( \mathcal{L} \), written in terms of the transformed Dirac fields, is

\[
\mathcal{J}_{\mu\nu} = -g_{\mu\nu} \xi_0 + \sigma_0^2 k_\mu k_\nu + \langle \bar{\psi} \gamma_\mu (p_\nu - \frac{1}{2} k_\nu \gamma_5 T_3) \psi \rangle
\]

\[
\mathcal{L}_0 = \langle \mathcal{L} \rangle = -\frac{1}{2} \sigma_0^2 k^2 + f \frac{m^2}{\pi} \sigma_0 \cos kx - \frac{1}{4} \lambda (\sigma_0^2 - \nu^2)^2 + \frac{1}{2} \left( \frac{g_\omega}{m_\omega} \right)^2 \rho^2.
\]

The oscillating term space-averages to zero in the condensed state \((k \neq 0)\).

The axial symmetry of the fermion momentum space can be shown to imply that the space-like components of the \( \omega \)-field vanish, \( \omega_k = 0 \), and that the time-like component is related to the density by

\[
\omega_0 = \frac{g_\omega}{m_\omega} \langle \bar{\psi} \gamma_0 N \psi \rangle = \frac{g_\omega}{m_\omega} \rho.
\]

The ground state expectation values of the source currents can be calculated using the propagator technique detailed in the appendix of Ref. 3. The result for the stress-energy tensor, after exploiting the axial symmetry of the occupied momentum space, is for pure neutron matter,
The time-like component $\mathcal{J}_{00}$ is the energy density and the space-like components are the pressures. The structure of $\mathcal{J}_{\mu\nu}$ shows that $P_1 = P_2$ while $P_3$ is different in form. It could happen, however, that, when the energy is minimized with respect to the pion condensate wave number $k$, the three components are equal. So this point is now addressed. This is most easily done if we go to the limit of large $\lambda$, and second if we expand all quantities in $1/m$, since this permits the integrals to be evaluated in closed form. From the field equation for the pion one finds that for large $\lambda$, $\sigma_0 = v + O(1/\lambda)$ and that the potential term in $\mathcal{L}_0$ is $O(1/\lambda)$. Thus as $\lambda \to \infty$, $\sigma_0$ becomes a constant, independent of $k$. This greatly facilitates solving $(d\varepsilon/dk)_\rho = 0$ for $k$.

In the normal state ($k \equiv 0 \to \pi \equiv 0$), the energy density and pressure are found from $\mathcal{J}_{\mu\nu}$ to be

$$\varepsilon_0 = -f_{\pi} m_\omega^2 \sigma_0 + m_\rho + \frac{(3m_\rho^2)^{5/3}}{10n^m} + \frac{1}{2} \left( \frac{g_\omega}{m_\omega} \right)^2 \rho^2 + O(\frac{1}{(m^3)})$$

$$P_0 = f_{\pi} m_\omega^2 \sigma_0 + \frac{(3m_\rho^2)^{5/3}}{15n^2m} + \frac{1}{2} \left( \frac{g_\omega}{m_\omega} \right)^2 \rho^2 + O(\frac{1}{(m^3)}).$$

In the condensed state the energy density is given by the following expression if only the $E_-$ eigenstates are occupied:

$$\varepsilon = m_\rho + \frac{(6m_\rho^2)^{5/3}}{20n^2m} + \frac{1}{2} \left( \frac{g_\omega}{m_\omega} \right)^2 \rho^2 + \frac{1}{2} \sigma_0^2 k^2 - k \left( \frac{\rho}{2} - \frac{(6m_\rho^2)^{5/3}}{60n^2m^2} \right) - O\left( \frac{1}{(m^3)} \right)$$

The condition that only the $E_-$ states are occupied is $\rho < (2m)^{3/2}/[6\pi^2(1 + k/2m)]$, which turns out to be satisfied over the range of interesting densities.
The possibility for extra binding in the condensed state is due to the \(-k\rho/2\) term in \(\epsilon\). The value of \(k\) must be determined by minimizing the energy at fixed density. The pressures in the transverse directions are
\[
P_1 = P_2 = -\frac{1}{2} \sigma_0 k^2 + \frac{1}{2} \left( \frac{g_{\omega}}{m_\omega} \right)^2 \rho^2 + \frac{6\pi^2 \rho}{30\pi^2 m} \left( 1 + \frac{k}{3m} \right) + 0\left( \frac{1}{m^3} \right)
\]
while at the equilibrium value of \(k\), the longitudinal pressure is
\[
P_3 = P_1 + \frac{k^3 \rho}{8m^2} + 0\left( \frac{1}{m^3} \right).
\]

This last expression achieves the first goal of this paper: it shows that \(P_3 \neq P_1\) at the equilibrium condition and, for the reason mentioned earlier, the difference in pressures is of order \(1/m^2\).

The energy density and pressures \(P_1\) and \(P_3\) are shown for the condensed phase, at densities higher than the critical point, in Table I. For this calculation, the axial vector current was renormalized along the lines discussed in ref. 2.

It is interesting now to consider possible implications for pulsars of the anisotropic pressure of dense neutron matter in the pion condensed phase. The prolate deformation due to this anisotropy can be estimated crudely by supposing that the density of the star is uniform (which is approximately true for stars with mass \(M \gtrsim M_\odot\)). Then the central pressure is related to the radius of the star through the condition for hydrostatic equilibrium, which yields \(P(0) \propto R^2\). Thus the radius that can be supported by \(P\) is \(\propto \sqrt{P}\). So we estimate the deformation as \(\delta \approx (\sqrt{P}_3 - \sqrt{P}_1)/\sqrt{P}\), where \(P\) denotes the average. These deformations are shown in Table I.

Gravity waves are radiated by the nonspherical flow of energy. This will be the case for a quadrupole deformed star if it rotates about any
axis, not its symmetry axis. The amount of mass energy contained in the
pole caps of a star of mass $M$ with prolate deformation $\delta = (R_3 - R_1)/R$ is
\(\sim M\delta^2\) and the moment of inertia attributable to this is $I_\delta = \delta MR^2$. Hence the
rate at which energy flows in a nonspherical way, if the star rotates about an
axis perpendicular to its symmetry axis, with angular velocity $\omega$, is
$L = \left(\frac{1}{2} I_\delta \omega^2\right) \omega$. The power radiated in gravity waves can be estimated, according
to Misner, Thorne and Wheeler\(^5\) from
\[ L_{GW} = L^2 L_0 = \frac{1}{4} \frac{\delta^2 M^2 R^2 \omega^6}{\omega L_0} \]
where $L_0$ is a constant with dimension of power.

Now we translate this radiated power into the damping of the star's
rotation, through the energy balance, which yields
\[ \dot{T} = 10^4 \frac{4 \delta^2 M R^2}{T^3 L_0} \approx \frac{\delta^2}{2} \left(\frac{M}{M_\odot}\right) \left(\frac{R}{10 \text{ km}}\right)^2 \frac{10^{-11}}{(T/\text{sec})^3} \]

For a typical neutron star, $M = M_\odot$, $R = 10$ km, and $\rho \approx 10\rho_0$. From Table I,
the corresponding deformation is $\delta \sim 6 \times 10^{-3}$. Hence $\dot{T} \sim 2 \times 10^{-16}/(T/\text{sec})^3$. Thus such a star will have a very stable period. Measurements of $\dot{T}$, referred to as breaking, have been made for many pulsars and are shown as a
function of period in Fig. 1.\(^6\) There also the evolution of such a star
as described here is indicated by the lower diagonal line.

If the star described here is identified as a pulsar, then it also
produces electromagnetic radiation. That is how pulsars are observed.
Current thinking is that pulsars have a large dipole magnetic field
inclined at an angle to the rotation axis which produces the beacon-like
signal. Thus the law for breaking of a pulsar in the pion condensed state
is at least as complicated as
\[ \dot{T} = \frac{a}{T^3} + \frac{b}{T} \]
If at birth the second term dominates the first, a pulsar will evolve along a line with slope -1 in a plot of log $\dot{T}$ vs log $T$; i.e., parallel to the top line in Fig. 1. If, on the other hand, gravity waves dominate the energy loss, the star will evolve along a line with slope -3, parallel to the bottom line. Otherwise, it will evolve along lines that begin with slope -3 and evolve to slope -1. As an added variation, if the magnetic field decays with time, as expected, then a star starting out with a slope -1 will evolve to a slope of -3, depending on the decay rate. The whole pattern shifts vertically proportional to $\delta^2$. Thus, it appears that the evolution of pulsars into the broadly dispersed region shown in Fig. 1 can be accounted for in terms of energy loss due to gravitational and electromagnetic dipole radiation, the gravitational radiation being a consequence of the quadrupole deformation induced by the pion condensate state of dense matter.

It might be objected that the condensate could form in various unaligned zones. However, the star is contained by gravity. Consequently such a configuration would be unstable by virtue of the discontinuity in pressure across the boundaries of the misaligned zones and would achieve stability only when the zones aligned themselves in a common direction. Gravity, in this case, enforces the long range order.

Now it is appropriate to mention a reservation concerning the quantitative results. The degree of pressure anisotropy was estimated using the chiral $\sigma$-model. The model has serious drawbacks, in that it does not possess a normal saturated state of symmetric nuclear matter. A theory of neutron-rich matter possessing the correct saturation, compressibility, symmetry energy and in $\beta$-stability and charge neutral has been formulated, and the numerical aspect is currently being explored.
When this calculation is complete, it will provide a much more accurate estimate of the deformation of neutron stars. In the meantime it is worth stressing that the existence of the pressure anisotropy will be a common feature of any theory for which the nucleon Fermi sea is axially distorted through the interaction with the pions.

I have been stimulated by conversations with several people, L. Van Hove, A. Lumbroso, D. N. Schramm and W. J. Swiatecki. This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract W-7405-ENG-48.

References
1. N. K. Glendenning and A. Lumbroso, Pion Condensation, Density Isomers and Anisotropic Pressures, invited paper at Hirschegg Workshop 1981 (LBL-12108)
Figure Captions

Fig. 1. The period derivative $\dot{T}$ as a function of period $T$ for 87 pulsars. Neutron stars, with periods damped by gravitational radiation alone would evolve along lines parallel to the lowest one. A pulsar starting at $t = 0$ with period $T = 0.1$ sec and having typical mass and radius $M = M$ and $R = 10$ km would evolve along the lines shown, depending on the weighting of gravitational and dipole contributions to their energy loss (parameterized by $x$) if it has a quadrupole deformation $\delta = 0.006$. The times taken to evolve are also shown.

Table I: Energy density, pressures and typical star deformations as function of density. Constants of the theory as in Ref. 2 with $g_A^* = g_A (1 + S)^{1/2} (1 - \gamma)^{1/2}$, $S = 0.8$, $\gamma = 0.6$ as the axial vector renormalization. (Recall $\rho_0 = 0.145$ fm$^{-3}$.)

<table>
<thead>
<tr>
<th>$\rho$(fm$^{-3}$)</th>
<th>$\epsilon$(fm$^{-4}$)</th>
<th>$P_1$(fm$^{-4}$)</th>
<th>$P_3$(fm$^{-4}$)</th>
<th>$\delta$</th>
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<td>25.36</td>
<td>14.21</td>
<td>14.45</td>
<td>0.009</td>
</tr>
</tbody>
</table>
\[ \dot{T} = \left( \frac{1}{T^3} + \frac{x}{T} \right) a \]

\[ a = 2 \times 10^{-16} \text{ sec}^3 \]
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