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CONSIDERATION OF THE
RESOLUTION OF A MAGNETIC-PARTICLE SPECTROMETER

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In performing any physical measurement the measuring apparatus
normally gives only an approximate value of the desired quantity.
Frequently the measuring apparatus is made less accurate than it
can be in order to aid in the ease or speed of measurement. This
is especially true of instruments that measure the flux of particles
or photons. Astronomical spectrographs for wave-length measurement
often do not measure wave length as accurately as some other
spectrographs, one reason being that it is necessary to limit film
exposures to reasonable times. Nuclear cross sections are normally
measured over a range of energies or range of angles to allow
reasonable counting rates. In designing equipment this battle between
ease of experimental measurement and accuracy of measurement must
be fought. In interpreting any measurement the instrumental resolu-
tion should be considered, and if possible, "unfolded" from the real
value of the quantity. Let us consider, for example, a magnetic
particle spectrometer used to separate charged particles of different
energies. Such an instrument is currently in use at the Radiation
Laboratory.
Slits (1) and (2) define the direction of the beam of particles that enter the magnet. Slit (3) is the device for detecting the particles. In the actual instrument used the orbit shown might correspond to a 100-Mev proton. If all the slits were infinitely narrow, the energy of the orbit would be well defined at exactly 100 Mev, but owing to the widths of the slits the energy interval actually defined by the slits was roughly 5 Mev.
Let us analyze a somewhat simpler instrument. Consider a two-slit system for collimating a beam of light from a broad source. How do the widths of the slits affect the collimation of the beam of light?

![Diagram of a two-slit system](image)

**Fig. 2**

The intensity of the beam at the angle $\theta$, $I(\theta)$, is determined by the size of the bundle of rays that get through the slits. This is given by

$$I(\theta) = \int_{-s/2}^{s/2} \frac{dx}{s - t}, \quad s < a$$

This is true only if

$$s << a.$$
If we plot this we get

![Diagram](image)

**Fig. 3**

The collimator does not perfectly define the beam direction but gives a beam out to an angle of $\theta = s/a$. As the slit gets narrower, the beam direction is better defined.

If the two slits are not completely transparent, but have transparencies of $T(x)$ and $U(x)$, the intensity becomes

$$I(\theta) = \int_{-a}^{a} T(x) U(x + t) \, dx, \quad \theta = \frac{t}{a}$$

This expression is very similar to the integral known as the Faltung, which is

$$F(t) = \int_{-\infty}^{\infty} A(x) B(x - t) \, dx$$

The only difference between the Faltung and the $I(\theta)$ integral is the fact that in the Faltung $t$ changes sign. This means that when the
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Faltung is used the t axis is reversed. The limits of integration are essentially the same, since in the I(θ) integral T(x) and U(x) are nonzero only over the limits covered. The properties of the Faltung may profitably be used in problems involving the resolution of experimental equipment.

The expression for I(θ), Eq. 2, could be used to give the flux of particles getting through slit 2 of the magnetic particle spectrometer previously mentioned. These particles now travel through the magnetic field. Use of the cyclotron equation,

$$\frac{mv^2}{R} = \text{Hev,}$$

(4)

enables one to see that particles have different radii of curvature depending on their momenta:

$$R = \frac{mv}{\text{He}} = \frac{P}{\text{He}}.$$  

(5)

Because of this, particles will leave the magnet at different points depending on their momenta. It is easy to see that if the beam of particles from slit 2 is not perfectly defined in direction this will confuse the separation of particles according to momentum at the exit end of the magnet.

Let us consider a beam of particles from slit 2 having a unique momentum, Po, but a spread in direction given by I(θ) as shown in Fig. 3. This spread in direction will produce a spread in position at the exit end of the magnet. But the function of the spectrometer is to separate particles of different momentum according to position
so this spread in position is equivalent to a spread in momentum -

\[ I(\theta) \rightarrow I(\text{position}) = I(P). \]

This says then that we would get a momentum spread resulting from this spread in direction looking like the following.

![Diagram](image)

Fig. 4

If the particle detector, slit 3, were infinitely narrow, it would measure the momentum spectrum given in Fig. 4 for a monoenergetic beam of particles, but if slit 3 has a finite physical width, and thereby a momentum width of \( \Delta P \), then it measures
\[ I(p) = \int_{p - \frac{\Delta P}{2}}^{p + \frac{\Delta P}{2}} I(P) dP. \quad (6) \]

This again is a use of the Faltung simplified by the fact that the slit "transparency" has been taken to be unity.

If we had called \( V(P) \) the "transparency" of the detector ("transparency" here could be interpreted to be the detection efficiency for particles of different momenta), then we could write for \( I(p) \) from Eq. 2:

\[
I(p) = \int_{p - \frac{\Delta P}{2}}^{p + \frac{\Delta P}{2}} \left[ \int_{-\frac{s}{2}}^{\frac{s}{2}} T(x) U(x + t) \, dx \right] V(P) \, dP.
\]

This process can be visualized by the following Fig. 5.
The area under the product curve $V(P) I(P)$ would be the yield of particles at $P$. This is given by the $I(p)$ integral above, Eq. 7.

Changing the variable $P \rightarrow (P - p)$, we get

$$I(p) = \int_{-\Delta P/2}^{+\Delta P/2} \left[ \int_{-\frac{S}{2}}^{\frac{S}{2}} T(x) U(x + t) \, dx \right] V(P - p) \, dP$$

(8)

This can be seen to be the Faltung integral performed twice. In many problems this integral is used to combine resolution parameters.

The $I(p)$ spectrum might now look like

The spectrum now does not need to be symmetrical if $\Delta P$ is a function of $P$:

This is the spectrum of momenta measured by the spectrometer when the incident beam of particles has a unique momentum. Because of this fact the curve of Fig. 5 could be called the momentum resolution of the spectrometer. The three slit widths that determine the shape
of the momentum resolution can be called resolution parameters. Although the widths are actually physical widths, all three can be expressed as equivalent momentum widths. This can be done by taking one slit width and assuming the other widths are zero and determining the spread in resolution due to each slit independently.* An example of this is shown in Fig. 7.

Fig. 7

* This process was followed in Gladis, Hess, and Moyer, Phys. Rev. 87, 425 (1952).
We could now write Eq. 8 as

\[ I(p) \int \left[ \frac{T(P_1)}{U(P_1 - P_2) \, dP_1} \right] V(P_2 - p) \, dP_2 \]  

Now let us consider what this instrumental momentum resolution does to an actual momentum spectrum in the beam of the incident particles. Let us consider \( F(P) \) the incident spectrum. Repeating the process shown in Fig. 5, but now substituting \( F(P) \) for \( I(P) \), we can write the following integral for the experimentally determined momentum spectrum:

\[ I(p) = \int \left\{ \int \left[ \frac{F(P)}{T(P - P_1) \, dP} \right] U(P_1 - P_2) \, dP_1 \right\} V(P_2 - p) \, dP_2. \]  

This is simply a triple Faltung integral. Quite generally it can be stated that when the resolution parameters are known the resolution of the instrument can be obtained by using the Faltung to combine the parameters. Similarly when the curve to be measured is known (as for example the momentum spectrum considered above) then the curve given by the instrument can be obtained by combining the known actual curve with the resolution parameters by using the Faltung. These processes are known as folding.

Method of Folding for Magnetic Particle Spectrometer

A consideration of this instrument shows that each resolution parameter can be considered to be rectangular, having an energy width \( \Delta E \) and a height \( h \). (The small-angle scattering parameter should be considered gaussian, not rectangular, but in any case is small enough to be neglected.) The figure below shows where
the various parameters originate.

\[ \Delta E_d \text{ due to small-angle scattering all along path} \]

\[ \Delta E_b \text{ due to target width} \]

\[ \Delta E_c \text{ due to slit proportional counter width} \]

\[ \Delta E_a \text{ due to energy loss in target} \]

\[ \Delta E_e \text{ due to GM tube width} \]

Fig. 8

Parameters b, c, and e are the 3 slits from the previous discussion. Item a is a real energy width introduced by the target thickness. Particles scattered from the front or back of the target have different energy losses. Parameter d, which is the result of small-angle scattering, is neglected in the calculations.

Considering parameters a, b, and c: these are rectangles of energy width \( \Delta E \), where \( \Delta E \) varies with energy. But the area under these resolution parameters does not vary with energy. This
results from the fact that this area represents a number of particles; by considering these parameters separately, we see that the number of particles getting through the instrument does not depend on the energy widths of these parameters. This means that the height of the resolution-parameter rectangles is \( h = \frac{1}{\Delta E} \) to give constant area. The area is taken to be unity for simplicity.

This is not the case with parameter \( e \). The energy width \( \Delta E \) varies and the height remains constant (it will be taken equal to unity), and the area increases linearly with \( \Delta E \). This can be seen roughly by considering that all parameters except 3 are equal to zero, and then having a flat energy spectrum of particles be incident on slit 1. Therefore a continuous distribution in position of particles will emerge from the magnet. As the physical width of slit 3, the detector, is increased, more particles will be detected, or—what is equivalent—as the energy width of slit 3 is increased, the area under the resolution parameter \( e \) increases linearly.

Now let us consider the folding of any one parameter of the group \( a, b, \) and \( c \) with the actual incident spectrum,

\[
G(\epsilon) = \frac{\Delta E}{2} \int_{-\Delta E/2}^{+\Delta E/2} F(E) h(E - \epsilon) \, dE = \frac{1}{E} \int_{-\Delta E/2}^{+\Delta E/2} F(E) \, dE
\]  

(11)

The folded spectrum \( G(\epsilon) \) can be seen to be in this case simply related to \( F(E) \). A point on the curve \( G(\epsilon) \) is the average height of the curve \( F(E) \) averaged over the interval \( \Delta E \) centered on the point \( \epsilon \). This is illustrated by the following figure.
From the two examples shown of the result of this folding process, it can be seen that the folding tends to smooth out curves or peaks in $F(E)$. The curve $G(\epsilon)$ is obtained by repeating this folding process several times. One point on the curve $G(\epsilon)$ is obtained as the result of each averaging process. When the curve $G(\epsilon)$ is constructed by using one of the three parameters $a$, $b$, or $c$, then this curve is in turn folded by the second of these parameters, by the same process as that given above, to give a new curve, which is in turn folded by the third parameter. This curve is then folded in the same manner by parameter $e$ to give the final experimentally determined energy spectrum.
Since resolution parameter e is not constant in area, this last step must be explained. This fold could be given by (using \( h = 1 \))
\[
I(\epsilon) = \int_{-\Delta E/2}^{+\Delta E/2} H(E) \ h(E - \epsilon) \ \text{d}E = \int_{-\Delta E/2}^{+\Delta E/2} H(E) \ \text{d}E
\]

But now, in presenting this curve, we want it to be an energy spectrum; therefore if the detectors do not all have the same energy widths the points on the final curve should be normalized so that they represent particles per unit energy interval after the normalization. This means multiplying each point by \( \frac{1}{\Delta E} \), which gives the final curve,
\[
I'(\epsilon) = \frac{1}{\Delta E} \int_{-\Delta E/2}^{+\Delta E/2} H(E) \ \text{d}E
\]

This is seen to be the same result as that given for resolution parameters a, b, and c, and therefore the folding process for parameter e is the same as for a, b, and c.
A CONSIDERATION IN THE DESIGN OF EXPERIMENTS

In considering the resolution of experimental equipment it is evident that we must reach a compromise between the amount of information accepted by the machine (flux) and the resolution of the machine.

If, in the design of the machine, several design parameters enter (for example, the three slits of the spectrometer mentioned earlier), it is important to arrive at a set of these parameters which, considered as a whole, gives the maximum flux through the machine for a given resolution. This concept can be illustrated by returning to the resolution of a two-slit system. Let us ask the question: for a given resolution what should be the ratio of slit widths to give maximum flux through these two slits?

For the system shown we have

\[ I(\theta) = \int_{\left(\frac{s_2}{2} - t\right)}^{\left(\frac{s_2}{2} + t\right)} dx = s_2, \quad \theta = \frac{t}{a} \]
However, this holds only as long as the bottom ray of the system hits the front slit. As $\theta$ is increased the bottom ray changes from hitting the front slit ($= s_2$) to the back slit. This happens when

$$t = \frac{s_1 - s_2}{2}.$$

At that time the limits of the integral change and we get

$$I(\theta) = \int_{s_1/2}^{s_2/2} dx = \frac{s_1 + s_2}{2} - t, \quad \theta = \frac{t}{a}.$$

If we plot this intensity pattern we get

Fig. 11
The trapezoid becomes a triangle in the limiting case when the slit widths are equal.

Now let us return to the question of optimum resolution.

We want

\[
\frac{\text{FLUX}}{\text{RESOLUTION}} = \text{MAXIMUM}
\]

In the two-slit system the flux is the area under the \( I(\theta) \) curve.

A measure of the resolution of the two-slit system is the width of the \( I(\theta) \) curve, which at the base is \((s_1 + s_2)\). If the resolution is held constant, the maximum flux is obtained when \( s_1 = s_2 \), or when the trapezoid becomes a triangle. This says the best design of the two-slit system is to have the slit widths equal. This may seem a somewhat obvious result, but the concept can be used profitably in more complicated systems. For instance, in the three-slit spectrometer the first slit should be wider than the second slit.

A quite general conclusion can be drawn from the two-slit problem. Usually the resolution of an instrument is a minimum if the different components that go to make up the over-all resolution are equal. In the energy resolution of the three-slit spectrometer the energy widths (not the physical widths) of the three slits should be equal. This rule is valid only if the different components have the same shape. In the two-slit problem the two components \( T(x) \) and \( U(x) \) are rectangles which, when folded, give a trapezoid. If one of the components, \( T(x) \), had a different shape—for instance a triangle—then \( U(x) \) should not be the total width of the base of the triangle, but rather half the width, to give optimum resolution.