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THE TWO-FIREBALL PHENOMENON AND THE MULTIPERIPHERAL MODEL

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ABSTRACT

In the context of the Chew-Snider version of the ABFST pion-exchange model, we investigate certain multiperipheral mechanisms that could account for those phenomena that have hitherto motivated the two-fireball model. These mechanisms are (1) double diffractive dissociation, (2) the presence of a sequence of neutral particles on the multiperipheral chain, and (3) statistical fluctuations in the log tan θ spectrum. At cosmic ray energies they are of equal importance.

The two-fireball model was proposed by various authors in the late 1950's to explain a remarkable property of cosmic ray events, observed in photographic emulsions, that the spectrum of secondary particles for some events, when presented in log tan θ_{lab} appeared to have large gaps, separating the secondary particles into "forward" and "backward" clusters. According to the model, secondary particles were thought to arise from two well-defined centers, one moving in the projectile direction and one in the target direction in the center of mass.
In interpreting these data in the context of the multiperipheral model, we shall stress what appears to us to be the best established empirical feature, namely the frequent presence of substantial gaps in the log tan $\theta_{\text{lab}}$ spectrum. We call this the "fireball effect." The decay distribution of the "fireballs," i.e., the forward and backward clusters, is empirically less well understood, and it is here that the multiperipheral model is at variance with the two-fireball model. (In our model the fireball is not a well-defined entity. Its average mass increases without bound as the overall energy increases.)

At first sight the fireball effect would seem to contradict predictions of the multiperipheral model. Consider a typical multiperipheral event described by the diagram in Fig. 1. All the two-particle subenergies $s_{ij}$ and momentum transfers $t_i$ are assumed to be small, compared with the overall energy. The final particles all have small transverse momenta $p_\perp$, and their longitudinal momenta in the laboratory frame $p_\parallel$ are arranged in an approximately sequential order, more or less uniformly spaced in the variable $\ln p_\parallel$. (Assuming all particles have the same $p_\perp$, the uniform distribution in $\ln p_\parallel$ transforms into a uniform distribution in log tan $\theta$.)

However, when the model permits a broad distribution in the subenergies so that a few can greatly exceed the average, individual events can deviate markedly from the above description. A large two-particle subenergy $s_{ij}$ will produce a "gap" in the $p_\parallel$ distribution between $p_i$ and $p_j$, dividing the momenta into two groups, thereby producing a "fireball-type event." In the ABFST multiperipheral model the two-particle subenergy distribution is proportional to the $\pi\pi$ elastic cross section, which is dominated by resonances (chiefly the $\rho$), but also has a small "high energy
tail" from Pomeranchuk exchange, extending to quite high energies. Thus there is a small probability that an abnormally large subenergy (hence a gap) will occur. This is the multiperipheral mechanism for double diffractive dissociation. A model employing this effect has been studied for other purposes by Chew and Snider; we will use their model as a basis for making quantitative predictions.

If one observes only charged secondaries, a sequence of neutral particles on the multiperipheral chain can also produce a "gap" in the momentum distribution. This is the second source of fireball-type events in the multiperipheral model.

Finally, for experiments in which only angles are measured and not momenta, there exists yet a third mechanism for producing gaps in log tan θ. Fluctuations in the transverse momentum $p_\perp$ can produce gaps in log tan θ, even for a regular distribution in $\ln p_{||}$. We will calculate the probability for this effect from the experimental $p_\perp$ distribution.

Before calculating the probability for producing "fireball-type events" from these three mechanisms, we must define our terms.

We offer two definitions of a "two-fireball event." (1) The first relies upon a knowledge of only the direction of the produced secondary, which is usually the only reliable datum in emulsion experiments. If the particle spectrum for an event in the variable $\log_{10} \tan \Theta_{\text{lab}}$ exhibits a gap larger than 1.3, say, then we define it to be a two-fireball event. (2) For experiments (mainly in the future) that measure the momenta of the produced particle as well, we propose a second, more specific definition of a two-fireball event. If the laboratory momenta of an event can be divided into two groups, fast and slow, so that the squared invariant mass of all
pairs of particles, one chosen from the fast group and one from the slow group, is greater than some minimum, say 3 GeV$^2$, then we call the event a two-fireball event with a "gap in momentum space." We will assume that, whenever the subenergy of an adjacent pair of particles on the chain exceeds such a large value, a gap in the momentum spectrum practically always results in accordance with our two-fireball criterion. Both definitions can be generalized in an obvious way for n-fireball events.

To obtain a quantitative prediction for the probability of fireball-type events due to the first mechanism, Pomeranchuk (P) exchange, we shall consider the model of the schizophrenic pomeron. In this model the kernel is approximated as a sum of two components, one with strength $g_R^4$ representing the low-subenergy resonances in the $\pi\pi$ elastic cross section, and the other with strength $g_P^4$ representing the "high energy tail" of that cross section. The strength of the high-subenergy part was determined in another paper where the $\pi\pi$ cross section, appearing in the kernel of the ABFST integral equation, was arbitrarily split into the two contributions above the $g$ meson peak at $s_{ij} = 3$ GeV$^2$, hence the choice of 3 GeV$^2$ in our definition of a two-fireball event.

The probability of single P exchange per event, when it is small, is well approximated by the average number of P exchanges per event. The latter is readily calculated by differentiating the logarithm of the total cross section with respect to $\log g_P^4$ just as the average number of pion pairs is found by differentiating with respect to $\log(g_R^4 + g_P^4)$. We are excluding the special class of single diffractive dissociation and elastic scattering...
events from the "two-fireball events." These events are associated with P exchange at the very ends of the chain. This exclusion can be accomplished by not differentiating with respect to the $g_p^2$ factor that corresponds to P exchange at the ends. After some algebra, using the parameters of Chew and Snider, we obtain the result plotted in Fig. 2 for the average number $\bar{n}_p$ of fireballs per event in a $\pi\pi$ collision. Shown also for comparison is the average multiplicity $\bar{n}$ of pions, i.e., the average number of pions produced per inelastic collision. It is possible to extend this result to $\pi p$ and $pp$ collisions by means of a crude scaling law. The plot $\bar{n}_p$ vs $\bar{n}$ in Fig. 3 is independent of this scale, however, and universal to all reactions in this model. For $\bar{n} \geq 7$, the curve is fit roughly by the expression

$$\bar{n}_p = 0.10(\bar{n} - 6).$$

To estimate the frequency of neutral-particle gaps, we first find the average number of sequential neutrals required to produce an apparent "gap" with a 3 GeV$^2$ "subenergy." To do this, we shall use an approximate expression derived by Chew and Pignotti, $^{10}$

$$s/s_0 \approx \prod_{\text{all subenergies}} (s_{ij}/c).$$

From the rate at which the average multiplicity of secondaries grows with energy, it is possible to estimate the average value of the ratio $s_{ij}/c$. Recent preliminary experimental results for secondary multiplicities in $pp$ collisions are fit with the expression

$$\bar{n} = a \ln s + b$$

for $a = 1.13$. $^{11}$ Since from Eq. (2)

$$\ln s/s_0 \approx \bar{n} \ln(s_{ij}/c),$$

for $n = a \ln s + b$. $^{11}$
we estimate that
\[ \ln(\frac{s_{ij}}{c}) \approx \frac{1}{a} = 0.9 \]  \hspace{1cm} (5)

Comparison of the CQA multiperipheral model with experimental data indicates that a typical value of the average subenergy is 0.5 GeV^2. With this value for \( s_{ij} \), we obtain \( c \approx 0.2 \text{ GeV}^2 \).

Corresponding to Eq. (2) there is an expression relating the subenergy of two nonadjacent particles \( i \) and \( \ell \) to the intervening adjacent-particle subenergies:
\[ \frac{s_{i\ell}}{c} \approx (\frac{s_{ij}}{c}) \cdots (\frac{s_{k\ell}}{c}) \]  \hspace{1cm} (6)
Thus when the number of intervening particles is two, the accumulated subenergy \( s_{i\ell} \) is already \( 3 \text{ GeV}^2 \) on the average. Hence only two successive neutral particles are required to produce the appearance of a large subenergy between adjacent particles. If it is further assumed that the probability of a neutral particle is \( 1/3 \) per particle and uncorrelated between adjacent particles, then we estimate the average multiplicity of neutral gaps to be \( (1/3)^2 \) times the average multiplicity, i.e.,
\[ \bar{n}_{II} = 0.1(\bar{n} - 3) \]  \hspace{1cm} (7)
(At least four produced particles are required to make two fireballs, each containing at least two particles, in addition to two neutral particles; hence the 3.)

These two mechanisms, \( P \) exchange and neutral gaps, constitute our model for producing fireball events, according to our second definition in terms of gaps in momentum space. Their combined probability is
\[ \bar{n}_P + \bar{n}_{II} = 0.2\bar{n} - 0.9 \]  \hspace{1cm} (8)

When only the angles of the secondaries are measured, one must resort to our first definition in terms of gaps in \( \log \tan \theta \). We show below that
for particles with average transverse momenta, the two definitions are equivalent. We only need add to the above probability the probability that an average subenergy and abnormal transverse momenta produce a gap in $\log \tan \theta$.

The average spacing in $\log \tan \theta$, i.e., the average value of 

$$\bar{k} = \log\left(\frac{p_{||j}}{p_{||i}} \cdot \frac{p_{||i}}{p_{||j}}\right)$$

is easily estimated from the expression for the average multiplicity. The total length of the $\log \tan \theta$ plot is $\log s + \text{const.}$ Therefore with $\bar{p}_{||j} \approx \bar{p}_{||i}$, $\bar{n} \bar{k} \approx \log s + \text{const.}$ Hence

$$\bar{k} \approx \log\left(\frac{p_{||j}}{p_{||i}}\right) \approx \frac{(\log_{10} e)}{a} = 0.38.$$ (9)

To estimate the spacing when $s_{ij} > 3 \text{ GeV}^2$, we can either use the rule of thumb, derived in the discussion of neutral-particle gaps, that three average gaps equal a $P$ gap, or we can calculate the spacing directly, using the expression for $s_{ij}$ in terms of the momenta, when $p_{||j} \gg p_{||i} \gg p_{||i}, p_{||j}$:

$$s_{ij} \approx \frac{p_{||j}}{p_{||i}^2 w_i^2},$$ (10)

where $w_i = (p_{||i}^2 + m_i^2)^{1/2}$. Then for $p_{||} \gg m_i$,

$$k \approx \log(s_{ij}/w_i w_j).$$ (11)

If we use a typical experimental value $\overline{p}_{||}^2 = 0.15$, then when $s_{ij} > 3 \text{ GeV}^2$, $k \approx 1.3$.

For an average subenergy $s_{ij} \approx 0.5$ the gap exceeds 1.3, only when $w_i w_j < 0.025$ [cf. Eq. (11)]. Assuming that the distribution in $p_{||}^2$ is approximately uncorrelated with $s_{ij}$ and given by

$$\frac{dN}{dp_{||}^2} = \exp\left(-p_{||}^2/0.15\right),$$ (12)

the probability of such an occurrence is about 5% per particle. However, this result is highly sensitive to the subenergy in this range. For $s_{ij} = 0.8$ (a value less likely than 0.5 by a factor of about two) the probability is 20%. In order of magnitude the average number of gaps from this source is
\[ \bar{n}_{III} \approx 0.1(\bar{n} - 1) \]  

The net probability for gaps in log tan \( \theta \) is then roughly \( 0.3\bar{n} - 1.0 \) for \( \bar{n} \geq 5 \).

Although the multiperipheral model predicts that for individual events there may be gaps in the distributions, the combined distribution of many events will not have dips. The gaps occur with equal frequency anywhere along the chain. We are encouraged by the observation reported by Dobrotin and Slavatinsky\(^1\) that in cosmic ray events at energies in the range \( 100-1000 \text{ GeV} \) a marked asymmetry is observed in the particle distribution in the center of mass for some events. Moreover, the multiperipheral model tends to agree with their observations that some of the secondaries in the "decay" of the fireball have abnormally large energies in the fireball center of mass. These would correspond to the left- and right-most particles in the fireball group on the multiperipheral chain.

Furthermore, our results are not necessarily in disagreement with the detailed statistical analysis of Gierula et al.\(^1\) for bimodality in the log tan \( \theta \) spectrum from emulsion experiments. A positive D-test for bimodality is not a positive test for two separate peaks. A trapezoidal distribution gives a positive D-test for bimodality.

Because of the attractiveness of the two-fireball idea, many experimenters have hitherto focused their attention on the structure of the fireball clusters and on the separation of the "centers" of the clusters. From the standpoint of the multiperipheral model, however, a frequency distribution of gap sizes would be a useful analytical tool.
FOOTNOTES AND REFERENCES

This work was supported in part by the U.S. Atomic Energy Commission.


2. The two-particle subenergy is the invariant $s_{ij} = (p_i + p_j)^2$.

3. For a discussion of this point, see C. E. DeTar, unpublished (1970).


7. Czyżewski and Krzywicki have shown that an idealized multiperipheral model, which treats final particles independently and gives a rectangular distribution in log tan $\theta$ on the average, can, through statistical fluctuations, account for the experimental data. [See O. Czyżewski and A. Krzywicki, Nuovo Cimento 30, 603 (1963).] We are concerned here with two-particle correlations.

8. The numbers 1.3 in log tan $\theta$ and 3 GeV$^2$ in subenergy were chosen so as to be roughly compatible when $p_\perp \approx 0.4$ GeV/c.

9. The multiperipheral model of the P. N. Lebedev Institute also allows for the production of fireballs. In their model, however, pion rather than P exchange divides the fireballs. Thus the gaps would presumably be quite small. [See V. N. Akimov, D. S. Chernavskii, I. M. Dremin, and


11. University of Michigan experiment by L. W. Jones, D. Lyon, et al. to be published in Phys. Rev. We thank D. Lyon for supplying us with this data before publication.

12. Z. Ajduk, L. Michejda, and W. Wójcik, Further Results from the CERN Model Applied to \( \pi p \rightarrow p5\pi \) Reaction at 8 GeV/c, INR (Warsaw) preprint 1088/VI/PH.

13. In comparing this with emulsion data, it must be kept in mind that multiplicities tend to be higher in collisions with nuclei.


15. J. Gierula, M. Mięsocwicz, and P. Zieliński, Nuovo Cimento 18, 102 (1960); J. Gierula and E. Wojner, INR (Warsaw) preprint 1126/VII/PH.

FIGURE CAPTIONS

Fig. 1. Multiperipheral diagram for the process \( A + B \rightarrow a + b + \cdots + z \).

Fig. 2. Right scale: Average multiplicity of Pomeranchuk exchange per inelastic event, \( \bar{n}_p \); left scale: average (produced) pion multiplicity per inelastic event, \( \bar{n} \) vs energy for \( \pi\pi \) collisions, as predicted by the schizophrenic pomeron model (see Ref. 6).

Fig. 3. Average multiplicity of Pomeranchuk exchange per inelastic event vs average (produced) pion multiplicity per inelastic event.
Fig. 3
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