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Author
Cronin, J.W.

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REPORT OF THE WORKING GROUP ON CP VIOLATION AND RARE DECAYS

J.W. Cronin
Department of Physics, University of Chicago, Chicago, IL 60637

N.G. Deshpande
Department of Physics, University of Oregon, Eugene, OR 97403

G.L. Kane
Department of Physics, University of Michigan, Ann Arbor, MI 48109

V.C. Luth and A.C. Odian
Stanford Linear Accelerator Center, P.O. Box 4349, Stanford, CA 94305

M.E. Machacek
Department of Physics, Northeastern University, Boston, MA 02115

F. Paige
Brookhaven National Laboratory, Upton, Long Island, NY 11973

M.P. Schmidt and J. Slaughter
Department of Physics, Yale University, New Haven, CT 06511

G.H. Trilling
Lawrence Berkeley Laboratory, University of California, Berkeley, CA 94720

M. Witherell
Department of Physics, University of California, Santa Barbara, CA 93106

S.G. Wojcicki
Department of Physics, Stanford University, Stanford, CA 94305

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I. General Introduction

It has been pointed out\(^1\) that, with its high energy and luminosity, the SSC may provide the best or only way in which CP violation in heavy meson decays or the rare decay modes of such mesons can be observed. The major problem in the exploitation of the high rates of heavy quark production is the identification of interesting decays in the midst of a large background of more conventional processes. There have been some optimistic reports on the feasibility of such experiments,\(^2\) but relatively little quantitative backup has been provided.

In the present report, we concentrate exclusively on B-meson decays. As is the case for K mesons, but not for charm or top decays, the favored modes are suppressed by the smallness of Cabibbo-Kobayashi-Maskawa angles, and therefore rare modes are relatively more frequent and potentially easier to observe.

Section II is theoretical. Part A gives a brief discussion of rare modes and part B provides a fairly detailed analysis of mixing and CP violation with particular emphasis on what can be measured. Section III discusses experimental issues, and Section IV gives a brief summary. Although the group listed above as authors participated in the discussions, the written material is largely due to G. Kane (Section II-A), M. Machacek (Section II-B), V. Luth and Jean Slaughter (Section III-A), F. Paige and G. Trilling (Sections III-B, C, D and IV). The brief discussion in III-E is based on work of J. Cronin which is described in more detail elsewhere in these proceedings.

II. Theoretical Considerations

A. Rare B Decays

There are three major categories of interesting decays; namely, (1) rare decays for which there is a standard model rate prediction, (2) decays forbidden by the standard model, and (3) decays which may allow the study of CP violation.

One should keep in mind that for all these classes of decays, effects much larger than those predicted by the Standard Model may enter. We briefly consider all of these categories in this section, and then provide in the next section a much more detailed discussion of CP violation phenomenology in B decay.

1. Rare Decays Allowed by the Standard Model

a) \(B_u \rightarrow \tau v\). This decay which proceeds via the annihilation diagram shown in Fig. 1a has a rate proportional to \(f_B^2 |U_{Bu}|^2\). The KM matrix element \(U_{Bu}\) is known to be \(< 0.006\)^3 The branching ratio is less than \(10^{-4}\), and there is uncertainty in \(f_B\) and possible background at some level from \(B_C \rightarrow \tau v\). However \(U_{Bu}\) is a fundamental parameter, and only this method, and the study of B production at large \(X\) in \(\tau v\) reactions, also difficult, are promising ways of measuring it.

b) \(B \rightarrow K^{*+} \pi^-\). The relevant diagram, shown in Fig. 1b, is an important one-loop correction in the Standard Model. The branching ratio is estimated to be \(10^{-5}\).

c) \(B \rightarrow \tau^+ \tau^-\). This mode is analogous to \(K_L \rightarrow \mu^+ \mu^-\). Since the rate is proportional to \(M_{\tau}^2\), it is enhanced by a factor \((M_\pi/M_{\tau})^2\approx 300\), and the expected branching ratio is about \(3 \times 10^{-6}\). The \(\mu^+ \mu^-\) and \(e^+ e^-\) final states are expected with Standard Model branching ratios of \(10^{-8}\) and \(10^{-12}\) respectively.

2. Forbidden Decays. We can list decays which, while forbidden by the Standard Model, occur at interesting levels in some model which goes beyond. Detection of any of them would mean the discovery of a new effect not presently observed in nature. Examples of such decays include:

\(B \rightarrow \mu e, \tau v\)

\(B \rightarrow \mu e, \tau v, K\mu, K\tau\)

3. CP Violation in B Decay. To study CP violation, one can aim for several kinds of effects.

a) Search for like-sign dileptons as a signature of \(B^+ \rightarrow \tau^+\) mixing, and compare \(\tau^+ \pi^-\) with \(e^+ \pi^-\) rates.

b) Study decays into exclusive modes to which both \(B^+\) and \(B^-\) can decay. Examples are \(K\pi K\pi\).

c) Within the Standard Model, one would expect equality of \((B \rightarrow D^{*}\pi)\) and \((B \rightarrow D\pi^*)\). However other ways of generating CP violation might lead to significant differences in these rates.

\[\text{b) } B \rightarrow K^{*+} \pi^-\]  
\[\text{c) } B \rightarrow \tau^+ \tau^-\]

Fig. 1. Relevant diagrams for (a) \(B \rightarrow \tau v\) and (b) \(B \rightarrow K^{*+} \pi^-\) decay.
B. Theoretical Overview of Mixing and CP Violation in the BB System

1. Introduction. We review the definitions of basic parameters necessary for the description of mixing and CP violation for B mesons to establish our notation. We also review estimates of these parameters in the standard Kobayashi-Maskawa (KM) model which take account of recent mixing angle measurements based on \( \frac{1}{r(b \rightarrow u)} < .05 \) and the long B lifetime, \( \tau_B = 10^{-12} \text{ sec.} \)

The standard model predictions provide a baseline for estimates of the size of mixing and CP violation effects in the BB system. We then discuss two experimental methods to search for these effects:

i) searches for like-sign dileptons from B meson semileptonic decays \( 4^5 \);

ii) searches for CP-violation effects through final-state interactions \( 5^6, 7 \).

We pay particular attention to the time dependence of asymmetry parameters in ii) where the effects are expected to be largest.

In direct analogy with the \( K^* - K^0 \) mesons, the \( B^* - B^0 \) mesons produced in hadron collisions by strong interactions are not eigenstates of the full Hamiltonian \( H \).

For each \( B^* - B^0 \) meson type the CPT theorem and hermiticity restrict the form of the resulting matrix, \( H \), as given by

\[
H(B^*) = \left( \begin{array}{c}
M - \frac{i}{2} \Gamma_B & M_{12} - \frac{i}{2} \Gamma_{12} \\
M_{12}^* - \frac{i}{2} \Gamma_{12}^* & M + \frac{i}{2} \Gamma_B
\end{array} \right)
\]

In principle, diagrammatic and operator analyses may be used to calculate these matrix elements from the underlying theory. Upon diagonalization of \( H \), the mass eigenstates \( B_1(B_2) \) with masses \( m_1(m_2) \) and decay rates \( \Gamma_1(\Gamma_2) \), respectively, are mixtures of \( B^* \) and \( B^0 \) parametrized by

\[
B_1,2(t) = \frac{(1+\epsilon_b)B^*(1-\epsilon_b)B^0}{\sqrt{2(1+|\epsilon_b|^2)}} \exp\left(\frac{-it(m_j+i\Gamma_j)}{2}\right)
\]

where \( \epsilon_b \) is the CP impurity parameter and

\[
\frac{(1+\epsilon_b)}{(1-\epsilon_b)} = \frac{M_{12}^* - i\Gamma_{12}^*}{M_{12} - i\Gamma_{12}}.
\]

If we denote an initially \( t=0 \) pure \( B^*(B^0) \) state which has evolved to some time \( t \) by \( |B^*(t)\rangle = |B^0(t)\rangle \), respectively, then,

\[
|B^*(t)\rangle = \frac{1}{\sqrt{2(1-|\epsilon_b|^2)}} \left\{ (1+\epsilon_b) \exp(-it(m_1+i\Gamma_1)) |B^1(t)\rangle + \exp(-it(m_2+i\Gamma_2)) |B^0(t)\rangle \right. \]

\[
\left. + (1-\epsilon_b) \exp(-it(m_1+i\Gamma_1)) |B^1(t)\rangle \right. \]

\[
\left. + \exp(-it(m_2+i\Gamma_2)) |B^0(t)\rangle \right. \]

\[\text{and} \]

\[
|B^0(t)\rangle = \frac{1}{\sqrt{2(1-|\epsilon_b|^2)}} \left\{ (1-\epsilon_b) \exp(-it(m_2+i\Gamma_2)) |B^1(t)\rangle \right. \]

\[
\left. + \exp(-it(m_1+i\Gamma_1)) |B^0(t)\rangle \right. \]

\[
\left. + (1-\epsilon_b) \exp(-it(m_1+i\Gamma_1)) |B^0(t)\rangle \right. \]

\[
\left. + \exp(-it(m_2+i\Gamma_2)) |B^1(t)\rangle \right. \]

Now define,

\[
\Delta m = m_1 - m_2,
\]

\[
m = m_1 + m_2,
\]

\[
\Delta \Gamma = \Gamma_1 - \Gamma_2
\]

and\( \Gamma_B = \Gamma_1 + \Gamma_2 \).

In the standard KM model \( 4^5.9 \) for the \( B_d-B_d \) system and \( \Delta \Gamma < 1 \) for the \( B_s-B_s \) system. Thus to good approximation these terms may be neglected and, in the following analysis, we assume \( \Delta \Gamma = 0 \). It is convenient to define mixing parameters \( x_d \) and \( x_s \) for the \( B_d \) and \( B_s \) neutral meson systems, respectively, where

\[
x_j = \frac{2(\Delta m)}{t_j}, \quad j = d, s.
\]

If all proper times are measured in units of the average B lifetime \( 2/\Gamma \), equations (4a-4b) take the simple form,

\[
|B^*(t)\rangle = \exp\left(\frac{-it}{2}\right) \cos\left(\frac{x_j t}{2}\right) |B^j(t)\rangle
\]

\[
+ \exp\left(\frac{-it}{2}\right) i\frac{(1-\epsilon_b)}{(1+\epsilon_b)} \sin\left(\frac{x_j t}{2}\right) |B^j(t)\rangle
\]

\[
|B^j(t)\rangle = \exp\left(\frac{-it}{2}\right) \left(\frac{(1+\epsilon_b)}{(1-\epsilon_b)} \sin\left(\frac{x_j t}{2}\right) |B^j(t)\rangle \right. \]

\[
+ \cos\left(\frac{x_j t}{2}\right) |B^j(t)\rangle \right. \]

The mixing parameters \( x_d \), \( x_s \) are strongly dependent on the evaluation of the hadronic matrix element. Estimates in the literature yield \( x_d \approx 1.4 - 4 \) and \( x_s \approx 5 - 1 \) where the larger mixing in each case is derived from the vacuum saturation approximation and the smaller mixing from the bag model hadronic wave functions.

2. Searches in Semileptonic B Meson Decays. One possible experimental means of probing the mixing and CP violation effects might be through a careful study of like-sign dileptons produced in semileptonic decays of BB pairs. Let \( N^+, N^- \), \( N^* \), and \( N^* \) denote the numbers of events in which the BB system decays into two leptons of the specified charges. Then,

\[
\Gamma_2 = \frac{N^{++} + N^{--}}{N^{++} + N^{--} + N^{*+} + N^{*-}}
\]

(7)

describes the amount of mixing in the system.
a \equiv \frac{N_+^-N_-^-}{N_+^+N_-^+} \quad (8)

is a direct measure of CP violation in the system, (provided that the rates of $B_i \bar{B}_i$ and $\bar{B}_i B_j$ are exactly equal) and the overall lepton asymmetry

$$A \equiv \frac{N_+^-N_-^-}{N_+^+N_-^+},$$  \quad (9)

with $N_\pm$ the total number of $\pm$ leptons, is a combination of both. We expect $b$ quarks to hadronize into $a u(bu), a d(bd)$, and $a s(bd)$ in the approximate ratio of $2:2:1$. Thus equations (6) should be used to evaluate equations (7)-(9) separately for each possible meson pairing, $B_i B_j$, $B_i B_j$, $\bar{B}_i B_j$, $B_i B_j$ $j=s,d$ and $B_i B_j + B_i B_j$. The mixing and asymmetry parameters vanish for $B_i B_j$ since non-neutral states do not mix. If the charge of the $a$ meson can be determined, then the asymmetry parameters for the combined $B_i B_j$ and $B_i B_j$ systems are, to good approximation,

$$r_2 = \sin^2 \left( \frac{x t}{2} \right),$$  \quad (10a)

$$a = \frac{-4R e B}{1 + |\epsilon| B^2},$$  \quad (10b)

and

$$A = \frac{-4R e B}{1 + |\epsilon| B^2} \sin^2 \left( \frac{x t}{2} \right),$$  \quad (10c)

where, in $A$, only leptons from $B_i$ and $\bar{B}_i$ are included and $t$ is the proper decay time (measured in units of average $B^-$ meson lifetime) of the neutral $\bar{B}_i (B_j)$ meson, and we have assumed equal production of $B_i \bar{B}_j$ and $B_i B_j$. For standard model estimates of $x_j$, like-sign dilepton pairs will equal unlike-sign dilepton pairs in this system for $t \geq 1.5$ lifetimes for $j=s$. The CP violation asymmetry $a$ is, however, independent of time and directly proportional to $R e B$. From equation (3) we see that $R e B$ vanishes in the limit that $M_{12}$ and $\Gamma_{12}$ have equal phases. In the standard KM model calculation the leading contributions to $M_{12}$ and $\Gamma_{12}$ have the same phase. $R e B$ is a higher order effect and thus small, $10^{-2}$ to $10^{-3}$ for $j=d$ and much smaller yet for $j=s$. Thus even for optimal decay time, the total lepton charge asymmetry $A$ is small. If only integrated rates are measured, the situation worsens

$$r_2 = \frac{x_j^2}{2(1 + x_j^2)} \approx -0.01 \pm 0.07 \quad j = d \quad (11a)$$

and

$$A = \frac{-2x_j^2 R e B}{(1 + x_j^2)(1 + |\epsilon| B)^2} \leq 10^{-3} \quad (11b)$$

For completely neutral $B \bar{B}$ systems we have the added complication that the meson pairs are produced in coherent C-even or C-odd states depending on the production mechanism. Thus the state function at any time $t$ is given by

$$|B_j(t), k; B_j(t), k' \rangle \times (-1)^C |B_j(t), k; B_j(t), k' \rangle \quad (12)$$

For example, in the $B_d - B_d$ meson system $r_2$ becomes

$$r_2 = \begin{cases} \sin^2 \left( \frac{x_d t}{2} \right) & C \text{ even} \\ \sin^2 \left( \frac{x_d (t_1 - t_2)}{2} \right) & C \text{ odd} \end{cases} \quad (13a)$$

where $t = \frac{1}{2} (t_1 + t_2)$ is the average decay time and $t_1 - t_2$ is the difference in decay times for the meson pair. The parameter $a$ is unchanged from the previous case and $A = ar_2$. When integrated over $t_1$ and $t_2$ we find

$$r_2 = \begin{cases} \frac{x_d^2}{2} \frac{3x_d^2}{(1 + x_d^2)^2} - 0.03 \pm 0.19 & C = \text{ even} \\ \frac{x_d^2}{2(1 + x_d^2)^2} - 0.01 \pm 0.07 & C = \text{ odd} \end{cases} \quad (13b)$$

which is comparable to equation (11). Therefore, while mixing may be visible through detection of like-sign dileptons from $B-\bar{B}$ pairs, the observation of CP violation in this channel requires an experiment of great precision. This may prove particularly difficult in pp machines, such as the SSC, where an initial state charge asymmetry already exists.

3. Searches for CP violation in Nonleptonic final state interactions. A more promising method to study CP violation is to exploit such effects originating in $B$ meson nonleptonic final states. The basic idea is to pick a final state $f$ common to both $B^-$ and $\bar{B}^-$ decays. Mixing causes the two amplitudes to interfere, making the CP violation in the final state interaction observable. Some possible common final states $f$ are

$$B_d (\bar{B}_d) \longrightarrow \psi K^-$$

$$D_0 (D^0) \psi ' s \longrightarrow K^- \psi ' s$$

$$D\bar{D}K^- + \psi ' s$$

$$F\bar{F}K^- + \psi ' s$$

and

$$B_s (\bar{B}_s) \longrightarrow \psi \phi$$

$$\psi K^-$$

$$\psi F$$

Although the following analysis applies to any such state $f$, we focus on the decay $B_d (\bar{B}_d) \psi K^-$ since this two-body mode permits a complete reconstruction of the final state. Following the notation of Bigi and Sanda we may then define

$$M_f = \left< f \mid H \mid B_d^* \right>$$

and

$$M_f = \left< f \mid H \mid \bar{B}_d^* \right> .$$

For CP violation, $M_f \neq \bar{M}_f$. The CP violation effect can be parametrized by a phase

$$\lambda = \frac{M_f (1 + |\epsilon|)}{M_f (1 - |\epsilon|)} = e^{-2i\phi}$$

with $|\lambda| = 1$. In the standard KM model this is

\begin{align*}
B_d (\bar{B}_d) & \longrightarrow \psi K^- \\
D_0 (D^0) \psi ' s & \longrightarrow K^- \psi ' s \\
D\bar{D}K^- & + \psi ' s \\
F\bar{F}K^- & + \psi ' s \\
B_s (\bar{B}_s) & \longrightarrow \psi \phi \\
\psi K^- & \\
\psi F & \\
\psi & 
\end{align*}
\[
\sin \phi_d = \frac{S_3 \sin \alpha}{\sqrt{S_3^2 + S_2^2 + 2S_2S_3 \cos \alpha}}
\]

and

\[
\sin \phi_s = 0 + 0 \left( \frac{m_c}{m_t}, S_2 \right)
\]

and

\[
\text{Im} \lambda_j = \begin{cases} 
0.3 & \text{if} \ j = d \\
0 & \text{if} \ j = s
\end{cases}
\]

As in section 2 we analyze each $B \bar{B}$ meson pair type independently. For $B_\mu \bar{B}_\mu$, there is obviously no mixing and thus no effect. For $B_j \bar{B}_j$ and $B_j \bar{B}_\mu$, the charge on $B_j (\bar{B}_j)$ prevents mixing with the neutral partner. Thus these pairs act like an incoherent source of $B_j$ or $\bar{B}_j$, $j=d,s$. The charged mode is tagged by its semileptonic decay at time $t_1$. This identifies its partner as $B_j$ or $\bar{B}_j$ at $t = 0$. This neutral partner decays to final state $f$ at time $t_2$. If we define an asymmetry parameter

\[
A_f = \frac{\sigma(\ell^-,f) - \sigma(\ell^+,f)}{\sigma(\ell^+,f)}
\]

and use equations (6) we find

\[
A_f = \text{Im} \lambda_j \sin(x_jt_2)
\]

In Fig. 2, we show the dependence of $\sigma(\ell^-,f)$ and $\sigma(\ell^+,f)$ on time, for the value $\text{Im} \lambda = 0.3, x = 0.4$.

For small $x$, $t \approx 2x$, a time which is very comfortable from the point of view of vertex detection. If the decay time $t_2$ is integrated from a minimum value $t_2 = 0$, the asymmetry parameter becomes

\[
A_f = \frac{\text{Im} \lambda_j}{(1 + x_j^2)} \cdot [x_j \cos(x_jt_2) + \sin(x_jt_2^*)]
\]

Indeed, it will generally be necessary to keep $t_1$ and $t_2$, the minimum allowable $B$ meson decay times greater than zero by an amount determined by the detector spatial resolution to establish that $B$ decay secondaries are being observed. In the KM model, the integrated asymmetry $(j=d)$ ranges from

\[
A_f = 0.03 - 0.06 \quad \text{for} \quad \lambda = 0.1
\]

and

\[
A_f = 0.1 - 0.2 \quad \text{for} \quad \lambda = 0.4
\]

and is negligible for $B_s$. 

Fig. 2: Relative rates for $\sigma(\ell^-,f), \sigma(\ell^+,f)$ for $\text{Im} \lambda = 0.3, x = 0.4$ (dashed) and $\text{Im} \lambda = 0$ (solid).
Again integrating from \( t_2 \) to \( \infty \), we find

\[
A_f = \text{Im} \left[ \frac{2x_j}{(1+x_j^2)^2} \cos(x_j(t_1^*+t_2)) \frac{1-2x_j^2}{(1+x_j^2)^2} \sin(x_j(t_1^*+t_2)) \right] \]

\[
\rightarrow \frac{2 \text{Im} x_j}{(1+x_j^2)^2} \text{ for C even,} \quad (19a)
\]

and

\[
A_f = \text{Im} \left[ \frac{x_j}{(1+x_j^2)^2} \sin(x_j(t_1^*-t_2^*)) \right] \]

\[
\rightarrow 0 \quad \text{for C odd.} \quad (19b)
\]

Representative values of the integrated asymmetries in the KM model for \( x_d=0.1 \) and 0.4 and for various values of \( t_1^*, t_2^* \) are listed in Table I.

<table>
<thead>
<tr>
<th>( t_1^* )</th>
<th>( t_2^* )</th>
<th>(( x_d=0.1 ))</th>
<th>C even (( x_d=0.1 ))</th>
<th>C odd (( x_d=0.4 ))</th>
<th>C even (( x_d=0.4 ))</th>
<th>C odd (( x_d=0.4 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.06</td>
<td>0</td>
<td>0.08</td>
<td>0</td>
<td></td>
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<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.09</td>
<td>0</td>
<td>0.24</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>0.10</td>
<td>-0.015</td>
<td>0.25</td>
<td>-0.06</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
<td>0.10</td>
<td>0.015</td>
<td>0.25</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.12</td>
<td>0</td>
<td>0.26</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

The treatment of the asymmetry parameter for the remaining neutral pairs, \( B_s^-B_s^+(-1)C B_d^-B_d^+ \), is similar to that for \( B_d^-B_d^+ \) except that two different sets of mixing parameters are involved. We assume that \( B_s \) and \( B_s \) decay into \( f \) are suppressed to a negligible level. The asymmetry parameters are then

\[
A_f = \text{Im} \frac{x_d}{(1+x_d^2)^2} \cos(x_d(t_2^*-x_1^*)) \sin(x_d(t_1^*+x_2^*)) \]

\[
\rightarrow 0 \quad \text{for C even,} \quad (20a)
\]

and

\[
A_f = \text{Im} \frac{x_d}{(1+x_d^2)^2} \sin(x_d(t_2^*-x_1^*)) \]

\[
\rightarrow 0 \quad \text{for C odd.} \quad (20b)
\]

Again integrating from \( t_1^* \) over the lepton decay times

\[
A_f = \text{Im} \frac{x_d}{(1+x_d^2)^2} \left[ \frac{x_d}{(1+x_d^2)^2} \cos(x_d(t_2^*+x_2^*)) \sin(x_d(t_2^*+x_2^*)) \right] \]

\[
\rightarrow \frac{2 \text{Im} x_d}{(1+x_d^2)^2} \text{ for C even,} \quad (21a)
\]

for C even.

\[
A_f = \frac{\text{Im} x_d}{(1+x_d^2)^2} \left[ -x_d \cos(x_d(t_2^*-x_1^*)) + \sin(x_d(t_2^*-x_1^*)) \right] \quad (21b)
\]

for C even.

Finally integrating over \( t_2 \) from \( t_2^* \) to \( \infty \) we complete this set of time-dependent asymmetry parameters with

\[
A_f = \frac{\text{Im} x_d}{(1+x_d^2)^2} \left[ -x_d \cos(x_d(t_2^*-x_1^*)) + \sin(x_d(t_2^*-x_1^*)) \right] + (1-x_d^2) \sin(x_d(t_2^*-x_1^*)) \]

\[
\rightarrow \frac{\text{Im} x_d}{(1+x_d^2)^2} \quad (22a)
\]

for C even, and

\[
A_f = \frac{\text{Im} x_d}{(1+x_d^2)^2} \left[ -x_d \cos(x_d(t_2^*-x_1^*)) + \sin(x_d(t_2^*-x_1^*)) \right] + (1-x_d^2) \sin(x_d(t_2^*-x_1^*)) \]

\[
\rightarrow \frac{\text{Im} x_d}{(1+x_d^2)^2} \quad (22b)
\]

for C odd. Representative values are listed in Table II.

<table>
<thead>
<tr>
<th>( t_1^* )</th>
<th>( t_2^* )</th>
<th>(( x_d=0.1 ))</th>
<th>(( x_d=0.4 ))</th>
<th>(( x_s=0.5 ))</th>
<th>(( x_s=1 ))</th>
</tr>
</thead>
<tbody>
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<td>0.142</td>
<td>-0.095</td>
<td>0.181</td>
<td>-0.078</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.203</td>
<td>-0.143</td>
<td>0.188</td>
<td>-0.128</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
<td>0.240</td>
<td>-0.194</td>
<td>0.138</td>
<td>-0.184</td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
<td>0.211</td>
<td>-0.131</td>
<td>0.173</td>
<td>-0.095</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.118</td>
<td>-0.184</td>
<td>0.107</td>
<td>-0.166</td>
</tr>
</tbody>
</table>

In all of the above calculations we have assumed that the different B meson types are experimentally distinguishable. In practice this may be difficult. We therefore calculate an average CP violating asymmetry \( A_f \) where each asymmetry \( A_f \) is weighted by the fraction of the relative production of a given meson pair type. We assume that C even and C odd states are equally likely and that hadronization with u, d and s quarks is in the ratio 2:2:1. From equations (15), (18), and (21) we find (integrating over \( t_1^* \))

\[
A_f = \frac{\text{Im} x_d}{(1+x_d^2)^2} \left[ -x_d \cos(x_d(t_2^*-x_1^*)) + \sin(x_d(t_2^*-x_1^*)) \right] + (1-x_d^2) \sin(x_d(t_2^*-x_1^*)) \]

\[
\rightarrow \frac{\text{Im} x_d}{(1+x_d^2)^2} \quad (23a)
\]

for C even.

Finally integrating over \( t_2 \) from \( t_2^* \) to \( \infty \), we complete this set of time-dependent asymmetry parameters with

\[
A_f = \frac{\text{Im} x_d}{(1+x_d^2)^2} \left[ -x_d \cos(x_d(t_2^*-x_1^*)) + \sin(x_d(t_2^*-x_1^*)) \right] + (1-x_d^2) \sin(x_d(t_2^*-x_1^*)) \]

\[
\rightarrow \frac{\text{Im} x_d}{(1+x_d^2)^2} \quad (23b)
\]
other sources of CP violation. Much larger effects based on the KM model of CP violation. If there are equality. For very small effects such as expected by the standard KM model. We have assumed exact equality of \( \sin \theta \) and CP violation. From gluon-gluon collisions would lead to the above values of the mixing angle. Taking \( \sin \theta = 0.3 \), and two choices of \( x_d \) and \( x_s \), we obtain the following estimates for \( A_f \):

\[
A_f = \frac{1}{4} \sin x_d \sin x_s [2(1+x_s^2)(1+x_s^2) + (1+x_s^2)(1+x_s^2)]
\]

Equation (24) has the same shape as the asymmetry depicted in Fig. 2, with an amplitude reduced by about a factor of 2. There is relatively little dependence of \( A_f \) on \( \Theta \) since the \( B_s \) production dominates the asymmetry. From (24), using \( \sin \theta = 0.3 \), and \( x_d \) and \( x_s \), we obtain the following estimates for \( A_f \):

\[
A_f = 0.28 \sin x_d \sin x_s = 0.088 \text{ at } t_2 = 1 \text{ lifetime for } x_d = 0.1, x_s = 0.5 \text{ and,}
A_f = 0.25 \sin x_d \sin x_s = 0.108 \text{ at } t_2 = 1 \text{ lifetime for } x_d = 0.4, x_s = 1.
\]

Finally, the average integrated CP violation asymmetry for \( f_{BB}K \) can be found in the same way as equation (23) from equations (16), (19), and (22). Representing roughly the effect of a vertex detector by taking \( t_2 = 1 \text{ lifetime for } x_d = 0.1, x_s = 0.5 \text{ and,}
A_f = 0.25 \sin x_d \sin x_s = 0.108 \text{ at } t_2 = 1 \text{ lifetime for } x_d = 0.4, x_s = 1.
\]

With respect to the SSC, these calculations suggest an a-priori two-order-of-magnitude advantage for operation in the collider mode as opposed to the fixed target mode. It is conceivable that the acceptance in a fixed-target experiment can be better for less cost, but it seems doubtful that the two orders of magnitude can be regained. For this reason, we have chosen to emphasize experiments in the SSC collider mode.

Table III. Produced \( BB \) Rates from Various Sources

<table>
<thead>
<tr>
<th>Source</th>
<th>Energy (GeV)</th>
<th>( \sigma ) (mb)</th>
<th>Total Rate (Hz)</th>
<th>( f_B )</th>
<th>( BB ) in ( 10^7 \text{ sec} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>TeV I</td>
<td>45</td>
<td>50</td>
<td>( 10^7 )</td>
<td>( 10^{-6} )</td>
<td>( 10^8 )</td>
</tr>
<tr>
<td>fixed target</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SSC</td>
<td>200</td>
<td>80</td>
<td>( 10^7 )</td>
<td>( 10^{-5} )</td>
<td>( 10^9 )</td>
</tr>
<tr>
<td>fixed target</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TeV I</td>
<td>2000</td>
<td>100</td>
<td>( 10^5 )</td>
<td>( 10^{-4} )</td>
<td>( 10^8 )</td>
</tr>
<tr>
<td>PP Collider</td>
<td>40000</td>
<td>200</td>
<td>( 10^7 )</td>
<td>( 10^{-3} )</td>
<td>( 10^{11} )</td>
</tr>
<tr>
<td>LEP/SLC</td>
<td>92</td>
<td>3 \times 10^{-5}</td>
<td>0.3</td>
<td>0.14</td>
<td>( 4 \times 10^5 )</td>
</tr>
<tr>
<td>( e^+e^- ) Collider</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

B. Some General Rate Considerations

We consider \( BB \) production by the collider in the central region which we define as the angular interval about the beam line,

\[ 30^\circ < \phi < 150^\circ \]

We assume the existence of a detector which covers this polar angle range and the full azimuth, and idealize the luminous region as a line source of transverse width, \( \sigma = 7 \text{ \mu m} \) and length a few cm. 

4. Concluding Remarks. In conclusion, we note the following:

a) The CP violating effects that we have discussed all require that both \( B \) and \( \bar{B} \) decays from the same production event be detected.

b) We have assumed exact equality of \( B_s \) production. This is undoubtedly violated at some level in an initial PP state although the dominant production from gluon-gluon collisions would lead to the above equality. For very small effects such as expected dilepton asymmetries the issue of how well the above assumption is fulfilled may become important.

c) All the CP violating effects discussed in this section require both significant mixing (\( x_d \)) and CP violation (\( R_{eq40} \) or \( |m| \)). The estimates given are based on the KM model of CP violation. If there are other sources of CP violation, much larger effects could arise.
We base our cross section information on the ISAJET program. Since the relevant x values for B \( B \) production are extremely small at 40 TeV, the theoretical predictions are very uncertain. However, the numbers seem plausible, and we use them.

The overall \( B \) \( B \) cross section at 40 TeV is estimated to be about 220 \( \mu \text{b} \). To provide a first level trigger, to have and to have \( B \) mesons which are not too soft (we have to detect their finite flight paths in a vertex detector), we require the transverse momentum of each \( B \)-jet to be greater than 10 GeV/c. Some relevant cross sections and multiplicities are given in Table IV. For the purposes of the discussion, we have treated \( D \) mesons as having zero lifetime, and given in Table IV only the stable charged multiplicity. It is clear from Table IV that the angular and \( p_T \) cuts already reduce the effective cross section by a factor of 200.

We now consider the problem of \( B \) recognition through use of a high resolution vertex detector. The actual reconstruction of separated vertices in a multi-hadron environment is an extremely difficult problem compounded in the case of \( B \) decay by the fact that usually out of a total average of 5 charged secondaries, half go with the \( B \) vertex and the other half with a separate \( D \) vertex. However, a more straightforward procedure is the observation of finite impact parameters for decay tracks with respect to the beam line. The distribution of impact parameters is very broad with a mean value of the order of 5 cm. For the purposes of the discussion, we have treated \( B \) mesons as having zero lifetime, and given in Table IV only the stable charged multiplicity. It is clear from Table IV that the angular and \( p_T \) cuts already reduce the effective cross section by a factor of 200.

More useful perhaps is the distribution of the ratio of impact parameters to error in impact parameter. This error can be written in the form

\[
\sigma^2 = \sigma^2 + (B/p)^2, \tag{3}
\]

where \( A \) and \( B \) are constants and \( p \) is the particle momentum. For \( A \), we have taken the quadratic combination of 5 \( \mu \text{m} \) solid-state detector resolution and 7 \( \mu \text{m} \) beam size. For \( B \), we have taken two choices - 20\( \mu \text{m} \) GeV/c corresponding to scattering from a 0.5\% radiator at 2 cm radius and 10 \( \mu \text{m} \) GeV/c corresponding to a 1 cm radius. This 0.5\% is the sum of the beam pipe and the closest silicon layer (with strips assumed to run parallel to the beam). Table V(a) shows the corresponding probabilities. For good separation of \( B \) secondaries from normal hadrons > 3\( \sigma \) signals are probably necessary. Their probabilities are 53\% and 62\%, respectively, per track, for each of the two error choices. Also of some interest is the minimum ratio of impact parameter to error for all the tracks of a given \( B \) decay. If this minimum is greater than 3, then all of the charged tracks are recognized and the charge of the \( B \) meson is established. As seen in Table V(c), this probability at the 3\( \sigma \) level is only 20\% for the larger

### Table V(a): Impact Parameter Distributions

<table>
<thead>
<tr>
<th>I.P. (( \mu \text{m} ))</th>
<th>Probability</th>
<th>I.P. (( \mu \text{m} ))</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 10</td>
<td>0.15</td>
<td>60 - 70</td>
<td>0.03</td>
</tr>
<tr>
<td>10 - 20</td>
<td>0.09</td>
<td>70 - 80</td>
<td>0.03</td>
</tr>
<tr>
<td>20 - 30</td>
<td>0.06</td>
<td>80 - 90</td>
<td>0.03</td>
</tr>
<tr>
<td>30 - 40</td>
<td>0.05</td>
<td>90 - 100</td>
<td>0.02</td>
</tr>
<tr>
<td>40 - 50</td>
<td>0.05</td>
<td>100 - 200</td>
<td>0.16</td>
</tr>
<tr>
<td>50 - 60</td>
<td>0.04</td>
<td>300 - 400</td>
<td>0.04</td>
</tr>
</tbody>
</table>

### Table V(b): Impact Parameter/Errors

<table>
<thead>
<tr>
<th>I.P./Error (2 cm)</th>
<th>Probability</th>
<th>I.P./Error (1 cm)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 1</td>
<td>0.26</td>
<td>0 - 1</td>
<td>0.20</td>
</tr>
<tr>
<td>1 - 2</td>
<td>0.13</td>
<td>1 - 2</td>
<td>0.11</td>
</tr>
<tr>
<td>2 - 3</td>
<td>0.08</td>
<td>2 - 3</td>
<td>0.07</td>
</tr>
<tr>
<td>3 - 4</td>
<td>0.07</td>
<td>3 - 4</td>
<td>0.06</td>
</tr>
<tr>
<td>4 - 5</td>
<td>0.04</td>
<td>4 - 5</td>
<td>0.05</td>
</tr>
<tr>
<td>&gt; 5</td>
<td>0.42</td>
<td>&gt; 5</td>
<td>0.51</td>
</tr>
</tbody>
</table>

### Table V(c): Minimum Impact Parameters/Errors

<table>
<thead>
<tr>
<th>I.P./Error (2 cm)</th>
<th>Probability</th>
<th>I.P./Error (1 cm)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 1</td>
<td>0.50</td>
<td>0 - 1</td>
<td>0.42</td>
</tr>
<tr>
<td>1 - 2</td>
<td>0.21</td>
<td>1 - 2</td>
<td>0.17</td>
</tr>
<tr>
<td>2 - 3</td>
<td>0.09</td>
<td>2 - 3</td>
<td>0.12</td>
</tr>
<tr>
<td>3 - 4</td>
<td>0.03</td>
<td>3 - 4</td>
<td>0.06</td>
</tr>
<tr>
<td>4 - 5</td>
<td>0.02</td>
<td>4 - 5</td>
<td>0.04</td>
</tr>
<tr>
<td>&gt; 5</td>
<td>0.15</td>
<td>&gt; 5</td>
<td>0.19</td>
</tr>
</tbody>
</table>

(a) Steven Errede - Private Communication
error and 29% for the smaller error. The fractions of B decays for which all charged products have significant (3σ) impact parameters are thus relatively small.

Finally, we add that for leptons of momentum greater than 3 GeV/c (required for efficient detectors), the distribution of impact parameter over error closely follows the first set of entries in Table V(b), almost independently of the choice of vertex detector radius (because of the relatively high momentum).

We conclude with a reminder that several approximations have been made. First, charm decay lifetimes have been neglected in calculating impact parameters. This is not expected to produce any great changes in Table V. Second, in calculating the multiple scattering errors, we have neglected the fact that the tracks usually are not normal to the scatterers. This underestimates the scattering effects, and the advantages of the small pipe radius are greater than suggested by the numbers of Table V.

C. Application to CP Violation Study in B Decay

As indicated in Section II.B, the detection of CP violation through lepton charge asymmetries is expected to be very difficult because of the very small effects expected, unless new phenomena greatly enhance these effects. Therefore we have chosen to study CP violation through the detection of final states f into which both B and B can decay. The appropriate phenomenology is given in Section II.B.3. For the state f, we have chosen Bb, B̅b+Ks, a completely reconstructible state with a distinctive signature. Although the connection of Ks with a separate vertex may be difficult, the dilepton decay products of ψ can be so associated through a high resolution vertex detector, and can then be combined with the Ks decay products to obtain the known B̅ b invariant mass.

We then study the processes,

\[ P + P \rightarrow B_a + B_b + X \]
\[ B_a \rightarrow \psi K_s \]
\[ B_b \rightarrow \ell^+ \ell^- + Y \]

where B_a, B_b are a BB pair of which at least one member is neutral, and \( \ell^\pm \) is an electron or muon. CP violation manifests itself through a non-zero value of the asymmetry parameters \( A_f \) already defined in Section II.B.3.

\[ A_f = \frac{(\psi K_s + \ell^-) - (\psi K_s + \ell^+)}{(\psi K_s + \ell^- + \psi K_s + \ell^+)} \]

whose value, on the basis of the standard model, is expected to be in the range of a few percent.

We now apply the rate considerations above to the study of these processes. The main ingredients which go into a rate calculation are the following:

a) Cross section for two-B jets, \( \sigma_{BB} \), with all secondaries seen: 1.3 \( \mu b \) (Table IV).

(Although for one of the decays only a lepton is required, it seems desirable to detect some of the other tracks to establish a B decay. There is at most a 1.4 factor to be gained by requiring only detection of the leptons).

b) We assume a \( \psi K^* \) branching ratio of 0.001. This is compatible with the -1% upper limit to inclusive from B decay set by the CLEO experiment.

c) The ψ is detected through its \( e^+e^- \) or \( \mu^+\mu^- \) decay modes with total branching ratio 14%. The \( K^- + K^+ \) sequence has probability of 33%.

d) We require both leptons from the ψ to have \( 3\sigma \) impact parameter signals. Assuming the 1 cm pipe radius, we get from Table V(c) 29% probability. Since only two tracks are involved, this is probably an underestimate. By just squaring the single track 3σ probability from Table V(b), we get 0.63 x 0.63 = 40% which we use.

e) We require that the leptons from the second B decay have a \( 3\sigma \) impact parameter, with 53% probability. (Table V(b) + comment at the end of Section IV.B.)

f) The lepton branching ratio from B decay is 24%. The probability that the lepton momentum be greater than 3 GeV/c is about 42%.

Thus, the effective cross section is:

\[ \sigma_{eff} = 1.3 \times 10^{-5} \text{ cm}^2 \]

For \( L = 10^{32} \text{ cm}^{-2} \text{ sec}^{-1} \), which appears at this time to be the optimistic maximum for vertex chamber experiments, we get \( \sigma_{eff} = 0.001 \text{ x} 0.14 \times 0.33 \times 0.53 \times 0.24 = 4 \text{ events/year}. \)

Since there will be at least three LEP and one SLC detector capable of doing this experiment (as opposed to probably only one SSC detector), the total rate is of order 16 events/year.

We have not discussed the difficult trigger problem at SSC which is nonexistent at LEP or SLC. We conclude that the SSC advantage is not, at this stage, very compelling, and the experiment in question is probably impossible at LEP and very difficult at SSC.

D. Comments on Detection of Rare B Decay Modes

We consider here only those decay modes such as B \( \rightarrow \mu e, K_S \mu e \) etc. for which all secondaries can be detected and complete reconstruction is possible. It is not clear that modes with missing neutrals can ever provide a sufficiently clean signature in a hadronic environment unless it turns out to be possible to make relatively clean B beams of useful intensity. We consider again detection in the central region already defined.

To be conservative we require that the secondaries of both B mesons be within the \( 30^\circ < \theta < 150^\circ \) angular interval corresponding to Table IV to a 1.3 \( \mu b \) cross section. The additional factors determining
the rate are the following:

a) All secondaries from the B decay under study to have \( \geq 3 \) impact parameter signals. We take this probability to be 0.40.

b) We require at least two secondaries from the second B decay to have \( \geq 3 \) impact parameter signals and assume a probability of unity for a typical hadronic decay with five charged secondaries.

Thus the typical cross-section is \( 1.3 \times 0.4 = 0.5 \) \( \mu b \), leading to 5 \( \times 10^8 \) detected decays for an integrated luminosity of 1039 \( \text{cm}^2 \). A branching ratio limit of \( 10^{-7} \) appears manageable if there are no other branching ratios (such as \( K_S \rightarrow \pi^+\pi^- \) in \( K_S \mu e \)) involved. If we have been unduly conservative and only one B decay need be detected, the potential rate is increased by about a factor of five, and a limit close to \( 10^{-8} \) may be possible. In this case, the gross LEP/SSC rate of \( 4 \times 10^5 \) times four detectors, may permit a branching ratio limit of order \( 10^{-5} \). Again the SSC wins by about two orders of magnitude, provided background and trigger problems can be solved.

E. \( B\bar{B} \) Detection in the Forward Region

J. Cronin has studied the detection of B meson pairs in the rapidity region \( 3 < y < 5 \) with high resolution silicon detectors arranged in planes placed at distances 1 to 3 meters downstream from the interaction point. Details are discussed in Cronin's paper, but we quote the result that one might expect to identify \( 10^5 \) double lepton events per year for studies of mixing and CP violation, and have \( 10^8-10^9 \) B mesons to search for rare two-body decay modes. These numbers are not terribly different from those expected for the central region detector discussed earlier, although the details of the detector design are of course quite different.

IV. Comments and Conclusions

We have examined the possibility of studying CP violation and rare decay modes of B mesons with the SSC. Although we have not considered in detail fixed-target experiments, it appears unlikely that the advantages of such experiments will outweigh the estimated factor of 100 reduction in rate (without obvious reduction of background) inherent in the lower center-of-mass energy. We can summarize our considerations in the following terms, assuming that we can operate tracking detectors and vertex devices at a luminosity of \( 10^{32} \text{cm}^2 \text{sec}^{-1} \):

1) The SSC produces \( B\bar{B} \) at a rate per year estimated to be three orders of magnitude larger than other hadron sources, and five orders of magnitude larger than LEP or SLC.

2) High resolution vertex detectors are essential for doing \( B\bar{B} \) physics, and are almost surely required at some level of the trigger. While this is true for any other hadron source, it is not true for LEP/SLC in which 14% of all events at the \( Z^0 \) are \( B\bar{B} \) pairs.

3) Rare B decay modes such as \( \mu^+\mu^- \) or \( \mu^+e^- \) which are completely reconstructable may be detectable at a branching ratio level of \( 10^{-7} \). Decay modes into non-reconstructable final states such as \( t\bar{t} \) or \( t\bar{t}t \) look very difficult.

4) The study of CP violation via mixing in \( B^0 \) looks very difficult unless the effects are much larger than predicted by the standard model. Unlike K decay, both B and \( B^0 \) decays from the same process must be detected. Lepton charge asymmetries are predicted to be very small (\( \lesssim 10^{-5} \)), and the asymmetric PP initial state will add systematic uncertainties to the statistical errors. CP violation in non-leptonic final states leads to larger expected asymmetries (\( 4 \) \% or percent) but the calculated event rates are at the level of \( 10^3 \) per year, probably too small to do definitive experiments.

5) The search for \( B^0\bar{B}^0 \) mixing effects (not including CP violation) is easier in that the mixing parameter \( r_2 \) (see Section II) is expected to be a few percent, and the dilepton rates are expected to be \( 10^5 \) per year. However the systematics of the asymmetric initial state may still be a serious problem; and, unless the mixing is very small, LEP/SLC may be a better bet to observe this mixing.

References

8. The reader should be warned that Hagelin in Ref. 4 uses the notation \( \Gamma = (\Gamma_1 + \Gamma_2)/2 \), whereas Bigi and Sanda in Ref. 5 define \( \Gamma = \Gamma_1 + \Gamma_2 \) as in this paper. However in Ref. 6, Bigi and Sanda use an undefined \( \Gamma \) and inconsistently carry over some formulas from Ref. 4 and others from Ref. 5.
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