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ASYMPTOTIC BEHAVIORS OF ELECTROPRODUCTION AMPLITUDES

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ABSTRACT

Asymptotic behaviors for deep inelastic electroproduction
and electroproduction of one hadron in the limit of $q^2 \to \infty$
(spacelike) with $(\text{laboratory energy})/q^2$ fixed large are derived
by means of the Bethe-Salpeter equation which takes account of
the vector property of the photon.

The structure functions of inelastic electron-nucleon scattering have
become of considerable experimental and theoretical interest. Bjorken\(^1\) has
put forth conjectures on the asymptotic behaviors of these functions in accord
with the experimental measurement.\(^2\) Combining his conjectures with Regge
asymptotic behaviors, Abarbanel, Goldberger and Treiman\(^3\) have argued that the
Pomeranchon in the Regge asymptotics also dominates in the Bjorken limit\(^4\) with
$mv/q^2$ fixed large, where $q^2$ and $v$ are the squared four momentum and the
laboratory energy of the photon, respectively.

Our purpose here is twofold:

(1) To confirm the scale invariance and the Pomeranchon dominance in the
$W_2$ amplitude and to derive the asymptotic behavior of the $W_1$ amplitude,
and further
(ii) To predict the behaviors of two-body electroproduction processes (like \( e + N \rightarrow e + N + \pi \)) in the same asymptotic limit and to show by unitarizing them that they are consistent with the behaviors of \( W_1 \) and \( W_2 \).

Our results are derived through the parametric representation of the solution to the Bethe-Salpeter equation with a scalar particle exchange in the ladder approximation which takes proper account of the vector property of the photon. The B-S equation we use is rather a simplified one, but the conclusions seem to remain valid even in more general cases.

We begin with the B-S equation of scalar rungs as drawn in Fig. 1a.

\[
(m^2 + k_{12}^2)(m^2 + k_{22}^2)f(k_{12}^2, k_{22}^2, s) = \frac{1}{\mu^2 - s - i\epsilon} + \frac{\lambda^2}{\pi i} \int_0^1 d\xi \frac{f(k_{12}^2, k_{12}^2, (p_1 + k_1')^2)}{\mu^2 + (k_1 - k_1')^2 - i\epsilon}
\]

where

\[
p_1^2 = p_2^2 = -m^2, \quad s = -(p_1 + k_1)^2
\]

and the dependence on the variable \( t = -(k_1 - k_2)^2 \) is suppressed. The solution is not obtained explicitly, but it is known to satisfy the following integral representation

\[
f(k_{12}^2, k_{22}^2, s) = \int_0^1 dy \int_{-1}^1 dz \int_0^\infty dy \phi(y, z, \gamma)
\]

\[
\times \left\{ \gamma + (1-y)\left[\frac{1}{2}(1+z)(k_{12}^2 + m^2) + \frac{1}{2}(1-z)(k_{22}^2 + m^2)\right] - y(s-m^2) - i\epsilon \right\}^{-3}
\]

where it is worth noting that the spectral function \( \phi(y, z, \gamma) \) does not depend on any of \( s, k_{12}^2 \) and \( k_{22}^2 \), but on \( t \). The function \( \phi(y, z, \gamma) \) satisfies an
integral equation\(^5\) which we shall not write down here. We find from the analytic property of the kernel of the integral equation for \(\phi(y,z,\gamma)\) that

(i) \(\phi(y,z,\gamma)\) is singular at \(y = 0\) like \([(1 - z^2)/y]^{\alpha(t)+1}\), if a Regge pole exists at \(\alpha(t)\),

(ii) it is regular elsewhere in \(y\) and \(z\) for \(y \in [0,1]\) and \(z \in [-1,1]\), and

(iii) it falls off rapidly like \(1/y\) as \(\gamma \to \infty\).

To construct the virtual Compton scattering amplitude out of \(f(k_1^2, k_2^2, s)\), we add one more rung, affix the photon lines to the extreme ends (see Fig. 1b) and supplement the Born terms and a possible seagull term. The forward scattering amplitude \((q_1 = q_2 = q, \quad p_1 = p_2 = p)\) thus obtained turns out to be, apart from the Born and seagull terms,

\[
T^{A}_1 = T_1(q^2, \nu)
\left(\delta_{\lambda\mu} - \frac{q_\lambda q_\mu}{q^2}\right) + \frac{T_2(q^2, \nu)}{m^2} \left(\frac{p_\lambda + \frac{\nu}{q^2} q_\lambda}{p_\mu + \frac{\nu}{q^2} q_\mu}\right),
\]

\[
T_1(q^2, \nu) = \int_0^1 dx \int_0^1 dy \int_{-1}^1 dz \int_0^\infty d\gamma (1 - x)^2 \phi(y,z,\gamma)(Q - i\epsilon)^{-1},
\]

\[
T_2(q^2, \nu) = m^2 \int_0^1 dx \int_0^1 dy \int_{-1}^1 dz \int_0^\infty d\gamma 2y^2(1-x)^4 \phi(y,z,\gamma)(Q - i\epsilon)^{-2},
\]

\[
Q = (1-x)y + x m^2 + (1-x)^2 y \mu^2 + (1-x)^2 (1-y) \mu^2 m^2
\]
\[
+ x(1-x)[(1-y)(q^2 + m^2) - y(s - \mu^2)],
\]

where \(\nu = -(pq)/m\), and \(T_1\) and \(T_2\) may be understood as amplitudes averaged over the nucleon target spin according to a remark in Footnote 6.
Since we are working in the ladder approximation which does not preserve the
gauge invariance properly, we have picked up only the gauge invariant
amplitudes above.

In the high-energy limit of $v \to \infty$ with $q^2$ fixed, $T_1$ and $T_2$
exhibit the usual Regge asymptotic behaviors of $v^\alpha$ and $v^{\alpha-2}$, respectively.
The main contribution to the integral in this limit comes from the region
$y \approx 0$.

We now go to the limit of $q^2 \to \infty$ with $mv/q^2$ fixed large.
Watching carefully the function $Q$ and keeping in mind the analytic
property of $\phi(y,z)\gamma$ listed above, we find that the leading asymptotic
behavior in this limit again comes out of the integration over $y$ near
$y = 0$. This implies that the leading Regge singularity still dominates in
this asymptotic region. Carrying out the integrations over the parameters
in Eqs. (4) and (5) and adding up the crossed ladder, we are led finally to

$$T_1(q^2, v) \sim \sum_i \frac{C_i}{q^2} \left(\frac{mv}{q^2}\right)^{\alpha_i^1} + o\left(\frac{1}{v}\right),$$  \hspace{1cm} (7)

$$T_2(q^2, v) \sim \sum_i \frac{D_i}{q^2} \left(\frac{mv}{q^2}\right)^{\alpha_i^2} + o\left(\frac{1}{v^2}\right),$$  \hspace{1cm} (8)

where $\alpha_i^1$ is the intercept at $t = 0$ of the $i$th Regge trajectory, and
$C_i$ and $D_i$ are constants independent of $q^2$ and $v$. The integrations
over $x$ near $x = 0$ and over $y$ near $y = 0$ give rise to $1/q^2$ and
the fractional powers of $mv/q^2$, respectively. Equation (8) confirms the
previous conjecture$^{1,3}$
\[ v W_2(q^2, v) \sim \left( \frac{mv}{q^2} \right)^{\alpha_T-1} \]
\[ \sim \text{const.,} \]

where \( W_i = \text{Im} \ T_i \) \((i = 1, 2)\) and \( P \) stands for the Pomeranchon. We thus find that the scale invariance holds for \( W_2 \) in agreement with Bjorken\(^1\) and also with the vector-meson dominance model by Sakurai.\(^8\) On the other hand Eq. (7) does not satisfy the scale invariance of Bjorken. This is in disagreement with the result of Abarbanel, Goldberger and Treiman.\(^3\) However, the appearance of the factor \( 1/q^2 \) in front is not merely kinematical, but deeper in origin. We therefore take seriously Eq. (7) as well as Eq. (8).

Comparing them with the formulas

\[
W_1(q^2, v) = \frac{1}{4\pi \alpha} \left( v - \frac{q^2}{2m} \right) \sigma_T(q^2, v),
\]

\[
W_2(q^2, v) = \frac{1}{4\pi \alpha} \left( v - \frac{q^2}{2m} \right) \frac{q^2}{v^2 + q^2} \left[ \sigma_T(q^2, v) + \sigma_L(q^2, v) \right],
\]

where \( \sigma_T \) and \( \sigma_L \) stand for the usual transverse and longitudinal cross sections, respectively, Eq. (7) and Eq. (8) lead us to

\[
\sigma_T(q^2, v) \sim \mathcal{O}\left(\frac{1}{q^2}\right)^2,
\]

\[
\sigma_L(q^2, v) \sim \mathcal{O}\left(\frac{1}{q^2}\right),
\]

in the limit of \( q^2 \to \infty \) with \( mv/q^2 \) fixed large.

Encouraged with the result for \( W_2 \) we proceed to analyze processes in which two hadrons come out in the final state, for instance,

\( \gamma(\text{virtual}) + N \to N + \pi \). It is of much theoretical interest to explore the
asymptotic behaviors of these processes in the sense that they are
intermediary between purely hadronic processes and the virtual Compton
scattering. Since we have no chance to make use of current algebra techniques
in processes where only the incident is highly virtual, our investigation
using the Bethe-Salpeter equation would make the most sense here.

The calculation goes through in the same way except that only one
photon should be affixed to the end and that \( q_2^2 \) should be put on the
mass-shell of a produced hadron. With the definition of the electro-pion-
production amplitudes from a target with spin averaged as

\[
M_\mu = M_1(q_1^2, \nu, t) \left( q_{2\mu} - \frac{(q_1 \cdot q_2)}{q_1^2} q_{1\mu} \right) + M_2(q_1^2, \nu, t) \left( p_\mu - \frac{(q_1 \cdot p)}{q_1^2} q_{1\mu} \right)
\]

(14)

where \( p_\mu = (p_1 + p_2)_\mu / 2 \), we obtain the asymptotic behaviors like

\[
M_1(q_1^2, \nu, t) \sim \sum_i \tilde{c}_i(t) \left( \frac{mv}{2q_1} \right)^{\alpha_i(t)} + O\left( \frac{1}{\nu^2} \right)
\]

(15)

\[
M_2(q_1^2, \nu, t) \sim \sum_i \tilde{d}_i(t) \left( \frac{mv}{2q_1} \right)^{\alpha_i(t)-1} + O\left( \frac{1}{\nu^2} \right)
\]

(16)

where \( \tilde{c}_i(t) \) and \( \tilde{d}_i(t) \) are functions of \( t \). In terms of the transverse
and longitudinal cross sections defined by Hand\(^{10}\) they are written as

\[
\sigma_T(q_1^2, \nu) \sim O\left[ \left( \frac{1}{q_1^2} \right)^4 \left( \frac{mv}{2q_1} \right)^{2\alpha(0)-2} \right],
\]

(17)

\[
\sigma_L(q_1^2, \nu) \sim O\left[ \left( \frac{1}{q_1^2} \right)^3 \left( \frac{mv}{2q_1} \right)^{2\alpha(0)-2} \right],
\]

(18)
in the limit of $q_1^2 \to \infty$ with $mv/q_1^2$ fixed large and with $t$ integrated over. The leading trajectory depends on a hadron produced in the final state. These will soon be tested with experiments now under preparation.

If one connects $M_1$ and $M_2$ by unitarity, one would obtain the lower bounds of $W_1$ and $W_2$, which are given for a large $mv/q^2$ as

$$W_1(q^2, v) \sim \left( \frac{1}{q^2} \right)^3 \left( \frac{mv}{q^2} \right)^{2\alpha_p - 1}$$

$$W_2(q^2, v) \sim \left( \frac{1}{q^2} \right)^3 \left( \frac{mv}{q^2} \right)^{2\alpha_p - 3}$$

apart from possible logarithmic factors. We find that these are consistent with Eqs. (7) and (8), respectively.

Finally we would like to make a few remarks in connection with the existing theoretical arguments. The asymptotic behaviors of $\sigma_T$ and $\sigma_L$ in Eqs. (12) and (13) are to be compared with the Callan-Gross predictions based on the equal-time commutators. Our asymptotic behaviors turn out to be consistent with the algebra of fields ($q^2 \sigma_T \to 0$), but not with the quark algebra ($q^2 \sigma_L \to 0$). The present results look quite similar to those of the vector-meson dominance model proposed by Sakurai\cite{8} at least in the high $q^2$ region, but they are quite different in origin. Our arguments are free from ambiguities in the mildness assumption which are inherent to the vector-meson dominance model.

We remark that the contributions from the "ordinary" trajectories fall off as slowly as that of the Pomeranchon when $q^2 \to \infty$. It has been argued by Harari\cite{12} that, since the finite-energy sum rules relate those ordinary trajectories to low-energy resonances which fall off rapidly
as $q^2 \to \infty$, the ordinary trajectory contributions must also fall off rapidly, say, like the square of the electromagnetic form factor. However, we are able to avoid this difficulty in the following way. When we write a finite-energy sum rule, we choose $m_{\text{cut}}/q^2$ as a large finite number, say, $L$ to assure the dominance of a few leading Regge exchanges. For a small value of $q^2$, $v_{\text{cut}} = q^2 L/m \nu$ is not very large. But, as $q^2$ increases, we must choose larger and larger values of $v_{\text{cut}}$ in order to maintain the same accuracy for the finite-energy sum rule. Since $s = 2m\nu - q^2 + m^2$, higher resonances contribute to the sum rule more and more as $v_{\text{cut}}$ increases. The transition form factors from the nucleon to higher resonances may be well expected to damp mildly enough to be consistent with our predictions. In the other way around, if one increases $q^2$ alone with $v_{\text{cut}}$ fixed in the sum rule, exchanges of lower Regge trajectories become more and more important so that the $q^2$ dependence of a single Regge term does not describe that of the whole amplitude. If this interpretation is correct, there is no need to modify the Adler—Dashen—Gell-Mann—Fubini sum rule either.

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FOOTNOTES AND REFERENCES


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5. Among others we shall use the representation by Nakanishi which is proved to hold to all orders of perturbation expansion. N. Nakanishi, Phys. Rev. 133, B214 (1964).

6. The nucleon is treated as a scalar particle while the photon as a vector particle. Strictly speaking, therefore, our conclusions apply to the virtual Compton scattering with, for instance, a pion target. But, when we are interested in amplitudes averaged over the target spin, we expect that this simplification will not affect our main conclusion.

7. This is explicitly confirmed in a more simplified version of the ladder model. See N. Nakanishi, Phys. Rev. 135, B1430 (1964).

9. The remark in Footnote 6 again applies here.

10. L. N. Hand, Phys. Rev. 129, 1834 (1963). Our $\sigma_T(q^2, \nu)$ and $\sigma_L(q^2, \nu)$ correspond to $\sigma_{\text{transverse}}(q^2, K)$ and $\sigma_{\text{scalar}}(q^2, K)$, respectively.


12. H. Harari, Phys. Rev. Letters 22, 1078 (1969). This also has suggested the Pomeranchon dominance in $\nu W_2$ in the limit of $q^2 \to \infty$ with $m\nu/q^2$ fixed.

FIGURE CAPTIONS

Fig. 1a. Ladder diagram of Feynman amplitude for hadron-hadron scattering.

Fig. 1b. Virtual Compton scattering.
Fig. 1a-1b.
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