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Radiative Models of Neutrino Mass, Dark Matter, and Related Phenomena

A Dissertation submitted in partial satisfaction
of the requirements for the degree of

Doctor of Philosophy

in

Physics

by

Oleg Popov

September 2017

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To my mother and all my teachers for all the support.
In this thesis I will summarize the research and the work I have contributed with, during the years of my PhD pursue. The focus of research includes the mysterious origin of Neutrino masses, the nature of Dark Matter, relation between existence of Dark Matter and the character of Neutrino mass mechanism, origin of the PontecorvoMakiNakagawaSakata (PMNS) matrix, CP violation in the leptonic sector and their relation to physics at high energy scale. Models simultaneously explaining several of the above have been developed. Study of these models’ phenomenology and possible discovery channels at Large Hadron Collider (LHC) and International Linear Collider (ILC) in the future have been performed as well. Study of new models unifying all fundamental forces have been also pursued, leading to the prediction of new exotic particles interacting through new forces of nature.
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Chapter 1

Introduction

The Standard Model of particle interactions have been successful for the last 50 years, it explains such things as the confinement of quarks into proton, neutron, it explains weak and electromagnetic interactions and there unification, since 2012 the newly discovered Higgs particles was the last part to complete the Standard Model, and which also seponsible for the particles’ masses and electroweak (EW) symmetry breaking. Despite all these achievements, there excist many open questions that require us to go beyond the Standard Model of particle interactions in order to attempt to resolve those questions. Some of this questions are the non-zero value of neutrino masses, excistance of Dark Matter in the Universe, unification of forces at high scales, the symmetry responsible for the generation of CKM and PMNS matrieses and the origin of structure in flavor sector, unification of quarks and leptons are higher scale, etc. These are just some of the many puzzles that are waiting there to be solved. In
this text we will attempt to touch on some of these questions.

Since 1979 it is known that the dimension-5 operator [19] to produce Majorana neutrino mass is $(\nu_i \phi^0)(\nu_j \phi^0)$. The simplest ways to realize this operator at the tree level were shown [20] 1998 for the first time. They are known as seesaw I,II,III type mechanisms [21, 22, 23]. Many higher loop order realizations of this operator also exist: radiatively at a one loop order [24, 20, 25, 1, 10, 7], or at higher orders [11, 12, 7]. Moreover, in order to achieve specific pattern of mixing angles in the lepton sector, there exist many models [1, 13, 14, 15, 16, 17] in the literature.

Another question to address is the unification of quarks and leptons of standard model of interactions at higher scales. Restoring symmetry between quarks and leptons, between right and left sectors of the standard model have been studied in many variations in the literature [29, 184, 185, 186, 169, 18].
A variation of the original 2006 radiative seesaw model of neutrino mass through dark matter is shown to realize the notion of inverse seesaw naturally. The dark-matter
candidate here is the lightest of three real singlet scalars which may also carry flavor.
In 1998, the simplest realizations of the dimension-five operator [19] for Majorana neutrino mass, i.e. $(\nu_i \phi^0)(\nu_j \phi^0)$, were discussed systematically [20] for the first time. Not only was the nomenclature for the three and only three tree-level seesaw mechanisms established: (I) heavy singlet neutral Majorana fermion $N$ [21], (II) heavy triplet Higgs scalar $(\xi^{++}, \xi^+, \xi^0)$ [22], and (III) heavy triplet Majorana fermion $(\Sigma^+, \Sigma^0, \Sigma^-)$ [23], the three generic one-loop irreducible radiative mechanisms involving fermions and scalars were also written down for the first time. Whereas one such radiative mechanism was already well-known since 1980, i.e. the Zee model [24], a second was not popularized until eight years later in 2006, when it was used [25] to link neutrino mass with dark matter, called scotogenic from the Greek scotos meaning darkness. The third remaining unused mechanism is the subject of this paper. It will be shown how it is a natural framework for a scotogenic inverse seesaw model of neutrino mass, as shown in Fig. 1. The new particles are three real singlet scalars.

Figure 2.1: One-loop generation of inverse seesaw neutrino mass.
\( s_{1,2,3}, \) and one set of doublet fermions \((E^0, E^-)_{L,R}\), and one Majorana singlet fermion \(N_L\), all of which are odd under an exactly conserved discrete symmetry \(Z_2\). This specific realization was designated T1-3-A with \(\alpha = 0\) in the compilation of Ref. [26]. Note however that whereas \((E^0, E^-)_L\) is not needed to complete the loop, it serves the dual purpose of (1) rendering the theory to be anomaly-free and (2) allowing \(E\) to have an invariant mass for the implementation of the inverse seesaw mechanism.

The notion of inverse seesaw [27, 28, 29] is based on an extension of the 2 \(\times\) 2 mass matrix of the canonical seesaw to a 3 \(\times\) 3 mass matrix by the addition of a second singlet fermion. In the space spanned by \((\nu, N, S)\), where \(\nu\) is part of the usual lepton doublet \((\nu, l)\) and \(N, S\) are singlets, all of which are considered left-handed, the most general 3 \(\times\) 3 mass matrix is given by

\[
M_\nu = \begin{pmatrix}
0 & m_2 & 0 \\
m_2 & m_N & m_1 \\
0 & m_1 & m_S \\
\end{pmatrix}.
\]

The zero \(\nu - S\) entry is justified because there is only one \(\nu\) to which \(N\) and \(S\) may couple through the one Higgs field \(\phi^0\). The linear combination which couples may then be redefined as \(N\), and the orthogonal combination which does not couple is \(S\). If \(m_{S,N}\) is assumed much less than \(m_1\), then the induced neutrino mass is

\[
m_\nu \simeq \frac{m_2^2 m_S}{m_1^2}.
\]

This formula shows that a nonzero \(m_\nu\) depends on a nonzero \(m_S\), and a small \(m_\nu\) is obtained by a combination of small \(m_S\) and \(m_2/m_1\). This is supported by the
consideration of an approximate symmetry, i.e. lepton number $L$, under which $\nu, S \sim 1$ and $N \sim -1$. Thus $m_{1,2}$ conserve $L$, but $m_S$ breaks it softly by 2 units. Note that there is also a finite one-loop contribution from $m_N$ [30, 31].

Other assumptions about $m_1, m_S, m_N$ are also possible [32]. If $m_2, m_N << m_1^2/m_S$ and $m_1 << m_S$, then a double seesaw occurs with the same formula as that of the inverse seesaw, but of course with a different mass hierarchy. If $m_1, m_2 << m_N$ and $m_1^2/m_N << m_S << m_1$, then a lopsided seesaw [32] occurs with $m_\nu \simeq -m_2^2/m_N$ as in the canonical seesaw, but $\nu - S$ mixing may be significant, i.e. $m_1m_2/m_S m_N$, whereas $\nu - N$ mixing is the same as in the canonical seesaw, i.e. $\sqrt{m_\nu/m_N}$. In the inverse seesaw, $\nu - N$ mixing is even smaller, i.e. $m_\nu/m_2$, but $\nu - S$ mixing is much larger, i.e. $m_2/m_1$, which is only bounded at present by about 0.03 [33]. In the double seesaw, the effective mass of $N$ is $m_1^2/m_S$, so $\nu - N$ mixing is also $\sqrt{m_\nu/m_N}$. Here $m_S >> m_N$, so the $\nu - S$ mixing is further suppressed by $m_1/m_S$.

In the original scotogenic model [25], neutrino mass is radiatively induced by heavy neutral Majorana singlet fermions $N_{1,2,3}$ as shown in Fig. 2. However, they may be replaced by Dirac fermions. In that case, a $U(1)_D$ symmetry may be defined [34], under which $\eta_{1,2}$ transform oppositely. If $Z_2$ symmetry is retained, then a radiative inverse seesaw neutrino mass is also possible [35, 36]. We discuss here instead the new mechanism of Fig. 1, based on the third one-loop realization of neutrino mass first presented in Ref. [20]. The smallness of $m_N$, i.e. the Majorana mass of $N_L$,
Figure 2.2: One-loop generation of seesaw neutrino mass with heavy Majorana $N$.

may be naturally connected to the violation of lepton number by two units, as in the original inverse seesaw proposal using Eq. (1). It may also be a two-loop effect as first proposed in Ref. [37], with a number of subsequent papers by other authors, including Refs. [38, 39, 40].

In our model, lepton number is carried by $(E^0, E^-)_{L,R}$ as well as $N_L$. This means that the Yukawa term $\bar{N}_L(E_R^0\phi^0 - E_R^-\phi^+)$ is allowed, but not $N_L(E_L^0\phi^0 - E_L^-\phi^+)$. In the $3 \times 3$ mass matrix spanning $(E_L^0, E_R^0, N_L)$, i.e.

$$M_{E,N} = \begin{pmatrix} 0 & m_E & m_D \\ m_E & 0 & 0 \\ m_D & 0 & m_N \end{pmatrix},$$

(2.3)

$m_E$ comes from the invariant mass term $(E_R^0E_L^0 + E_R^+E_L^-)$, $m_D$ comes from the Yukawa term given above connecting $N_L$ with $E_R^0$ through $\langle \phi^0 \rangle = v$, and $m_N$ is the soft lepton-number breaking Majorana mass of $N_L$. Assuming that $m_N << m_D, m_E$, the mass
eigenvalues of $M_{E,N}$ are

\[ m_1 = \frac{m_E^2 m_N}{m_E^2 + m_D^2}, \]
\[ m_2 = \sqrt{m_E^2 + m_D^2} + \frac{m_N^2 m_E}{2(m_E^2 + m_D^2)}, \]
\[ m_3 = -\sqrt{m_E^2 + m_D^2} + \frac{m_N^2 m_E}{2(m_E^2 + m_D^2)}. \]

In the limit $m_N \to 0$, $E_R^0$ pairs up with $E_L^0 \cos \theta + N_L \sin \theta$ to form a Dirac fermion of mass $\sqrt{m_E^2 + m_D^2}$, where $\sin \theta = m_D/\sqrt{m_E^2 + m_D^2}$. This means that the one-loop integral of Fig. 1 is well approximated by

\[ m_\nu = \frac{f^2 m_E^2 m_N}{16\pi^2(m_E^2 + m_D^2 - m_s^2)} \left[ 1 - \frac{m_s^2 \ln((m_E^2 + m_D^2)/m_s^2)}{(m_E^2 + m_D^2 - m_s^2)} \right]. \]

This expression is indeed of the form expected of the inverse seesaw.

The radiative mechanism of Fig. 1 is also suitable for supporting a discrete flavor symmetry, such as $Z_3$. Consider the choice

\[ (\nu_i, l_i)_L \sim 1, 1', 1'', \quad s_1 \sim 1, \quad (s_2 + is_3)/\sqrt{2} \sim 1', \quad (s_2 - is_3)/\sqrt{2} \sim 1'', \]

with mass terms $m_s^2 s_1^2 + m'_s (s_2^2 + s_3^2)$, then the induced $3 \times 3$ neutrino mass matrix is of the form

\[
\mathcal{M}_\nu = \begin{pmatrix}
  f_e & 0 & 0 \\
  0 & f_\mu & 0 \\
  0 & 0 & f_\tau
\end{pmatrix}
\begin{pmatrix}
  I(m_s^2) & 0 & 0 \\
  0 & I(m'_s^2) & 0 \\
  0 & 0 & I(m'_s^2)
\end{pmatrix}
\begin{pmatrix}
  f_e & 0 & 0 \\
  0 & f_\mu & 0 \\
  0 & 0 & f_\tau
\end{pmatrix}
\]

\[ = \begin{pmatrix}
  f_e f_\tau I(m_s^2) & 0 \\
  0 & f_\mu f_\tau I(m'_s^2) \\
  0 & f_\mu f_\tau I(m'_s^2)
\end{pmatrix}. \]
where $I$ is given by Eq. (7) with $f^2$ removed. Let $l_{iR} \sim 1, 1', 1''$, then the charged-lepton mass matrix is diagonal using just the one Higgs doublet of the standard model, in keeping with the recent discovery [41, 42] of the 125 GeV particle. To obtain a realistic neutrino mass matrix, we break $Z_3$ softly, i.e. with an arbitrary $3 \times 3$ mass-squared matrix spanning $s_{1,2,3}$, which leads to

$$
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1/\sqrt{2} & i/\sqrt{2} \\
0 & 1/\sqrt{2} & -i/\sqrt{2}
\end{pmatrix}
\begin{pmatrix}
I(m^2_{s1}) & 0 & 0 \\
0 & I(m^2_{s2}) & 0 \\
0 & 0 & I(m^2_{s3})
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1/\sqrt{2} & 1/\sqrt{2} \\
0 & i/\sqrt{2} & -i/\sqrt{2}
\end{pmatrix},
$$

where $O$ is an orthogonal matrix but not the identity, and there can be three different mass eigenvalues $m_{s1,s2,s3}$ for the $s_{1,2,3}$ sector. The assumption of Eq. (8) results in Eq. (10) and allows the following interesting pattern for the neutrino mass matrix $M_\nu$. The Yukawa couplings $f_{e,\mu,\tau}$ may be rendered real by absorbing their phases into the arbitrary relative phases between $E^0_R$ and $\nu_{e,\mu,\tau}$. If we further assume $f_\mu = f_\tau$, then $M_\nu$ is of the form [43]

$$
M_\nu = \begin{pmatrix}
A & C & C^* \\
C & D^* & B \\
C^* & B & D
\end{pmatrix},
$$

where $A$ and $B$ are real. Note that this pattern is protected by a symmetry first pointed out in Ref. [44], i.e. $e \rightarrow e$ and $\mu - \tau$ exchange with $CP$ conjugation, and appeared previously in Refs. [45, 46]. As such, it is also guaranteed to yield maximal $\nu_\mu - \nu_\tau$ mixing ($\theta_{23} = \pi/4$) and maximal $CP$ violation, i.e. $\exp(-i\delta) = \pm i$, whereas $\theta_{13}$ may be nonzero and arbitrary. Our scheme is thus a natural framework for this
possibility. Further, from Eq. (7), it is clear that it is also a natural framework for quasi-degenerate neutrino masses as well. Let

$$F(x) = \frac{1}{1-x} \left[ 1 + \frac{x \ln x}{1-x} \right],$$

(2.12)

where $x = m_s^2/(m_E^2 + m_D^2)$, then Eq. (7) becomes

$$m_\nu = \frac{f^2 m_D^2 m_N}{(m_E^2 + m_D^2)} F(x).$$

(2.13)

Since $F(0) = 1$ and goes to zero only as $x \to \infty$, this scenario does not favor a massless neutrino. If $f_{e,\mu,\tau}$ are all comparable in magnitude, the most likely outcome is three massive neutrinos with comparable masses.

Since the charged leptons also couple to $s_{1,2,3}$ through $E^-$, there is an unavoidable contribution to the muon anomalous magnetic moment given by [47]

$$\Delta a_\mu = \frac{(g-2)_\mu}{2} = \frac{f^2 m_\mu^2}{16\pi^2 m_E^2} \sum_i |U_{\mu i}|^2 G(x_i),$$

(2.14)

where

$$G(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x}{6(1-x)^4},$$

(2.15)

with $x_i = m_{s_i}^2/m_E^2$ and

$$U = O \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & i/\sqrt{2} & -i/\sqrt{2} \end{pmatrix}.$$  

(2.16)

To get an estimate of this contribution, let $x_i << 1$, then $\Delta a_\mu = f^2 m_\mu^2/96\pi^2 m_E^2$. For $m_E \sim 1$ TeV, this is of order $10^{-11} f^2$, which is far below the present experimental
sensitivity of $10^{-9}$ and can be safely ignored. The related amplitude for $\mu \to e\gamma$ is given by
\[ A_{\mu e} = \frac{e f_\mu f_e m_\mu}{32\pi^2 m_E^2} \sum_i U^*_{ei} U_{\mu i} G(x_i). \] (2.17)

Using the most recent $\mu \to e\gamma$ bound [48]
\[ B = \frac{12\pi^2 |A_{\mu e}|^2}{m_\mu^2 G_F^2} < 5.7 \times 10^{-13}, \] (2.18)
and the approximation $\sum_i U^*_{ei} U_{\mu i} G(x_i) \sim 1/36$ (based on tribimaximal mixing with $x_1 \sim 0$ and $x_2 \sim 1$) and $m_E \sim 1$ TeV, we find
\[ f_\mu f_e < 0.03. \] (2.19)

Let $f_{e,\mu,\tau} \sim 0.1$, $m_N \sim 10$ MeV, $m_D \sim 10$ GeV, $m_E \sim 1$ TeV, then the very reasonable scale of $m_\nu \sim 0.1$ eV in Eq. (7) is obtained, justifying its inverse seesaw origin.

Since $N_L$ is the lightest particle with odd $Z_2$, it is a would-be dark-matter candidate. However, suppose we add $N_R$ so that the two pair up to have a large invariant Dirac mass, then the lightest scalar (call it $S$) among $s_{1,2,3}$ is a dark-matter candidate. It interacts with the standard-model Higgs boson $h$ according to
\[ -\mathcal{L}_{int} = \frac{\lambda_h S}{2} v h S^2 + \frac{\lambda_h S}{4} h^2 S^2. \] (2.20)

If we assume that all its other interactions are suppressed, then the annihilations $SS \to h \to$ SM particles and $SS \to hh$ determine its relic abundance, whereas its elastic scattering off nuclei via $h$ exchange determines its possible direct detection in underground experiments. A detailed analysis [49] shows that the present limit of
the invisible width of the observed 125 GeV particle (identified as $h$) allows $m_S$ to be only within several GeV below $m_h/2$ or greater than about 150 GeV using the recent LUX data [50]. Note that the vector fermion doublet $(E^0, E^-)$ is not the usually considered vector lepton doublet because it is odd under $Z_2$ and cannot mix with the known leptons.

In conclusion, we have shown how neutrino mass and dark matter may be connected using a one-loop mechanism proposed in 1998. This scotogenic model is naturally suited to implement the notion of inverse seesaw for neutrino mass, allowing the scale of new physics to be 1 TeV or less. The imposition of a softly broken $Z_3$ flavor symmetry yields an interesting pattern of radiative neutrino mass, allowing for maximal $\theta_{23}$ and maximal $CP$ violation. The real singlet scalars in the dark sector carry lepton flavor, the lightest of which is absolutely stable. Our proposal provides thus a natural theoretical framework for this well-studied phenomenological possibility.

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Chapter 3

Neutrino Mixing and CP Phase Correlations[2]

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Abstract

A special form of the $3 \times 3$ Majorana neutrino mass matrix derivable from $\mu - \tau$ interchange symmetry accompanied by a generalized $CP$ transformation was
obtained many years ago. It predicts $\theta_{23} = \pi/4$ as well as $\delta_{CP} = \pm \pi/2$, with $\theta_{13} \neq 0$.

Whereas this is consistent with present data, we explore a deviation of this result which occurs naturally in a recent proposed model of radiative inverse seesaw neutrino mass.
A special form of the $3 \times 3$ Majorana neutrino mass matrix first appeared in 2002 [45, 46], i.e.
\[
\mathcal{M}_\nu = \begin{pmatrix}
A & C & C^* \\
C & D^* & B \\
C^* & B & D
\end{pmatrix},
\tag{3.1}
\]
where $A, B$ are real. It was shown that $\theta_{13} \neq 0$ and yet both $\theta_{23}$ and the $CP$ nonconserving phase $\delta_{CP}$ are maximal, i.e. $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$. Subsequently, this pattern was shown [44] to be protected by a symmetry, i.e. $e \to e$ and $\mu \leftrightarrow \tau$ exchange with $CP$ conjugation. All three predictions are consistent with present experimental data. Recently, a radiative (scotogenic) model of inverse seesaw neutrino mass has been proposed [51] which naturally obtains
\[
\mathcal{M}_\nu^\lambda = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & \lambda
\end{pmatrix} \mathcal{M}_\nu \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & \lambda
\end{pmatrix},
\tag{3.2}
\]
where $\lambda = f_\tau/f_\mu$ is the ratio of two real Yukawa couplings.

This model has three real singlet scalars $s_{1,2,3}$ and one Dirac fermion doublet $(E^0, E^-)$ and one Dirac fermion singlet $N$, all of which are odd under an exactly conserved (dark) $Z_2$ symmetry. As a result, the third one-loop radiative mechanism proposed in 1998 [20] for generating neutrino mass is realized, as shown below.

The mass matrix linking $(\bar{N}_L, \bar{E}_L^0)$ to $(N_R, E_R^0)$ is given by
\[
\mathcal{M}_{N,E} = \begin{pmatrix}
m_N & m_D \\
m_F & m_E
\end{pmatrix},
\tag{3.3}
\]
where $m_N, m_E$ are invariant mass terms, and $m_D, m_F$ come from the Higgs vacuum expectation value $\langle \phi^0 \rangle = v/\sqrt{2}$. As a result, $N$ and $E^0$ mix to form two Dirac fermions of masses $m_{1,2}$, with mixing angles

$$m_D m_E + m_F m_N = \sin \theta_L \cos \theta_L (m_1^2 - m_2^2), \quad (3.4)$$

$$m_D m_N + m_F m_E = \sin \theta_R \cos \theta_R (m_1^2 - m_2^2). \quad (3.5)$$

To connect the loop, Majorana mass terms $(m_L/2)N_L N_L$ and $(m_R/2)N_R N_R$ are assumed. Since both $E$ and $N$ may be defined to carry lepton number, these new terms violate lepton number softly and may be naturally small, thus realizing the mechanism of inverse seesaw [27, 28, 29] as explained in Ref. [51]. Using the Yukawa interaction $f s E_R^0 \nu_L$, the one-loop Majorana neutrino mass is given by

$$m_\nu = f^2 m_R \sin^2 \theta_R \cos^2 \theta_R (m_1^2 - m_2^2)^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 - m_2^2)(k^2 - m_1^2)} \frac{1}{(k^2 - m_1^2)^2} \frac{1}{(k^2 - m_2^2)^2}$$

$$+ f^2 m_L m_1^2 \sin^2 \theta_L \cos \theta_R \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_2^2)(k^2 - m_1^2)^2} \frac{1}{(k^2 - m_1^2)^2} \frac{1}{(k^2 - m_2^2)^2}$$
\[ + f^2 m_L m_2^2 \sin^2 \theta_R \cos^2 \theta_L \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - m_2^2)(k^2 - m_L^2)} \]
\[ - 2 f^2 m_L m_1 m_2 \sin \theta_L \sin \theta_R \cos \theta_L \cos \theta_R \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - m_2^2)(k^2 - m_1^2)(k^2 - m_L^2)} \]

It was also shown in Ref. [51] that the implementation of a discrete flavor $Z_3$ symmetry, which is softly broken by the $3 \times 3$ real scalar mass matrix spanning $s_{1,2,3}$, leads to $\mathcal{M}_\nu^\lambda$ of Eq. (2).

To explore how the predictions $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm \pi/2$ are changed for $\lambda \neq 1$, consider the general diagonalization of $\mathcal{M}_\nu$, i.e.

\[ \mathcal{M}_\nu = E_\alpha U G_\beta M_d E_\beta U^T E_\alpha, \]  

(3.7)

where

\[ E_\alpha = \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & e^{i\alpha_3} \end{pmatrix}, \quad E_\beta = \begin{pmatrix} e^{i\beta_1} & 0 & 0 \\ 0 & e^{i\beta_2} & 0 \\ 0 & 0 & e^{i\beta_3} \end{pmatrix}, \quad M_d = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \]  

(3.8)

Hence

\[ \mathcal{M}_\nu \mathcal{M}_\nu^\dagger = E_\alpha U M^2_d U^\dagger E_\alpha^\dagger. \]  

(3.9)

We then have

\[ \mathcal{M}_\nu^\lambda (\mathcal{M}_\nu^\lambda)^\dagger = E_\alpha U [1 + \Delta] M^2_{\lambda d} [1 + \Delta^\dagger] U^\dagger E_\alpha^\dagger, \]  

(3.10)

where

\[ \Delta = U^\dagger \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \lambda - 1 \end{pmatrix} U, \quad M^2_{\lambda d} = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & \lambda^2 m_3^2 \end{pmatrix}. \]  

(3.11)
We now diagonalize numerically

\[ [1 + \Delta] M_{\lambda d}^2 [1 + \Delta^\dagger] = O M_{\text{new}}^2 O^T, \]  

(3.12)

where \( O \) is an orthogonal matrix, and \( M_{\text{new}}^2 \) is diagonal with mass eigenvalues equal to the squares of the physical neutrino masses. Let us define

\[ A = (1 + \Delta)^{-1} O, \]  

(3.13)

then

\[ A M_{\text{new}}^2 A^\dagger = M_{\lambda d}^2. \]  

(3.14)

Since \( U \) is known with \( \theta_{23} = \pi/4 \) and \( \delta = \pm \pi/2 \), we know \( \Delta \) once \( \lambda \) is chosen. The orthogonal matrix \( O \) has three angles as parameters, so \( A \) has three parameters. In Eq. (14), once the three physical neutrino mass eigenvalues of \( M_{\text{new}}^2 \) are given, the three off-diagonal entries of \( M_{\lambda d}^2 \) are constrained to be zero, thus determining the three unknown parameters of \( O \). Once \( O \) is known, \( UO \) is the new neutrino mixing matrix, from which we can extract the correlation of \( \theta_{23} \) with \( \delta_{CP} \). There is of course an ambiguity in choosing the three physical neutrino masses, since only \( \Delta m_{32}^2 \) and \( \Delta m_{21}^2 \) are known. There are also the two different choices of \( m_1 < m_2 < m_3 \) (normal ordering) and \( m_3 < m_1 < m_2 \) (inverted ordering). We consider each case, and choose a value of either \( m_1 \) or \( m_3 \) starting from zero. We then obtain numerically the values of \( \sin^2(2\theta_{23}) \) and \( \delta_{CP} \) as functions of \( \lambda \neq 1 \). We need also to adjust the input values of \( \theta_{12} \) and \( \theta_{13} \), so that their output values for \( \lambda \neq 1 \) are the preferred experimental
We use the 2014 Particle Data Group values \([52]\) of neutrino parameters:

\[
\begin{align*}
\sin^2(2\theta_{12}) &= 0.846 \pm 0.021, \quad \Delta m_{21}^2 = (7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2, \quad (3.15) \\
\sin^2(2\theta_{23}) &= 0.999 \left( \begin{array}{c} +0.001 \\ -0.018 \end{array} \right), \quad \Delta m_{32}^2 = (2.44 \pm 0.06) \times 10^{-3} \text{ eV}^2 \quad (\text{normal}) \quad (3.16) \\
\sin^2(2\theta_{23}) &= 1.000 \left( \begin{array}{c} +0.000 \\ -0.017 \end{array} \right), \quad \Delta m_{32}^2 = (2.52 \pm 0.07) \times 10^{-3} \text{ eV}^2 \quad (\text{inverted}) \quad (3.17) \\
\sin^2(2\theta_{13}) &= (9.3 \pm 0.8) \times 10^{-2}. \quad (3.18)
\end{align*}
\]

We consider first normal ordering, choosing the three representative values \(m_1 = 0, 0.03, 0.06 \text{ eV}\). We then vary the value of \(\lambda > 1\). [The case \(\lambda < 1\) is equivalent to \(\lambda^{-1} > 1\) with \(\mu - \tau\) exchange.] Following the algorithm already mentioned, we obtain numerically the values of \(\sin^2(2\theta_{23})\) and \(\delta_{CP}\) as functions of \(\lambda\). Our solutions are fixed by the central values of \(\Delta m_{21}^2, \Delta m_{32}^2, \sin^2(2\theta_{12}),\) and \(\sin^2(2\theta_{13})\). In Figs. 2 and 3 we

---

**Figure 3.2:** \(\sin^2(2\theta_{23})\) versus \(\lambda\) in normal ordering.
plot $\sin^2(2\theta_{23})$ and $\delta_{CP}$ respectively versus $\lambda$. We see from Fig. 2 that $\lambda < 1.15$ is required for $\sin^2(2\theta_{23}) > 0.98$. We also see from Fig. 3 that $\delta_{CP}$ is not sensitive to $m_1$. Note that our scheme does not distinguish $\delta_{CP}$ from $-\delta_{CP}$. In Fig. 4 we plot $\sin^2(2\theta_{23})$ versus $\delta_{CP}$. We see that $\delta_{CP}/(\pi/2) > 0.95$ is required for $\sin^2(2\theta_{23}) > 0.98$.

We then consider inverted ordering, using $m_3$ instead of $m_1$. We plot in Figs. 5, 6, and 7 the corresponding results. Note that in our scheme, the effective neutrino mass $m_{ee}$ measured in neutrinoless double beta decay is very close to $m_1$ in normal ordering and $m_3 + \sqrt{\Delta m_{32}^2}$ in inverted ordering. We see similar constraints on $\sin^2(2\theta_{23})$ and $\delta_{CP}$. In other words, our scheme is insensitive to whether normal or inverted ordering is chosen. Finally, we have checked numerically that $\theta_{23} < \pi/4$ if $\lambda > 1$, and $\theta_{23} > \pi/4$ if $\lambda < 1$. As we already mentioned, the two solutions are related by the mapping
Figure 3.4: $\sin^2(2\theta_{23})$ versus $\delta_{CP}$ in normal ordering.

$\lambda \to \lambda^{-1}$. 
In conclusion, we have explored the possible deviation from the prediction of maximal $\theta_{23}$ and maximal $\delta_{CP}$ in a model of radiative inverse seesaw neutrino mass. We find that given the present $1\sigma$ bound of 0.98 on $\sin^2(2\theta_{23})$, $\delta_{CP}/(\pi/2)$ must be greater than about 0.95.

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Figure 3.6: $\delta_{CP}$ versus $\lambda$ in inverted ordering.

Figure 3.7: $\sin^2(2\theta_{23})$ versus $\delta_{CP}$ in inverted ordering.
Chapter 4

Type II Radiative Seesaw Model of Neutrino Mass with Dark Matter[3]

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Abstract

We consider a model of neutrino mass with a scalar triplet \((\xi^{++}, \xi^+, \xi^0)\) assigned lepton number \(L = 0\), so that the tree-level Yukawa coupling \(\xi^0 \nu_i \nu_j\) is not allowed. It is generated instead through the interaction of \(\xi\) and \(\nu\) with dark matter and the soft breaking of \(L\) to \((-1)^L\). We discuss the phenomenological implications of this model, including \(\xi^{++}\) decay and the prognosis of discovering the dark sector at the Large Hadron Collider.
4.1 Introduction

Nonzero neutrino mass is necessary to explain the well-established phenomenon of neutrino oscillations in many experiments. Theoretically, neutrino masses are usually assumed to be Majorana and come from physics at an energy scale higher than that of electroweak symmetry breaking of order 100 GeV. As such, the starting point of any theoretical discussion of the underlying theory of neutrino mass is the effective dimension-five operator [19]

$$L_5 = -\frac{f_{ij}}{2\Lambda}(\nu_i\phi^0 - l_i\phi^+)(\nu_j\phi^0 - l_j\phi^+) + H.c.,$$  \hspace{1cm} (4.1)

where $\nu_i, l_i, i = 1, 2, 3$ are the three left-handed lepton doublets of the standard model (SM) and $(\phi^+, \phi^0)$ is the one Higgs scalar doublet. As $\phi^0$ acquires a nonzero vacuum expectation value $\langle \phi^0 \rangle = v$, the neutrino mass matrix is given by

$$M_\nu' = \frac{f_{ij}v^2}{\Lambda}.$$  \hspace{1cm} (4.2)

Note that $L_5$ breaks lepton number $L$ by two units.

It is evident from Eq. (2) that neutrino mass is seesaw in character, because it is inversely proportional to the large effective scale $\Lambda$. The three well-known tree-level seesaw realizations [20] of $L_5$ may be categorized by the specific heavy particle used to obtain it: (I) neutral fermion singlet $N$, (II) scalar triplet $(\xi^+, \xi^0, \xi)$, (III) fermion triplet $(\Sigma^+, \Sigma^0, \Sigma^0)$. It is also possible to realize $L_5$ radiatively in one loop [20] with the particles in the loop belonging to the dark sector, the lightest neutral one being
the dark matter of the Universe. The simplest such example [25] is the well-studied
“scotogenic” model, from the Greek ‘scotos’ meaning darkness. The one-loop diagram
is shown in Fig. 1. The new particles are a second scalar doublet \((\eta^+, \eta^0)\) and three
neutral singlet fermions \(N_R\). The dark \(Z_2\) is odd for \((\eta^+, \eta^0)\) and \(N_R\), whereas all
SM particles are even. This is thus a Type I radiative seesaw model. It is of course
possible to replace \(N\) with \(\Sigma^0\), so it becomes a Type III radiative seesaw model [53].
What then about Type II?

Since \(\mathcal{L}_5\) is a dimension-five operator, any loop realization is guaranteed to be
finite. On the other hand, if a Higgs triplet \((\xi^{++}, \xi^+, \xi^0)\) is added to the SM, a
dimension-four coupling \(\xi^0 \nu_i \nu_j - \xi^+(\nu_i l_j + l_i \nu_j)/\sqrt{2} + \xi^{++} l_i l_j\) is allowed. As \(\xi^0\) obtains
a small vacuum expectation value [54] from its interaction with the SM Higgs doublet,
neutrinos acquire small Majorana masses, i.e. Type II tree-level seesaw. If an exact
symmetry is used to forbid this dimension-four coupling, it will also forbid any possible
loop realization of it. Hence a Type II radiative seesaw is only possible if the symmetry

\[ \begin{align*}
\phi^0 & \quad \phi^0 \\
\eta^0 & \quad \eta^0 \\
\nu_L & \quad N_R & \quad \nu_L
\end{align*} \]

Figure 4.1: One-loop \(Z_2\) scotogenic neutrino mass.
used to forbid the hard dimension-four coupling is softly broken in the loop, as recently proposed [55].

4.2 Type II Radiative Seesaw Neutrino Masses

The symmetry used to forbid the hard $\xi^0\nu\nu$ coupling is lepton number $U(1)_L$ under which $\xi \sim 0$. The scalar trilinear $\bar{\xi}^0\phi^0\phi^0$ term is allowed and induces a small $\langle \xi^0 \rangle$, but $\nu$ remains massless. To connect $\xi^0$ to $\nu\nu$ in one loop, we add a new Dirac fermion doublet $(N, E)$ with $L = 0$, together with three complex neutral scalar singlets $s$ with $L = 1$. The resulting one-loop diagram is shown in Fig. 2. Note that the hard terms $\xi^0 N N$ and $s\bar{\nu}_L N_R$ are allowed by $L$ conservation, whereas the $ss$ terms break $L$ softly by two units to $(-1)^L$. A dark $Z_2$ parity, i.e. $(-1)^{L+2j}$, exists under which $N, E, s$ are odd and $\nu, l, \xi$ are even. Hence the lightest $s$ is a possible dark-matter candidate. The three $s$ scalars are the analogs of the three right-handed sneutrinos in

![Figure 4.2: One-loop neutrino mass from $L = 0$ Higgs triplet.](image-url)
supersymmetry, and \((N,E)_{L,R}\) are the analogs of the two higgsinos. However, their interactions are simpler here and less constrained.

The usual understanding of the Type II seesaw mechanism is that the scalar trilinear term \(\mu \xi^{\dagger} \Phi \Phi\) induces a small vacuum expectation value \(\langle \xi^0 \rangle = u\) if either \(\mu\) is small or \(m_\xi\) is large or both. More precisely, consider the scalar potential of \(\Phi\) and \(\xi\).

\[
V = m^2 \Phi^{\dagger} \Phi + M^2 \xi^{\dagger} \xi + \frac{1}{2} \lambda_1 (\Phi^{\dagger} \Phi)^2 + \frac{1}{2} \lambda_2 (\xi^{\dagger} \xi)^2 + \lambda_3 |2\xi^{++} \xi^0 - \xi^+ \xi^+|^2 \\
+ \lambda_4 (\Phi^{\dagger} \Phi)(\xi^{\dagger} \xi) + \frac{1}{2} \lambda_5 [\sqrt{2} \xi^{++} \phi^- + \xi^+ \phi^0]|^2 + |\xi^+ \phi^- + \sqrt{2} \xi^0 \phi^0|^2]
+ \mu (\bar{\phi}^0 \phi^0 + \sqrt{2} \phi^0 \phi^+ + \xi^- \phi^+ + \xi^- \phi^+ + H.c.).
\]

(4.3)

Let \(\langle \phi^0 \rangle = v\), then the conditions for the minimum of \(V\) are given by [54]

\[
m^2 + \lambda_1 v^2 + (\lambda_4 + \lambda_5)u^2 + 2\mu u = 0, \tag{4.4}
\]

\[
u[M^2 + \lambda_2 u^2 + (\lambda_4 + \lambda_5)v^2] + \mu v^2 = 0. \tag{4.5}
\]

For \(\mu \neq 0\) but small, \(u\) is also naturally small because it is approximately given by

\[
u \simeq \frac{-\mu v^2}{M^2 + (\lambda_4 + \lambda_5)v^2}, \tag{4.6}
\]

where \(v^2 \simeq -m^2/\lambda_1\). The physical masses of the \(L = 0\) Higgs triplet are then given by

\[
m^2(\xi^0) \simeq M^2 + (\lambda_4 + \lambda_5)v^2, \tag{4.7}
\]

\[
m^2(\xi^+) \simeq M^2 + (\lambda_4 + \frac{1}{2}\lambda_5)v^2, \tag{4.8}
\]

\[
m^2(\xi^{++}) \simeq M^2 + \lambda_4 v^2. \tag{4.9}
\]
Since the hard term $\xi^0 \nu \nu$ is forbidden, $u$ by itself does not generate a neutrino mass. Its value does not have to be extremely small compared to the electroweak breaking scale. For example $u \sim 0.1$ GeV is acceptable, because its contribution to the precisely measured $\rho$ parameter $\rho_0 = 1.00040 \pm 0.00024$ [56] is only of order $10^{-6}$. With the soft breaking of $L$ to $(-1)^L$ shown in Fig. 2, Type II radiative seesaw neutrino masses are obtained. Let the relevant Yukawa interactions be given by

$$L_Y = f_s s \bar{\nu}_L N_R + \frac{1}{2} f_R \xi^0 N_R N_R + \frac{1}{2} f_L \xi^0 N_L N_L + H.c.,$$

(4.10)

together with the allowed mass terms $m_E(\bar{N}N + EE)$, $m_s^2 s^* s$, and the $L$ breaking soft term $(1/2)(\Delta m^2_s) s^2 + H.c.$, then

$$m_\nu = \frac{f^2 s u r x}{16\pi^2} [f_R F_R(x) + f_L F_L(x)],$$

(4.11)

where $r = \Delta m^2_s/m^2_s$ and $x = m^2_s/m^2_E$, with

$$F_R(x) = \frac{1 + x}{(1 - x)^2} + \frac{2x \ln x}{(1 - x)^3},$$

(4.12)

$$F_L(x) = \frac{2}{(1 - x)^2} + \frac{(1 + x) \ln x}{(1 - x)^3}.$$

(4.13)

Using for example $x \sim f_R \sim f_L \sim 0.1$, $r \sim f_s \sim 0.01$, we obtain $m_\nu \sim 0.1$ eV for $u \sim 0.1$ GeV. This implies that $\xi$ may be as light as a few hundred GeV and be observable, with $\mu \sim 1$ GeV. For $f_s \sim 0.01$ and $m_E$ a few hundred GeV, the new contributions to the anomalous muon magnetic moment and $\mu \rightarrow e\gamma$ are negligible in this model.

In the case of three neutrinos, there are of course three $s$ scalars. Assuming that the $L$ breaking soft terms $|\Delta m^2_s|_{ij} << |m^2_{s_i} - m^2_{s_j}|$ for $i \neq j$, then the $3 \times 3$
neutrino mass matrix is diagonal to a very good approximation in the basis where
the $s$ mass-squared matrix is diagonal. This means that the dark scalars $s_j$ couples
to $U_{ij}l_i$, where $U_{ij}$ is the neutrino mixing matrix linking $e, \mu, \tau$ to the neutrino mass
eigenstates $\nu_{1,2,3}$.

4.3 Doubly Charged Higgs Production and Decay

The salient feature of any Type II seesaw model is the doubly charged Higgs
boson $\xi^{++}$. If there is a tree-level $\xi^{++}l_i^-l_j^-$ coupling, then the dominant decay of $\xi^{++}$
is to $l_i^+l_j^+$. Current experimental limits [57] on the mass of $\xi^{++}$ into $e\mu$, $\mu\mu$, and $ee$
final states are about 490 to 550 GeV, assuming for each a 100% branching fraction.
In the present model, since the effective $\xi^{++}l_i^-l_j^-$ coupling is one-loop suppressed,
$\xi^{++} \rightarrow W^+W^+$ should be considered [58] instead, for which the present limit on
$m(\xi^{++})$ is only about 84 GeV [59]. A dedicated search of the $W^+W^+$ mode in the
future is clearly called for.

If $m(\xi^{++}) > 2m_E$, then the decay channel $\xi^{++} \rightarrow E^+E^+$ opens up and will
dominate. In that case, the subsequent decay $E^+ \rightarrow l^+s$, i.e. charged lepton plus
missing energy, will be the signature. The present experimental limit [60] on $m_E$, assuming
electroweak pair production, is about 260 GeV if $m_s < 100$ GeV for a 100%
branching fraction to $e$ or $\mu$, and no limit if $m_s > 100$ GeV. There is also a lower
threshold for $\xi^{++}$ decay, i.e. $m(\xi^{++})$ sufficiently greater than $2m_s$, for which $\xi^{++}$
decays through a virtual $E^+E^+$ pair to $ssl^+l^+$, resulting in same-sign dileptons plus missing energy.

Figure 4.3: LHC Production cross section of $\xi^{++}\xi^{--}$ at 13 TeV.

In Fig. 3 we plot the pair production cross section of $\xi^{++}\xi^{--}$ at the Large Hadron Collider (LHC) at a center-of-mass energy of 13 TeV. We assume that $\xi^+$ and $\xi^0$ are heavier than $\xi^{++}$ so that we can focus only on the decay products of $\xi^{\pm\pm}$. The $W^\pm W^\pm$ mode is always possible and should be looked for experimentally in any case. However, as already noted, a much more interesting possibility is the case $m(\xi^{++}) > 2m_E$, with the subsequent decay $E^+ \to l^+s$. This would yield four charged leptons plus missing energy, and depending on the linear combination of charged leptons coupling to $s$, there could be exotic final states which have very little SM background, becoming thus excellent signatures to search for. Suppose $s_1$ is the lightest scalar, and $s_{2,3}$ are heavier than $E^+$, then $E^+$ decays to $s_1 \sum U_{i1} l_i^+$. Hence the decay of $\xi^{++}\xi^{--}$ could
yield for example $e^+e^+\mu^-\mu^-$ plus four $s_1$ (missing energy) in the final state.

Recent LHC searches for multilepton signatures at 8 TeV by CMS [61] and ATLAS [62] are consistent with SM expectations, and are potential restrictions on our model. In particular, the CMS study includes rare SM events such as $e^+e^+\mu^-\mu^-$ and $e^+e^+\mu^-$. Due to the absence of opposite-sign, same-flavor (OSSF) $l^+l^-$ pairs, both events are classified as OSSF0 where lepton $l$ refers to electron or muon. Leptonic tau decays contribute to the electron and muon counts, and this determines the OSSF$n$ category. Details from CMS are shown in Table 1. The CMS study estimates a neg-

<table>
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<th>signal regions</th>
<th>$H_T &gt; 200$ GeV</th>
<th>$H_T &lt; 200$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>≥ 4 leptons</td>
<td>$E_T^*$(GeV)</td>
<td>Obs.</td>
</tr>
<tr>
<td>SR1</td>
<td>$(100, \infty)$</td>
<td>0</td>
</tr>
<tr>
<td>SR2</td>
<td>$(50, 100)$</td>
<td>0</td>
</tr>
<tr>
<td>SR3</td>
<td>$(0, 50)$</td>
<td>0</td>
</tr>
<tr>
<td>3 leptons</td>
<td>$E_T^*$ (GeV)</td>
<td>Obs.</td>
</tr>
<tr>
<td>SR4</td>
<td>$(100, \infty)$</td>
<td>5</td>
</tr>
<tr>
<td>SR5</td>
<td>$(50, 100)$</td>
<td>3</td>
</tr>
<tr>
<td>SR6</td>
<td>$(0, 50)$</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 4.1: Events observed by CMS at 8 TeV with integrated luminosity 19.5 fb$^{-1}$. 

34
ligible SM background for SR1-SR3, and in our simulation we use the same selection criteria. We impose the cuts on transverse momentum $p_T > 10$ GeV and pseudorapidity $|\eta| < 2.4$ for each charged lepton, with at least one lepton $p_T > 20$ GeV. In order to be isolated, each lepton with $p_T$ must satisfy $\sum_i p_{Ti} < 0.15 p_T$, where the sum is over all objects within a cone of radius $\Delta R = 0.3$ around the lepton direction.

We implement our model with FeynRules 2.0 [63]. Using the CTEQ6L1 parton distribution functions, we generate events using MadGraph5 [64], which includes the Pythia package for hadronization and showering. MadAnalysis [65] is then used with the Delphes card designed for CMS detector simulation. Generated events initially have 4 leptons. About half are detected as 3 lepton events, but the constraints from signal regions SR4-SR6 are less restrictive than SR1-SR3. The number of detected events in the OSSF0 ≥ 4 lepton category is almost the same as $e^\pm e^\pm \mu^\mp \mu^\mp 2s_1^2s_2^*$ with very few additional leptons from showering or initial/final state radiation.

To examine the production of $e^\pm e^\pm \mu^\mp \mu^\mp$ we take the mass of $s_1$ to be 130 GeV, which allows $s_1$ to be dark matter as discussed in the next section. We use the values $f_R = f_L = 0.1$ and $f_s = 0.01$, although the results are not sensitive to the exact values due to on-shell production and decay. The effects due to $u \sim 0.1$ GeV may be neglected.

For our model, we scan the mass range of $\xi^{++}$ and $E^+$. In Fig. 4 we plot contours showing the expected number of detected events in the OSSF0 ≥ 4 lepton category for 13 TeV at luminosity 100 fb$^{-1}$ assuming a negligible background as for the 8 TeV
A similar analysis performed for 8 TeV at 19.5 fb$^{-1}$ excludes the region with more than about 15 events in Fig. 4.

Figure 4.4: Number of $e^\pm e^\mp \mu^\pm \mu^\mp 2s_1 2s_1^*$ events for 13 TeV at luminosity 100 fb$^{-1}$.

4.4 Dark Matter Properties

The lightest $s$, say $s_1$, is dark matter. Its interaction with leptons is too weak to provide a large enough annihilation cross section to explain the present dark matter
relic density $\Omega_M$ of the Universe. However, it also interacts with the SM Higgs boson through the usual quartic coupling $\lambda_s s^* s \Phi^i \Phi$. For a value of $\lambda_s$ consistent with $\Omega_M$, the direct-detection cross section in underground experiments is determined as a function of $m_s$. A recent analysis [66] for a real $s$ claims that the resulting allowed parameter space is limited to a small region near $m_s < m_h/2$.

In our model, we can evade this constraint by evoking $s_{2,3}$. The mass-squared matrix spanning $s_i^* s_j$ is given by

$$ (\mathcal{M}_s^2)_{ij} = m_{ij}^2 + \lambda_{ij} v^2, \quad (4.14) $$

whereas the coupling matrix of the one Higgs $h$ to $s_i^* s_j$ is $\lambda_{ij} v \sqrt{2}$. Upon diagonalizing $\mathcal{M}_s^2$, the coupling matrix will not be diagonal in general. In the physical basis, $s_1$ will interact with $s_2$ through $h$. This allows the annihilation of $s_1 s_1^*$ to $hh$ through $s_2$ exchange, and contributes to $\Omega_M$ without affecting the $s_1$ scattering cross section off nuclei through $h$. This mechanism restores $s_1$ as a dark-matter candidate for $m_s > m_h$.

To demonstrate the scale of the values involved, we consider the simplifying case when $m_{s_2} = m_{s_3}$ and $\lambda_{12} = \lambda_{13}$. The additional choice $m_{s_{2,3}}^2 = m_{s_1}^2 + m_h^2$ ensures that $s_{2,3}$ are heavier than $s_1$, and is convenient because then the relic abundance requirement no longer depends explicitly on $m_{s_{2,3}}^2$. Taking into account that $s_1$ is a complex scalar, we use $\sigma \times v_{rel} = 4.4 \times 10^{-26} \text{cm}^3\text{s}^{-1}$ [67] and in Fig. 5 we plot the allowed values for $\lambda_{12}$ and $m_{s_1}$ taking $\lambda_{11} = 0$ for simplicity to satisfy the LUX data.
Another possible scenario is to add a light scalar $\chi$ with $L = 0$, which acts as a mediator for $s$ self-interactions. This has important astrophysical implications [68, 45, 69, 70, 71, 72]. In this case, $s_1 s_1^*$ annihilating to $\chi \chi$ becomes possible.

4.5 Conclusion

We have studied a new radiative Type II seesaw model of neutrino mass with dark matter [55], which predicts a doubly charged Higgs boson $\xi^{++}$ with suppressed decay to $l^+ l^+$, thereby evading the present LHC bounds of 490 to 550 GeV on its mass. In
this model, $\xi^{++}$ may decay to two charged heavy fermions $E^+ E^+$, each with odd dark parity. The subsequent decay of $E^+$ is into a charged lepton $l^+$ and a scalar $s$ which is dark matter. Hence there is the interesting possibility of four charged leptons, such as $\mu^- \mu^- e^+ e^+$, plus large missing energy in the final state. We show that the LHC at 13 TeV will be able to probe such a doubly charged Higgs boson with a mass of the order 400 to 500 GeV.

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Chapter 5

Phenomenology of the Utilitarian Supersymmetric Standard Model[4]

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Abstract

We study the 2010 specific version of the 2002 proposed $U(1)_X$ extension of the supersymmetric standard model, which has no $\mu$ term and conserves baryon number and lepton number separately and automatically. We consider in detail the scalar sector as well as the extra $Z_X$ gauge boson, and their interactions with the necessary extra color-triplet particles of this model, which behave as leptoquarks. We show how the diphoton excess at 750 GeV, recently observed at the LHC, may be explained within this context. We identify a new fermion dark-matter candidate and discuss its properties. An important byproduct of this study is the discovery of relaxed supersymmetric constraints on the Higgs boson’s mass of 125 GeV.
5.1 Introduction

Since the recent announcements [73, 74] by the ATLAS and CMS Collaborations at the Large Hadron Collider (LHC) of a diphoton excess around 750 GeV, numerous papers [75] have appeared explaining its presence or discussing its implications. In this paper, we study the phenomenology of a model proposed in 2002 [45], which has exactly all the necessary and sufficient particles and interactions for this purpose. They were of course there for solving other issues in particle physics. However, the observed diphoton excess may well be a first revelation [76] of this model, including its connection to dark matter.

This 2002 model extends the supersymmetric standard model by a new $U(1)_X$ gauge symmetry. It replaces the $\mu$ term with a singlet scalar superfield which also couples to heavy color-triplet superfields which are electroweak singlets. The latter are not ad hoc inventions, but are necessary for the cancellation of axial-vector anomalies. It was shown in Ref. [45] how this was accomplished by the remarkable exact factorization of the sum of eleven cubic terms, resulting in two generic classes of solutions [77]. Both are able to enforce the conservation of baryon number and lepton number up to dimension-five terms. As such, the scalar singlet and the vector-like quarks are indispensible ingredients of this 2002 model. They are thus naturally suited for explaining the observed diphoton excess. In 2010 [78], a specific version was discussed, which will be the subject of this paper as well. An important byproduct of
this study is the discovery of relaxed supersymmetric constraints on the Higgs boson’s mass of 125 GeV. This is independent of whether the diphoton excess is confirmed or not.

5.2 Model

Consider the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ with the particle content of Ref. [45]. For $n_1 = 0$ and $n_4 = 1/3$ in Solution (A), the various superfields transform as shown in Table 1. There are three copies of $Q, u^c, d^c, L, e^c, N^c, S_1, S_2$; two copies of $U, U^c, S_3$; and one copy of $\phi_1, \phi_2, D, D^c$. The only allowed terms of the superpotential are thus trilinear, i.e.

\begin{align}
Qu^c\phi_2, & \quad Qd^c\phi_1, & \quad Le^c\phi_1, & \quad LN^c\phi_2, & \quad S_3\phi_1\phi_2, & \quad N^cN^cS_1, \quad (5.1) \\
S_3UU^c, & \quad S_3DD^c, & \quad u^cN^cU, & \quad u^ce^cD, & \quad d^cN^cD, & \quad QLD^c, \quad S_1S_2S_3. \quad (5.2)
\end{align}

The absence of any bilinear term means that all masses come from soft supersymmetry breaking, thus explaining why the $U(1)_X$ and electroweak symmetry breaking scales are not far from that of supersymmetry breaking. As $S_{1,2,3}$ acquire nonzero vacuum expectation values (VEVs), the exotic $(U, U^c)$ and $(D, D^c)$ fermions obtain Dirac masses from $\langle S_3 \rangle$, which also generates the $\mu$ term. The singlet $N^c$ fermion gets a large Majorana mass from $\langle S_1 \rangle$, so that the neutrino $\nu$ gets a small seesaw mass in the usual way. The singlet $S_{1,2,3}$ fermions themselves get Majorana masses from their
scalar counterparts $\langle S_{1,2,3} \rangle$ through the $S_1S_2S_3$ terms. The only massless fields left are the usual quarks and leptons. They then become massive as $\phi^0_{1,2}$ acquire VEVs, as in the minimal supersymmetric standard model (MSSM).

Because of $U(1)_X$, the structure of the superpotential conserves both $B$ and $(-1)^L$, with $B = 1/3$ for $Q, U, D$, and $B = -1/3$ for $u^c, d^c, U^c, D^c$; $(-1)^L$ odd for $L, e^c, N^c, U, U^c, D, D^c$, and even for all others. Hence the exotic $U, U^c, D, D^c$ scalars are leptoquarks and decay into ordinary quarks and leptons. The $R$ parity of the MSSM is defined here in the same way, i.e. $R \equiv (-)^{2j+3B+L}$, and is conserved. Note also that the quadrilinear terms $QQQL$ and $u^c u^c d^c e^c$ (allowed in the MSSM) as well as $u^c d^c d^c N^c$ are forbidden by $U(1)_X$. Proton decay is thus strongly suppressed. It may proceed through the quintilinear term $QQQLS_1$ as the $S_1$ fields acquire VEVs, but this is a dimension-six term in the effective Lagrangian, which is suppressed by two powers of a very large mass, say the Planck mass, and may safely be allowed.

5.3 Gauge Sector

The new $Z_X$ gauge boson of this model becomes massive through $\langle S_{1,2,3} \rangle = u_{1,2,3}$, whereas $\langle \phi^0_{1,2} \rangle = v_{1,2}$ contribute to both $Z$ and $Z_X$. The resulting $2 \times 2$ mass-squared matrix is given by [79]

$$
\mathcal{M}_{Z,Z_X}^2 = \begin{pmatrix}
(1/2)g_2^2(v_1^2 + v_2^2) & (1/2)g_gg_X(v_2^2 - v_1^2) \\
(1/2)g_gg_X(v_2^2 - v_1^2) & 2g_X^2[(1/9)u_1^2 + (4/9)u_2^2 + u_3^2 + (1/4)(v_1^2 + v_2^2)]
\end{pmatrix}.
$$

(5.3)
Since precision electroweak measurements require $Z - Z_X$ mixing to be very small \cite{80}, $v_1 = v_2$, i.e. $\tan \beta = 1$, is preferred. With the 2012 discovery \cite{41, 42} of the 125 GeV particle, and identified as the one Higgs boson $h$ responsible for electroweak symmetry breaking, $\tan \beta = 1$ is not compatible with the MSSM, but is perfectly consistent here, as shown already in Ref. \cite{78} and in more detail in the next section.

Consider the decay of $Z_X$ to the usual quarks and leptons. Each fermionic partial width is given by

$$\Gamma(Z_X \rightarrow \bar{f}f) = \frac{g_X^2 M_{Z_X}^2}{24\pi} [c_L^2 + c_R^2], \quad (5.4)$$

where $c_{L,R}$ can be read off under $U(1)_X$ from Table 1. Thus

$$\frac{\Gamma(Z_X \rightarrow \bar{t}t)}{\Gamma(Z_X \rightarrow \mu^+\mu^-)} = \frac{\Gamma(Z_X \rightarrow \bar{b}b)}{\Gamma(Z_X \rightarrow \mu^+\mu^-)} = \frac{27}{5}. \quad (5.5)$$

This will serve to distinguish it from other $Z'$ models \cite{81}.

At the LHC, limits on the mass of any $Z'$ boson depend on its production by $u$ and $d$ quarks times its branching fraction to $e^-e^+$ and $\mu^-\mu^+$. In a general analysis of $Z'$ couplings to $u$ and $d$ quarks,

$$\mathcal{L} = \frac{g'_V}{2} Z'_\mu \bar{f} \gamma_\mu (g_V - g_A \gamma_5) f, \quad (5.6)$$

where $f = u, d$. The $c_u, c_d$ coefficients used in an experimental search \cite{57, 60} of $Z'$ are then given by

$$c_u = \frac{g_V^2}{2} [(g_V^u)^2 + (g_A^u)^2] B(Z' \rightarrow l^-l^+), \quad c_d = \frac{g_V^2}{2} [(g_V^d)^2 + (g_A^d)^2] B(Z' \rightarrow l^-l^+), \quad (5.7)$$
where \( l = e, \mu \). In this model

\[
c_u = c_d = \frac{g_X^2}{4} B(Z' \to l^- l^+). \tag{5.8}
\]

To estimate \( B(Z' \to l^- l^+) \), we assume \( Z_X \) decays to all SM quarks and leptons with effective zero mass, all the scalar leptons with effective mass of 500 GeV, all the scalar quarks with effective mass of 800 GeV, the exotic \( U, D \) fermions with effective mass of 400 GeV (needed to explain the diphoton excess), and one pseudo-Dirac fermion from combining \( \tilde{S}_{1,2} \) (the dark matter candidate to be discussed) with mass of 200 GeV. We find \( B(Z' \to l^- l^+) = 0.04 \), and for \( g_X = 0.53 \), a lower bound of 2.85 TeV on \( m_{Z_X} \) is obtained from the LHC data based on the 7 and 8 TeV runs.

### 5.4 Scalar Sector

Consider the scalar potential consisting of \( \phi_{1,2} \) and \( S_{1,2,3} \). Whereas there are 2 copies of \( S_3 \) and 3 copies each of \( S_{1,2} \), we can choose one copy each to be the one with nonzero vacuum expectation value. We then assume that the superpotential linking them is given by

\[
W = f S_3 \phi_1 \phi_2 + h S_3 S_2 S_1, \tag{5.9}
\]

which is of course missing some terms. We have neglected them for simplicity. Its contribution to the scalar potential is

\[
V_F = f^2(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2)S_3^* S_3 + h^2(S_1^* S_1 + S_2^* S_2)S_3^* S_3 + |f \Phi_1^\dagger \Phi_2 + h S_1 S_2|^2, \tag{5.10}
\]

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where $\phi_1$ has been redefined to $\Phi_1 = (\phi_1^+, \phi_1^0)$. The gauge contribution is

$$V_D = \frac{1}{8} g_2^2 [(\Phi_1^d \Phi_1)^2 + (\Phi_2^d \Phi_2)^2 + 2(\Phi_1^d \Phi_1)(\Phi_2^d \Phi_2) - 4(\Phi_1^d \Phi_2)(\Phi_2^d \Phi_1)]$$

$$+ \frac{1}{8} g_2^2 [- (\Phi_1^d \Phi_1) + (\Phi_2^d \Phi_2)]^2$$

$$+ \frac{1}{2} g_X^2 \left[ -\frac{1}{2} \Phi_1^d \Phi_1 - \frac{1}{2} \Phi_2^d \Phi_2 - \frac{1}{3} S_1^* S_1 - \frac{2}{3} S_2^* S_2 + S_3^* S_3 \right]^2. \quad (5.11)$$

The soft supersymmetry-breaking terms are

$$V_{soft} = \mu_1^2 \Phi_1^d \Phi_1 + \mu_2^2 \Phi_2^d \Phi_2 + m_3^2 S_3^* S_3 + m_2^2 S_2^* S_2 + m_1^2 S_1^* S_1$$

$$+ \left[ m_{12} S_2^* S_1^2 + A_{f f} S_3^* \Phi_1^d \Phi_2 + A_{h h} S_3 S_2 S_1 + H.c. \right]. \quad (5.12)$$

In addition, there is an important one-loop contribution from the $t$ quark and its supersymmetric scalar partners:

$$V_t = \frac{1}{2} \lambda_2 (\Phi_2^d \Phi_2)^2, \quad (5.13)$$

where

$$\lambda_2 = \frac{6G_F^2 m_t^4}{\pi^2} \ln \left( \frac{m_{i_1} m_{i_2}}{m_t^2} \right) \quad (5.14)$$

is the well-known correction which allows the Higgs mass to exceed $m_Z$.

Let $\langle \phi_{1,2}^0 \rangle = v_{1,2}$ and $\langle S_{1,2,3} \rangle = u_{1,2,3}$, we study the conditions for obtaining a minimum of the scalar potential $V = V_F + V_D + V_{soft} + V_t$. We look for the solution $v_1 = v_2 = v$ which implies that

$$\mu_1^2 = \mu_2^2 + \lambda_2 v^2 \quad (5.15)$$

$$0 = \mu_1^2 + A_{f f} u_3 + f^2 (u_3^2 + v^2) + \frac{1}{2} g_X^2 \left( v^2 + \frac{1}{3} u_1^2 + \frac{2}{3} u_2^2 - u_3^2 \right) + f h u_1 u_2 \quad (5.16)$$
We then require that this solution does not mix the $\text{Re}(\phi_{1,2})$ and $\text{Re}(S_{1,2,3})$ sectors.

The additional conditions are

$$0 = Af + (2f^2 - g_X^2)u_3, \quad (5.17)$$
$$0 = \frac{1}{3}g_X^2u_1 + fh u_2, \quad (5.18)$$
$$0 = \frac{2}{3}g_X^2u_2 + fh u_1. \quad (5.19)$$

Hence

$$u_1 = \sqrt{2}u_2, \quad fh = \frac{-\sqrt{2}g_X^2}{3}. \quad (5.20)$$

The $2 \times 2$ mass-squared matrix spanning $[\sqrt{2}\text{Re}(\phi_1^0), \sqrt{2}\text{Re}(\phi_2^0)]$ is

$$\mathcal{M}_\phi^2 = \begin{pmatrix}
\kappa + g_X^2v^2/2 & -\kappa + g_X^2v^2/2 + 2f^2v^2 \\
-\kappa + g_X^2v^2/2 + 2f^2v^2 & \kappa + g_X^2v^2/2 + 2\lambda_2v^2
\end{pmatrix}, \quad (5.21)$$

where

$$\kappa = (2f^2 - g_X^2)u_3^2 + \frac{2}{3}g_X^2u_2^2 + \frac{1}{2}(g_1^2 + g_2^2)v^2. \quad (5.22)$$

For $\lambda_2v^2 << \kappa$, the Higgs boson $h \simeq \text{Re}(\phi_1^0 + \phi_2^0)$ has a mass given by

$$m_h^2 \simeq \left(g_X^2 + 2f^2 + \lambda_2\right)v^2, \quad (5.23)$$

whereas its heavy counterpart $H \simeq \text{Re}(-\phi_1^0 + \phi_2^0)$ has a mass given by

$$m_H^2 \simeq (4f^2 - 2g_X^2)u_3^2 + \frac{4}{3}g_X^2u_2^2 + (g_1^2 + g_2^2 - 2f^2 + \lambda_2)v^2. \quad (5.24)$$

The conditions for obtaining the minimum of $V$ in the $S_{1,2,3}$ directions are

$$0 = m_3^2 + g_X^2u_3^2 + \left(3h^2 - \frac{4}{3}g_X^2\right)u_2^2 + \frac{\sqrt{2}A_h hu_3^2}{u_3}, \quad (5.25)$$
$$0 = m_2^2 + 2m_{12}u_2 + \left(2h^2 + \frac{8}{9}g_X^2\right)u_2^2 + \left(h^2 - \frac{2}{3}g_X^2\right)u_3^2 + \frac{\sqrt{2}A_h hu_3}{u_3}, \quad (5.26)$$
$$0 = m_1^2 + 2m_{12}u_2 + \left(h^2 + \frac{4}{9}g_X^2\right)u_2^2 + \left(h^2 - \frac{1}{3}g_X^2\right)u_3^2 + \frac{\sqrt{2}A_h hu_3}{u_3}. \quad (5.27)$$
The $3 \times 3$ mass-squared matrix spanning $[\sqrt{2}Re(S_1), \sqrt{2}Re(S_2), \sqrt{2}Re(S_3)]$ is given by

$$m_{11}^2 = \frac{4}{9} g_X^2 u_2^2 - \frac{1}{\sqrt{2}} A_h h u_3 + \frac{1}{3} g_X^2 v^2, \quad m_{22}^2 = 2 m_{11}^2 - 2 m_{12} u_2, \quad (5.28)$$

$$m_{12}^2 = m_{21}^2 = 2 \sqrt{2} m_{12} u_2 + A_h h u_3 + 2 \sqrt{2} \left( h^2 + \frac{2}{9} g_X^2 \right) u_2^2 - \frac{\sqrt{2}}{3} g_X^2 v^2, \quad (5.29)$$

$$m_{33}^2 = 2 g_X^2 u_3^2 - \sqrt{2} A_h h u_2^2 / u_3 + (2 f^2 - g_X^2) v^2, \quad (5.30)$$

$$m_{13}^2 = m_{31}^2 = A_h h u_2 + 2 \sqrt{2} \left( h^2 - \frac{1}{3} g_X^2 \right) u_3 u_2, \quad (5.31)$$

$$m_{23}^2 = m_{32}^2 = \sqrt{2} A_h h u_2 + 2 \left( h^2 - \frac{2}{3} g_X^2 \right) u_3 u_2. \quad (5.32)$$

The $5 \times 5$ mass-squared matrix spanning $[\sqrt{2}Im(\phi_1^0), \sqrt{2}Im(\phi_2^0), \sqrt{2}Im(S_1), \sqrt{2}Im(S_2), \sqrt{2}Im(S_3)]$ has two zero eigenvalues, corresponding to the would-be Goldstone modes

$$(1,1,0,0,0) \quad \text{and} \quad (v/2, -v/2, -\sqrt{2}u_2/3, -2u_2/3, u_3), \quad (5.33)$$

for the $Z$ and $Z_X$ gauge bosons. One exact mass eigenstate is $A_{12} = [2Im(S_1) - \sqrt{2}Im(S_2)]/\sqrt{3}$ with mass given by

$$m_{A_{12}}^2 = -6 m_{12} u_2. \quad (5.34)$$

Assuming that $v^2 << u_2^2$, the other two mass eigenstates are $A \simeq -Im(\phi_1^0) + Im(\phi_2^0)$ and $A_S \simeq [u_3 Im(S_1) + \sqrt{2}u_3 Im(S_2) + \sqrt{2}u_2 Im(S_3)]/\sqrt{u_2^2 + 3u_3^2}/2$ with masses given by

$$m_A^2 \simeq (4f^2 - 2g_X^2) u_3^2 + \frac{4}{3} g_X^2 u_2^2, \quad (5.35)$$

$$m_{A_S}^2 \simeq -A_h h \left( \frac{3u_3 + \sqrt{2}u_2}{u_3} \right), \quad (5.36)$$

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respectively. The charged scalar $H^\pm = (-\phi_1^\pm + \phi_2^\pm)/\sqrt{2}$ has a mass given by

$$m_{H^\pm}^2 = (4f^2 - 2g_X^2)u_3^2 + \frac{4}{3}g_X^2u_2^2 + (g_2^2 - 2f^2)v^2.$$  

(5.37)

### 5.5 Physical Scalars and Pseudoscalars

In the MSSM without radiative corrections,

$$m_{H^\pm}^2 = m_A^2 + m_W^2,$$  

(5.38)

$$m_{h,H}^2 = \frac{1}{2} \left( m_A^2 + m_Z^2 \mp \sqrt{(m_A^2 + m_Z^2)^2 - 4m_Z^2m_A^2\cos^22\beta} \right).$$  

(5.39)

where $\tan\beta = v_2/v_1$. For $v_1 = v_2$ as in this model, $m_h$ would be zero. There is of course the important radiative correction from Eq. (14), but that alone will not reach 125 GeV. Hence the MSSM requires both large $\tan\beta$ and large radiative correction, but a significant tension remains in accommodating all data. In this model, as Eq. (23) shows, $m_h^2 \simeq (g_X^2 + 2f^2 + \lambda_2)v^2$, where $v = 123$ GeV. This is a very interesting and important result, allowing the Higgs boson mass to be determined by the gauge $U(1)_X$ coupling $g_X$ in addition to the Yukawa coupling $f$ which replaces the $\mu$ parameter, i.e. $\mu = fu_3$. There is no tension between $m_h = 125$ GeV and the superparticle mass spectrum. Since $\lambda_2 \simeq 0.25$ for $\tilde{m}_t \simeq 1$ TeV, we have the important constraint

$$\sqrt{g_X^2 + 2f^2} \simeq 0.885.$$  

(5.40)

For illustration, we have already chosen $g_X = 0.53$. Hence $f = 0.5$ and for $u_3 = 2$ TeV, $fu_3 = 1$ TeV is the value of the $\mu$ parameter of the MSSM. Let us choose $u_2 = 4$
TeV, then $m_{Z_X} = 2.87$ TeV, which is slightly above the present experimental lower bound of 2.85 TeV using $g_X = 0.53$ discussed earlier.

As for the heavy Higgs doublet, the four components $(H^\pm, H, A)$ are all degenerate in mass, i.e. $m^2 \simeq (4f^2 - 2g_X^2)u_3^2 + (4/3)g_X^2 u_2^2$ up to $v^2$ corrections. Each mass is then about 2.78 TeV. In more detail, as shown in Eq. (37), $m_{H^\pm}^2$ is corrected by $g_2^2 v^2 = m_W^2$ plus a term due to $f$. As shown in Eq. (24), $m_H^2$ is corrected by $(g_1^2 + g_2^2) v^2 = m_Z^2$ plus a term due to $f$ and $\lambda_2$. These are exactly in accordance with Eqs. (38) and (39).

In the $S_{1,2,3}$ sector, the three physical scalars are mixtures of all three $Re(S_i)$ components, whereas the physical pseudoscalar $A_{12}$ has no $Im(S_3)$ component. Since only $S_3$ couples to $UU^c$, $DD^c$, and $\phi_1 \phi_2$, a candidate for the 750 GeV diphoton resonance must have an $S_3$ component. It could be one of the three scalars or the pseudoscalar $A_S$, or the other $S_3$ without VEV. In the following, we will consider the last option, specifically a pseudoscalar $\chi$ with a significant component of this other $S_3$. This allows the $\chi UU^c$, $\chi DD^c$ and $\chi \phi_1 \phi_2$ couplings to be independent of the masses of $U$, $D$, and the charged higgsino. The other scalars and pseudoscalars are assumed to be much heavier, and yet to be discovered.
5.6 Diphoton Excess

In this model, other than the addition of $N^c$ for seesaw neutrino masses, the only new particles are $U,U^c,D,D^c$ and $S_{1,2,3}$, which are exactly the ingredients needed to explain the diphoton excess at the LHC. The allowed $S_3UU^c$ and $S_3DD^c$ couplings enable the one-loop gluon production of $S_3$ in analogy to that of $h$. The one-loop decay

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig1.png}
\caption{One-loop production of $S_3$ by gluon fusion.}
\end{figure}

of $S_3$ to two photons comes from these couplings as well as $S_3\phi_1\phi_2$. In addition, the

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig2.png}
\caption{One-loop decay of $S_3$ to two photons.}
\end{figure}
direct \(S_1S_2S_3\) couplings enable the decay of \(S_3\) to other final states, including those of the dark sector, which contribute to its total width. The fact that the exotic \(U,U^c,D,D^c\) scalars are leptoquarks is also very useful for understanding \[82\] other possible LHC flavor anomalies. In a nutshell, a desirable comprehensive picture of possible new physics beyond the standard model is encapsulated by this existing model. In the following, we assume that the pseudoscalar \(\chi\) is the 750 GeV particle, and show how its production and decay are consistent with the present data.

The production cross section through gluon fusion is given by

\[
\hat{\sigma}(gg \to \chi) = \frac{\pi^2}{8m_\chi} \Gamma(\chi \to gg) \delta(\hat{s} - m_\chi^2).
\]

For the LHC at 13 TeV, the diphoton cross section is roughly \[83\]

\[
\sigma(gg \to \chi \to \gamma\gamma) \simeq (100 \text{ pb}) \times (\lambda_g \text{ TeV})^2 \times B(\chi \to \gamma\gamma),
\]

where \(\lambda_g\) is the effective coupling of \(\chi\) to two gluons, normalized by

\[
\Gamma(\chi \to gg) = \frac{\lambda_g^2}{8\pi} m_\chi^3.
\]

Let the \(\chi Q\bar{Q}\) coupling be \(f_Q\), where \(Q\) is a leptoquark fermion, then

\[
\lambda_g = \frac{\alpha_s}{\pi m_\chi} \sum_Q f_Q F(m_Q^2/m_\chi^2),
\]

where \[84\]

\[
F(x) = 2\sqrt{x} \left[ \arctan \left( \frac{1}{\sqrt{4x - 1}} \right) \right]^2,
\]
which has the maximum value of \( \pi^2/4 = 2.47 \) as \( x \to 1/4 \). Let \( f_{Q/4}^2/4\pi = 0.21 \) and \( F(m_Q^2/m_\chi^2) = 2.0 \) (i.e. \( m_Q = 380 \text{ GeV} \)) for all \( Q = U, U, D \), then \( \lambda_g = 0.49 \text{ TeV}^{-1} \).

For the corresponding

\[
\Gamma(\chi \to \gamma\gamma) = \frac{\lambda_\gamma^2}{64\pi} m_\chi^3, \tag{5.46}
\]

the \( \phi^\pm \) higgsino contributes as well as \( U, D \). However, its mass is roughly \( f_u = 1 \) TeV, so \( F(x_\phi) = 0.394 \), and

\[
\lambda_\gamma = \frac{2\alpha}{\pi m_\chi} \sum_\psi N_\psi Q_\psi^2 f_\psi F(x_\psi), \tag{5.47}
\]

where \( \psi = U, U, D, \phi^\pm \) and \( N_\psi \) is the number of copies of \( \psi \). Using \( f_{\phi}^2/4\pi = 0.21 \) as well, \( \lambda_\gamma = 0.069 \text{ TeV}^{-1} \) is obtained. We then have \( \Gamma(\chi \to \gamma\gamma) = 10 \text{ MeV} \) and \( \Gamma(\chi \to gg) = 4.0 \text{ GeV} \). If \( B(\chi \to \gamma\gamma) = 2.5 \times 10^{-4} \), then \( \sigma = 6 \text{ fb} \), and the total width of \( \chi \) is 40 GeV, in good agreement with data [73, 74].

Note the important fact that we have considered 380 GeV for the mass of the leptoquark fermions. If they are leptoquark scalars, then their mass would be constrained by LHC data to be above 1 TeV or so. As fermions, \( Q \) has odd \( R \) parity, and must decay into the lightest supersymmetric particle, which is discussed in more detail below. We assume 200 GeV for this particle, hence there is no useful bound on \( m_Q \) at present.

As mentioned earlier, there are 2 copies of \( S_3 \) and 3 copies each of \( S_{1,2} \). In addition to the ones with VEVs in their scalar components, there are 5 other superfields. One pair \( \tilde{S}_{1,2} \) may form a pseudo-Dirac fermion, and be the lightest particle with odd \( R \).
parity. It will couple to $\chi$, say with strength $f_S$ which is independent of all other
couplings that we have discussed, then the tree-level decay $\chi \rightarrow \tilde{S}_1 \tilde{S}_2$ dominates the
total width of $\chi$ and is invisible.

\[ \Gamma(\chi \rightarrow \tilde{S}_1 \tilde{S}_2) = \frac{f_S^2}{8\pi} \sqrt{m_\chi^2 - 4m_S^2}. \quad (5.48) \]

For $m_\chi = 750$ GeV and $m_S = 200$ GeV, we find $\Gamma = 36$ GeV if $f_S = 1.2$. These
numbers reinforce our numerical analysis to support the claim that $\chi$ is a possible
candidate for the 750 GeV diphoton excess. Note also that $\lambda_g$ and $\lambda_\gamma$ have scalar
contributions which we have not considered. Adding them will allow us to reduce the
fermion contributions we have assumed and still get the same final results.
If we disregard the decay to dark matter \((f_S = 0)\), then the total width of \(\chi\) is dominated by \(\Gamma(\chi \rightarrow gg)\), which is then less than a GeV. Assuming that the cross section for the diphoton resonance is \(6.2 \pm 1\) fb \([83]\), we plot the allowed values of \(f_Q^2/4\pi\) versus \(m_Q\) for both \(f_S = 1.2\) which gives a total width of about 40 GeV for \(\chi\).
and $f_S = 0$ which requires much smaller values of $f_Q^2/4\pi$. Since $\chi$ must also decay into two gluons, we show the diject exclusion upper limits ($\sim 2$ pb) from the 8 TeV data in each case as well. Our choice of the pseudoscalar $\chi$ to be the 750 GeV diphoton resonance is motivated by the necessity of large couplings to $U, D$ leptoquark fermions for explaining the large width of about 40 GeV observed by ATLAS. If we take the evidence of CMS that this width is narrow, then as Fig. 3 shows, we can have much smaller couplings and much greater masses for $U, D$. In that case, we can use a physical scalar, with mass-squared matrix given in Eqs.(28) to (32), which is directly associated with the $\mu$ term.

5.7 Scalar Neutrino and Neutralino Sectors

In the neutrino sector, the $2 \times 2$ mass matrix spanning $(\nu, N^c)$ per family is given by the well-known seesaw structure:

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix},$$

(5.49)

where $m_D$ comes from $v_2$ and $m_N$ from $u_1$. There are two neutral complex scalars with odd $R$ parity per family, i.e. $\tilde{\nu} = (\tilde{\nu}_R + i\tilde{\nu}_I)/\sqrt{2}$ and $\tilde{N}^c = (\tilde{N}_R + i\tilde{N}_I)/\sqrt{2}$. The
$4 \times 4$ mass-squared matrix spanning ($\tilde{\nu}_R, \tilde{\nu}_I, \tilde{N}^c_R, \tilde{N}^c_I$) is given by

$$
\mathcal{M}^2_{\tilde{\nu}, \tilde{N}^c} = \begin{pmatrix}
    m_{\tilde{\nu}}^2 & 0 & A_D m_D & 0 \\
    0 & m_{\tilde{\nu}}^2 & 0 & -A_D m_D \\
    A_D m_D & 0 & m_{\tilde{N}^c}^2 + A_N m_N & 0 \\
    0 & -A_D m_D & 0 & m_{\tilde{N}^c}^2 - A_N m_N
\end{pmatrix}.
$$

(5.50)

In the MSSM, $\tilde{\nu}$ is ruled out as a dark-matter candidate because it interacts elastically with nuclei through the $Z$ boson. Here, the $A_N$ term allows a mass splitting between the real and imaginary parts of the scalar fields, and avoids this elastic-scattering constraint by virtue of kinematics. However, we still assume their masses to be heavier than that of $\tilde{S}_{1,2}$, discussed in the previous section.

In the neutralino sector, in addition to the $4 \times 4$ mass matrix of the MSSM spanning ($\tilde{\tilde{B}}, \tilde{\tilde{W}}_3, \tilde{\phi}^0_1, \tilde{\phi}^0_2$) with the $\mu$ parameter replaced by $fu_3$, i.e.

$$
\mathcal{M}_0 = \begin{pmatrix}
    M_1 & 0 & -g_1 v_1/\sqrt{2} & g_1 v_2/\sqrt{2} \\
    0 & M_2 & g_2 v_1/\sqrt{2} & -g_2 v_2/\sqrt{2} \\
    -g_1 v_1/\sqrt{2} & g_2 v_1/\sqrt{2} & 0 & -fu_3 \\
    g_1 v_2/\sqrt{2} & -g_2 v_2/\sqrt{2} & -fu_3 & 0
\end{pmatrix},
$$

(5.51)

there is also the $4 \times 4$ mass matrix spanning ($\tilde{\tilde{X}}, \tilde{\tilde{S}}_3, \tilde{\tilde{S}}_2, \tilde{\tilde{S}}_1$), i.e.

$$
\mathcal{M}_S = \begin{pmatrix}
    M_X & \sqrt{2} g_X u_3 & -2\sqrt{2} g_X u_2/3 & -\sqrt{2} g_X u_1/3 \\
    \sqrt{2} g_X u_3 & 0 & h_u_1 & h_u_2 \\
    -2\sqrt{2} g_X u_2/3 & h_u_1 & 0 & h_u_3 \\
    -\sqrt{2} g_X u_1/3 & h_u_2 & h_u_3 & 0
\end{pmatrix}.
$$

(5.52)
The two are connected through the $4 \times 4$ matrix

$$
\mathcal{M}_{0S} = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
-g_x v_1 / \sqrt{2} & -f v_2 & 0 & 0 \\
-g_x v_2 / \sqrt{2} & -f v_1 & 0 & 0 \\
\end{pmatrix}.
$$

(5.53)

These neutral fermions are odd under $R$ parity and the lightest could in principle be a dark-matter candidate. To avoid the stringent bounds on dark matter with the MSSM alone, we assume again that all these particles are heavier than $\tilde{S}_{1,2}$, as the dark matter discussed in the previous section.

## 5.8 Dark Matter

The $5 \times 5$ mass matrix spanning the 5 singlet fermions ($\tilde{S}_1, \tilde{S}_2, \tilde{S}_1, \tilde{S}_2, \tilde{S}_3$), corresponding to superfields with zero VEV for their scalar components, is given by

$$
\mathcal{M}_{\tilde{S}} = \begin{pmatrix}
0 & m_0 & 0 & 0 & m_{13} \\
m_0 & 0 & 0 & 0 & m_{23} \\
0 & 0 & 0 & M_3 & M_2 \\
m_{13} & m_{23} & M_2 & M_1 & 0 \\
\end{pmatrix}.
$$

(5.54)

Note that the $4 \times 4$ submatrix spanning ($\tilde{S}_1, \tilde{S}_2, \tilde{S}_1, \tilde{S}_2$) has been diagonalized to form two Dirac fermions. We can choose $m_0$ to be small, say 200 GeV, and $M_{1,2,3}$ to be large, of order TeV. However, because of the mixing terms $m_{13}, m_{23}$, the light Dirac
The dark matter with odd $R$ parity is the lighter of the two Majorana fermions, call it $\tilde{S}$, contained in the pseudo-Dirac fermion formed out of $\tilde{S}_{1,2}$ as discussed in Sec. 6. It couples to the $Z_X$ gauge boson, but in the nonrelativistic limit, its elastic scattering cross section with nuclei through $Z_X$ vanishes because it is Majorana. It also does not couple directly to the Higgs boson $h$, so its direct detection at underground search experiments is very much suppressed. However, it does couple to $A_S$ which couples also to quarks through the very small mixing of $A_S$ with $A$. This is further suppressed because it contributes only to the spin-dependent cross section. To obtain a spin-independent cross section at tree level, the constraint of Eqs. (17) to (19) have to be relaxed so that $h$ mixes with $S_{1,2,3}$.

Let the coupling of $h$ to $\tilde{S}\tilde{S}$ be $\epsilon$, then the effective interaction for elastic scattering of $\tilde{S}$ with nuclei through $h$ is given by

$$
L_{\text{eff}} = \epsilon \frac{f_q}{m_h} \bar{\tilde{S}} \tilde{S} \bar{q} q,
$$

(5.55)

where $f_q = m_q/2v = m_q/(246 \text{ GeV})$. The spin-independent direct-detection cross section per nucleon is given by

$$
\sigma^{SI} = \frac{4\mu_{DM}^2}{\pi A^2} \left[ \lambda_p Z + (A - Z) \lambda_n \right]^2,
$$

(5.56)

where $\mu_{DM} = m_{DM} M_A/(m_{DM} + M_A)$ is the reduced mass of the dark matter. Us-
\[
\lambda_N = \left[ \sum_{u,d,s} f_q^N + \frac{2}{27} \left( 1 - \sum_{u,d,s} f_q^N \right) \right] \frac{e m_N}{(246 \text{ GeV}) m_h^2},
\]  
(5.57)

with
\[
f_u^p = 0.023, \quad f_d^p = 0.032, \quad f_s^p = 0.020, 
\]
(5.58)
\[
f_u^n = 0.017, \quad f_d^n = 0.041, \quad f_s^n = 0.020, 
\]
(5.59)
we find \( \lambda_p \simeq 3.50 \times 10^{-8} \text{ GeV}^{-2} \), and \( \lambda_n \simeq 3.57 \times 10^{-8} \text{ GeV}^{-2} \). Using \( A = 131 \), \( Z = 54 \), and \( M_A = 130.9 \) atomic mass units for the LUX experiment [87], and \( m_{DM} = 200 \) GeV, we find for the upper limit of \( \sigma^{SI} < 1.5 \times 10^{-45} \text{ cm}^2 \), the bound \( \epsilon < 6.5 \times 10^{-4} \).

We have already invoked the \( \chi \bar{S}_1 \bar{S}_2 \) coupling to obtain a large invisible width for \( \chi \). Consider now the fermion counterpart of \( \chi \), call it \( \bar{S}' \), and the scalar counterparts of \( \bar{S}_{1,2} \), then the couplings \( \bar{S}' \bar{S}_1 \bar{S}_2 \) and \( \bar{S}' \bar{S}_2 \bar{S}_1 \) are also \( f_S = 1.2 \). Suppose one linear combination of \( S_{1,2} \), call it \( \zeta \), is lighter than 200 GeV, then the thermal relic abundance of dark matter is determined by the annihilation \( \bar{S} \bar{S} \rightarrow \zeta \zeta \), with a cross section times relative velocity given by
\[
\sigma \times v_{\text{rel}} = \frac{f_{\zeta}^4 m_{S'}^2 \sqrt{1 - m_{\zeta}^2/m_{S'}^2}}{16\pi (m_{S'}^2 + m_S^2 - m_{\zeta}^2)^2}. 
\]
(5.60)
Setting this equal to the optimal value [88] of \( 2.2 \times 10^{-26} \text{ cm}^3/\text{s} \), we find \( f_{\zeta} \simeq 0.62 \) for \( m_{S'} = 1 \text{ TeV} \), \( m_S = 200 \text{ GeV} \), and \( m_{\zeta} = 150 \text{ GeV} \). Note that \( \zeta \) stays in thermal equilibrium through its interaction with \( h \) from a term in \( V_D \). It is also very difficult to be produced at the LHC, because it is an SM singlet, so its mass of 150 GeV is allowed.
5.9 Conclusion

The utilitarian supersymmetric $U(1)_X$ gauge extension of the Standard Model of particle interactions proposed 14 years ago [45] allows for two classes of anomaly-free models which have no $\mu$ term and conserve baryon number and lepton number automatically. A simple version [78] with leptoquark superfields is especially interesting because of existing LHC flavor anomalies.

The new $Z_X$ gauge boson of this model has specified couplings to quarks and leptons which are distinct from other gauge extensions and may be tested at the LHC. On the other hand, a hint may already be discovered with the recent announcements by ATLAS and CMS of a diphoton excess at around 750 GeV. It may well be the revelation of the singlet scalar (or pseudoscalar) $S_3$ predicted by this model which also predicts that there should be singlet leptoquarks and other particles that $S_3$ must couple to. Consequently, gluon fusion will produce $S_3$ which will then decay to two photons together with other particles, including those of the dark sector. This scenario explains the observed diphoton excess, all within the context of the original model, and not an invention after the fact.

Since $S_3$ couples to leptoquarks, the $S_3 \rightarrow l_i^+ l_j^-$ decay must occur at some level. As such, $S_3 \rightarrow e^+ \mu^-$ would be a very distinct signature at the LHC. Its branching fraction depends on unknown Yukawa couplings which need not be very small. Similarly, the $S_3$ couplings to $\phi_1 \phi_2$ as well as leptoquarks imply decays to $ZZ$ and $Z\gamma$ with rates
comparable to $\gamma\gamma$.

An important byproduct of this study is the discovery of relaxed supersymmetric constraints on the Higgs boson’s mass of 125 GeV. It is now given by Eq. (23), i.e. $m_h^2 \simeq (g_X^2 + 2f^2 + \lambda_2)v^2$, which allows it to be free of the tension encountered in the MSSM. This prediction is independent of whether the diphoton excess is confirmed or not.

Most importantly, since $S_3$ replaces the $\mu$ parameter, its association with the 750 GeV excess implies the existence of supersymmetry. If confirmed and supported by subsequent data, it may even be considered in retrospect as the first evidence for the long-sought existence of supersymmetry.

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Table 5.1: Particle content of proposed model.

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<th>$SU(2)_L$</th>
<th>$U(1)_Y$</th>
<th>$U(1)_X$</th>
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<td>1/6</td>
<td>0</td>
</tr>
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<td>-2/3</td>
<td>1/2</td>
</tr>
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<td>1</td>
<td>1/3</td>
<td>1/2</td>
</tr>
<tr>
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<td>2</td>
<td>-1/2</td>
<td>1/3</td>
</tr>
<tr>
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<td>1</td>
<td>1</td>
<td>1/6</td>
</tr>
<tr>
<td>$N^c$</td>
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<td>1</td>
<td>0</td>
<td>1/6</td>
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<td>$\phi_1$</td>
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<td>2</td>
<td>-1/2</td>
<td>-1/2</td>
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<tr>
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<td>1/2</td>
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</tr>
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<td>1</td>
<td>0</td>
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Chapter 6

Gauge $B - L$ Model of Radiative Neutrino Mass with Multipartite Dark Matter[5]

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Abstract

We propose an extension of the standard model of quarks and leptons to include gauge $B - L$ symmetry with an exotic array of neutral fermion singlets for anomaly cancellation. With the addition of suitable scalars also transforming under $U(1)_{B-L}$, this becomes a model of radiative seesaw neutrino mass with possible multipartite dark matter. If leptoquark fermions are added, necessarily also transforming under $U(1)_{B-L}$, the diphoton excess at 750 GeV, recently observed at the Large Hadron Collider, may also be explained.
6.1 Introduction

It is well-known that a gauge $B - L$ symmetry is supported by a simple extension of the standard model (SM) of quarks and leptons with the addition of one singlet right-handed neutrino per family, so that the theory is anomaly-free. For convenience in notation, let these three extra neutral fermion singlets $N$ be left-handed, then their charges under $U(1)_{B-L}$ are $(1,1,1)$. Their additional contributions to the axial-vector anomaly and the mixed gauge-gravitational anomaly are respectively

$$(1)^3 + (1)^3 + (1)^3 = 3, \quad (1) + (1) + (1) = 3,$$

which cancel exactly those of the SM quarks and leptons. On the other hand, it has been known for some time [89] that another set of charges are possible, i.e.

$$(-5)^3 + (4)^3 + (4)^3 = 3, \quad (-5) + (4) + (4) = 3.$$  \hspace{1cm} (6.1)

Adding also three pairs of neutral singlet fermions with charges $(1,-1)$, naturally small seesaw Dirac masses for the known three neutrinos may be obtained [95], and a residual global $U(1)$ symmetry is maintained as lepton number. A further extension in the scalar sector allows for the unusual case of $Z_3$ lepton number [90] with the appearance of a scalar dark-matter candidate which is unstable but long-lived and decays to two antineutrinos. Here we consider another set of possible charges for the neutral fermion singlets, such that tree-level neutrino masses are forbidden. New scalar particles transforming under $U(1)_{B-L}$ are then added to generate one-loop
Majorana neutrino masses. The breaking of $B - L$ to $Z_2$ results in lepton parity and thus $R$ parity or dark parity [55] which is odd for some particles, the lightest neutral one being dark matter. A closer look at the neutral fermion singlets shows that one may be a keV sterile neutrino, and two others are heavy and stable, thus realizing the interesting scenario of multipartite dark matter. If color-triplet fermions with both $B$ and $L$ are added, the diphoton excess [73, 74] at 750 GeV, recently observed at the Large Hadron Collider (LHC), may also be explained.

6.2 Model

The extra left-handed neutral singlet fermions have charges $(2, 2, 2, -1, -1, -3)$, so that

$$4(2)^3 + 2(-1)^3 + (-3)^3 = 3, \quad 4(2) + 2(-1) + (-3) = 3.$$  \hfill (6.3)

Since there is no charge +1 in the above, there is no connection between them and the doublet neutrinos $\nu$ with charge $-1$ through the one Higgs doublet $\Phi$ which has charge zero. Neutrinos are thus massless at tree level. To generate one-loop Majorana masses, the basic mechanism of Ref. [25] is adopted, using the four fermions with charge +2, but because of the $U(1)_{B-L}$ gauge symmetry, we need both a scalar doublet $(\eta^+, \eta^0)$ and a scalar singlet $\chi^0$. The $U(1)_{B-L}$ gauge symmetry itself is broken by $\rho_2^0$ with charge $-2$ and by $\rho_4^0$ with charge $-4$. The leptoquark fermions $D_{1,2}$ and $D'_{1,2}$ are not necessary for neutrino mass, but are natural extensions of this model.
if the diphoton excess at 750 GeV requires an explanation. The complete particle content of this model is shown in Table 1.

### 6.3 Radiative Neutrino Mass

Using the four $N$'s, radiative Majorana masses for the three $\nu$'s are generated as shown in Fig. 1. [A systematic study of this mechanism under $B - L$ (with only one fermion and three scalars) has recently appeared [91] but does not include our case, which has four scalars.] Note that $N, \eta, \chi$ all have odd $R$ parity, so that

\[
\begin{array}{ccc}
\nu_i & N_k & \nu_j \\
\eta_0 & & \eta_0 \\
\rho_0^2 & & \chi_0 \\
\end{array}
\]

Figure 6.1: Radiative generation of neutrino mass through dark matter.

the lightest neutral particle among them is a dark-matter candidate. This is the scotogenic mechanism, from the Greek 'scotos' meaning darkness. In addition to the $\eta^i\Phi\chi$ trilinear coupling used in Fig. 1, there is also the $\eta^i\Phi\chi^\dagger\rho_2$ quadrilinear coupling, which may also be used to complete the loop. There are 4 real scalar fields, spanning
\[ \sqrt{2} \text{Re}(\eta^0), \sqrt{2} \text{Im}(\eta^0), \sqrt{2} \text{Re}(\chi^0), \sqrt{2} \text{Im}(\chi^0). \]

We denote their mass eigenstates as \( \zeta^0_l \) with mass \( m_l \). Let the \( \nu_i N_k \eta^0 \) coupling be \( h_{ik}^\nu \), then the radiative neutrino mass matrix is given by [25]

\[
(M_{\nu})_{ij} = \sum_k \frac{h_{ik}^\nu h_{jk}^\nu M_k}{16\pi^2} \sum_l [(y_l^R)^2 F(x_{lk}) - (y_l^I)^2 F(x_{lk})],
\]

where \(\sqrt{2} \text{Re}(\eta^0) = \sum_l y_l^R \zeta^0_l, \sqrt{2} \text{Im}(\eta^0) = \sum_l y_l^I \zeta^0_l, \) with \( \sum_l (y_l^R)^2 = \sum_l (y_l^I)^2 = 1, \)

\( x_{lk} = m_l^2/M_k^2, \) and the function \( F \) is given by

\[
F(x) = \frac{x \ln x}{x - 1}.
\]

### 6.4 Multipartite Dark Matter

Since the only neutral particles of odd \( R \) parity are \( N, \eta^0, \chi^0 \), there appears to be only one dark-matter candidate. However as shown below, there could be two or even four, all within the context of the existing model.

First note that \( \rho_{2,4}^0 \) have exactly the right \( U(1)_{B-L} \) charges to make the \((S,S,S')\) fermions massive. The corresponding \( 3 \times 3 \) mass matrix is of the form

\[
M_S = \begin{pmatrix}
m_{S1} & 0 & m_{13} \\
0 & m_{S2} & m_{23} \\
m_{13} & m_{23} & 0
\end{pmatrix},
\]

where \( m_{S1}, m_{S2} \) come from \( \langle \rho_2^0 \rangle = u_2 \) and \( m_{13}, m_{23} \) from \( \langle \rho_4^0 \rangle = u_4 \). If all these entries are of order 100 GeV to a few TeV, then there are three extra heavy singlet neutrinos in this model which also have even \( R \) parity. They do not mix with the
light active neutrinos $\nu$ at tree level, but do so in one loop. For example, $S'$ mixes with $\nu$ as shown in Fig. 2. Similarly $S$ will also mix with $\nu$, using the $SN\chi^0$ Yukawa coupling. However, these terms are negligible compared to the assumed large masses for $(S, S, S')$ and may be safely ignored.

Consider now the possibility that $m_{13}, m_{23} < m_{S1}, m_{S2}$ in $\mathcal{M}_S$, then $S'$ obtains a small seesaw mass given by

$$m_{S'} \simeq -\frac{m_{13}^2}{m_{S1}} - \frac{m_{23}^2}{m_{S2}}. \quad (6.7)$$

Let this be a few keV, then $S'$ is a light sterile neutrino which mixes with $\nu$ only slightly through Fig. 2. Hence it is a candidate for warm dark matter. Whereas the usual sterile neutrino is an ad hoc invention, it has a natural place here in terms of its mass as well as its suppressed mixing with the active neutrinos.

We now have the interesting scenario where part of the dark matter of the Universe
is cold, and the other is warm. This hybrid case was recently also obtained in a different radiative model of neutrino masses [92]. Within the present context, there is a third possibility. If we assign an extra $Z_2$ symmetry, under which $S_{1,2}$ are odd and all other particles even, then the only interactions involving $S_{1,2}$ come from their diagonal $U(1)_{B-L}$ gauge couplings and the diagonal Yukawa terms $f_1 S_1 S_1 (\rho_2^0)^*$ and $f_2 S_2 S_2 (\rho_2^0)^*$. This means that both $S_1$ and $S_2$ are stable and their relic abundances are determined by their annihilation cross sections to SM particles. In this scenario, dark matter has four components [93].

Since $S_{1,2}$ are now separated from $S'$, the $m_{13}$ and $m_{23}$ terms in $\mathcal{M}_S$ are zero and there is no tree-level mass for $S'$. However, there is a one-loop mass as shown in Fig. 3. This makes it more natural for $S'$ to be light. A detailed study of the dark-matter phenomenology of this multipartite scenario will be given elsewhere.

Figure 6.3: Radiative generation of $S'$ mass.


6.5 Scalar Sector for Symmetry Breaking

In this model, there is only one Higgs doublet $\Phi$ which breaks the $SU(2)_L \times U(1)_Y$ electroweak symmetry, whereas there are two Higgs singlets $\rho_2$ and $\rho_4$ which break $U(1)_{B-L}$ to $Z_2$. The most general Higgs potential consisting of $\Phi, \rho_2, \rho_4$ is given by

$$V = \mu_0^2 \Phi^\dagger \Phi + \mu_2^2 \rho_2^* \rho_2 + \mu_4^2 \rho_4^* \rho_4 + \frac{1}{2} \mu_24 [\rho_2^2 \rho_4^* + H.c.] + \frac{1}{2} \lambda_0 (\Phi^\dagger \Phi)^2 + \frac{1}{2} \lambda_2 (\rho_2^* \rho_2)^2 + \frac{1}{2} \lambda_4 (\rho_4^* \rho_4)^2 + \frac{1}{2} \lambda_0^2 (\Phi^\dagger \Phi) (\rho_2^* \rho_2) + \lambda_04 (\Phi^\dagger \Phi) (\rho_4^* \rho_4) + \lambda_24 (\rho_2^* \rho_2) (\rho_4^* \rho_4).$$  \hspace{1cm} (6.8)

Let $\langle \phi^0 \rangle = v$, $\langle \rho_2 \rangle = u_2$, $\langle \rho_4 \rangle = u_4$, then the minimum of $V$ is determined by

$$0 = \mu_0^2 + \lambda_0 v^2 + \lambda_2 u_2^2 + \lambda_4 u_4^2,$$  \hspace{1cm} (6.9)

$$0 = \mu_2^2 + \lambda_0 v^2 + \lambda_2 u_2^2 + \lambda_4 u_4^2,$$  \hspace{1cm} (6.10)

$$0 = u_4 (\mu_4^2 + \lambda_0 v^2 + \lambda_2 u_2^2 + \lambda_4 u_4^2) + \frac{1}{2} \mu_24 u_2^2.$$  \hspace{1cm} (6.11)

The would-be Goldstone bosons are $\phi^\pm$, $\sqrt{2} Im(\phi^0)$, corresponding to the breaking of $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$, and $\sqrt{2} [u_2 Im(\rho_2) + 2u_4 Im(\rho_4)]/\sqrt{u_2^2 + 4u_4^2}$, corresponding to the breaking of $U(1)_{B-L}$ to $Z_2$. The linear combination orthogonal to the latter is a physical pseudoscalar $A$, with a mass given by

$$m_A = \frac{-\mu_24 (u_2^2 + 4u_4^2)}{2u_4}.$$  \hspace{1cm} (6.12)

The $3 \times 3$ mass-squared matrix of the physical scalars $[\sqrt{2} Re(\phi^0), \sqrt{2} Re(\rho_2), \sqrt{2} Re(\rho_4)]$ is given by

$$M^2 = \begin{pmatrix}
2\lambda_0 v^2 & 2\lambda_0 v u_2 & 2\lambda_04 v u_4 \\
2\lambda_0 v u_2 & 2\lambda_2 u_2^2 & u_2 (2\lambda_2 u_4 + \mu_24) \\
2\lambda_04 v u_4 & u_2 (2\lambda_2 u_4 + \mu_24) & 2\lambda_4 u_4^2 - \mu_24 u_2^2/2u_4
\end{pmatrix}. \hspace{1cm} (6.13)$$
For $v^2 << u_{2,4}^2$, $\sqrt{2}Re(\phi^0) = h$ is approximately a mass eigenstate which is identified with the 125 GeV particle discovered at the LHC.

### 6.6 Gauge Sector

Since $\phi^0$ does not transform under $U(1)_{B-L}$ and $\rho_{2,4}$ do not transform under $SU(2)_L \times U(1)_Y$, there is no tree-level mixing between their corresponding gauge bosons $Z$ and $Z_{B-L}$. In our convention, $M_{Z_{B-L}}^2 = 8g_{B-L}^2(u_2^2 + 4u_4^2)$. The LHC bound on $M_{Z_{B-L}}$ comes from the production of $Z_{B-L}$ from $u$ and $d$ quarks and its subsequent decay to $e^-e^+$ and $\mu^-\mu^+$. If all the particles listed in Table 1 are possible decay products of $Z_{B-L}$ with negligible kinematic suppression, then its branching fraction to $e^-e^+$ and $\mu^-\mu^+$ is about 0.061. The $c_u,d$ coefficients used in the LHC analysis [57, 60] are then

$$c_u = c_d = \left[ \left( \frac{1}{3} \right)^2 + \left( \frac{1}{3} \right)^2 \right] g_{B-L}^2 \times B(Z_{B-L} \rightarrow e^-e^+,\mu^-\mu^+) = 1.36 \times 10^{-2} g_{B-L}^2.$$  

(6.14)

From LHC data based on the 7 and 8 TeV runs, a bound of about 2.5 TeV would correspond to $g_{B-L} < 0.24$. 

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6.7 Leptoquark Fermions

The singlet leptoquark fermions $D_{1,2}$ have charge $-1/3$ and the following possible interactions:

$$D_1 d^c \chi^*, \quad D_2 d^c \chi, \quad D_1 D_2^c \rho^*_2, \quad D_2 D_1^c \rho_2.$$  \hfill (6.15)

Hence they mix in a $2 \times 2$ mass matrix linking $D_{1,2}$ to $D_{1,2}^c$ with $\langle \rho_2 \rangle = u_2$, and decay to $d$ quarks + $\chi(\chi^*)$. Now $\chi$ mixes with $\eta^0$, so it decays to neutrinos ($\nu$) and dark matter ($N$), which are invisible. The search for $D_{1,2}$ at the LHC would be similar to the search for scalar quarks which decay to quarks + missing energy. However, if we assume that $N$ has a mass of about 200 GeV, then there is no useful limit at present on the mass of $D_{1,2}$ from the LHC.

Consider now the pseudoscalar $A$ of Eq. (12). Let the two mass eigenstates in the $(D_{1,2}, D_{1,2}^c)$ sector be $\psi_{1,2}$, then $A$ couples to them according to

$$\mathcal{L}_{\text{int}} = f_1 \bar{\psi}_1 \gamma_5 \psi_1 + f_2 \bar{\psi}_2 \gamma_5 \psi_2,$$  \hfill (6.16)

where $f_{1,2}$ are rearranged from their original $D_1 D_2^c \rho_2^*$ and $D_2 D_1^c \rho_2$ couplings. Hence $A$ decays to two gluons as well as to two photons in one loop through $\psi_{1,2}$. It may also decay to dark matter, say $NN$, at tree level. It is thus a possible candidate for explaining the 750 GeV diphoton excess recently observed [73, 74] at the LHC. The numerical analysis of this model runs parallel to that of a recent proposal [94], and will not be repeated here. Note again that these leptoquark fermions are not essential for the radiative generation of neutrino masses based on $B - L$. 

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6.8 Conclusion

Using gauge $U(1)_{B-L}$ symmetry, we have proposed a new anomaly-free solution with exotic fermion singlets, such that neutrino mass is forbidden at tree level. We add a number of new scalars so that neutrino masses are obtained in one loop through dark matter, i.e. the scotogenic mechanism. Because of the structure of the new singlets required for anomaly cancellation, we find a possible dark-matter scenario with four components. Three are stable cold Weakly Interaction Massive Particles (WIMPs) and one a keV singlet neutrino, i.e. warm dark matter with a very long lifetime. If leptoquark fermions are added, transforming under $U(1)_{B-L}$, the recently observed 750 GeV diphoton excess may also be explained.

6.9 Acknowledgement

This work was supported in part by the U. S. Department of Energy Grant No. DE-SC0008541.
Table 6.1: Particle content of proposed model.

<table>
<thead>
<tr>
<th>Particle</th>
<th>$SU(3)_C$</th>
<th>$SU(2)_L$</th>
<th>$U(1)_Y$</th>
<th>$B$</th>
<th>$L$</th>
<th>$B - L$</th>
<th>copies</th>
<th>$R$ parity</th>
</tr>
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<td>2</td>
<td>1/6</td>
<td>1/3</td>
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<td>1/3</td>
<td>3</td>
<td>+</td>
</tr>
<tr>
<td>$u^c$</td>
<td>$3^*$</td>
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<td>$-2/3$</td>
<td>$-1/3$</td>
<td>0</td>
<td>$-1/3$</td>
<td>3</td>
<td>+</td>
</tr>
<tr>
<td>$d^c$</td>
<td>$3^*$</td>
<td>1</td>
<td>$1/3$</td>
<td>$-1/3$</td>
<td>0</td>
<td>$-1/3$</td>
<td>3</td>
<td>+</td>
</tr>
<tr>
<td>$L = (\nu, e)$</td>
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<td>$-1/2$</td>
<td>0</td>
<td>1</td>
<td>$-1$</td>
<td>3</td>
<td>+</td>
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<tr>
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<td>3</td>
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<td>0</td>
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<td>−</td>
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<td>0</td>
<td>0</td>
<td>3</td>
<td>$-3$</td>
<td>1</td>
<td>+</td>
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<td>0</td>
<td>1</td>
<td>+</td>
</tr>
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<td>2</td>
<td>$1/2$</td>
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<td>−</td>
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<td>1</td>
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<td>−</td>
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<tr>
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<td>0</td>
<td>2</td>
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<td>+</td>
</tr>
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<td>0</td>
<td>4</td>
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<td>+</td>
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<td>4/3</td>
<td>1</td>
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<td>1</td>
<td>−</td>
</tr>
<tr>
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<td>$3^*$</td>
<td>1</td>
<td>$1/3$</td>
<td>$-1/3$</td>
<td>1</td>
<td>$-4/3$</td>
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<td>−</td>
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Chapter 7

Pathways to Naturally Small Dirac Neutrino Masses[6]

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Department of Physics and Astronomy,

University of California, Riverside, California 92521, USA
Abstract

If neutrinos are truly Dirac fermions, the smallness of their masses may still be natural if certain symmetries exist beyond those of the standard model of quarks and leptons. We perform a systematic study of how this may occur at tree level and in one loop. We also propose a scotogenic version of the left-right gauge model with naturally small Dirac neutrino masses in one loop.
7.1 Introduction

If neutrinos are Majorana fermions, then it has been known [19] since 1979 that they are described by a unique dimension-five operator beyond the standard model of quarks and leptons, i.e.

$$L_5 = -\frac{f_{ij}}{2\Lambda}(\nu_i\phi^0 - l_i\phi^+)(\nu_j\phi^0 - l_j\phi^+) + H.c. \quad (7.1)$$

Neutrino masses are then proportional to $v^2/\Lambda$, where $v = \langle \phi^0 \rangle$ is the vacuum expectation value of the Higgs doublet $(\phi^+, \phi^0)$. This formula is necessarily seesaw because $\Lambda$ has already been assumed to be much greater than $v$ in the first place. It has also been known [20] since 1998 that there are three specific tree-level realizations (denoted as Types I,II,III) and three generic one-particle-irreducible one-loop realizations.

If neutrinos are Dirac fermions, then the term $m_D\bar{\nu}_L\nu_R\phi^0$ is desired but $(m_N/2)\nu_R\nu_R$ must be forbidden. This requires the existence of a symmetry, usually taken to be global $U(1)_L$ lepton number. This may be the result of a spontaneously broken $U(1)_{B-L}$ gauge symmetry where the scalar which breaks the symmetry carries three [95] and not two units of $B-L$ charge. On the other hand, global $U(1)_L$ is not the only possibility. The notion of lepton number itself may in fact be discrete. It cannot of course be $Z_2$, then $m_N$ would be allowed and neutrino masses are Majorana. However, it may be $Z_3$ [90, 96] or $Z_4$ [97, 98, 99], but then new particles must appear to legitimize this discrete lepton symmetry. Since there are three neutrinos, a flavor symmetry may also be used to forbid the $\nu_R\nu_R$ terms [100].
To obtain a naturally small $m_D$, there must be another symmetry which forbids the dimension-four $\bar{\nu}_L \nu_R \bar{\phi}^0$ term, but this symmetry must also be softly or spontaneously broken, so that an effective $m_D$ appears, at tree level or in one loop, suppressed by large masses. The symmetry used to achieve this is model-dependent. Nevertheless generic conclusions may be obtained regarding the nature of the necessary particles involved, as shown below.

### 7.2 Four specific tree-level realizations

Assume a symmetry $\mathcal{S}$ under which $\nu_L$ and $\phi^0$ do not transform, but $\nu_R$ does. There are then four and only four ways to connect them at tree level through the soft breaking of this symmetry.

- Insert a Dirac fermion singlet $N$ which does not transform under $\mathcal{S}$, then break $\mathcal{S}$ softly by the dimension-three $\bar{\nu}_R N_L$ term.

$\nu_L \quad N_R \quad N_L \quad \bar{\nu}_R$

Figure 7.1: Dirac neutrino mass with a Dirac singlet fermion insertion.
• Insert a Dirac fermion triplet \((\Sigma^+, \Sigma^0, \Sigma^-)\) which does not transform under \(\mathcal{S}\), then break \(\mathcal{S}\) and \(SU(2)_L \times U(1)\) together spontaneously to obtain the dimension-three \(\bar{\nu}_R \Sigma^0_L\) term.

\[
\begin{array}{c}
\phi^0 \\
\downarrow \\
\nu_L \Sigma^0_R \Sigma^0_L \nu_R
\end{array}
\]

Figure 7.2: Dirac neutrino mass with a Dirac triplet fermion insertion.

• Insert a Dirac fermion doublet \((E^0, E^-)\) which transforms as \(\nu_R\) under \(\mathcal{S}\), then break \(\mathcal{S}\) softly by the dimension-three \((\bar{E}^0 \nu_L + E^+ e^-)\) term.

\[
\begin{array}{c}
\phi^0 \\
\downarrow \\
\nu_L E^0_R E^0_L \nu_R
\end{array}
\]

Figure 7.3: Dirac neutrino mass with a Dirac doublet fermion insertion.
• Insert a scalar doublet \((\eta^+, \eta^0)\) which transforms as \(\nu_R\) under \(S\), then break \(S\) softly by the dimension-two \((\eta^- \phi^+ + \eta^0 \phi^0)\) term.

\[
\begin{array}{c}
\nu_R \\
\eta^0 \\
\phi^0 \\
\nu_L \\
\end{array}\]

Figure 7.4: Dirac neutrino mass with a doublet scalar insertion.

In Figs. 1 to 3, the mechanism which makes \(m_D\) small is the Dirac seesaw [101]. The \(2 \times 2\) mass matrix linking \((\bar{\nu}_L, \bar{\psi}_L)\) to \((\nu_R, \psi_R)\), where \(\psi = N, \Sigma^0, E^0\), is of the form

\[
M_{\nu\psi} = \begin{pmatrix}
0 & m_1 \\
m_2 & M_\psi
\end{pmatrix}.
\]  

(7.2)

Since \(M_\psi\) is an invariant mass, it may be assumed to be large, whereas \(m_{1,2}\) come from either electroweak symmetry breaking or \(S\) breaking and may be assumed small in comparison. Hence \(m_D \simeq m_1 m_2 / M_\psi\) is naturally small as desired.

In Fig. 4, the mechanism is also seesaw but in the scalar sector, as first pointed out in Ref. [102]. Using the small soft \(S\) breaking term \(\bar{\eta}^0 \phi^0\) together with a large mass for \(\eta\), a small vacuum expectation value \(\langle \eta^0 \rangle\) is induced to obtain \(m_D\) [103]. This may also be accomplished by extending the gauge symmetry [104, 105].
7.3 Two generic one-loop realizations

Suppose the new particles considered previously for connecting $\nu_L$ with $\nu_R$ at tree level are not available, then a Dirac neutrino mass may still occur in one loop. Assuming that this loop consists of a fermion line and a scalar line, then the external Higgs boson must couple to either the scalar line or the fermion line, yielding two generic diagrams.

- Consider the one-loop connection shown below. Since $\nu_R$ transforms under $S$ and $\nu_L$ and $\phi^0$ do not, a Dirac neutrino mass is only generated if $S$ is broken softly by either the dimension-three $\bar{\psi}_L \psi_R$ term or the dimension-three $\bar{\eta} \chi \phi^0$ term. There are an infinite number of solutions for the new fermion $\psi$ and the new scalars $\eta$ and $\chi$. Under the electroweak $SU(2)_L \times U(1)_Y$, the three simplest solutions are listed in Table 1. Note that solutions also exist with $\psi, \chi, \eta$ all carrying color. Let $S$ be $Z_2$ as an example, then the assignments
Table 7.1: $SU(2)_L \times U(1)_Y$ assignments of $\psi$, $\eta$, and $\chi$.

<table>
<thead>
<tr>
<th>solution</th>
<th>$\psi$</th>
<th>$\eta$</th>
<th>$\chi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(1,0)</td>
<td>(2, $-1/2$)</td>
<td>(1,0)</td>
</tr>
<tr>
<td>B</td>
<td>(2, $1/2$)</td>
<td>(1,0)</td>
<td>(2, $1/2$)</td>
</tr>
<tr>
<td>C</td>
<td>(2, $-1/2$)</td>
<td>(1, $-1$)</td>
<td>(2, $-1/2$)</td>
</tr>
</tbody>
</table>

of $\eta$, $\psi_R$, $\psi_L$, and $\chi$ under $S$ are given in Table 2. The solutions A1 and B1

Table 7.2: $S = Z_2$ assignments of $\eta$, $\psi_R$, $\psi_L$, and $\chi$.

<table>
<thead>
<tr>
<th>solution</th>
<th>$\eta$</th>
<th>$\psi_R$</th>
<th>$\psi_L$</th>
<th>$\chi$</th>
<th>$\bar{\psi}_L\psi_R$</th>
<th>$\bar{\eta}\chi\phi^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>A2</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
<tr>
<td>B1</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>B2</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
<tr>
<td>C1</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
</tr>
<tr>
<td>C2</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
</tbody>
</table>

must be discarded, because $\chi$ and $\eta$ are neutral scalar singlets which are even under $Z_2$ respectively. As such, they will acquire vacuum expectation values from interactions with $\Phi$. From the trilinear coupling $\bar{\eta}\chi\phi^0$, this in turn would induce a vacuum expectation value for $\eta$ and $\chi$ in A1 and B1 respectively.
Hence the loop of Fig. 5 would collapse to a tree as shown in Figs. 1 and 3. The solutions $A2(B2)$ should also be discarded because $\psi_{R,L}$ transform exactly as $\nu_{R,L}$, thus collapsing to Fig. 4. However, these solutions could be reinstated with the scotogenic mechanism to be discussed later.

- Consider now the other possible connection. The only soft term here is the quadratic $\bar{\eta}\chi$ term which must be odd under $S = Z_2$. If $\eta$ and $\chi$ are neutral, then again they must have vacuum expectation values, thus collapsing the loop of Fig. 6 to a tree. Hence $\eta$ and $\chi$ must be charged or colored, and if $\chi \sim \pm$ under $S$, then $\psi_{L,R}, \eta \sim \mp$. An example of such a model is Ref. [106]. It has also been implemented in left-right gauge models many years ago [29, 107].

In the above, there must be of course also a symmetry which maintains lepton number. This symmetry may propagate along the fermion line in the loop, which is the conventional choice, but it may also propagate along the scalar line in the loop. If the

![Figure 7.6: Dirac neutrino mass in one loop with quadratic scalar mixing.](image)
latter, then lepton number may serve as the stabilizing symmetry of dark matter [55].

The reason is very simple. For the lightest scalar, say $\eta$, having lepton number which is conserved, it can only decay into a lepton plus a fermion which has no lepton number, say $\psi$, and vice versa. Hence the lightest $\psi$ or the lightest $\eta$ is dark matter. This means that the loop diagrams of Figs. 5 and 6 could be naturally scotogenic, from the Greek 'scotos' meaning darkness. This mechanism was invented 10 years ago [25]. The unconventional assignment of lepton number to scalars and fermions also reinstates the solutions A,B considered earlier, because now the particles in the loop have odd dark parity, as discussed in Ref. [108, 109]. This application of the one-loop diagram for Dirac neutrino mass using scalars carrying lepton number is actually well-known in supersymmetry, where the exchange of sleptons and neutralinos contributes to charged-lepton masses. Here we show that the generic idea is also applicable without supersymmetry.

7.4 Scotogenic Dirac neutrino mass in left-right model

The absence of a tree-level Dirac neutrino mass may be due to the underlying gauge symmetry and the scalar particle content. Consider the following left-right gauge mode based on $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X$ together with a discrete $Z_2$ symmetry. It extends the standard model (SM) to include heavy charged quarks...
and leptons which are odd under $Z_2$, but no scalar bidoublet [110] as shown in Table 3. Its fermion content is identical to a recent proposal [111] at this point.

Table 7.3: Particle content of proposed left-right gauge model.

<table>
<thead>
<tr>
<th>particles</th>
<th>$SU(3)_C$</th>
<th>$SU(2)_L$</th>
<th>$SU(2)_R$</th>
<th>$U(1)_X$</th>
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<td>$u_R$</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>$2/3$</td>
<td>$+$</td>
</tr>
<tr>
<td>$d_R$</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>$-1/3$</td>
<td>$+$</td>
</tr>
<tr>
<td>$(\nu,e)_L$</td>
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<td>1</td>
<td>$-1/2$</td>
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<tr>
<td>$e_R$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$-1$</td>
<td>$+$</td>
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<td>2</td>
<td>$1/6$</td>
<td>$-$</td>
</tr>
<tr>
<td>$(\nu,E)_R$</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>$-1/2$</td>
<td>$-$</td>
</tr>
<tr>
<td>$U_L$</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>$2/3$</td>
<td>$-$</td>
</tr>
<tr>
<td>$D_L$</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>$-1/3$</td>
<td>$-$</td>
</tr>
<tr>
<td>$E_L$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$-1$</td>
<td>$-$</td>
</tr>
<tr>
<td>$(\phi^+_L, \phi^0_L)$</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>$1/2$</td>
<td>$+$</td>
</tr>
<tr>
<td>$(\phi^+_R, \phi^0_R)$</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>$1/2$</td>
<td>$+$</td>
</tr>
</tbody>
</table>

The breaking of $SU(2)_{L,R}$ is accomplished by the Higgs doublets $\Phi_{L,R}$. The SM quarks and charged leptons obtain masses from $\Phi_L$. The heavy quarks and charged leptons obtain masses from $\Phi_R$. They are separated by the $Z_2$ symmetry and do not
Table 7.4: Scotogenic additions to the proposed left-right gauge model.

<table>
<thead>
<tr>
<th>particles</th>
<th>$SU(3)_C$</th>
<th>$SU(2)_L$</th>
<th>$SU(2)_R$</th>
<th>$U(1)_X$</th>
<th>$Z_2$</th>
<th>$Z_2^D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{L,R}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>$(\eta_0^+, \eta_0^0)$</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1/2</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>$(\eta_0^+, \eta_0^0)$</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1/2</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>$\chi_L$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>+</td>
<td>−</td>
</tr>
<tr>
<td>$\chi_R$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>$\chi_0$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>−</td>
<td>+</td>
</tr>
</tbody>
</table>

mix. Both $\nu_L$ and $\nu_R$ are massless and separated by $Z_2$. To link them with a Dirac mass, this $Z_2$ has to be broken. This is implemented as shown in Table 4 using the unbroken symmetry $Z_2^D$ for dark matter, under which $N, \eta_{L,R}, \chi_{L,R}$ are odd and all others are even. The symmetry $Z_2$ is assumed to be respected by all dimension-three terms as well, so there is no $\bar{Q}_L q_R$ or $\bar{E}_L e_R$ term. Hence $V_{CKM}$ remains unitary as in the SM. It is broken only by the unique dimension-two term $\chi_L \chi_R$. The resulting scotogenic diagram for Dirac neutrino mass is shown in Fig. 7. The connection between the heavy fermions of the $SU(2)_R$ sector and the SM fermions is $\chi_0$ with the allowed dimension-four Yukawa couplings $\chi_0 \bar{Q}_L q_R$ and $\chi_0 \bar{E}_L e_R$. Now $\chi_0$ mixes only radiatively with the SM Higgs boson, a phenomenon discovered only recently [76], and decays to SM particles but its lifetime may be long. At the Large Hadron Collider,
the heavy $SU(2)_R$ quarks are easily produced if kinematically allowed. The lightest will decay to a SM quark and $\chi_0$ which may escape the detector as missing energy. This has the same signature as dark matter. The true dark matter is of course the lightest neutral fermion or boson with odd $Z_2^D$.

### 7.5 Concluding remarks

The notion that neutrino masses are Dirac is still viable in the absence of incontrovertible experimental proof of the existence of neutrinoless double beta decay. The theoretical challenge is to understand why. In this paper we study systematically how the smallness of Dirac neutrino masses may be achieved at tree level (four specific cases) and in one loop (two generic cases). We also propose a scotogenic left-right gauge model.
7.6 Acknowledgement

This work was supported in part by the U. S. Department of Energy Grant No. DE-SC0008541.
Chapter 8

One Leptoquark to unify them?

Neutrino masses and unification in

the light of \((g - 2)\mu\), \(R_{D(\ast)}\) and \(R_K\)

anomalies[7]

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Abstract

Leptoquarks have been proposed as a possible explanation of anomalies in $B \rightarrow D^* \tau \bar{\nu}$ decays, the apparent anomalies in $(g - 2)_\mu$ experiments and a violation of lepton universality. Motivated by this, we examine other motivations of leptoquarks: radiatively induced neutrino masses in the presence of a discrete symmetry that prevents a tree level see-saw mechanism, gauge coupling unification, and vacuum stability at least up to the unification scale. We present a new model for radiatively generating a neutrino mass which can significantly improve gauge coupling unification at one loop. We discuss this, and other models in the light of recent work on flavour anomalies.
One Leptoquark to unify them? Neutrino masses and unification in the light of \((g - 2)_{\mu}\), \(R_{D(\ast)}\) and \(R_{K}\) anomalies

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Abstract

Leptoquarks have been proposed as a possible explanation of anomalies in \(\bar{B} \rightarrow D^*\tau\bar{\nu}\) decays, the apparent anomalies in \((g - 2)_{\mu}\) experiments and a violation of lepton universality. Motivated by this, we examine other motivations of leptoquarks: radiatively induced neutrino masses in the presence of a discrete symmetry that prevents a tree level see-saw mechanism, gauge coupling unification, and vacuum stability at least up to the unification scale. We present a new model for radiatively generating a neutrino mass which can significantly improve gauge coupling unification at one loop. We discuss this, and other models in the light of recent work on flavour anomalies.

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I. INTRODUCTION

Recently there has been a lot of interest in leptoquarks as a possible explanation for some striking deviations from the standard model [1–12] including anomalous B decays observed in BaBar [13, 14], Belle [15] and LHCb [16–18], a violation in lepton universality [19] and a deviation from the standard model prediction of $(g - 2)_\mu$ [20]. A particularly interesting claim was that all three anomalies could be explained via the addition of a single leptoquark [4]. Specifically they chose a single $\sim$ TeV scale leptoquark, $\phi$, that is a colour triplet and an SU(2)$_L$ singlet with hyper charge $-1/3$. The correction to the standard model Lagrangian due to the existence of this leptoquark is Lagrangian

$$\mathcal{L} \ni (D_\mu \phi)^\dagger D^\mu \phi - M^2 \phi \phi - g h \phi |\Phi|^2 |\phi|^2$$

$$+ \bar{Q}^c \lambda_L \tau_2 L \phi^* + \bar{u}^c \lambda_R \tau_2 R \phi^* + \text{h. c.}$$

where $\Phi$ is the standard model Higgs doublet. We will denote such a leptoquark by its gauge quantum numbers $(3, 1, -1/3)$. The attempt to explain lepton universality with such a leptoquark was brought into significant doubt recently [21] (who explain the anomaly with a leptoquark with quantum numbers $(3, 2, 1/6)$). However it has been recently demonstrated that the $(3, 1, -1/3)$ leptoquark indeed can explain all three anomalies [22].

Leptoquarks are also of theoretical interest as the tend to appear in various grand unified theories (GUTs). Indeed the particular choice of leptoquark used in ref. [4] appears in E6 GUTs [23, 24] and the $(3, 2, 1/6)$ leptoquark arises for example in an
SU(5) GUT. Leptoquarks have also been proposed as a non-supersymmetric catalyst of gauge coupling unification [25–28] and a cause of neutrino masses via radiative corrections [27, 29–39]. These explanations of the neutrino mass are phenomenologically attractive as unlike the see saw mechanism, these models have predictions at much lower energies.

It is of interest to us whether using these leptoquarks to explain these flavour anomalies is compatible with some of these theoretical motivations and if not what is the minimal extension needed for such compatibility. Specifically we look at radiatively induced neutrino masses, gauge coupling unification, and vacuum stability. We will consider both the \((3, 1, -1/3)\) and \((3, 2, 1/6)\) representations and propose a model for each that radiatively induces Weinberg dim-5 operator \((LLHH/\Lambda)\) to produce Majorana neutrino mass at one and two loops respectively. This requires the introduction of new particles for both cases. For the 1-loop case the addition of heavy d-type quarks is needed, 1 per family. We show a sample parameter space that avoids experimental bounds and gives neutrino masses of the correct order. As for 2 loop scenario with \((3, 1, -1/3)\) leptoquark we include heavy scalar diquarks with different isospin structure for the loop completion. Proton decay is also of interest when one considers unification. However, the \((3, 2, 1/6)\) easily avoids constraints on proton decay and the \((3, 1, -1/3)\) Leptoquarks are similarly safe in the presence of a discrete symmetry where SM leptons and the leptoquark have opposite parity.

For gauge coupling unification we find two paths - first through the \((3, 2, 1/6)\) case without the extra particles required to radiatively induce a neutrino mass - and
second through the introduction of both the \((3, 1, -1/3)\) leptoquark and new particles required to generate the neutrino mass at one loop. A third more difficult path also becomes manifest in the case of a \((3, 1, -1/3)\) leptoquark through large leptoquark couplings. However, this puts constraints on the leptoquark couplings which partly conflict with the constraints on leptoquark coupling given in [4] as they had \(\lambda^R_\ell << \lambda^L_\ell\) which is generically the opposite condition to what gauge coupling unification requires. One in principle has enough freedom left to achieve gauge coupling unification although this requires some couplings to be brought close to the perturbativity bound. This makes the calculation susceptible to high theoretical error. Most of this tension is due to one of the anomalies - the observed violation of lepton universality. This is precisely the anomaly that is difficult to explain via the \((3, 1, -1/3)\) leptoquark [21]. By contrast, vacuum stability is substantially boosted due to the improved running of the Higgs quartic coupling when the portal coupling is non-negligible. This is true for both leptoquark representations. Furthermore the extra fermions used to generate a neutrino mass at 1 loop will only contribute to the running of the Higgs quartic at two loop. For the case where gauge coupling unification is achieved through large leptoquark couplings one requires a larger Higgs portal coupling to maintain stability.

The structure of this paper is as follows. We discuss minimal modifications to achieve neutrino masses aside from the tree level term in section II. In sections III and IV we discuss the modification to the running of the gauge coupling constants and the Higgs quartic coupling respectively, thereby discussing the viability of gauge coupling
unification and vacuum stability in this model. We then discuss how compatible these constraints are with collider constraints and the need to simultaneously explain violations of lepton universality, anomalies in $B$ decays and the measurement of $(g - 2)_\mu$ experiments in section V. We conclude in section VI.

II. POSSIBLE EXTENSIONS OF THE LEPTOQUARK MODEL FOR THE NEUTRINO MASS GENERATION

To generate Neutrino mass using the leptoquark model discussed above, we need to go beyond this model and include more particles/fields. Depending on the content added, a neutrino mass can be generated at 1 or 2 loop level. There are several examples in the literature that generate neutrino mass through the use of multiple leptoquarks fields [27, 29–39]. If Dirac Neutrino Mass is generated then potentially additional symmetries, global or gauged, are needed to forbid tree level Neutrino mass generation. Here we briefly discuss 2 extensions of the leptoquark model to generate the effective Majorana neutrino mass at the 1 and 2 loop order through an effective d-5 Weinberg operator[56] $LLHH/\Lambda$ with only one leptoquark field in the model.

A. 2 loop Majorana Neutrino Mass

Generating a Majorana neutrino mass at 2 loop order from the leptoquark model mentioned in the present work requires the addition of extra colored charged scalar multiplets. Figure 1 shows 2 different variants of Majorana neutrino mass with an
FIG. 1: 2 loop Majorana Neutrino Mass through effective d-5 Weinberg operator.

effective Weinberg operator. In the first diagram, external Higgs fields are coupled symmetrically at the center vertex to the charged scalar line, and another one having one of Higgs field coupled to the leptoquarks at the top vertex and the second Higgs field coupled at the center. Both diagrams require the LQ$\phi$ coupling, fields added to complete the loops do not couple to the Leptons and Quarks simultaneously and so do not effect the results of the anomaly calculations done in [4] at the leading order calculations. The first diagram requires the minimal addition of 2 diquarks, which have the following representations (6,1, -2/3), (6,3,-1/3) which are all color sextets. The charged scalar field coupled to the leptoquarks must be an iso-singlet where as the charged scalar field coupled to SM quark doublets is a triplet under SU(2)$_L$. Lepton
number is broken at the soft dim-3 $\phi \phi \chi^*$ term by 2 units. The second diagram of Fig.1 is similar to the first diagram but now $\chi \sim (6,2,1/6)$ is a doublet under SU(2)$_L$ and $\rho \sim (6,3,2/3)$ is an-isotriplet as in the first diagram. The Lepton number is broken at the only dim-3 soft term by 2 units.

If the $\phi \sim (3,2,1/6)$ leptoquark representation is used instead of $(3,1,-1/3)$, then we get $\bar{d}_R L \phi$ operator instead of $LQ \phi$. Then the required fields for the first 2-loop completion are $\chi \sim (6,3,1/3)$ and $\rho \sim (6,1,2/3)$. For the second 2-loop diagram the required scalar diquark fields are $\chi \sim (6,2,7/6)$ and $\rho \sim (6,1,2/3)$.

B. 1 loop Majorana Neutrino Mass through Lepotoquark and $d_R$ mixing

\[
\begin{align*}
\text{FIG. 2: 1 loop Neutrino Mass Diagram through d quark mixing with effective d-5 Weinberg operator.}
\end{align*}
\]

Here we briefly discuss another extension of the leptoquark model mentioned above to generate Majorana Neutrino Mass at 1 loop level shown in Fig. 2. Table I shows the field contents of the model. Besides the SM and a leptoquark field, we add 3 generations of heavy vectorlike quark doublets with a hypercharge $-5/6$ to complete
TABLE I: Particle content of the model generating Neutrino mass at 1 loop order.

The new Lagrangian terms are shown below. To break the Lepton number softly we need the right-handed heavy quark fields to carry 2 units of L number and the Left-handed heavy quark fields to carry no L number.

\[
\mathcal{L}_{\text{new,4D}}^Y \subset y_1 \overline{Q}_L^c L \phi^* + y_2 \overline{u}_R^c e_R \phi^* \\
+ y_3 \overline{A}_R L \phi + y_e \overline{d}_R^c A_L H + h.c.
\]  

\[
\mathcal{L}_{3D} \subset \mathcal{M}_A \overline{A} A
\]

\[
V(H, \phi) = -m_1^2 |H|^2 + \frac{\lambda_1}{4} |H|^4 + m_2^2 |\phi|^2 \\
+ \frac{\lambda_2}{4} |\phi|^4 + g_{h\phi} (H^\dagger H) |\phi|^2
\]  

The \( y_e \) Yukawa term is the mixing of SM d type quarks and heavy quarks and required
to be small. The effective mass matrix for the d type quark mixing is shown below, where the top left entry is the usual Higgs mass of the SM and the bottom right entry is the invariant mass of the heavy quarks, where as the off-diagonal term is the source of mixing of SM quarks and the heavy quarks.

\[
\mathcal{L}_{\text{eff mix}} = \left( \begin{array}{c} d_L \ a_L^{-1/3} \\ d_R \ a_R^{-1/3} \end{array} \right) M_{da} \left( \begin{array}{c} d_R \\ a_R^{-1/3} \end{array} \right) + h.c.
\]

Diagonalizing this mass matrix, we obtain the following mass eigenstates

\[
m_{D1} = c_L c_R y_d \nu / \sqrt{2} + s_L c_R y_e \nu / \sqrt{2} + s_L s_R M_A,
\]

\[
m_{D2} = c_L c_R M_A + s_L s_R y_d \nu / \sqrt{2} - c_L s_R y_e \nu / \sqrt{2},
\]

where the c/s stand for Cos/Sin and L/R subscripts stand for Left/Right mixing angles of the Left/Right chiral components of the fermion fields. The mixing is given by following relations
FIG. 3: Deviation of 2 mass eigenvalues from their unmixed values for $M_A=1\text{TeV}$ and $M_A=10\text{TeV}$ with Yukawa’s set to 1 and $m_\phi=1\text{TeV}$.

Fig. 3 shows the deviations of mass eigenstates, light d quark and heavy $a^{-1/3}$ state, from their unmixed values for different values of $y_e$ and $M_A$ for Yukawa’s set to 1 and $m_\phi=1\text{TeV}$.

1 loop radiative neutrino mass given in Fig. 2 can be evaluated to
\[ m_\nu = \frac{y_1 y_3 m_\phi s_{R_L}}{16\pi^2} x_1 \left[ 2\log \left( \frac{x_2}{x_1} \right) + f(x_2) - f(x_1) \right] \] (14)

where \( x_i = m_{Di}/m_\phi \) and \( f(x) = \frac{\log(x)}{x^2 - 1} \) (15)

FIG. 4: Neutrino mass spectrum for different values of \( y_\epsilon \) and \( M_A \) with Yukawa’s set to 1 and \( m_\phi = 1\)TeV.

Fig. 4 shows a sample parameter space that gives the correct neutrino mass order for different values of \( y_\epsilon \) and \( M_A \). Yukawa’s are set to 1, \( m_\phi = 1\)TeV.

The heavy quark searches set limits on the mixing of d-type quarks and the masses of the heavy quarks. Leptoquark searches set limits on the leptoquark couplings to LQ and Leptoquark masses. Possible CP violating phases might occur in the 6×6 d-type mass matrix.
III. GAUGE COUPLING UNIFICATION

When one examines how the three gauge coupling constants of the standard model run, they come close to intersecting at a high energy scale. It is of course now known that they do not unify without some new physics entering in at a lower scale, often at a testable scale just above the weak scale. The Leptoquarks change the running of gauge coupling constants and it was first suggested that a weak scale Leptoquark could cause gauge coupling unification to manifest in ref. [25] and as mentioned in the introduction this has become a major motivation and attraction for considering low scale leptoquarks. It is therefore of interest whether this is feasible in a model where a leptoquark is used to simultaneously explain flavour anomalies and neutrino masses.

We will find that by itself the $(3, 1, -1/3)$ representation makes gauge coupling unification worse unless the leptoquark couplings are large enough to make a difference at two loops. This however puts criteria on the leptoquark couplings that are in tension with both perturbativity and constraints needed to explain flavour anomalies. This is also in contrast with the $(3, 2, 1/6)$ leptoquark. It was recently shown that lepton flavour universality and gauge coupling unification are compatible via the $(3, 2, 16)$ leptoquark [40]. However, when one adds the particles required to radiatively generate a neutrino mass, we find that such particles tend to make unification worse for the $(3, 2, 1/6)$ leptoquark. In contrast the additional fermions required to generate a neutrino mass at one loop, $A_{L,R}$, can easily improve gauge coupling unification for the $(3, 1, -1/3)$ representation.
The one loop beta functions changes when one extends the standard model by a leptoquark. Defining the parameter $b_i$ as

$$\frac{\partial g_i}{\partial \mu} = -\frac{b_i}{16\pi^2} g_i^3$$

(16)

it is straightforward to derive a measure of unification [58]

$$\delta U = \frac{\alpha_3^{-1}(\mu) - \alpha_2^{-1}(\mu)}{\alpha_2^{-1}(\mu) - \alpha_1^{-1}(\mu)} \cdot \frac{b_2 - b_3}{b_1 - b_2}$$

(17)

where a low value of $|\delta U|$ indicates one is close to unification. Choosing $\mu = 1$ TeV one has for the standard model $\delta U = 0.19$. It is of course well known that the $(3,2,1/6)$ leptoquark improves unification. Introducing such leptoquarks at 1 TeV and running the standard model couplings up to the TeV scale using the numerical package SARAH [41] gives a value of $\delta U = (0.13, 0.06, -0.02)$ for $(1, 2, 3)$ generations respectively. On the other hand, adding the leptoquark defined in equation 1 modifies these values of $b_i$ such that $b_1 = -25/6$ $b_3 = 41/6$ and $b_2$ remains unchanged. Again running the standard model couplings up to the TeV scale one finds that for the leptoquark extension one has $\delta U = 0.21$, worse than the standard model. The reason for this is that one needs to change $b_2$ faster than one changes $\frac{1}{2}(b_1 + b_3)$ in order to achieve unification but the leptoquark is an isospin singlet.

In principle the 2-loop corrections to the beta functions can drive gauge coupling unification. The leptoquark couplings $\lambda_L$ and $\lambda_R$ affect the two loop beta functions for gauge coupling constants at two loop level as follows
\[ \beta_{g_1} \equiv \frac{1}{(4\pi)^2} \frac{25g_1^3}{6} - \frac{1}{(4\pi)^4} \frac{30\text{Tr}[\lambda_L\lambda_L^\dagger]}{g_1^3} \]
\[ \beta_{g_2} \equiv - \frac{1}{(4\pi)^2} \frac{19g_2^3}{6} - \frac{1}{(4\pi)^4} \frac{3\text{Tr}[\lambda_L\lambda_L^\dagger]}{g_2^3} \]
\[ \beta_{g_3} \equiv - \frac{1}{(4\pi)^2} \frac{41g_3^3}{6} - \frac{1}{(4\pi)^4} \frac{30\text{Tr}[\lambda_L\lambda_L^\dagger] + 15\text{Tr}[\lambda_R\lambda_R^\dagger]}{g_3^3}. \]  

(18)

Note that the Higgs portal coupling does not effect the running. The Higgs portal coupling is the major player in achieving a boost to the stability of the vacuum so two constraints - vacuum stability and gauge coupling unification - are only moderately correlated. Typically raising the left handed coupling constants takes one further away from coupling unification whereas raising the right handed coupling gets one closer. If all left handed couplings are null then gauge coupling unification is achieved when

\[ \text{Tr} \left[ \lambda_R\lambda_R^\dagger \right] \sim 4. \]  

(19)

This number rises if the left handed leptoquark couplings rises.

If the leptoquark is used to explain the apparent violation of lepton universality, it turns out to be difficult to avoid at least one leptoquark coupling being large, namely the \( \lambda_{e\mu}^L \) coupling. This raises the required value of the right handed leptoquark couplings. Fig. 5 shows the running of the gauge couplings for leptoquark couplings chosen to satisfy all phenomenology constraints as well as explaining all three flavour anomalies. As before, we take all standard model couplings at weak scale and use the standard model RGEs defined to two loop to run all parameters from the weak scale to the mass of the leptoquark (which we set to be 1 TeV). In principle, there is
enough freedom left over from these constraints to achieve gauge coupling unification if some couplings are made sufficiently large. The results however should be taken with a heavy grain of salt because the running of the leptoquark couplings are such that they increase and approach the perturbativity bound. Therefore the calculation is expected to have a high amount of uncertainty.

Next let us turn to the extra particle content required to produce a neutrino mass at one and two loop respectively. For the two loop case we need two additional particles including an isospin triplet. This is true for both representations of leptoquark that we consider. This particle makes too dramatic a change to $b_2$ to assist unification unless its mass is quite high $\approx \mathcal{O}(10^{13})$ GeV. For the case where one induces a neutrino mass at one loop, the new particles change the values of $b_i$ as follows

$$\delta b_1 = \left(-\frac{5}{6}\right) N_g$$

$$\delta b_2 = (-2) N_g$$

$$\delta b_3 = \left(-\frac{4}{3}\right) N_g$$

where $N_g$ is the number of generations active at that mass scale. For a single generation active from 1 TeV up to around the GUT scale one essentially has unification $\delta U = 0.015$. Similarly one achieves unification if two generations of $A_{L,R}$ with masses around $10 - 1000$ TeV and the third generation near the GUT scale. The masses of the lightest two generations can be within a wide range of values however. Varying their masses between 10 and 1000 TeV one finds that $|\delta U|$ ranges between $0.05 \lesssim |\delta U| \lesssim 0.08$. If one has three generations of these fermions it is unavoidable
that at least one generation is very heavy. By contrast, the analogous fermions used to generate a neutrino mass at one loop for the $(3, 2, 1/6)$ leptoquark makes gauge coupling unification worse because they are isospin singlets.

Let us conclude this section with a summary of this section. The extra A particles make unification at 1 loop fairly straightforward for the $(3, 1, 1/6)$ model. Without such particles one can in principle achieve unification at 2 loops with very large couplings, a result that is in some tension with flavour anomalies but not stability. By contrast, while the $(3, 2, 1/6)$ model achieves unification without a radiatively induced neutrino mass, introducing such extra particles is in tension with unification.

IV. VACUUM STABILITY

The recent discovery of a Higgs like Boson at 125 GeV [42, 43] has led to the realization that the standard model vacuum is likely metastable [44–47]. This is because the Higgs quartic coupling becomes negative at large energy scales. This has motivated research on how to improve the stability of the electroweak vacuum through addition to the standard model. There are two ways of getting a boost improve the stability of the vacuum

• improved running through corrections to the standard model RGEs. This has been done numerously in singlet extensions to the standard model [48–52]

• extended scalar sector, for adding a real singlet that acquires a vacuum expectation value. The result can be that the Higgs quartic coupling can be larger
than its standard model value [53].

The leptoquark cannot have a vacuum expectation value at zero temperature so the only option is through improved running [54]. The beta functions for the Higgs quartic coupling will get a correction compared to the standard model which we denote $\delta \lambda_1$ and $\delta \lambda_2$ for one and two loop contributions respectively. The one loop correction to the standard model RGE for the higgs quartic coupling receives contributions from the portal coupling only

$$\frac{\beta_\lambda}{(4\pi)^2} \supseteq \frac{6}{16\pi^2} g_{H\phi}^2 H_\phi^2.$$ (23)

Note that the one loop correction is positive which makes a boost to stability possible in the first place. So if the portal coupling is large then the leptoquark contributes to improving the stability of the vacuum. This is true of both leptoquark representations that we consider. The two loop correction can be significant enough to justify a careful analysis. For the sake of simplicity the leptoquark coupling to one generation dominates. This is simply to show the structure of the two loop beta functions more clearly - our numerical calculations consider all generations. We can then write the dominant contribution as

$$\frac{\beta_{\lambda}}{(4\pi)^4} \supseteq \frac{1}{256\pi^4}$$
$$\times \left\{ \frac{6}{5} g_{H\phi} g_1^4 - 30 \lambda g_{H\phi}^2 g_1^2 g_3^2 g_{H\phi}^2 + 64 g_3^2 g_{H\phi}^2 - 24 g_{H\phi}^3 ight.$$ 
$$+ g_{H\phi}^2 \lambda_{L,3}^2 \left( 24 + 9 \lambda y_t^2 - 12 y_b^4 - 12 y_t^4 ight)$$
$$+ 9 \lambda y_t^2 - 24 y_t^2 y_t^2 - 12 y_t^4 \right\} + g_{H\phi}^2 \lambda_{R,3} \left( 12 - y_t^4 + 9 \lambda y_t^2 - 24 y_t^2 y_t^2 - 12 y_t^4 \right) \} \right.$$ (24)
Typically this is strongly negative if the leptoquark couplings are large which drives up the value of $g_{h\phi}$ needed to make the higgs quartic coupling remain non-negative up to the unification scale. To achieve gauge coupling unification with all left handed leptoquark couplings set to zero, right handed leptoquark couplings need to be large. Also, one of the left handed leptoquark coupling constants has to be quite large to explain the violation of lepton universality. This means that several right handed leptoquark couplings have to be quite large to make gauge coupling unification manifest which pushes up the required value of $g_{h\phi}$ to a moderate value. In Fig. 6 we again the numerical package SARAH [41] to run standard model parameters up to the scale of the leptoquark mass (which we choose to be 1 TeV) and then run all parameters including the leptoquark parameters up to the a high scale. For a value of $g_{\phi} \sim 0.5$ the Higgs quartic coupling never goes negative. Finally we note that the extra fermions introduced to radiatively induce a neutrino mass at one loop only affect the running of the Higgs quartic at 2 loops.

V. LEPTOQUARKS AND FLAVOUR ANOMALIES

In this section we consider the constraints on the parameter space due to both collider constraints and the proposal to have such a leptoquark explaining the aforementioned anomalies. We should note that some doubt has been raised on whether this leptoquark representation can provide an explanation for the $R_K$ anomaly [21]. However, further inspection does indeed seem to confirm the original claim that this leptoquark can explain the apparent violation of lepton flavour universality [22] and
recent work has indeed argued that it can indeed provide an explanation for the 
$R_{D^*}$ anomaly and the $(g-2)_\mu$ anomaly [57]. We will focus on briefly reviewing the 
parameter space proposed in [4] and find that there is tension between the parameter 
space they propose and unification. The parameter space discussed in [22] will has 
similar features to the parameter space in [4].

Let us begin by cataloguing the constraints from the anomalies. The constraints 
are written in terms of the leptoquark couplings written in the mass basis. To convert 
from the mass basis to the weak basis we make use of both the CKM matrix and the 
PMNS matrix as well as the relations

$$
\lambda_{ue}^L = U_u^T \lambda_L U_e, \quad \lambda_{\nu\nu}^L = U_d^T \lambda_L, \quad \lambda_{ue}^R = V_u^T \lambda_R V_e
$$

We will begin with the requirement that the leptoquark provide an explanation for 
anomalous B decays which gives the following condition on the leptoquark couplings

$$
\lambda_{e\tau}^L \lambda_{b\tau}^L = 3.5 \times 10^{-7} M_\phi^2 \quad (26)
$$

$$
\lambda_{e\tau}^R \lambda_{b\tau}^L = -3 \times 10^{-8} M_\phi^2 \quad (27)
$$

These constraints cause two left handed couplings to be around 0.6 and the right 
handed coupling $\lambda_{e\tau}^R$ about 0.05. Since we require the trace of the right handed 
leptoquark coupling matrix to be significantly larger than the trace of the left handed 
leptoquark matrix this already constrains the parameter space somewhat. Next we 
turn to the anomalous measurement of $(g-2)_\mu$. This leads to the condition

$$
(1 + 0.17 \ln 10^{-3} M_\phi) \text{Re} (\lambda_{e\mu}^R \lambda_{\tau\tau}^L) 
$$

$$
+ 20.7 (1 + 1.06 \ln 10^{-3} M_\phi) \text{Re} (\lambda_{\tau\mu}^R \lambda_{\tau\mu}^L) \approx 8 \times 10^{-8} M_\phi^2 \quad (28)
$$
This does not put much constraint on the right handed couplings as we can just tune the left handed couplings down. The apparent violation of lepton universality, by contrast, puts the most severe constraint on the parameters as it inevitably leads to some coupling constants being large. Specifically the constraints are

$$\sum_i |\lambda^L_{ui\mu}|^2 \text{Re} \frac{\sum_{j} \lambda^L_{brij} \lambda^L_{sij}}{V_{tb} V_{ts}^*} - 1.74 |\lambda^L_{tij}|^2 \approx 1.25 \times 10^{-5} M^2 \phi$$  \hspace{1cm} (29)

$$\frac{\sum_{j} \lambda^L_{brij} \lambda^L_{sij}}{V_{tb} V_{ts}^*} \approx (1.87 + 0.45i) \times 10^{-3} M \phi$$  \hspace{1cm} (30)

The second constraint is straightforward to satisfy because it requires that the sum of a set of left handed couplings must come to the small value of 0.076 which is consistent with gauge coupling unification. Combining this with the first constraint though says that the square of two couplings, $|\lambda_{u\mu}|^2 + |\lambda_{c\mu}|^2$ must add to about 6.7. This is obviously impossible to satisfy without making at least one of these couplings very large. It can be more dangerous phenomenologically to have large leptoquark couplings to first generation quarks or leptons, so in reality this will lead to the coupling $|\lambda_{c\mu}| \approx 2.4$ for a leptoquark mass of around a TeV. This puts a very sharp constraint on the ability of this leptoquark to simultaneously give rise to gauge coupling unification and explain the apparent violation of lepton universality. Note that only the violation of lepton univerality requires a condition that conflicts with gauge coupling unification.

Let us next turn our attention to the phenomenological constraints. We can get constraints on the leptoquark mass from reference [55]. The lower bound for leptoquarks that decay into bottom quarks is 625 GeV and the lower bound for leptoquarks de-
caying into muons with a significant branching ratio is 850 GeV. The latter bound is more relevant if one is indeed desiring to explain an observed violation of lepton universality. For convenience we will catalogue all other relevant collider constraints in a single list

\[ -1.2 \times 10^{-6} M_\phi^2 < \text{Re} \sum_j \frac{\lambda^L_{\mu j} \lambda^L_{\mu j}^*}{V_{\mu L} V_{t s}^*} < 2.25 \times 10^{-6} M_\phi^2 \]

\[ \sqrt{\left| \lambda^L_{\mu \mu} \right|^2 + \left| \lambda^R_{\mu \mu} \right|^2} < 1.2 \times 10^{-9} M_\phi^2 \]

\[ \left| \lambda^L_{\mu \mu} \lambda^L_{\mu \mu}^* + \lambda^R_{\mu \mu} \lambda^R_{\mu \mu}^* \right| < 5.1 \times 10^{-8} \]

\[ \sqrt{\left| \lambda^L_{\mu \mu} \right|^2 + \left| \lambda^L_{\mu \mu} \right|^2} < \frac{3.24 \times 10^{-3} M_\phi}{\sqrt{1 + 0.76 \ln 10^{-3} M_\phi}} \]

\[ \left| \lambda^L_{\mu \mu} \right| < \frac{1.22 \times 10^{-3} M_\phi}{\sqrt{1 + 0.76 \ln 10^{-3} M_\phi}} \]

\[ \left[ (1 + 0.17 \ln 10^{-3} M_\phi) \lambda^R_{\mu \mu} \lambda^L_{\mu \mu}^* \right. \]

\[ + 20.7 (1 + 1.06 \ln 10^{-3} M_\phi) \lambda^R_{\mu \mu} \lambda^L_{\mu \mu}^* - 0.015 \sum_i \lambda^L_{u i \mu} \lambda^L_{u i \tau} \left| \right|^2 \]

\[ + (L \leftrightarrow R) \right]^{1/2} < 1.7 \times 10^{-8} M_\phi^2 \]  (31)

The constraints in the above list are, in order of their appearance

- \( B^- \to K^- \nu \bar{\nu} \) and \( B^- \to K^-* \nu \bar{\nu} \) decays.

- Next two are from the bound on branching ratio of \( D^0 \to \mu^+ \mu^- \).

- Next two are from constraints on the partial width of \( Z \to \mu^+ \mu^- \).

- Last is a bound on the branching ratio of \( \tau \to \mu \gamma \)

The constraint from the \( B^- \to K^- \nu \bar{\nu} \) and \( B^- \to K^-* \nu \bar{\nu} \) decays put no constraints on right handed couplings but can be satisfied with arbitrarily small left handed
couplings. The constraint from the branching ratio of $D^0 \rightarrow \mu^+\mu^-$ forces one $\Lambda^R$ to be small. It is satisfied if the left handed couplings are very small, the right handed couplings if they are equal can be $\sim 0.23$. Note that lepton universality means that the $\lambda^L_{c\mu}$ coupling has to be $\approx 2.5$ which is fine if $\lambda^L_{u\mu}$ is small. The constraints due to the partial width of $Z \rightarrow \mu^+\mu^-$ leads to no constraint on the right handed couplings but is satisfied for arbitrarily small left handed couplings. Lepton universality requires that $\lambda^L_{c\mu}$ left handed coupling to be large, $\sim 2.5$, but this is compatible with the constraint so long as other left handed couplings appearing in the constraint are suppressed which we desire anyway.

The last constraint due to the branching ratio of $\tau \rightarrow \mu\gamma$ is the most dangerous if one desires the addition of a TeV scale leptoquark to lead to gauge coupling unification at two loops. There is a relative sign in the equation which means in principle that one can find a fine tuned region to the parameter space where a fortuitous cancellation occurs. But let us concentrate on the non-fine tuned region of parameter space. Recalling that the apparent violation of lepton universality leads to $\lambda^L_{c\mu} \sim 2.4$, this leads to $\lambda^R_{\mu\nu_i}$ and $\lambda^R_{\tau\nu_i}$ needing to be have a value of approximately $\sim 0.08$. So we have 7 right handed couplings set to be small in principle. However, a loophole is just to set 3 of them to be very small. To give maximum freedom set right handed $\lambda^R_{c\tau}$, $\lambda^R_{c\mu}$, $\lambda^R_{u\mu}$ to be small as well as $\lambda^R_{t\mu}$. This will satisfy all of the above constraints without significant fine tuning. Finally let us conclude this section by summarizing one of the recent works which called into question whether this leptoquark can explain the observed violation of lepton universality. Ref. [57] found that using this type of
leptoquark to explain the $R_{D^{(*)}}$ anomaly requires $\lambda^2_{Lj}$

$$\frac{\lambda^3_{Lj} \lambda^{23*}_{R}}{2M_{LQ}/GeV} = -0.26 \times 2^{3/2} G_F V_{cb} \quad (32)$$

$$\frac{\lambda^{33}_{Lj} \lambda^{23*}_{R}}{2M^2_{LQ}/GeV^2} = \pm 0.64 \times 2^{3/2} G_F V_{cb} \quad (33)$$

which is in serious tension with the requirement that $\lambda^L_{c\mu}$ is large.

VI. DISCUSSION AND CONCLUSION

In this work we have considered some of the most intriguing experimental signatures that suggest a departure from standard model physics and attempted to see how a minimal explanation for them fits into the bigger picture of Unification. Improvement in vacuum stability via improved running is straightforward to realize. However, the attempt to explain a violation of lepton universality has some tension with using this leptoquark to achieve gauge coupling unification.

One way or another, we find that new physics is probably needed before the GUT scale. Achieving gauge coupling unification requires pushing leptoquark couplings up such that they are near the perturbativity bound at a high scale and neutrino masses cannot be achieved through this leptoquark alone. Both require additional particle content (although if a tree level right handed neutrino mass is not forbidden by a discrete symmetry, the usual see-saw mechanism is of course sufficient). The single leptoquark model is insufficient to neutrino masses at any loop level. We proposed some minimal extensions by either introducing some gauge multiplets to generate a Majorana mass at two loops or including a heavy quark doublet with a hyper charge
of $-5/6$ to generate such a mass at one loop. The latter is through d type mixing.

**ACKNOWLEDGMENTS**

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[8] B. Dumont, K. Nishiwaki and R. Watanabe, LHC constraints and prospects for $S_1$ scalar leptoquark explaining the $\bar{B} \to D^* \tau^+ \bar{\nu}$ anomaly, Phys. Rev. D, 94 034001 (2016).


FIG. 5: Gauge coupling unification ($\alpha^{-1}$) achieved via strong right handed leptoquark couplings. With a single generation the couplings get precariously close to the perturbativity bound, a problem accentuated if one wishes to explain a violation in lepton universality, adding in a great deal of theoretical uncertainty which can be alleviated with extra generations of leptoquarks.
FIG. 6: Vacuum stability achieved through improved running of the Higgs quartic coupling due to moderately large portal couplings \( g_{h\phi} = 0.5 \). This is compatible with low energy phenomenology as well as explaining anomalies in the value of \((g - 2)_\mu\), B decays and violation of lepton universality.
Chapter 9

Quartified Leptonic Color, Bound States, and Future Electron-Positron Collider[8]

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Abstract

The $[SU(3)]^4$ quartification model of Babu, Ma, and Willenbrock (BMW), proposed in 2003, predicts a confining leptonic color $SU(2)$ gauge symmetry, which becomes strong at the keV scale. It also predicts the existence of three families of half-charged leptons (hemions) below the TeV scale. These hemions are confined to form bound states which are not so easy to discover at the Large Hadron Collider (LHC). However, just as $J/\psi$ and $\Upsilon$ appeared as sharp resonances in $e^- e^+$ colliders of the 20th century, the corresponding 'hemionium' states are expected at a future $e^- e^+$ collider of the 21st century.
9.1 Introduction

Fundamental matter consists of quarks and leptons, but why are they so different? Both interact through the $SU(2)_L \times U(1)_Y$ electroweak gauge bosons $W^\pm, Z^0$ and the photon $A$, but only quarks interact through the strong force as mediated by the gluons of the unbroken (and confining) color $SU(3)$ gauge symmetry, called quantum chromodynamics (QCD). Suppose this is only true of the effective low-energy theory. At high energy, there may in fact be three 'colors' of leptons transforming as a triplet under a leptonic color $SU(3)$ gauge symmetry. Unlike QCD, only its $SU(2)_l$ subgroup remains exact, thus confining only two of the three 'colored' leptons, called 'hemions' in Ref. [169] because they have $\pm 1/2$ electric charges, leaving the third ones free as the known leptons.

The notion of leptonic color was already discussed many years ago [170, 171], and its incorporation into $[SU(3)]^4$ appeared in Ref. [172], but without full unification. Its relevance today is threefold. (1) The $[SU(3)]^4$ quartification model [169] of Babu, Ma, and Willenbrock (BMW) is non-supersymmetric, and yet achieves gauge-coupling unification at $4 \times 10^{11}$ GeV without endangering proton decay. This unification of gauge couplings is only possible if the three families of hemions have masses below the TeV scale. Given the absence of experimental evidence for supersymmetry at the Large Hadron Collider (LHC) to date, this alternative scenario deserves a closer look. (2) The quartification scale determines the common gauge coupling for the $SU(2)_l$
symmetry. Its extrapolation to low energy predicts that it becomes strong at the keV scale, in analogy to that of QCD becoming strong at somewhat below the GeV scale. This may alter the thermal history of the Universe and allows the formation of gauge-boson bound states, the lightest of which is a potential warm dark-matter candidate [173]. (3) The hemions (called 'liptons' previously [171]) have \( \pm 1/2 \) electric charges and are confined to form bound states by the \( SU(2)_l \) 'stickons' in analogy to quarks forming hadrons through the \( SU(3)_C \) gluons. They have been considered previously [174] as technifermions responsible for electroweak symmetry breaking. Their electroweak production at the LHC is possible [175] but the background is large. However, in a future \( e^-e^+ \) collider (ILC, CEPC, FCC-ee), neutral vector resonances of their bound states (hemionia) would easily appear, in analogy to the observations of quarkonia \( (J/\psi, \Upsilon) \) at past \( e^-e^+ \) colliders.

### 9.2 The BMW model

Under the \([SU(3)]^4 \) quartification gauge symmetry, quarks and leptons transform as \((3, \bar{3})\) in a moose chain linking \( SU(3)_q \) to \( SU(3)_L \) to \( SU(3)_l \) to \( SU(3)_R \) back to \( SU(3)_q \) as depicted in Fig. 1.
Figure 9.1: Moose diagram of $[SU(3)]^4$ quartification.

Specifically,

\[
q \sim (3, \bar{3}, 1, 1) \sim \begin{pmatrix} d & u & h \end{pmatrix}, \quad l \sim (1, 3, \bar{3}, 1) \sim \begin{pmatrix} y_1 & y_2 & e \\ z_1 & z_2 & N \end{pmatrix}, \quad (9.1)
\]

\[
l^c \sim (1, 1, 3, 3) \sim \begin{pmatrix} x_1^c & y_1^c & z_1^c \end{pmatrix}, \quad q^c \sim (\bar{3}, 1, 1, 3) \sim \begin{pmatrix} u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix}. \quad (9.2)
\]

Below the TeV energy scale, the gauge symmetry is reduced [169] to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_Y$ with the particle content given in Table 1. The electric charge $Q$ is given by $Q = I_3L + Y$ as usual. The exotic $SU(2)_L$ doublets $x, y$ have $\pm 1/2$ charges, hence the name hemions. Whereas the quarks and charged leptons must obtain masses through electroweak symmetry breaking, the hemions have invariant mass terms, i.e. $x_1Ly_2L - x_2Ly_1L$ and $x_1Ry_2R - x_2Ry_1R$. This is important because they are then allowed to be heavy without disturbing the electroweak oblique parameters $S, T, U$ which are highly constrained experimentally. In the following, the
Table 9.1: Particle content of proposed model.

<table>
<thead>
<tr>
<th>particles</th>
<th>$SU(3)_C$</th>
<th>$SU(2)_L$</th>
<th>$SU(2)_L$</th>
<th>$U(1)_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(u, d)_L$</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>1/6</td>
</tr>
<tr>
<td>$u_R$</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2/3</td>
</tr>
<tr>
<td>$d_R$</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>−1/3</td>
</tr>
<tr>
<td>$(x, y)_L$</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>$x_R$</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>$y_R$</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>−1/2</td>
</tr>
<tr>
<td>$(ν, l)_L$</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>−1/2</td>
</tr>
<tr>
<td>$ν_R$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$l_R$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>−1</td>
</tr>
<tr>
<td>$(φ^+, φ^0)$</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1/2</td>
</tr>
</tbody>
</table>

Mass terms from electroweak symmetry breaking, i.e. $\bar{x}_L x_R \bar{φ}^0$ and $\bar{y}_L y_R φ^0$, will be assumed negligible.
9.3 Gauge coupling unification and the leptonic color confinement scale

The renormalization-group evolution of the gauge couplings is dictated at leading order by

\[
\frac{1}{\alpha_i(\mu)} - \frac{1}{\alpha_i(\mu')} = \frac{b_i}{2\pi} \ln \left( \frac{\mu'}{\mu} \right),
\]

(9.3)

where \(b_i\) are the one-loop beta-function coefficients,

\[
b_C = -11 + \frac{4}{3}N_F, \quad (9.4)\\
b_L = \frac{-22}{3} + \frac{4}{3}N_F, \quad (9.5)\\
b_L = \frac{-22}{3} + 2N_F + \frac{1}{6}N_\Phi, \quad (9.6)\\
b_Y = \frac{13}{9}N_F + \frac{1}{12}N_\Phi. \quad (9.7)
\]

The number of families \(N_F\) is set to three, and the number of Higgs doublets \(N_\Phi\) is set to two, as in the original BMW model. Here we make a small adjustment by separating the three hemion families into two light ones at the electroweak scale \(M_Z\) and one at a somewhat higher scale \(M_X\). We then input the values [176]

\[
\alpha_C(M_Z) = 0.1185, \quad (9.8)\\
\alpha_L(M_Z) = (\sqrt{2}/\pi)G_F M_W^2 = 0.0339, \quad (9.9)\\
\alpha_Y(M_Z) = 2\alpha_L(M_Z) \tan^2 \theta_W = 0.0204, \quad (9.10)
\]
where $\alpha_Y$ has been normalized by a factor of 2 (and $b_Y$ by a factor of 1/2) to conform to $[SU(3)]^4$ quartification. We find

$$M_U = 4 \times 10^{11} \text{ GeV}, \quad \alpha_U = 0.0301, \quad M_X = 486 \text{ GeV}. \quad (9.11)$$

We then use $b_l$ to extrapolate back to $M_Z$ and obtain $\alpha_l(M_Z) = 0.0469$. Below the electroweak scale, the evolution of $\alpha_l$ comes only from the stickons and it becomes strong at about 1 keV. Hence 'stickballs' are expected at this confinement mass scale.

Unlike QCD where glueballs are heavier than the $\pi$ mesons so that they decay quickly, the stickballs are so light that they could decay only to lighter stickballs or to photon pairs through their interactions with hemions.

\section*{9.4 Thermal history of stickons}

At temperatures above the electroweak symmetry scale, the hemions are active and the stickons ($\zeta$) are in thermal equilibrium with the standard-model particles. Below the hemion mass scale, the stickon interacts with photons through $\zeta\zeta \rightarrow \gamma\gamma$ scattering with a cross section

$$\sigma \sim \frac{9\alpha^2\alpha_l^2T^6}{16M_{\text{eff}}^8}. \quad (9.12)$$

The decoupling temperature of $\zeta$ is then obtained by matching the Hubble expansion rate

$$H = \sqrt{\frac{(8\pi/3)G_N(\pi^2/30)g_*T^4}{9\alpha^2\alpha_l^2}} \quad (9.13)$$
to \([6\zeta(3)/\pi^2]T^3\langle\sigma v\rangle\).

Hence

\[
T^{14} \sim \frac{2^8}{3^8} \left(\frac{\pi^7}{5(\zeta(3))^2}\right) \frac{G_N g_4 M_{\text{eff}}^{16} \alpha^4 \alpha_4^4}{\alpha_4^4},
\]

(9.14)

where \(6M_{\text{eff}}^{-4} = \sum(M_{xyi}^{-4})\). For \(M_{\text{eff}} = 110\) GeV and \(g_4 = 92.25\) which includes all particles with masses up to a few GeV, \(T \sim 6.66\) GeV. Hence the contribution of stickons to the effective number of neutrinos at the time of big bang nucleosynthesis (BBN) is given by [177]

\[
\Delta N_\nu = \frac{8}{7}(3) \left(\frac{10.75}{92.25}\right)^{4/3} = 0.195,
\]

(9.15)

compared to the value \(0.50 \pm 0.23\) from a recent analysis [178]. The most recent PLANCK measurement [179] coming from the cosmic microwave background (CMB) is

\[
N_{\text{eff}} = 3.15 \pm 0.23.
\]

(9.16)

However, at the time of photon decoupling, the stickons have disappeared, hence \(N_{\text{eff}} = 3.046\) as in the SM. This is discussed in more detail below.

### 9.5 Formation and decay of stickballs

As the Universe further cools below a few keV, leptonic color goes through a phase transition and stickballs are formed. If the lightest stickball \(\omega\) is stable, it may be a candidate for warm dark matter. It has strong self-interactions and the \(3 \rightarrow 2\) process determines its relic abundance. Following Ref. [180] and using Ref. [173], we estimate
that it is overproduced by a factor of about 3. However, \( \omega \) is not absolutely stable. It is allowed to mix with a scalar bound state of two hemions which would decay to two photons. We assume this mixing to be \( f_\omega m_\omega / M_{xy} \), so that its decay rate is given by

\[
\Gamma(\omega \to \gamma\gamma) = \frac{9\alpha^2 f_\omega^2 m_\omega^5}{64\pi^3 M_{\text{eff}}^4},
\] (9.17)

where \( M_{\text{eff}} \) is now defined by \( 6M_{\text{eff}}^{-2} = \sum(M_{xy}^i)^{-2} \). Setting \( m_\omega = 5 \text{ keV} \) to be above the astrophysical bound of 4 keV from Lyman \( \alpha \) forest observations [181] and \( M_{\text{eff}} = 150 \text{ GeV} \), its lifetime is estimated to be \( 4 \times 10^{17} \text{s} \) for \( f_\omega = 1 \). This is exactly the age of the Universe, and it appears that \( \omega \) may be a candidate for dark matter after all. However, CMB measurements constrain [182] a would-be dark-matter lifetime to be greater than about \( 10^{25} \text{s} \), and x-ray line measurements in this mass range constrain [183] it to be greater than \( 10^{27} \text{s} \), so this scenario is ruled out. On the other hand, if \( m_\omega = 10 \text{ keV} \), then the \( \omega \) lifetime is \( 1.4 \times 10^{16} \text{s} \), which translates to a fraction of \( 2 \times 10^{-14} \) of the initial abundance of \( \omega \) to remain at the present Universe. Compared to the upper bound of \( 10^{-10} \) for a lifetime of \( 10^{16} \text{s} \) given in Ref. [182], this is easily satisfied, even though \( \omega \) is overproduced at the leptonic color phase transition by a factor of 3.

At the time of photon decoupling, the \( SU(2)_l \) sector contributes no additional relativistic degrees of freedom, hence \( N_{\text{eff}} \) remains the same as in the SM, i.e. 3.046, coming only from neutrinos. In this scenario, \( \omega \) is not dark matter. However, there are many neutral scalars and fermions in the BMW model which are not being considered.
here. They are naturally very heavy, but some may be light enough and stable, and be suitable as dark matter.

\section{Revelation of leptonic color at future $e^-e^+$ colliders}

Unlike quarks, all hemions are heavy. Hence the lightest bound state is likely to be at least 200 GeV. Its cross section through electroweak production at the LHC is probably too small for it to be discovered. On the other hand, in analogy to the observations of $J/\psi$ and $\Upsilon$ at $e^-e^+$ colliders of the last century, the resonance production of the corresponding neutral vector bound states (hemionia) of these hemions is expected at a future $e^-e^+$ collider (ILC, CEPC, FCC-ee) with sufficient reach in total center-of-mass energy. Their decays will be distinguishable from heavy quarkonia (such as toponia) experimentally.

The formation of hemion bound states is analogous to that of QCD. Instead of one-gluon exchange, the Coulomb potential binding a hemion-antihemion pair comes from one-stickon exchange. The difference is just the change in an SU(3) color factor of 4/3 to an SU(2) color factor of 3/4. The Bohr radius is then $a_0 = [(3/8)\bar{\alpha}_l m]^{-1}$, and the effective $\bar{\alpha}_l$ is defined by

$$\bar{\alpha}_l = \alpha_l(a_0^{-1}).$$

(9.18)
Using Eqs. (3) and (5), and $\alpha_l(M_Z) = 0.047$ with $m = 100$ GeV, we obtain $\bar{\alpha}_l = 0.059$ and $a_0^{-1} = 2.2$ GeV. Consider the lowest-energy vector bound state $\Omega$ of the lightest hemion of mass $m = 100$ GeV. In analogy to the hydrogen atom, its binding energy is given by

$$E_b = \frac{1}{4} \left( \frac{3}{4} \right)^2 \bar{\alpha}_l^2 m = 0.049 \text{ GeV},$$

and its wavefunction at the origin is

$$|\psi(0)|^2 = \frac{1}{\pi a_0^3} = 3.4 \text{ GeV}^3.$$

Since $\Omega$ will appear as a narrow resonance at a future $e^-e^+$ collider, its observation depends on the integrated cross section over the energy range $\sqrt{s}$ around $m_\Omega$:

$$\int d\sqrt{s} \sigma(e^-e^+ \to \Omega \to X) = \frac{6\pi^2 \Gamma_{ee} \Gamma_X}{m_\Omega^2 \Gamma_{\text{tot}}},$$

where $\Gamma_{\text{tot}}$ is the total decay width of $\Omega$, and $\Gamma_{ee}, \Gamma_X$ are the respective partial widths.

Since $\Omega$ is a vector meson, it couples to both the photon and $Z$ boson through its constituent hemions. Hence it will decay to $W^-W^+, q\bar{q}, l^-l^+$, and $\nu\bar{\nu}$. Using

$$\langle 0 | \bar{x} \gamma^\mu x | \Omega \rangle = e_{\Omega}^\mu \sqrt{8m_\Omega} |\psi(0)|,$$

the $\Omega \to e^-e^+$ decay rate is given by

$$\Gamma(\Omega \to \gamma, Z \to e^-e^+) = \frac{2m_\Omega^2}{3\pi} (|C_V|^2 + |C_A|^2) |\psi(0)|^2,$$

where

$$C_V = \frac{e^2(1/2)(-1)}{m_\Omega^2} + \frac{g_Z^2(-\sin^2 \theta_W/4)[(-1 + 4\sin^2 \theta_W)/4]}{m_\Omega^2 - M_Z^2},$$

and

$$C_A = \frac{g_Z^2(-\sin^2 \theta_W/4)(1/4)}{m_\Omega^2 - M_Z^2}.$$
In the above, $\Omega$ is assumed to be composed of the singlet hemions $x_R$ and $y_R$ with invariant mass term $x_1 R y_2 R - x_2 R y_1 R$ (case A). Hence $\Gamma_{ee} = 43$ eV. If $\Omega$ comes instead from $x_L$ and $y_L$ with invariant mass term $x_1 L y_2 L - x_2 L y_1 L$ (case B), then the factor $(-\sin^2 \theta_W / 4)$ in $C_V$ and $C_A$ is replaced with $(\cos^2 \theta_W / 4)$ and $\Gamma_{ee} = 69$ eV. Similar expressions hold for the other fermions of the Standard Model (SM).

For $\Omega \rightarrow W^- W^+$, the triple $\gamma W^- W^+$ and $ZW^- W^+$ vertices have the same structure. The decay rate is calculated to be

$$\Gamma(\Omega \rightarrow \gamma, Z \rightarrow W^- W^+) = \frac{m_\Omega^2 (1 - r)^{3/2}}{6 \pi r^2} \left(4 + 20r + 3r^2\right) C_W^2 |\psi(0)|^2,$$

(9.26)

where $r = 4M_W^2/m_\Omega^2$ and

$$C_W = \frac{e^2 (1/2)}{m_\Omega^2} + \frac{g_Z^2 (-\sin^2 \theta_W / 4)}{m_\Omega^2 - M_Z^2}$$

(9.27)

in case A. Because of the accidental cancellation of the two terms in the above, $C_W$ turns out to be very small. Hence $\Gamma_{WW} = 3.2$ eV. In addition to the $s$–channel decay of $\Omega$ to $W^- W^+$ through $\gamma$ and $Z$, there is also a $t$–channel electroweak contribution in case B because $x_L$ and $y_L$ form an electroweak doublet. Replacing $(-\sin^2 \theta_W / 4)$ with $(\cos^2 \theta_W / 4)$ in $C_W$, and adding this contribution, we obtain

$$\Gamma(\Omega \rightarrow W^- W^+) = \frac{m_\Omega^2 (1 - r)^{3/2}}{6 \pi r^2} \left[(4 + 20r + 3r^2) C_W^2 - 2r(10 + 3r)C_W D_W + r(8 - r) D_W^2\right]|\psi(0)|^2,$$

(9.28)

where

$$D_W = \frac{-g^2}{4(m_\Omega^2 - 2M_W^2)}.$$  

(9.29)
Thus a much larger $\Gamma_{WW} = 190$ eV is obtained. For $\Omega \rightarrow ZZ$, there is only the $t$–channel contribution, i.e.
\[
\Gamma(\Omega \rightarrow ZZ) = \frac{m_{\Omega}^2(1 - r_Z)^{5/2}}{3\pi r_Z} D_Z^2 |\psi(0)|^2; \quad (9.30)
\]
where $r_Z = 4M_Z^2/m_{\Omega}^2$ and $D_Z = g_Z^2 \sin^4 \theta_W / [4(m_{\Omega}^2 - 2m_Z^2)]$ in case A, with $\sin^4 \theta_W$ replaced by $\cos^4 \theta_W$ in case B. Hence $\Gamma_{ZZ}$ is negligible in case A and only 2.5 eV in case B.

The $\Omega$ decay to two stickons is forbidden by charge conjugation. Its decay to three stickons is analogous to that of quarkonium to three gluons. Whereas the latter forms a singlet which is symmetric in $SU(3)_C$, the former forms a singlet which is antisymmetric in $SU(2)_l$. However, the two amplitudes are identical because the latter is symmetrized with respect to the exchange of the three gluons and the former is antisymmetrized with respect to the exchange of the three stickons. Taking into account the different color factors of $SU(2)_l$ versus $SU(3)_C$, the decay rate of $\Omega$ to three stickons and to two stickons plus a photon are
\[
\begin{align*}
\Gamma(\Omega \rightarrow \zeta\zeta\zeta) & = \frac{16}{27}(\pi^2 - 9) \frac{\alpha_s^3}{m_{\Omega}^2} |\psi(0)|^2, \quad (9.31) \\
\Gamma(\Omega \rightarrow \gamma\zeta\zeta) & = \frac{8}{9}(\pi^2 - 9) \frac{\alpha_s^2}{m_{\Omega}^2} |\psi(0)|^2. \quad (9.32)
\end{align*}
\]
Hence $\Gamma_{\zeta\zeta\zeta} = 4.5$ eV and $\Gamma_{\gamma\zeta\zeta} = 1.1$ eV. The integrated cross section of Eq. (21) for $X = \mu^-\mu^+$ is then $3.8 \times 10^{-33}$ cm$^2$-keV in case A and $2.1 \times 10^{-33}$ cm$^2$-keV in case B. For comparison, this number is $7.9 \times 10^{-30}$ cm$^2$-keV for the $\Upsilon(1S)$. At a
high-luminosity $e^-e^+$ collider, it should be feasible to make this observation. Table 2 summarizes all the partial decay widths.

We should point out that the generic idea of $SU(2)_l$ leptonic color [170, 171, 172] is applicable to our discussion in this section. What distinguishes the BMW model [169] is its insistence that the four fundamental gauge couplings be unified. This in turn requires the existence of three hemion families below the TeV scale and that the leptonic color confining scale to be keV. Without these constraints, there is no guarantee that hemionia would be observable at a future $e^-e^+$ collider, but then there is also no reason to forbid them. Note also that the value of $\alpha_l$ is predicted in the BMW model, whereas in a generic leptonic color model, it is not.

9.7 Discussion and outlook

There are important differences between QCD and QHD (quantum hemiodynamics). In the former, because of the existence of light $u$ and $d$ quarks, it is easy to pop up $\bar{u}u$ and $\bar{d}d$ pairs from the QCD vacuum. Hence the production of open charm in an $e^-e^+$ collider is described well by the fundamental process $e^-e^+ \rightarrow c\bar{c}$. In the latter, there are no light hemions. Instead it is easy to pop up the light stickballs from the QHD vacuum. As a result, just above the threshold of making the $\Omega$ resonance, the many-body production of $\Omega +$ stickballs becomes possible. This cross section is presumably also well described by the fundamental process $e^-e^+ \rightarrow x\bar{x}$. In case A,
the cross section is given by

\[
\sigma(e^-e^+ \to x\bar{x}) = \frac{2\pi\alpha^2}{3} \sqrt{1 - \frac{4m^2}{s}} \left[ \frac{(s + 2m^2)}{s^2} + \frac{x_W^2}{2(1 - x_W)^2(s - m_Z^2)} \right. \\
+ \left. \frac{x_W}{(1 - x_W)s(s - m_Z^2)} - \frac{(1 - 4x_W)}{4(1 - x_W)s(s - m_Z^2)} \right].
\]

(9.33)

where \(x_W = \sin^2 \theta_W\) and \(s = 4E^2\) is the square of the center-of-mass energy. In case B, it is

\[
\sigma(e^-e^+ \to x\bar{x}) = \frac{2\pi\alpha^2}{3} \sqrt{1 - \frac{4m^2}{s}} \left[ \frac{(s + 2m^2)}{s^2} + \frac{(s - m^2)}{2(s - m_Z^2)^2} \right. \\
- \left. \frac{(s - m^2)}{s(s - m_Z^2)} + \frac{(1 - 4x_W)}{4x_W\frac{m^2}{s(s - m_Z^2)}} \right].
\]

(9.34)

Using \(m = 100\ \text{GeV}\) and \(s = (250\ \text{GeV})^2\) as an example, we find these cross sections to be 0.79 and 0.44 pb respectively.

In QCD, there are \(q\bar{q}\) bound states which are bosons, and \(qqq\) bound states which are fermions. In QHD, there are only bound-state bosons, because the confining symmetry is \(SU(2)_l\). Also, unlike baryon (or quark) number in QCD, there is no such thing as hemion number in QHD, because \(y\) is effectively \(\bar{x}\). This explains why there are no stable analog fermion in QHD such as the proton in QCD.

The SM Higgs boson \(h\) couples to the hemions, but these Yukawa couplings could be small, because hemions have invariant masses themselves as already explained. So far we have assumed these couplings to be negligible. If not, then \(h\) may decay to two photons and two stickons through a loop of hemions. This may show up in precision Higgs studies as a deviation of \(h \rightarrow \gamma\gamma\) from the SM prediction. It will also imply
a partial invisible width of $h$ proportional to this deviation. Neither would be large effects and that is perfectly consistent with present data.

The absence of observations of new physics at the LHC is a possible indication that fundamental new physics may not be accessible using the strong interaction, i.e. quarks and gluons. It is then natural to think about future $e^-e^+$ colliders. But is there some fundamental issue of theoretical physics which may only reveal itself there? and not at hadron colliders? The BMW model is one possible answer. It assumes a quartification symmetry based on $[SU(3)]^4$. It has gauge-coupling unification without supersymmetry, but requires the existence of new half-charged fermions (hemions) under a confining $SU(2)_l$ leptonic color symmetry, with masses below the TeV scale. It also predicts the $SU(2)_l$ confining scale to be keV, so that stickball bound states of the vector gauge stickons are formed. These new particles have no QCD interactions, but hemions have electroweak couplings, so they are accessible in a future $e^-e^+$ collider, as described in this paper.

9.8 Acknowledgement

This work was supported in part by the U. S. Department of Energy Grant No. DE-SC0008541.
Table 9.2: Partial decay widths of the hemionium $\Omega$.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Width (A)</th>
<th>Width (B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu\bar{\nu}$</td>
<td>11 eV</td>
<td>123 eV</td>
</tr>
<tr>
<td>$e^-e^+$</td>
<td>43 eV</td>
<td>69 eV</td>
</tr>
<tr>
<td>$\mu^-\mu^+$</td>
<td>43 eV</td>
<td>69 eV</td>
</tr>
<tr>
<td>$\tau^-\tau^+$</td>
<td>43 eV</td>
<td>69 eV</td>
</tr>
<tr>
<td>$u\bar{u}$</td>
<td>50 eV</td>
<td>175 eV</td>
</tr>
<tr>
<td>$c\bar{c}$</td>
<td>50 eV</td>
<td>175 eV</td>
</tr>
<tr>
<td>$d\bar{d}$</td>
<td>10 eV</td>
<td>147 eV</td>
</tr>
<tr>
<td>$s\bar{s}$</td>
<td>10 eV</td>
<td>147 eV</td>
</tr>
<tr>
<td>$b\bar{b}$</td>
<td>10 eV</td>
<td>147 eV</td>
</tr>
<tr>
<td>$W^-W^+$</td>
<td>3.2 eV</td>
<td>190 eV</td>
</tr>
<tr>
<td>$ZZ$</td>
<td>0.02 eV</td>
<td>2.5 eV</td>
</tr>
<tr>
<td>$\zeta\zeta\zeta$</td>
<td>4.5 eV</td>
<td>4.5 eV</td>
</tr>
<tr>
<td>$\zeta\zeta\gamma$</td>
<td>1.1 eV</td>
<td>1.1 eV</td>
</tr>
<tr>
<td>sum</td>
<td>279 eV</td>
<td>1319 eV</td>
</tr>
</tbody>
</table>
Chapter 10

Dark Gauge U(1) Symmetry for an Alternative Left-Right Model[9]

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Abstract

An alternative left-right model of quarks and leptons, where the $SU(2)_R$ lepton doublet $(\nu, l)_R$ is replaced with $(n, l)_R$ so that $n_R$ is not the Dirac mass partner of

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$\nu_L$, has been known since 1987. Previous versions assumed a global $U(1)_S$ symmetry to allow $n$ to be identified as a dark-matter fermion. We propose here a gauge extension by the addition of extra fermions to render the model free of gauge anomalies, and just one singlet scalar to break $U(1)_S$. This results in two layers of dark matter, one hidden behind the other.
10.1 Introduction

The alternative left-right model [29] of 1987 was inspired by the $E_6$ decomposition to the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry through an $SU(2)_R$ which does not have the conventional assignments of quarks and leptons. Instead of $(u,d)_R$ and $(\nu,l)_R$ as doublets under $SU(2)_R$, a new quark $h$ and a new lepton $n$ per family are added so that $(u,h)_R$ and $(n,e)_R$ are the $SU(2)_R$ doublets, and $h_L$, $d_R$, $n_L$, $\nu_R$ are singlets.

This structure allows for the absence of tree-level flavor-changing neutral currents (unavoidable in the conventional model), as well as the existence of dark matter. The key new ingredient is a $U(1)_S$ symmetry, which breaks together with $SU(2)_R$, such that a residual global $S'$ symmetry remains for the stabilization of dark matter. Previously [184, 185, 186], this $U(1)_S$ was assumed to be global. We show in this paper how it may be promoted to a gauge symmetry. To accomplish this, new fermions are added to render the model free of gauge anomalies. The resulting theory has an automatic discrete $Z_2$ symmetry which is unbroken, as well as the global $S'$, which is now broken to $Z_3$. Hence dark matter has two components [93]. They are identified as one Dirac fermion (nontrivial under both $Z_2$ and $Z_3$) and one complex scalar (nontrivial under $Z_3$).
10.2 Model

The particle content of our model is given in Table 1, where the scalar $SU(2)_L \times SU(2)_R$ bidoublet is given by

$$\eta = \begin{pmatrix} \eta_1^0 & \eta_2^+ \\ \eta_1^- & \eta_2^0 \end{pmatrix},$$ (10.1)

with $SU(2)_L$ transforming vertically and $SU(2)_R$ horizontally. Without $U(1)_S$ as a gauge symmetry, the model is free of anomalies without the addition of the $\psi$ and $\chi$ fermions. In the presence of gauge $U(1)_S$, the additional anomaly-free conditions are all satisfied by the addition of the $\psi$ and $\chi$ fermions. The $[SU(3)_C]^2 U(1)_S$ anomaly is canceled between $(u, h)_R$ and $h_L$; the $[SU(2)_L]^2 U(1)_S$ anomaly is zero because $(u, d)_L$ and $(\nu, l)_L$ do not transform under $U(1)_S$; the $[SU(2)_R]^2 U(1)_S$ and $[SU(2)_R]^2 U(1)_X$ anomalies are both canceled by summing over $(u, h)_R$, $(n, l)_R$, $(\psi_1^0, \psi^-)_R$, and $(\psi^+_2, \psi^0_2)_R$; the addition of $\chi^+_R$ renders the $[U(1)_X]^2 U(1)_S$, $U(1)_X[U(1)_S]^2$, $[U(1)_X]^3$, and $U(1)_X$ anomalies zero; and the further addition of $\chi^0_1_R$ and $\chi^0_2_R$ kills both the $[U(1)_S]^3$ and $U(1)_S$ anomalies, i.e.

$$0 = 3[6(-1/2)^3 - 3(-1)^3 + 2(1/2)^3 - (1)^3] + 2(2)^3 + 2(1)^3 + 2(-3/2)^3 + (-1/2)^3 + (-5/2)^3,$$ (10.2)

$$0 = 3[6(-1/2) - 3(-1) + 2(1/2) - (1)] + 2(2) + 2(1) + 2(-3/2) + (-1/2) + (-5/2).$$ (10.3)

Under $T_{3R} + S$, the neutral scalars $\phi^0_R$ and $\eta^0_2$ are zero, so that their vacuum
expectation values do not break $T_{3R} + S$ which remains as a global symmetry. However, $\langle \sigma \rangle \neq 0$ does break $T_{3R} + S$ and gives masses to $\psi^0_1 \psi^0_2 - \psi^-_1 \bar{\psi}^-_2 R, \chi^+_R \chi^-_R$, and $\chi^0_1 \chi^0_2 R$. These exotic fermions all have half-integral charges [187] under $T_{3R} + S$ and only communicate with the others with integral charges through $W^\pm R, \sqrt{2} Re(\phi^0_R), \zeta$, and the two extra neutral gauge bosons beyond the $Z$. Some explicit Yukawa terms are

\begin{align}
\psi^0_1 \phi^0_R - \psi^-_1 \bar{\phi}^0_R \chi^+_R, & \quad \psi^+_1 \bar{\phi}^0_R - \psi^0_2 \phi^+_R \chi^-_R, \\
(\psi^0_1 \phi^0_R - \psi^-_1 \bar{\phi}^0_R) \chi^0_2 R, & \quad (\psi^+_1 \bar{\phi}^0_R + \psi^0_2 \phi^+_R) \chi^0_1 R.
\end{align}

(10.4)

(10.5)

This dichotomy of particle content results in an additional unbroken symmetry of the Lagrangian, i.e. discrete $Z_2$ under which the exotic fermions are odd. Hence dark matter has two layers: those with nonzero $T_{3R} + S$ and even $Z_2$, i.e. $n, h, W_R^\pm, \phi^+_R, \eta_1^0, \eta_1^1, \bar{\eta}^0_1, \zeta$, and the underlying exotic fermions with odd $Z_2$. Without $\zeta$, a global $S'$ symmetry remains. With $\zeta$, because of the $\zeta^3 \sigma^+ \chi^0_1 \chi^0_2 R \zeta$ terms, the $S'$ symmetry breaks to $Z_3$.

Let

\begin{align}
\langle \phi^0_L \rangle = v_1, \quad \langle \eta^0_1 \rangle = v_2, \quad \langle \phi^0_R \rangle = v_R, \quad \langle \sigma \rangle = v_S,
\end{align}

(10.6)

then the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X \times U(1)_S$ gauge symmetry is broken to $SU(3)_C \times U(1)_Q$ with $S'$, which becomes $Z_3$, as shown in Table 2 with $\omega^3 = 1$. The discrete $Z_2$ symmetry is unbroken. Note that the global $S'$ assignments for the exotic fermions are not $T_{3R} + S$ because of $v_S$ which breaks the gauge $U(1)_S$ by 3 units.
10.3 Gauge sector

Consider now the masses of the gauge bosons. The charged ones, $W_L^\pm$ and $W_R^\pm$, do not mix because of $S'(Z_3)$, as in the original alternative left-right models. Their masses are given by

$$M_{W_L}^2 = \frac{1}{2} g_L^2 (v_1^2 + v_2^2), \quad M_{W_R}^2 = \frac{1}{2} g_R^2 (v_R^2 + v_2^2). \quad (10.7)$$

Since $Q = I_{3L} + I_{3R} + X$, the photon is given by

$$A = \frac{e}{g_L} W_{3L} + \frac{e}{g_R} W_{3R} + \frac{e}{g_X} X, \quad (10.8)$$

where $e^{-2} = g_L^{-2} + g_R^{-2} + g_X^{-2}$. Let

$$Z = (g_L^2 + g_Y^2)^{-1/2} \left( g_L W_{3L} - \frac{g_Y}{g_R} W_{3R} - \frac{g_Y}{g_X} X \right), \quad (10.9)$$

$$Z' = (g_R^2 + g_X^2)^{-1/2} (g_R W_{3R} - g_X X), \quad (10.10)$$

where $g_Y^2 = g_R^{-2} + g_X^{-2}$, then the $3 \times 3$ mass-squared matrix spanning $(Z, Z', S)$ has the entries:

$$M_{ZZ}^2 = \frac{1}{2} (g_L^2 + g_Y^2) (v_1^2 + v_2^2), \quad (10.11)$$

$$M_{ZZ'}^2 = \frac{1}{2} (g_R^2 + g_X^2) v_R^2 + \frac{g_Y v_1^2 + g_X v_2^2}{2 (g_R^2 + g_X^2)}, \quad (10.12)$$

$$M_{SS}^2 = 18 g_S^2 v_S^2 + \frac{1}{2} g_S^2 (v_R^2 + v_2^2), \quad (10.13)$$

$$M_{ZZ'}^2 = \frac{\sqrt{g_L^2 + g_Y^2}}{2 \sqrt{g_R^2 + g_X^2}} (g_X v_1^2 - g_R v_2^2), \quad (10.14)$$

$$M_{ZS}^2 = \frac{1}{2} g_S \sqrt{g_L^2 + g_Y^2} v_2, \quad (10.15)$$

$$M_{Z'S}^2 = -\frac{1}{2} g_S \sqrt{g_R^2 - g_X^2} v_2 - \frac{g_S g_R v_2^2}{2 \sqrt{g_R^2 + g_X^2}}, \quad (10.16)$$
Their neutral-current interactions are given by

\[
\mathcal{L}_{NC} = eA_{\mu}j_{Q}^{\mu} + g_{Z}Z_{\mu}(j_{3L}^{\mu} - \sin^{2}\theta_{W}j_{Q}^{\mu}) + (g_{R}^{2} + g_{X}^{2})^{-1/2}Z_{\mu}(g_{R}j_{3R}^{\mu} - g_{X}^{2}j_{X}^{\mu}) + gsS_{\mu}j_{S}^{\mu},
\]  

(10.17)

where \(g_{Z}^{2} = g_{L}^{2} + g_{Y}^{2}\) and \(\sin^{2}\theta_{W} = g_{Y}^{2}/g_{Z}^{2}\).

In the limit \(v_{1,2}^{2} \ll v_{R}^{2}, v_{S}^{2}\), the mass-squared matrix spanning \((Z', S)\) may be simplified if we assume

\[
\frac{v_{S}^{2}}{v_{R}^{2}} = \frac{(g_{R}^{2} + g_{X}^{2} + g_{S}^{2})^{2}}{36g_{S}^{2}(g_{R}^{2} + g_{X}^{2} - g_{S}^{2})},
\]  

(10.18)

and let

\[
\tan\theta_{D} = \sqrt{\frac{g_{R}^{2} + g_{X}^{2} - g_{S}}{g_{R}^{2} + g_{X}^{2} + g_{S}}},
\]  

(10.19)

then

\[
\begin{pmatrix}
D_{1} \\
D_{2}
\end{pmatrix} = \begin{pmatrix}
\cos\theta_{D} & \sin\theta_{D} \\
-\sin\theta_{D} & \cos\theta_{D}
\end{pmatrix} \begin{pmatrix}
Z' \\
S
\end{pmatrix},
\]  

(10.20)

with mass eigenvalues given by

\[
M_{D_{1}}^{2} = \sqrt{g_{R}^{2} + g_{X}^{2}} \sqrt{g_{R}^{2} + g_{X}^{2} + g_{S}^{2}} \frac{v_{R}^{2}}{2\sqrt{2}\cos\theta_{D}},
\]  

(10.21)

\[
M_{D_{2}}^{2} = \sqrt{g_{R}^{2} + g_{X}^{2}} \sqrt{g_{R}^{2} + g_{X}^{2} + g_{S}^{2}} \frac{v_{R}^{2}}{2\sqrt{2}\sin\theta_{D}}.
\]  

(10.22)

In addition to the assumption of Eq. (18), let us take for example

\[
2gs = \sqrt{g_{R}^{2} + g_{X}^{2}},
\]  

(10.23)

then \(\sin\theta_{D} = 1/\sqrt{10}\) and \(\cos\theta_{D} = 3/\sqrt{10}\). Assuming also that \(g_{R} = g_{L}\), we obtain

\[
\frac{g_{X}^{2}}{g_{Z}^{2}} = \frac{\sin^{2}\theta_{W}\cos^{2}\theta_{W}}{\cos 2\theta_{W}}, \quad \frac{gs}{g_{z}} = \frac{\cos^{2}\theta_{W}}{2\sqrt{\cos 2\theta_{W}}},
\]  

(10.24)

\[
\frac{v_{S}^{2}}{v_{R}^{2}} = \frac{25}{108}, \quad M_{D_{2}}^{2} = 3M_{D_{1}}^{2} = \frac{5\cos^{4}\theta_{W}}{4\cos 2\theta_{W}}g_{z}^{2}v_{R}^{2}.
\]  

(10.25)
The resulting gauge interactions of $D_{1,2}$ are given by

\[ L_{D} = \frac{gZ}{\sqrt{10/\cos 2\theta_W}} \{ [3 \cos 2\theta_W j_{3R}^\mu - 3 \sin^2 \theta_W j_X^\mu + (1/2) \cos^2 \theta_W j_S^\mu] D_{1\mu} \\
+ [- \cos 2\theta_W j_{3R}^\mu + \sin^2 \theta_W j_X^\mu + (3/2) \cos^2 \theta_W j_S^\mu] D_{2\mu} \} \]. \ (10.26)

Since $D_2$ is $\sqrt{3}$ times heavier than $D_1$ in this example, the latter would be produced first in $pp$ collisions at the Large Hadron Collider (LHC).

### 10.4 Fermion sector

All fermions obtain masses through the four vacuum expectation values of Eq. (6) except $\nu_R$ which is allowed to have an invariant Majorana mass. This means that neutrino masses may be small from the usual canonical seesaw mechanism. The various Yukawa terms for the quark and lepton masses are

\[-L_Y = \frac{m_u}{v_2} [\bar{u}_R(u_L \eta^0_2 - d_L \eta^+_2) + \bar{h}_R(-u_L \eta^-_2 + d_L \eta^0_1)] \\
+ \frac{m_d}{v_1} (\bar{u}_L \phi^+_L + \bar{d}_L \phi^0_L) d_R + \frac{m_h}{v_R} (\bar{u}_R \phi^+_R + \bar{h}_R \phi^0_R) h_L \\
+ \frac{m_l}{v_2} [(\bar{\nu}_L \eta^0_2 + \bar{l}_L \eta^-_2) n_R + (\bar{\nu}_L \eta^+_2 + \bar{l}_L \eta^0_1) l_R] \\
+ \frac{m_D}{v_1} \bar{\nu}_R(\nu_L \phi^0_L - l_L \phi^+_L) + \frac{m_n}{v_R} \bar{n}_L(n_R \phi^0_R - l_R \phi^-_R) + H.c. \] \ (10.27)

These terms show explicitly that the assignments of Tables 1 and 2 are satisfied.

As for the exotic $\psi$ and $\chi$ fermions, they have masses from the Yukawa terms of Eqs. (4) and (5), as well as

\[(\phi^0_{1R} \psi^0_{2R} - \psi^-_{1R} \psi^+_{2R})^\ast, \quad \bar{\chi}_R \chi^+_R \sigma, \quad \chi^0_{1R} \chi^0_{2R} \sigma, \] \ (10.28)
As a result, two neutral Dirac fermions are formed from the matrix linking \( \chi_{1R}^0 \) and \( \psi_{1R}^0 \) to \( \chi_{2R}^0 \) and \( \psi_{2R}^0 \). Let us call the lighter of these two Dirac fermions \( \chi_0 \), then it is one component of dark matter of our model. The other will be the scalar \( \zeta \), to be discussed later. Note that \( \chi_0 \) communicates with \( \zeta \) through the allowed \( \chi_{1R}^0 \chi_{1R}^0 \zeta \) interaction. Note also that the allowed Yukawa terms

\[
\bar{d}_R h_L \zeta, \quad \bar{n}_L \nu_R \zeta
\]

enable the dark fermions \( h \) and \( n \) to decay into \( \zeta \).

### 10.5 Scalar sector

Consider the most general scalar potential consisting of \( \Phi_{L,R}, \eta, \) and \( \sigma \). Let

\[
\eta = \begin{pmatrix} \eta_1^0 & \eta_2^+ \\ \eta_1^- & \eta_2^0 \end{pmatrix}, \quad \tilde{\eta} = \sigma \eta^* \sigma
\]

\[
\tilde{\eta} = \begin{pmatrix} \eta_2^- & -\eta_1^- \\ -\eta_2^+ & \eta_1^- \end{pmatrix}
\]

then

\[
V = -\mu_L^2 \Phi_L^\dagger \Phi_L - \mu_R^2 \Phi_R^\dagger \Phi_R - \mu^2 \sigma^* \sigma - \mu_\eta^2 \text{Tr}(\eta^\dagger \eta) + [\mu_3 \Phi_L^\dagger \eta \Phi_R + \text{H.c.}]
\]

\[
+ \frac{1}{2} \lambda_L (\Phi_L^\dagger \Phi_L)^2 + \frac{1}{2} \lambda_R (\Phi_R^\dagger \Phi_R)^2 + \frac{1}{2} \lambda_\sigma (\sigma^* \sigma)^2 + \frac{1}{2} \lambda_\eta \text{Tr}(\eta^\dagger \eta) \text{Tr}(\eta^\dagger \eta) + \frac{1}{2} \lambda_\eta \text{Tr}(\eta^\dagger \eta) \text{Tr}(\eta^\dagger \eta)
\]

\[
+ \lambda_{LR}(\Phi_L^\dagger \Phi_L)(\Phi_R^\dagger \Phi_R) + \lambda_L \sigma (\Phi_L^\dagger \Phi_L)(\sigma^* \sigma) + \lambda_R \sigma (\Phi_R^\dagger \Phi_R)(\sigma^* \sigma) + \lambda_\eta (\sigma^* \sigma) \text{Tr}(\eta^\dagger \eta)
\]

\[
+ \lambda_\eta \Phi_L^\dagger \eta \Phi_L + \lambda'_L \eta \Phi_L^\dagger \Phi_L + \lambda_{R\eta} \Phi_R^\dagger \eta \Phi_R + \lambda'_{R\eta} \Phi_R^\dagger \Phi_R - \text{Tr}(\eta^\dagger \eta) \text{Tr}(\eta^\dagger \eta).
\]

Note that

\[
2 |\text{det}(\eta)|^2 = |\text{Tr}(\eta^\dagger \eta)|^2 - \text{Tr}(\eta^\dagger \eta^\dagger \eta),
\]
\[(\Phi_L^* \Phi_L) Tr(\eta^* \eta) = \Phi_L \eta \eta^* \Phi_L + \Phi_L \eta^* \Phi_L, \quad (10.33)\]

\[(\Phi_R^* \Phi_R) Tr(\eta^* \eta) = \Phi_R \eta \eta^* \Phi_R + \Phi_R \eta^* \Phi_R. \quad (10.34)\]

The minimum of \(V\) satisfies the conditions

\[
\mu_L^2 = \lambda_L v_1^2 + \lambda_{L\eta} v_2^2 + \lambda_{LR} v_R^2 + \lambda_{L\sigma} v_S^2 + \mu_3 v_R v_1/v_1, \quad (10.35)\]

\[
\mu_\eta^2 = (\lambda_\eta + \lambda_\eta') v_2^2 + \lambda_{L\eta} v_1^2 + \lambda_{R\eta} v_R^2 + \lambda_{\sigma\eta} v_S^2 + \mu_3 v_1 v_R/v_2, \quad (10.36)\]

\[
\mu_R^2 = \lambda_{R\eta} v_1^2 + \lambda_{LR} v_R^2 + \lambda_{R\eta} v_R^2 + \lambda_{R\sigma} v_S^2 + \mu_3 v_1 v_R/v_2, \quad (10.37)\]

\[
\mu_\sigma^2 = \lambda_{\sigma} v_1^2 + \lambda_{L\sigma} v_1^2 + \lambda_{\sigma\eta} v_2^2 + \lambda_{R\sigma} v_R^2. \quad (10.38)\]

The 4 \times 4 mass-squared matrix spanning \(\sqrt{2} Im(\phi_L^0, \eta_2^0, \phi_R^0, \sigma)\) is then given by

\[
\mathcal{M}_1^2 = \mu_3 \begin{pmatrix}
-v_2 v_R/v_1 & v_R & v_2 & 0 \\
v_R & -v_1 v_R/v_2 & v_1 & 0 \\
v_2 & v_1 & -v_1 v_2/v_R & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}. \quad (10.39)
\]

and that spanning \(\sqrt{2} Re(\phi_L^0, \eta_2^0, \phi_R^0, \sigma)\) is

\[
\mathcal{M}_R^2 = \mathcal{M}_1^2 + 2 \begin{pmatrix}
\lambda_L v_1^2 & \lambda_{L\eta} v_1 v_2 & \lambda_{LR} v_1 v_R & \lambda_{L\sigma} v_1 v_S \\
\lambda_{L\eta} v_1 v_2 & (\lambda_\eta + \lambda_\eta') v_2^2 & \lambda_{R\eta} v_2 v_R & \lambda_{\sigma\eta} v_2 v_S \\
\lambda_{LR} v_1 v_R & \lambda_{R\eta} v_2 v_R & \lambda_R v_R^2 & \lambda_{R\sigma} v_R v_S \\
\lambda_{L\sigma} v_1 v_S & \lambda_{\sigma\eta} v_2 v_S & \lambda_{R\sigma} v_R v_S & \lambda_\sigma v_S^2
\end{pmatrix}. \quad (10.40)
\]

Hence there are three zero eigenvalues in \(\mathcal{M}_1^2\) with one nonzero eigenvalue \(-\mu_3[v_1 v_2/v_R + v_R(v_1^2 + v_2^2)/v_1 v_2]\) corresponding to the eigenstate \((-v_1^{-1}, v_2^{-1}, v_R^{-1}, 0)/\sqrt{v_1^{-2} + v_2^{-2} + v_R^{-2}}\).

In \(\mathcal{M}_R^2\), the linear combination \(H = (v_1, v_2, 0, 0)/\sqrt{v_1^2 + v_2^2}\), is the standard-model
Higgs boson, with

\[ m_H^2 = 2[\lambda_L v_1^4 + (\lambda_\eta + \lambda'_\eta) v_2^4 + 2\lambda_L \eta_1 v_2^2 v_1^2] / (v_1^2 + v_2^2). \]  

(10.41)

The other three scalar bosons are much heavier, with suppressed mixing to \( H \), which may all be assumed to be small enough to avoid the constraints from dark-matter direct-search experiments. The addition of the scalar \( \zeta \) introduces two important new terms:

\[ \zeta^3 \sigma^+ , \ (\eta_1^0 \eta_2^0 - \eta_1^- \eta_2^+) \zeta. \]  

(10.42)

The first term breaks global \( S' \) to \( Z_3 \), and the second term mixes \( \zeta \) with \( \eta_1^0 \) through \( v_2 \). We assume the latter to be negligible, so that the physical dark scalar is mostly \( \zeta \).

### 10.6 Present phenomenological constraints

Many of the new particles of this model interact with those of the standard model. The most important ones are the neutral \( D_{1,2} \) gauge bosons, which may be produced at the LHC through their couplings to \( u \) and \( d \) quarks, and decay to charged leptons \((e^- e^+ \) and \( \mu^- \mu^+)\). As noted previously, in our chosen example, \( D_1 \) is the lighter of the two. Hence current search limits for a \( Z' \) boson are applicable [57, 60]. The \( c_u,d \) coefficients used in the data analysis are

\[ c_u = (g_{uL}^2 + g_{uR}^2) B = 0.0273 \ B, \quad c_d = (g_{dL}^2 + g_{dR}^2) B = 0.0068 \ B, \]  

(10.43)
where $B$ is the branching fraction of $Z'$ to $e^-e^+$ and $\mu^-\mu^+$. Assuming that $D_1$ decays to all the particles listed in Table 2, except for the scalars which become the longitudinal components of the various gauge bosons, we find $B = 1.2 \times 10^{-2}$. Based on the 2016 LHC 13 TeV data set, this translates to a bound of about 4 TeV on the $D_1$ mass.

The would-be dark-matter candidate $n$ is a Dirac fermion which couples to $D_{1,2}$ which also couples to quarks. Hence severe limits exist on the masses of $D_{1,2}$ from underground direct-search experiments as well. The annihilation cross section of $n$ through $D_{1,2}$ would then be too small, so that its relic abundance would be too big for it to be a dark-matter candidate. Its annihilation at rest through s-channel scalar exchange is $p$-wave suppressed and does not help. As for the $t$-channel diagrams, they also turn out to be too small. Previous studies where $n$ is chosen as dark matter are now ruled out.

10.7 Dark sector

Dark matter is envisioned to have two components. One is a Dirac fermion $\chi_0$ which is a mixture of the four neutral fermions of odd $Z_2$, and the other is a complex scalar boson which is mostly $\zeta$. The annihilation $\chi_0\bar{\chi}_0 \rightarrow \zeta\zeta^*$ determines the relic abundance of $\chi_0$, and the annihilation $\zeta\zeta^* \rightarrow HH$, where $H$ is the standard-model Higgs boson, determines that of $\zeta$. The direct $\zeta\zeta^*H$ coupling is assumed small to
avoid the severe constraint in direct-search experiments.

Let the interaction of $\zeta$ with $\chi_0$ be $f_0 \zeta \chi_0 R \chi_0 R + H.c.$, then the annihilation cross section of $\chi_0 \bar{\chi}_0$ to $\zeta \zeta^*$ times relative velocity is given by

$$\langle \sigma \times v_{\text{rel}} \rangle_\chi = \frac{f_0^4}{4\pi m_{\chi_0}} \frac{(m_{\chi_0}^2 - m_\zeta^2)^{3/2}}{(2m_{\chi_0}^2 - m_\zeta^2)^2}. \quad (10.44)$$

Let the effective interaction strength of $\zeta \zeta^*$ with $HH$ be $\lambda_0$, then the annihilation cross section of $\zeta \zeta^*$ to $HH$ times relative velocity is given by

$$\langle \sigma \zeta \times v_{\text{rel}} \rangle_\zeta = \frac{\lambda_0^2}{16\pi} \frac{(m_\zeta^2 - m_H^2)^{1/2}}{m_\zeta^3}. \quad (10.45)$$

Note that $\lambda_0$ is the sum over several interactions. The quartic coupling $\lambda_\zeta H$ is assumed negligible, to suppress the trilinear $\zeta \zeta^* H$ coupling which contributes to the elastic $\zeta$ scattering cross section off nuclei. However, the trilinear couplings $\zeta \zeta^* Re(\phi_0^R)$ and $Re(\phi_0^R)HH$ are proportional to $v_R$, and the trilinear couplings $\zeta \zeta^* Re(\sigma)$ and $Re(\sigma)HH$ are proportional to $v_S$. Hence their effective contributions to $\lambda_0$ are proportional to $v_R^2/m^2[\sqrt{2} Re(\phi_0^R)]$ and $v_S^2/m^2[\sqrt{2} Re(\sigma)]$, which are not suppressed.

As a rough estimate, we will assume that

$$\langle \sigma \times v_{\text{rel}} \rangle_\chi^{-1} + \langle \sigma \zeta \times v_{\text{rel}} \rangle_\zeta^{-1} = (4.4 \times 10^{-26} \text{ cm}^3/\text{s})^{-1} \quad (10.46)$$

to satisfy the condition of dark-matter relic abundance [88] of the Universe. For given values of $m_\zeta$ and $m_{\chi_0}$, the parameters $\lambda_0$ and $f_0$ are thus constrained. We show in Fig. 1 the plots of $\lambda_0$ versus $f_0$ for $m_\zeta = 150$ GeV and various values of $m_{\chi_0}$. Since
Figure 10.1: Relic-abundance constraints on $\lambda_0$ and $f_0$ for $m_\zeta = 150$ GeV and various values of $m_{\chi_0}$.

$m_\zeta$ is fixed at 150 GeV, $\lambda_0$ is also fixed for a given fraction of $\Omega_\zeta/\Omega_{DM}$. To adjust for the rest of dark matter, $f_0$ must then vary as a function of $m_{\chi_0}$ according to Eq. (44).

As for direct detection, both $\chi_0$ and $\zeta$ have possible interactions with quarks through the gauge bosons $D_{1,2}$ and the standard-model Higgs boson $H$. They are suppressed by making the $D_{1,2}$ masses heavy, and the $H$ couplings to $\chi_0$ and $\zeta$ small. In our example with $m_\zeta = 150$ GeV, let us choose $m_{\chi_0} = 500$ Gev and the relic abundances of both to be equal. From Fig. 1, these choices translate to $\lambda_0 = 0.12$.
and $f_0 = 0.56$.

Consider first the $D_{1,2}$ interactions. Using Eq. (26), we obtain

$$g_u^V(D_1) = 0.0621, \quad g_d^V(D_1) = 0.0184, \quad g_\zeta(D_1) = 0.1234, \quad (10.47)$$

$$g_u^V(D_2) = -0.1235, \quad g_d^V(D_2) = -0.0062, \quad g_\zeta(D_2) = 0.3701. \quad (10.48)$$

The effective $\zeta$ elastic scattering cross section through $D_{1,2}$ is then completely determined as a function of the $D_1$ mass (because $M_{D_2} = \sqrt{3}M_{D_1}$ in our example), i.e.

$$\mathcal{L}_{\zeta q}^V = \frac{(\zeta^* \partial_\mu - \zeta \partial_\mu \zeta^*)}{M_{D_1}^2} [(-7.57 \times 10^{-3}) \bar{u} \gamma^\mu u + (1.51 \times 10^{-3}) \bar{d} \gamma^\mu d]. \quad (10.49)$$

Using the latest LUX result [188] and Eq. (25), we obtain $v_R > 35$ TeV which translates to $M_{D_1} > 18$ TeV, and $M_{W_R} > 16$ TeV.

The $\bar{\chi}_0 \gamma_\mu \chi_0$ couplings to $D_{1,2}$ depend on the $2 \times 2$ mass matrix linking $(\chi_1, \psi_1)$ to $(\chi_2, \psi_2)$ which has two mixing angles and two mass eigenvalues, the lighter one being $m_{\chi_0}$. By adjusting these parameters, it is possible to make the effective $\chi_0$ interaction with xenon negligibly small. Hence there is no useful limit on the $D_1$ mass in this case.

Direct search also constrains the coupling of the Higgs boson to $\zeta$ (through a possible trilinear $\lambda_{\zeta H} \sqrt{2} v_H \zeta^* \zeta$ interaction) or $\chi_0$ (through an effective Yukawa coupling $\epsilon$ from $H$ mixing with $\sigma_R$ and $\phi^0_R$). Let their effective interactions with quarks through
$H$ exchange be given by

$$L_{\xi q}^S = \frac{\lambda_{\xi H} m_q}{m_H^2} \xi^* q q + \frac{\epsilon f_q}{m_H^2} \bar{\chi}_0 \chi_0 q q,$$  

(10.50)

where $f_q = m_q / \sqrt{2} v_H = m_q / (246 \text{ GeV})$. The spin-independent direct-detection cross section per nucleon in the former is given by

$$\sigma^{SI} = \frac{\mu_{\xi}^2}{\pi A^2} [\lambda_p Z + (A - Z) \lambda_n]^2,$$  

(10.51)

where $\mu_{\xi} = m_{\xi} M_A / (m_{\xi} + M_A)$ is the reduced mass of the dark matter, and [85]

$$\lambda_N = \left[ \sum_{u,d,s} f_q^N + \frac{2}{27} \left( 1 - \sum_{u,d,s} f_q^N \right) \right] \frac{\lambda_{\xi H} m_N}{2 m_{\xi} m_H^2},$$  

(10.52)

with [86]

$$f_u^p = 0.023, \quad f_d^p = 0.032, \quad f_s^p = 0.020,$$  

(10.53)

$$f_u^n = 0.017, \quad f_d^n = 0.041, \quad f_s^n = 0.020.$$  

(10.54)

For $m_{\xi} = 150 \text{ GeV}$, we have

$$\lambda_p = 2.87 \times 10^{-8} \lambda_{\xi H} \text{ GeV}^{-2}, \quad \lambda_n = 2.93 \times 10^{-8} \lambda_{\xi H} \text{ GeV}^{-2}.$$  

(10.55)

Using $A = 131$, $Z = 54$, and $M_A = 130.9$ atomic mass units for the LUX experiment [188], and twice the most recent bound of $2 \times 10^{-46} \text{ cm}^2$ (because $\xi$ is assumed to account for only half of the dark matter) at this mass, we find

$$\lambda_{\xi H} < 9.1 \times 10^{-4}.$$  

(10.56)

As noted earlier, this is negligible for considering the annihilation cross section of $\xi$ to $H$. 

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For the $H$ contribution to the $\chi_0$ elastic cross section off nuclei, we replace $m_\zeta$ with $m_{\chi_0} = 500$ GeV in Eq. (51) and $\lambda_{\zeta H}/2m_\zeta$ with $\epsilon/\sqrt{2}v_H$ in Eq. (52). Using the experimental data at 500 GeV, we obtain the bound.

$$\epsilon < 9.6 \times 10^{-4}. \quad (10.57)$$

From the above discussion, it is clear that our model allows for the discovery of dark matter in direct-search experiments in the future if these bounds are only a little above the actual values of $\lambda_{\zeta H}$ and $\epsilon$.

### 10.8 Conclusion and outlook

In the context of the alternative left-right model, a new gauge $U(1)_S$ symmetry has been proposed to stabilize dark matter. This is accomplished by the addition of a few new fermions to cancel all the gauge anomalies, as shown in Table 1. As a result of this particle content, an automatic unbroken $Z_2$ symmetry exists on top of $U(1)_S$ which is broken to a conserved residual $Z_3$ symmetry. Thus dark matter has two components. One is the Dirac fermion $\chi_0 \sim (\omega, -)$ and the other the complex scalar $\zeta \sim (\omega, +)$ under $Z_3 \times Z_2$. We have shown how they may account for the relic abundance of dark matter in the Universe, and satisfy present experimental search bounds.

Whereas we have no specific prediction for discovery in direct-search experiments, our model will be able to accommodate any positive result in the future, just like
many other existing proposals. To single out our model, many additional details must also be confirmed. Foremost are the new gauge bosons $D_{1,2}$. Whereas the LHC bound is about 4 TeV, the direct-search bound is much higher provided that $\zeta$ is a significant fraction of dark matter. If $\chi_0$ dominates instead, the adjustment of free parameters of our model can lower this bound to below 4 TeV. In that case, future $D_{1,2}$ observations are still possible at the LHC as more data become available.

Another is the exotic $h$ quark which is easily produced if kinematically allowed. It would decay to $d$ and $\zeta$ through the direct $\bar{d}_R h_L \zeta$ coupling of Eq. (29). Assuming that this branching fraction is 100%, the search at the LHC for 2 jets plus missing energy puts a limit on $m_h$ of about 1.0 TeV, as reported by the CMS Collaboration [189] based on the $\sqrt{s} = 13$ TeV data at the LHC with an integrated luminosity of 35.9 fb$^{-1}$ for a single scalar quark.

If the $\bar{d}_R h_L \zeta$ coupling is very small, then $h$ may also decay significantly to $u$ and a virtual $W_R^-$, with $W_R^-$ becoming $\bar{n}l^-$, and $\bar{n}$ becoming $\bar{\nu}\zeta^*$. This has no analog in the usual searches for supersymmetry or the fourth family because $W_R$ is heavy ($> 16$ TeV). To be specific, the final states of 2 jets plus $l_1^- l_2^+$ plus missing energy should be searched for. As more data are accumulated at the LHC, such events may become observable.
10.9 Acknowledgement

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Table 10.1: Particle content of proposed model of dark gauge $U(1)$ symmetry.

<table>
<thead>
<tr>
<th>particles</th>
<th>$SU(3)_C$</th>
<th>$SU(2)_L$</th>
<th>$SU(2)_R$</th>
<th>$U(1)_X$</th>
<th>$U(1)_S$</th>
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<td>2</td>
<td>1</td>
<td>1/6</td>
<td>0</td>
</tr>
<tr>
<td>$(u, h)_R$</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>1/6</td>
<td>−1/2</td>
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<tr>
<td>$d_R$</td>
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<td>1</td>
<td>1</td>
<td>−1/3</td>
<td>0</td>
</tr>
<tr>
<td>$h_L$</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>−1/3</td>
<td>−1</td>
</tr>
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<td>2</td>
<td>1</td>
<td>−1/2</td>
<td>0</td>
</tr>
<tr>
<td>$(n, l)_R$</td>
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<td>1</td>
<td>2</td>
<td>−1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>$\nu_R$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$n_L$</td>
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<td>1</td>
<td>0</td>
<td>1</td>
</tr>
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<td>1</td>
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<td>$\sigma$</td>
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Table 10.2: Particle content of proposed model under \((T_{3R} + S) \times Z_2\).

<table>
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<tr>
<th>particles</th>
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<th>global (S')</th>
<th>(Z_3)</th>
<th>(Z_2)</th>
</tr>
</thead>
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<td>0</td>
<td>1</td>
<td>+</td>
</tr>
<tr>
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<td>0</td>
<td>1</td>
<td>+</td>
</tr>
<tr>
<td>(n, \phi^+_R, \zeta)</td>
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<td>1</td>
<td>(\omega)</td>
<td>+</td>
</tr>
<tr>
<td>(h, (\eta_1^0, \eta_1^-))</td>
<td>(-1)</td>
<td>(-1)</td>
<td>(\omega^2)</td>
<td>+</td>
</tr>
<tr>
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<td>1</td>
<td>(-)</td>
</tr>
<tr>
<td>(\psi_{1R}^-, \chi_{2R}^-)</td>
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<td>0</td>
<td>1</td>
<td>(-)</td>
</tr>
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<td>(1, -1)</td>
<td>(\omega, \omega^2)</td>
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<td>(\chi_{1R}^0, \chi_{2R}^0)</td>
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<td>(\omega, \omega^2)</td>
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<tr>
<td>(\sigma)</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>+</td>
</tr>
</tbody>
</table>
Chapter 11

Conclusion

In conclusion, many different studies has been presented here which study various expentions of the standard model in order to incorporate several new physics effects into current state of known theoretical high energy physics. In one of the studies, the radiative inverse neutrino model is explained. It illustrates how standard model of elementary particles and interactions is enlarged to generate naturally small Majorana neutrino masses. The neutrino mass generating mechanism here naturally includes dark matter candidates which are phenomenologically accessible at the current, Large Hadron Collider, and future colliders. In this work another interesting open question was also addressed. This study introduces totally new and intriguing flavor symmetry, Cobimaximal symmetry, in the context of $Z_3$ symmetry, which could be responsible for the origin of PMNS matrix, mixing matrix in the lepton sector. According to cobimaximal mixing, Atmospheric mixing angle, $\theta_{23}$ and CP violating phase is predicted
to be maximal, in agreement with current experimental data.

In one of the following works, in the context of the study discussed in the paragraph above, the deviation from cobimaximal scenario was analysed. It was shown that there exists a correlation between the deviations of $\theta_{23}$ and CP phase from the cobimaximal case. This will allow us to determine the second, if one of this observables is measured in the future experiments.

In one of another studies, a new radiative Type II seesaw model with dark matter generating a neutrino mass has been presented. It has been shown in the context of this model, that this could lead to a unique discovery signatures, same flavour same sign 4 lepton in the final state channel, at the Large Hadron Collider.

Other project focuses on the study of different generations of Dirac neutrino mass in the case that double beta decay data disfavors Majorana case in the future. In the scope of this work, it has been shown that there exists 4 and only 4 tree level realizations of Dirac neutrino mass and only 2 one loop level mechanisms. The study also includes the one loop scotogenic Dirac neutrino mass generation in the context of left-right gauge model.

In the Quartification study the question of unification of all gauge couplings and its low energy phenomenology was addressed. In this study, by restoring the symmetry between standard model quarks and leptons, in the context of non-supersymmetric $[SU(3)]^4$ gauge symmetry group, all gauge coupling constants are shown to unify at the unification scale. As a consequence, the particles called Hemions are introduced.
These heavy hemions confine at the keV scale into bound states called hemionium, which might be light and show themselves as resonances in the future $e^+e^-$ collider. In one of the recent studies, we have explored the new variation of long studied left-right models. A new gauged version of alternative left-right model of quarks and leptons, where right-handed electron is paired into a right-handed doublet with a new particle $n$. As a consequence of this gauge symmetry, new particles are predicted which transform non-trivially under new Dark symmetry. This symmetry stabilize dark matter. In order to render this dark gauge symmetry anomaly free, new exotic fermions are predicted, which leads to another residual symmetry. As a result 2 separate discrete symmetries, $Z_2$ and $Z_3$, allow for the existence of 2 components of dark matter which would transform differently under discrete symmetries mentioned. The observation of newly predicted gauge particles at the Large Hadron Collider, or lack of it, will favor or disfavor this model in the future experiments.
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