Analogical Problem Solving: Insights from Verbal Reports

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Abstract

Problem solving is a complex cognitive activity that involves the construction of sequences of actions to reach a given goal. One powerful strategy is to identify analogies between the problem at hand and previously encountered ones. Relevant similarities between problems can be detected more easily if there is a high resemblance on the surface or with respect to structure. Earlier theoretical observations and performance data have pointed to two distinct kinds of analogical reasoning, direct solution transfer (transformational analogy) and the creation of a new solution based on adapted past reasoning processes (derivational analogy). In order to gain insights about the cognitive processes involved, we extend this work by an analysis of different kinds of verbal data. Planning protocols were collected prior to problem solving, and retrospective reports, evaluations, and instructions were elicited after the task was completed. Results show that the different kinds of analogical reasoning involved different degrees of analogy awareness, as reflected by the verbalizations. Derivational analogy involved problem solving on a more detailed and structured step-by-step basis than the more superficial transformational strategy, in which a simple matching procedure was employed.

Keywords: analogical problem solving, verbal reports, cognitive discourse analysis

Introduction

People frequently encounter problem situations in their daily lives. Most of these are instantly solved, as humans are well equipped with numerous problem solving strategies. One powerful strategy is the adaptation of previous experiences to solve the newly encountered problem. Analogical reasoning is not only involved in problem solving but also in a number of other human activities, such as use of metaphor, scientific reasoning, humor, and empathy. Gentner, Holyoak & Kokinov (2001) therefore argue that analogical reasoning is at the core of cognition. Hofstadter (2001) supports this view by arguing that all concepts that are used to understand recurrent and new situations are packages of analogies.

Even though analogical reasoning is assumed to be a ubiquitous and efficient problem solving strategy, participants seldom apply it in experimental settings, unless the analogous nature is directly salient (Gick & Holyoak, 1980; Schmid, Wirth & Polkeln, 1999). If analogies are used in problem solving the transfer can be on different levels of specificity, leading to different cognitive solution steps and strategies. In this paper, we pursue these issues by an analysis of verbal data collected while solving an analogy problem. In the following, we will first discuss previous work on analogical problem solving, with a brief look at the role of language. Next, we introduce a previous study by Schelhorn, Griego, and Schmid (2007) that served as our starting point. Our current study is then presented and discussed in the remainder of this paper.

Analogical Problem Solving

Analogies are based on shared relations between base and target problem (Gentner, 1983; Clement & Gentner, 1991). By highlighting shared relational structures, analogies connect domains and problems that may appear only marginally similar on the surface. This process involves structural alignment as a crucial component of analogical reasoning. While similarity centers on shared attributes, analogy concerns the alignment of relational structures at a deeper level. According to the systematicity principle, the structural relations are connected by one-to-one correspondences (Gentner & Markman, 1997).

As proposed by Carbonell (1986), analogical problem solving can be performed on different levels of abstraction. Transformational analogy is based on direct solution transfer, i.e. the solution to a previous problem is slightly altered in a transformation process to solve the new problem. The solution transfer process contains three basic processes. First, the initial partial matching process determines if two problems share similar aspects based on state information and operator sequences. Second, the sequence of actions from the retrieved solution is transferred to the new situation in an analogical mapping process. Third, the retrieved solution is copied and altered in a
heuristically guided manner to finally satisfy the given constraints (Carbonell, 1986).

Derivational analogy follows the same processes of analogical thinking. However, the accessed information is different, since it is based on the preservation and reconstruction of past reasoning processes. In the initial partial matching process, significant aspects are considered analogous if they share the same reasoning steps, i.e. the same issues are considered and equivalent decisions are made. Second, in transfer of earlier derivation, significant aspects of the reasoning process are recreated. Finally, the retrieved derivation is applied to the current situation “by ‘replaying’ the problem solving episode, checking for each step if the derivation is still applicable in the new problem solving context” (Schmid & Carbonell, 1999: 116). To summarize, in transformational analogy the solution is slightly altered to fit the new problem. In derivational analogy, in contrast, previous reasoning processes are applied and adapted to find a solution. As a result, new solutions are likely to be different from previous ones.

While Carbonell introduced derivational analogy as an artificial intelligence model, humans have also been shown to use this strategy (Schmid & Carbonell, 1999). Experiments showed that a high saliency of analogous elements fosters the use of transformational analogy (Schelhorn et al., 2007). Furthermore, participants were more successful in solving novel problems when they studied examples by using instructions fostering derivational analogy (Kleinbeck et al., 2001).

Verbal Reports in Studying Problem Solving

Measuring solution times is common in problem solving research (e.g. Funke & Sperling, 2006); the analysis is based on assumptions about the time which different processes take. However, solution times do not contribute information about cognitive processes at work during problem solving. A combination with other measures such as verbal or behavioral data can lead to more detailed insights.

The elicitation and analysis of verbal reports is an establish method to study the processes involved in human problem solving (e.g. Newell & Simon, 1972; Gick & Holyoak, 1980; Ericsson & Simon, 1984). Ericsson & Simon (1984) outline the different processes that can be accessed by different kinds of verbal reports. They argue that think-aloud protocols and retrospective reports do not modify cognitive processes; the task-oriented processes determine what information is heeded and verbalized. Most information in these reports is still held in short term memory. If information is retrieved from long-term memory in retrospective reports, some information might already be missing or erroneous, for instance with regard to difficulties encountered while solving the problem.

Most studies investigating verbal data along with problem solving focus on the content level, identifying the explicit statements elicited from participants during (or following) a problem solving process. Few studies analyze verbal reports on a deeper linguistic level, identifying more precisely how the different processes are described (Caron & Caron-Pargue, 1987; Wedman, Wedman & Folger, 1996), or which linguistic differences can be found between reports of successful and unsuccessful problem solvers (Roth, 1985).

In this study, the analysis of performance data is supplemented by a content and linguistic analysis of different kinds of verbal reports. By combining those measures we aim to

- identify the processes described in the verbal reports and match those to the different processes proposed for transformational and derivational analogy,
- explore how transformational and derivational analogy can be linguistically distinguished in verbal reports, and
- confirm that a high saliency of analogous elements fosters transformational analogy.

The Effect of High vs. Low Guidance on the Selected Analogue Transfer

Our experiment is based on a study by Schelhorn et al. (2007) in which the saliency of the correspondences between entities in base and target problem was varied. These authors used the following design to address the influence of saliency on the selected analogical strategy. First, in order to prime participants for analogical reasoning, they were given two example problems (‘The Fortress’ and ‘Radiation’, cf. Gick & Holyoak, 1980) and their solutions, where the second was solved analogously to the first. While those two problems were purely conceptual, the base and target tasks in the study by Schelhorn et al. concerned a type of path-finding problem called “Eulierian Trail” where a path needs to be found that visits every edge of the graph exactly once. After participants were presented with the base problem (Boat; see Schelhorn et al., 2007: Appendix), the solution was explained step-by-step (visually supplemented by a graph). Then participants were given the target problem (Birthday) and asked to find a solution. In the high guidance condition, the initial letters of the five objects that represented the edges of the graph were identical (cities in the base problem and peoples’ names in the target problem). This was not the case in the low guidance condition. After giving the solution, participants were asked to map objects from the base to those of the target. Mapping times were recorded. Participants then completed a Strategy Assessment Questionnaire (SAQ), which contained 16 statements that participants should agree or disagree with (see Schelhorn et al., 2007: Appendix). Five statements corresponded to the derivational strategy (e.g. The “boat” and the “birthday” problem seemed similar but I could not figure out how the solutions were related), and five corresponded to the transformational strategy (e.g. It was simple to use the “boat” solution to solve the “birthday” problem by replacing the names of the towns with the names of the people). The remaining statements were fillers. Finally, biographical information was collected.

Schelhorn et al. (2007) found that participants transferred knowledge from the base to the target problem even in the
The current study supplements the quantitative analysis of performance data by a qualitative analysis of verbal data. We expected protocols to show transformational and derivational strategies, reflecting the processes proposed by Carbonell (1986). In particular:

- Participants in the high guidance version were expected to state explicitly that they noticed the analogy.
- We assumed that participants using the transformational strategy would explicitly state that they used the base solution for solving the target problem. On a linguistic level, they might use explicit markers of correspondence, such as “the same as”, “similar”, and “analogous”.
- Reports by participants using derivational analogy should include descriptions of different stages of the problem solving process, such as visualizing the different connections (relationships) between the different points (people). Furthermore, unspecific terms (in general) and structuring devices, i.e. ordinal numbers and temporal connectors were expected to be more frequent.

The Eulerian Trail: Empirical Study

Hypotheses

The current study supplements the quantitative analysis of performance data by a qualitative analysis of verbal data. Since the derivation process takes longer than direct solution transfer, participants in the high guidance version should be faster. However, as participants may solve the problem while reading the task instructions, or take time to re-read parts of the instruction when facing difficulties, solution times as such did not seem to be an accurate measure. Mapping times seemed more informative since participants in the low guidance condition needed to map the entities from the base problem to those of the target problem in a separate step to solve the mapping task. Results showed that, as expected, mapping times were significantly longer in the low guidance condition. Furthermore, participants in the low guidance condition agreed with more SAQ questions that corresponded to the derivational strategy than participants in the high guidance condition, indicating that low correspondence hampers direct solution transfer.

In a comparable study, Schmid & Carbonell (1999) report similar performance results. Additionally, they briefly report how the two analogical strategies were expressed in think aloud protocols. In preparation for the analysis of verbal reports in our current study, we revisited the set of 14 think aloud protocols to identify linguistic markers of cognitive processes. This analysis revealed the following general structure in 10 of the protocols:

1. construct the graphs representing cities and locks,
2. connect cities satisfying the given constraints, and
3. check the solution during a final evaluation.

The final stage of evaluation was missing in the remaining four protocols. An analysis of the verbs occurring in the protocols revealed that participants were engaged in a number of mental activities: satisfying constraints (have to, need to), forming hypotheses (should, could), gaining insight (I see), planning (want to, going to), and recalling (seen before). In those data, the distribution of verbs (categorized following Halliday & Matthiessen, 1999) could to some extent be associated with use of derivational and transformational strategies. For instance, participants using derivational analogy used more verbs of “doing” (go, start) than participants using transformational analogy.

Design

In addition to replicating the study by Schelhorn et al. (2007) (original condition), we collected different kinds of verbal reports in two further conditions, yielding a 2x3 design (high vs. low guidance, original vs. planning vs. retrospective). In the planning condition, participants were asked to write down how they would solve the problem (planning protocol) after going through the example problem and viewing the target problem for the first time. Furthermore, they were asked to evaluate their plans after completing the mapping task. Participants in the retrospective condition were asked to write a report on how they solved the target problem, and subsequently to write an instruction for a friend to solve this problem. Since these verbal reports were collected at different times relative to the problem solving process, different kinds of information were gathered. We expected that planning protocols and retrospective reports would be most likely to include descriptions of the actual problem solving process. Planning protocols were expected to contain information on spontaneous transfer from base to target problem. Retrospective reports may further include memories of detours and fresh starts, and possibly information on the mapping task and meta-information on the study or the problem solving process. This kind of meta-information may also be reported in evaluation protocols. Instructions, on the other hand, would be highly structured and include generalized steps to solve the given problem.

Procedure

Participants were recruited by various means, e.g. by a call for participation on LinguistList.org and among students at the University of Bremen. As a consequence, the age range was very wide (22 to 73 years, mean 38.1 years). Participants were randomly assigned to conditions. Performance was analyzed with regard to solution and mapping correctness, mapping times, and the answers given in the SAQ. The elicited verbal reports were analyzed qualitatively with regard to their content, the overall structure, and linguistic markers such as verbs, nouns, structuring devices, and other keywords.

Results

69 participants (35 female and 34 male) took part in the web-based study. 33 of these were given the high guidance condition by the system, and 36 saw the low guidance
condition. We collected 20 planning protocols, 21 evaluations, 22 retrospective reports, and 21 instructions (evenly distributed between the two guidance conditions). **Performance and Strategy Assessment.** Solutions to the target problem did not differ significantly between the high (57.6%) and low guidance version (66.7%). However, the mapping task was solved significantly better in the high (90.9%) than the low guidance version (63.9%), $\chi^2 = 7.01; p < .01$. Also, mapping times in the high guidance condition ($n = 33; M = 76.9$ s; $SD = 64.5$) were significantly shorter ($W = 947, p < .001$) than in the low guidance condition ($n = 36; M = 217.0$ s; $SD = 206.7$).

The answers to the SAQ were analyzed using Mann-Whitney U tests to compare the amount of positively ranked questions belonging to the derivational strategy to those that belong to the transformational analogy. Participants with high guidance ($n = 33$) agreed with significantly more statements corresponding to transformational analogy than participants with low guidance ($W = 854.4, p < .001$). Statements that corresponded to derivational analogy were significantly more often confirmed by participants in the low guidance ($n = 35$) than by those in the high guidance condition ($W = 771.5, p < .01$) (Figure 1). One participant in the low guidance condition did not complete the SAQ.

![Figure 1: Mean number of agreed SAQ questions.](image)

Running a Kruskal-Wallis test no interaction was found between the verbal elicitation task and mapping times ($H = 2.8207, p = .244$). For this reason, and because the results of the performance measures confirmed the findings by Schelhorn et al. (2007), we assume that the elicited verbal data did not affect performance in any substantial way and can therefore contribute to the study of differences in derivational and transformational analogy.

**Verbal Data Analysis.** First we investigated the general structure of the collected 21 planning protocols. In 12 of these, participants stated noticing the analogy between base and target problem in the beginning. In 7 protocols this insight was followed by a description of the graph and a subsequent description of the problem solving process. 17 participants stated the solution at the end of the protocol. Similar structures were also found in the retrospective reports, which additionally contained meta-information such as remarks on the study design, the problem solving process, and background knowledge. 7 participants (2 high guidance, 5 low guidance) furthermore reported how they solved the mapping task. They reported different strategies, viz. redrawing the graph (4 cases), aligning the entities (once), or matching functions of entities (twice).

88% of all planning protocols in the high guidance condition contained statements of noticing the analogy, as compared to 40% in the low guidance condition. A similar trend emerged in the retrospective reports; 73% of the high guidance reports explicitly mentioned noticing the analogies, as compared to 27% in the low guidance version. With regard to success, it could be observed that those participants who reported noticing the analogy in their planning protocols succeeded in finding the solution three times more often than those who did not mention the analogy. In retrospective reports no such effect of noticing the analogy could be found. A closer analysis showed that some participants reported solving the example problem themselves; if they came up with a wrong solution there, noticing the analogy and transferring the solution to the target would result in a wrong solution.

A closer look at the nature of the descriptions of the analogies revealed a systematic difference. In the high guidance version they reflected abstract, general observations of the following kind: “When I read the birthday problem, I recognized that it was the exact same problem as the boat problem”. Here, no alignment of structures or entities is provided. Only protocols collected in the low guidance version contained more detailed representations such as: “I recognized right away that the messages were analogous to the locks.” Here, the entities of the target problem are matched to those of the base problem.

The descriptions of the problem solving processes could be divided into four subcategories. Descriptions of abstract, generalized steps were classified as general strategy. If the solution of the base problem was directly transferred to the target problem, this was called direct solution transfer. If the strategy (rather than the solution) was described as being transferred, this was categorized as direct strategy transfer. If instances of the solution process were specified, the description was classified as step-by-step. These categories showed different linguistic markers as illustrated by the following examples:

1. I worked counterclockwise and connected as many of the people along the edges as possible before working on the connections that cut across the middle of my shape. (general strategy)
2. I simply copied and pasted the solution from the third problem onto the forth. (direct solution transfer)
3. I solved it the same way. (direct strategy transfer)
4. Starting from S, I first connected the group of 3 persons that know each other (S, B, E), coming back to S. Then from S to R, and from R to the other group of 3 persons that know each other (R, M, B), coming back to R. Finally from R to E. (step-by-step)

In detail, the category general strategy included:
- general statements of the kind ‘find pattern/mapping’,
- the generalized strategy ‘go through the graph’, and
• the verbs ‘connect’ and ‘draw’ (6 occurrences as compared to 3 in step-by-step descriptions).
The two categories that described direct transfer both contained the markers ‘same’ and ‘again’ (6 times) and the verb ‘solve’ (5 occurrences). But the verb ‘copy’ was solely used in two protocols to describe direct solution transfer. And the nouns ‘strategy’ and ‘template’ only occurred in protocols displaying direct strategy transfer. Protocols in the category step-by-step contained
• first letters or names of entities,
• references to connections drawn by the participants (5 out of 7 cases), and
• a detailed description of the process of drawing the graph, listing the connections and finding a way to satisfy the task constraints (2 cases).

Structuring devices were most frequent in general strategy or step-by-step descriptions, viz. temporal connectors (13 compared to 3 occurrences in the two transfer categories) and the conjunction ‘and’ (27 compared to 2 occurrences).

26 out of 33 planning protocols and retrospective reports contained strategy descriptions. Of these, 15 were categorized as general strategy descriptions and 7 occurrences were found for each of the other three categories respectively. Step-by-step descriptions were most frequently used in the low guidance version (5 as compared to 2 times). Direct solution transfer was more often described in the high guidance version (6 as compared to 1 occurrence).

Instructions were found to be more structured (structuring devices were used in 50% of the instructions) and more general in describing the steps to be taken (12 out of 20 protocols). Those steps can be summarized as ‘draw the graph & find a pattern by satisfying the given constraints’. Eight instructions included references to task-specific entities (e.g. lock, messenger puzzle). One participant refused to write an instruction. 16 instructions contained an advice for a specific strategy. The same strategy categories as outlined before could be used for the analysis. A comparison between the strategy described in the instruction and (by the same participant) in the retrospective report revealed that people advised a more general strategy (6 cases) or the same strategy (9). No differences with regard to high or low guidance was found. 15 out of 21 evaluations stated if the planned strategy was used; this was mostly the case (10 of 15).

Discussion
We set out in our study to extend findings previously published by Schelhorn et al. (2007) concerning the use of transformational and derivational strategies in analogical problem solving. Our performance results successfully replicate the earlier findings in that participants given high guidance were more likely to use transformational analogy, while participants given low guidance could be associated with derivational analogy. The assumption that participants would have to map base and target entities in a separate step in order to solve the mapping task was confirmed by the descriptions found in 8 protocols.

The analysis of verbal data furthermore provides a range of insights about the cognitive processes involved in analogical problem solving. As a tendency, participants reported recognizing the analogy more often when it was highly salient. Equally unsurprisingly, those recognizing the analogy appeared more likely to succeed in giving the right solution. However, as the retrospective reports revealed, participants working their way through the example problem by themselves may give the wrong solution although they noticed the analogy. This observation might explain the low performance in solution correctness of the target problem. The distinctively better performance of participants in the high guidance version on the mapping task supports this view. This interesting possibility could not be detected by performance data alone.

Our qualitative analysis of strategy categories suggested that participants in the high guidance version described direct solution transfer more often than participants in the low guidance version, as opposed to more detailed step-by-step descriptions that were associated with low guidance. Together these tendencies support the idea that high guidance fosters a transformational strategy involving more superficial and less intricate cognitive processes.

A comparison of the processes identified in our verbal data with the processes hypothesized for transformational and derivational analogy by Carbonell (1986) reveals the following. For transformational analogy, which is associated with high guidance, the initial partial matching process is expressed in descriptions of noticing the analogy. Since the descriptions exhibit a very abstract level of representation, no conclusions can be drawn about the nature of the aspects that are considered analogous. Quite possibly, participants did not need to consider the matter in any more depth (leading to the lack of more detailed alignment descriptions), since a superficial transformation was sufficient. The analogical matching process in which knowledge from the base problem is transferred to the target problem is evident in the descriptions of graph representation. The following example is representative for the transfer of the sequence of actions: “making nodes for each person and drawing lines between acquaintances”. The process of alteration of the retrieved solution is straightforwardly expressed in descriptions of direct solution transfer.

In low guidance protocols that are associated with the use of derivational analogy, the initial step of matching reasoning processes of base and target problem is also verbally expressed by noticing the analogy. These descriptions show a more detailed representation (matching base and target entities), but do not contain information on individual reasoning steps. The transfer of significant aspects of the reasoning process and traces of replaying the problem solving episode can be observed in step-by-step descriptions.
Our analysis of verbal reports thus enabled the identification of different cognitive processes involved in analogical problem solving, along with linguistic markers, depending on the degree of guidance which led to the different problem solving strategies previously described as transformational and derivational (Carbonell, 1986). These findings illustrate that the analysis of verbal data contributes to a more detailed understanding of the processes at work during analogical problem solving.

Our elicitation of written data can be regarded as a first broad exploration of the kinds of verbalizations that might be expected along with complex analogical problem solving tasks. Quite typically for free production tasks and low participant numbers that allow for a more or less complete comprehension of the descriptions (rather than performing quantitative computations), the resulting numbers of occurrences of specific phenomena (as reported in this paper) were too small for statistical validation. Nevertheless, the distribution of contents and linguistic markings were both inspiring and suggestive in light of the theoretical background of this study, and thus open up some avenues for further research. In particular, we suggest the following:

- Of the four types of elicited verbalization the planning protocols seemed to be the most informative. Quite unexpectedly, participants seemed to already solve the task while writing up how they would do this, rendering the descriptions rather similar to think aloud data (elicited during, rather than before, problem solving). If this observation can be supported by further studies, it would open up interesting ways of collecting verbal data much more efficiently than possible with think aloud recording.

- The tendency for low-guidance participants to produce more detailed procedural descriptions of a derivational problem solving process calls for further exploration. Focusing on this particular aspect, a more controlled elicitation of a larger amount of verbal data should highlight how these matching processes develop over time, as well as the extent to which the two proposed analogical reasoning strategies (transformational and derivational) are systematically distinct. The linguistic markers identified in the present study can serve as a first indication of the ways in which language represents these distinctions.

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