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OPTICAL MODEL SOLUTIONS FOR PIONS IN NUCLEAR MATTER

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Abstract

Solutions for damped plane pion waves in homogeneous nuclear matter are deduced from optical potential parameters, taken from pion scattering and pionic atom data.

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Pionic modes and pion propagation in baryonic matter are of considerable interest concerning pion production in nuclei and heavy-ion collisions and more exotic phenomena like pion condensation in nuclear and neutron matter. Sound reviews of the pion spectrum and the pion self-energy involving the $\Delta$-isobar can be found, for example, in ref. 1 and 2. The purpose of this letter is to point out to which extent information about propagation and attenuation of the "pionic" branch of the pion-spectrum in nuclear matter (in contrast to the $\Delta$-isobar branch or the acoustical branch) can be deduced from optical model parameters, which are compiled from scattering and pionic atom data.

The (complex) pion momentum $k = k_1 + ik_2$ in the medium is related to the (also complex) optical potential $U_{\text{opt}}(k, \omega)$ by the expression

$$k^2 - k_0^2 = -2\omega U_{\text{opt}}(k, \omega)$$

where $k_0^2 = \omega^2 - 1$ is the free pion momentum. The usual parametrization of $U_{\text{opt}}$ for homogeneous nuclear matter of the density $\rho$ is given by

$$2\omega U_{\text{opt}} = -4\pi [p_1 b_0 \rho + p_2 B_0 \rho^2] - 4\pi [\frac{1}{p_1} c_0 \rho (1 + 4\pi g' c_0 \rho)^{-1} + \frac{1}{p_2} c_0 \rho^2] k^2$$

with $p_1 = 1 + \frac{\omega}{M}$, $p_2 = 1 + \frac{\omega}{2M}$, nucleon mass $M = 6.7$.

\textsuperscript{+}We set $m_\pi = 1$ in all expressions.
The Landau Fermi-liquid parameter $g'$ represents short-range
correlations, whereas $b_0$ and $B_0$ are the s-wave, and $c_0$ and $C_0$
are the p-wave contributions to $U_{\text{opt}}$. Equation (2) can be
contracted to

$$2\omega U_{\text{opt}}(k, \omega) = -2\omega [\text{Re}U + \text{Re}Uk^2 + i\text{Im}U + \text{Im}Uk^2]$$

which is easily solved by

$$k_2 = [-x + (\frac{x^2}{4} + y^2)^{1/2}]^{1/2} \quad \text{with} \quad x_1 = -2\omega \text{Re}U + k_0^2 - 4\omega^2 \text{Im}U \text{Im}U(1 + 2\omega \text{Re}U)^{-1}$$

$$y_1 = 1 + 2\omega \text{Re}U + 4\omega^2(\text{Im}U)^2 (1 + 2\omega \text{Re}U)^{-1}$$

$$x = \frac{x_1}{y_1} = k_1^2 - k_2^2$$

$$y = (-\omega \text{Im}U - \omega \text{Im}Ux)(1 + 2\omega \text{Re}U)^{-1} = k_1 k_2$$

The imaginary part $k_2$ is connected with the pion mean free path

$$\lambda = \frac{1}{2k_2} = \frac{k_1}{2\omega \text{Im}U_{\text{opt}}}$$

Solutions of eq. (1) exist for all values of $\text{Re}U$, $\text{Re}U$, $\text{Im}U$, $\text{Im}U$. The
so-called Kisslinger-catastrophe $^2$ occurs for $1 + 2\omega \text{Re}U \rightarrow 0$ with
$\text{Im}U = \text{Im}U = 0$. This does not happen in reality, since $\text{Im}U$
is substantially different from zero for the pionic branch because of
$\text{Im}C_0$ and $\text{Im}B_0$. These arise from pion annihilation, which is still
active at threshold ($\omega = m_\pi$). For the far off-shell, undamped
($k_2 = 0$) branch with $\omega = 0$ (or $\omega = \mu_n - \mu_p$ in neutron matter), the
imaginary parts $\text{Im}U$ and $\text{Im}U$ are zero. The optical potential
parameters, extracted from experiment, however, do not serve in this case, and \( \text{Re} U \) has to be replaced by the \( k \)-dependent self-energy \( \Pi(k, \omega) \), which prevents the Kisslinger-catastrophe.

The appearance of a complex momentum \( k \) in \( U_{\text{opt}} \) has no obvious interpretation in terms of many-body diagrams, since typical particle-hole and \( \Delta \)-hole diagrams (fig. 1) depend on \( k_2 \) independently from the real part \( k_1 \). A nucleon-hole or \( \Delta \)-hole excitation with a complex pion momentum and consequently complex nucleon (or \( \Delta \)-) energies has a dubious meaning. The imaginary part \( k_2 \) does not describe the coupling of a pion to a NN- or \( \Delta N \)-vertex, but merely represents a reduction of the pion flux, which results from the imaginary part of all diagrams describing particle-hole excitations induced by pions. In view of this, the use of only the real part \( k_1 \) in \( U_{\text{opt}} \) appears to be more appropriate. With the abbreviations

\[
A = 1 + 2 \omega \text{Re} U - \omega^2 \text{Im} U^2
\]

\[
B = 2 \omega \text{Re} U - k_0^2 - 2 \omega^2 \text{Im} U \text{Im} U
\]

\[
C = \omega^2 \text{Im} U^2
\]

the solution of eq. (1) with \( U_{\text{opt}}(k_1, \omega) \) takes the form

\[
k_1^2 = \frac{-B}{2A} \pm \sqrt{\left[ \frac{B}{2A} \right]^2 + \frac{C}{A}}.
\]
Obviously, $k_1$ is not real for all possible values of the components of $U_{opt}$, since $A$ can be negative. This is more than a remote mathematical possibility. The parameters of the optical model, given in Table 1, are an interpolation between the results from inelastic pion scattering and pionic atom data$^{3,4}$. Above $k_0 \sim 2.4$, sufficient data are missing. The Fermi liquid parameter $g' = 1/3$ is probably too small; however, it has to be kept for consistency with the other parameters. The solutions of eq. (1) with $U_{opt}(k,\omega)$ and $U_{opt}(k_1,\omega)$ are also shown. For several values of $k_0$ with the choice $U_{opt}(k_1,\omega)$, no solutions exist for a real $k_1$. The value of $Rec_0 \sim 0.4$ may be reckoned as slightly too large, but confining the $Rec_0$ below $\sim 0.25$ does not save this situation either, since then the same problem emerges at densities slightly larger than $\rho_0$. Also, the Kisslinger-catastrophe occurs at $A = 0$, even if $\text{Im}U \neq 0$. This fact and the occasional nonexistence of solutions refute the ansatz $U_{opt}(k_1,\omega)$ as insufficient.

The result for the mean free path $\lambda$ for a complex $k$ in $U_{opt}(k,\omega)$ has a strange character: it is rather flat between $1 \lesssim k_0 \lesssim 2.4$.

From the estimate

$$\lambda \sim \frac{1}{\rho_0^{\text{tot}} \pi N}$$

(7)

one should expect a pronounced minimum at resonance. Also, for $\rho = 2\rho_0$ (not shown in Table 1) the mean free path is larger than at $\rho = \rho_0$, which is rather unreasonable. Furthermore, a $\lambda$ between 0.6 and 1 fm seems extraordinarily small. One could adopt the view that, since eq. (3) is a phenomenological ansatz, it is unnecessary to have a clear identification of the $p$-wave part of $U_{opt}$ (with a
complex k) with a specific many-body diagram. But even then, the above results for $\lambda$ with this ansatz prove unsatisfactory.

A different approach would be the application of the free pion momentum $k_0$ in $U_{opt}(k_0, \omega)$ instead of $k$, which provides a different outcome. A distinct minimum of the mean free path at the resonance is produced, fitting well to the estimate eq. (7) with an isospin-averaged $\sigma_{\pi N}^{tot} = 140$ mb; the variation of $\lambda$ with $k_0$ seems more acceptable; for $\rho = 2\rho_0$ it is smaller throughout than for $\rho = \rho_0$; and the result is (in the isobar-dominated region) in good agreement with Ginocchio's results. Also, the pion spectrum in low energy heavy-ion collisions is highly nonthermal, which advocates a rather large mean free path of the pions emanating from the reaction zone, which--for the central collisions with the highest pion productivity--lies inside the fireball. The solution with $U_{opt}(k, \omega)$ gives a short mean free path ($\sim 1$ fm), which would result in a thermalization.

We conclude that a "self-consistent" solution of the pion momentum $k$ in the medium turns out to be unsatisfactory in comparison with the approximation $k = k_0$ in the optical potential $U_{opt}$, at least if the parameters of $U_{opt}$ are deduced from experimental data. The reason for this is not entirely clear; probably the abilities of the optical model are overstressed. One argument could be that pion scattering and pionic atom data probe too much of the nuclear surface and do not yield sufficient information about the interior which is more like nuclear matter. However, calculations of $\lambda$--with the parameters $c_0, C_0,$ and $B_0$ taken from calculations, not from experiment--have been performed for nuclear matter. The results for the mean free path $\lambda$ agree rather well with those for the above
empirical values of $c_0$, $B_0$ and $C_0$, indicating that optical model parameters, abstracted from experiments with finite nuclei, provide already a good description of pion absorption in nuclear matter, at least for pion momenta up to the $\Delta$-resonance.

Finally, we would like to mention that the often-adopted expression $\lambda = \frac{1}{v_{\text{free}}(2 \text{Im}U_{\text{opt}})}$, where $v_{\text{free}}$ is the velocity of the free pion, can be erroneous. It clearly gives wrong results for $k_0 = 0$ but is also a bad approximation for larger $k_0$, except in the resonance (see Fig. 2a). One could think of the group velocity $v_{\text{gr}}$ in the medium instead of $v_{\text{free}}$ as a more suitable quantity to describe the propagation of a wave packet in the optical potential. Unfortunately there is no unique way to determine the group velocity for strong absorption and dispersion.

The expression $\frac{d\omega}{dk_1} = v_{\text{gr}}$ does not work well, since the $\omega(k_1)$-curve (Fig. 2b) has a backbending at the resonance. Hence, $\frac{d\omega}{dk_1} \rightarrow \infty$ at the turning point, reflecting the breakdown of the Taylor-expansion of $\omega(k_1)$. Apart from this, the result for $\lambda = \frac{1}{dk_1} (2 \text{Im}U_{\text{opt}})^{-1}$ for momenta below the resonance is even less suitable than that for $v_{\text{free}}$ (Fig. 2a). The only proper expression is $\lambda = k_1 (2\omega \text{Im}U_{\text{opt}})^{-1}$.

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References

Figure Captions

Fig. 1. Typical nucleon-hole and Δ-hole excitations by a pion of momentum k and energy ω.

Fig. 2a. The mean free path λ for λ = k₁(2ωImUₜₜ)⁻¹ (full curve), λ = v_free(2ImUₜₜ)⁻¹ (dashed curve), and λ = dω/dk₁(2ImUₜₜ)⁻¹ (dotted curve) as a function of the free pion momentum k₀; Uₜₜ = Uₜₜ(k₀, ω)

Fig. 2b. The energy ω as a function of the real part k₁ of the pion momentum in the medium for Uₜₜ = Uₜₜ(k₀, ω)
Table 1. The optical model parameters from ref. 3 and 4 with \( g' = 1/3 \), together with the solutions for \( U_{\text{opt}}(k, \omega) \); \( U_{\text{opt}}(k_1, \omega) \) and \( U_{\text{opt}}(k_0, \omega) \) in eq. (1). The density is \( \rho = \rho_0 = 0.5 \, \text{m}^3 \).

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<th>( k_0 ) [m]</th>
<th>( b_0 ) [m(^{-1})]</th>
<th>( c_0 ) [m(^{-3})]</th>
<th>( B_0 ) [m(^{-4})]</th>
<th>( C_0 ) [m(^{-6})]</th>
<th>( U_{\text{opt}}(k, \omega) )</th>
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