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Technical Report of the Betatron Design Study

GENERATION OF MOMENTUM SPREAD WITH A CARBON GRATING*

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August 1981

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# Generation of Momentum Spread with a Carbon Grating

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**Abstract:**

Momentum spread is produced in a relativistic electron beam by passing it through a carbon grating. At high power levels the beam radius must be kept large enough to avoid damage to the grating by heating. This requirement competes with the desire to keep emittance growth low.
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Introduction

The negative mass instability is suppressed by sufficiently large momentum spread in the circulating beam. For electrons with \( I_b = 10 \text{ kA} \) and \( E_0 = 50 \text{ Mev} \) the required spread has been estimated to be \( \Delta P/P_0 = 7.2 \) percent (full width - flat distribution), which considerably exceeds the intrinsic width expected from an induction linac injector. A simple method for introducing this spread is to pass the pulse through a grating prior to injection into the betatron (see figure), however, this produces a net energy loss, current loss and an emittance increase. There is also a possibility of breaking or melting the grating by the sudden deposition of heat.

The problem of heat deposition is reduced by increasing the beam's radius at the grating, but the emittance produced by scattering is thereby increased. It is found that there is a narrow window of beam radii within which these effects are tolerable for the contemplated injection parameters. We assume the grating is made of carbon (graphite), which is sturdy up to about \( 3000^\circ \text{C} \). Sequential pulses of a burst may pass through the grating at different transverse positions, so we only need to examine the effect of a single pulse.

It appears that there will be a severe problem of spallation if a single thick (- .5 cm) grating is used, but this may be overcome with a more
complex design -- e.g., a series of thin gratings or construction from many separated fibers. The estimates given here are nearly independent of design details.

**Thickness and Energy Loss**

The stopping power and density of carbon are $E' = 1.90 \text{ Mev cm}^2/\text{gm}$ and $\rho = 2.25 \text{ gm/cm}^3$. To achieve a $\pm 3.6$ percent spread we use thickness

$$t_1 = \frac{3.6 \text{ Mev}}{E' \rho} = .84 \text{ cm},$$

and to hold the grating together we insert an extra thickness

$$t_0 = \frac{t_1}{20} = .042 \text{ cm}.$$

Energy and momentum are essentially equivalent since the electrons are highly relativistic, so the resulting spread is flat with

$$46.2 \text{ Mev} \leq E \leq 49.8 \text{ Mev},$$

Mean Energy $<E> = 48.0 \text{ Mev},$

Mean thickness $<t> = \frac{(t_1 + 2t_0)}{2} = .46 \text{ cm}$

**Bremsstrahlung and Current Loss**

Scattering by carbon nuclei causes energy loss in the form of gamma rays. However, the dominant effect is loss of current because a beam particle which loses more than $\sim 5 \text{ Mev}$ will be lost in transport or will not be accepted by the betatron. To see this we note that the probability per electron for the emission of a photon of energy $\epsilon$ in the interval $d\epsilon$ is
Where $\lambda_R = 23.5$ cm is the radiation length of 50 Mev electrons in carbon. This formula is valid for $\varepsilon$ greater than a few hundred ev up to $E_0$, and the thin target distribution can be assumed since $<t> < \lambda_R$. Mean energy loss to photons is

$$- <\Delta E> \text{ rad} \approx \int_0^{E_0} \frac{d\varepsilon}{\varepsilon} \frac{<t> \varepsilon}{\lambda_R}$$

$$= \frac{<t> E_0}{\lambda_R} = \frac{(0.46)(50)}{(23.5)} = 0.98 \text{ Mev}$$

Since the energy loss spectrum [$\varepsilon P(\varepsilon)$] is independent of $\varepsilon$, 90 percent goes to photons with $\varepsilon > 5$ Mev. The electrons which suffer such a loss do not reach the betatron so we have current loss

$$-\frac{\Delta I}{I} \text{ rad} = 5 \int \frac{d\varepsilon}{\varepsilon} \frac{<t>}{\lambda_R} = \frac{<t>}{\lambda_R} \ln (10)$$

$$= \frac{(0.46) \ln (10)}{(23.5)} = 0.045$$

This is a marginally significant value. The mean energy loss (to radiation) of the accepted particles is a negligible 0.098 Mev. 

Temperature Rise

The specific heat of carbon rises from a value of 0.712 J/gm-°C at 25° C
to 2.08 J/gm-°C for T ~ 1500°C. We use the mean value $C_v = 1.40$
J/gm-°C. Then the temperature rise produced by a pulse of length $\tau_p$ is

$$\Delta T = \frac{I_b \tau_p E^\prime}{\pi a^2 C_v},$$

where (a) is the beam edge radius. Since the current density may be peaked
on axis we allow a maximum increment $\Delta T_{\text{max}} = 1500°C$ [computed from this
formula] instead of the actual limit of 3000°C. We therefore require

$$a > \left( \frac{I_b \tau_p E^\prime}{\pi C_v (\Delta T)_{\text{max}}} \right)^{1/2}$$

$$= \left( \frac{10^4 \times 60 \times 10^{-9} \times 1.9 \times 10^6}{\pi \times 1.40 \times 1500} \right)^{1/2} = .416 \text{ cm}$$

The pulse is carefully focused down to this radius on the grating.

**Emittance Increase**

The pulse radius is made as small as possible to prevent an
unacceptable increase of emittance by scattering. Factors of two are
important here, so we must be careful with definitions. The usual l-d
emittance is

$$\pi Q = \pi x_{\text{max}} (dx/ds)_{\text{max}}$$

On passing through the grating the mean squared angle of the electron
trajectories (total x-y projections) is increased by
\[ \Delta \langle \Theta^2 \rangle = \Delta \langle (dx/ds)^2 + (dy/ds)^2 \rangle = \]

\[ \sum \frac{16 \pi l_{1/2}^2}{\rho_0^2} = \frac{8\pi \rho <t> Z(Z+1)re^2}{M_b^4 \gamma_0^2 \ln \left( \frac{\Theta_{\text{max}}}{\Theta_{\text{min}}} \right)} \]

where \( Z = 6 \) and \( M = 12.01 \) amu are the nuclear charge and mass, \( r_e = 2.82 \times 10^{-13} \) cm is the classical electron radius, and \( (\beta, \gamma) \) are the relativistic factors. The logarithmic factor is evaluated using the thin target formula:

\[ \left( \frac{\Theta_{\text{max}}}{\Theta_{\text{min}}} \right) = \frac{Z^2/3 <t>/M)^{1/2} 2 \pi}{\sqrt{\pi} m \beta c} = 103.4 \]

for the present case. We find

\[ \Delta \langle \Theta^2 \rangle = 2.08 \times 10^{-3} \text{(rad)}^2 \]

The relations between rms quantities and maximal \( (x, dx/ds) \) projections for a flat profile are

\[ x_{\text{max}}^2 = a^2 = 2 <x^2 + y^2>, \]

\[ (dx/ds)_\text{max}^2 = 2 \Theta^2 \]

The increment of \( Q^2 \) is therefore
\[(\Delta Q^2)_{\text{scat}} = 2a^2 \Delta \phi^2\]
\[= 2 \times (0.416)^2 \times (2.08 \times 10^{-13}) = 0.720 \times 10^{-3} \ (r\text{-cm})^2 \].

This (increment)^2 is to be added to the value Q^2 has before reaching the grating.

The induction linac ETA produces normalized emittance
\[\pi Q_N = \pi \beta \lambda Q = (0.5) \pi \ r\text{-cm} \]
at 5 Mev, and this value will presumably be preserved in ATA up to 50 Mev. We thus expect

\[\text{initial } Q_N^2 \approx 0.25 \ (r\text{-cm})^2, \]

which is smaller by more than two orders of magnitude than required. Scattering produces the increment

\[\beta_0^2 \gamma_0^2 (\Delta Q^2)_{\text{scat}} \ (98.8)^2 (0.720 \times 10^{-3}) = 7.027 \ (r\text{-cm})^2. \]

This increment dominates over the initial value, but it is still somewhat less than the allowable value of 36 (r\text{-cm})^2. This pulse radius could thus be increased by a factor of up to two at the grating if necessary.

**Pinched Radius**

It is of interest to calculate here the achievable pinched radius which is consistent with the computed
\[ Q_N^2 = (\text{Initial value})^2 + (\text{scattered part})^2 \]
\[ = 0.25 + 7.027 = 7.277 \text{ (r-cm)}^2 \]

The Bennett pinch condition for equilibrium between the azimuthal field and transverse thermal pressure is

\[ \langle \varphi^2 \rangle = \frac{I}{178y kA}. \]

We have

\[ a = \frac{Q}{\sqrt{2\langle \varphi^2 \rangle}} = \frac{Q_N}{8y} \frac{178y kA}{\sqrt{21}} \]
\[ = (0.250 \text{cm}) \left[ \left( \frac{Q_N}{7.277} \right) \left( \frac{98.9}{8y} \right) \left( \frac{10kA}{I} \right) \right]^{1/2} \]

A further ten-fold increase of energy would therefore allow sub-millimeter radii.
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