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THE ELLIPTICAL SHAPE OF THE COMA CLUSTER

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Received

ABSTRACT

The elliptical shape of the Coma cluster is examined quantitatively. The degree of ellipticity is high and depends to some extent on the radial distance of the sample from the Coma center as well as on the brightness of the sample. The elliptical shape does not appear to be caused by rotation; other possible causes are briefly discussed.

Running title: Coma cluster ellipticity

Subject headings: Galaxies: Clusters of
The ellipticity of the Coma cluster of galaxies is obvious at a glance. It has often been remarked upon (Abell 1962, 1975; Rood et al. 1972; Bahcall 1973; Gregory and Tifft 1976a, b), but most investigations have, in fact, treated the cluster as spherically symmetric. Angular variations in density are worth studying, however, because they affect the dynamics of the cluster and also because variations in the mass density of luminous matter might occasion corresponding variations in the emissivity of the intergalactic gas.

In the present paper we shall examine the ellipticity of the Coma cluster quantitatively. Is it statistically significant? Does the ellipticity consist merely of an elongated clumping in the central regions, or does it extend to all radial distances? Is it confined to the bright galaxies that catch our eye, or do the fainter galaxies also show an elliptical distribution? Finally, how can we reconcile the elliptical shape of the Coma cluster with its lack of rotation?
I. TESTS FOR ELLIPTICITY

A simple measure of ellipticity has been described by Treanor (1958). We divide our sample of \( n \) galaxies into 12 sectors of equal angular size, and fit the number of galaxies in sector \( i \) to the formula

\[
N_i = \bar{N} \left( 1 + A \cos 2\theta_i + B \sin 2\theta_i \right),
\]

where \( \theta_i \) is the position angle bisecting sector \( i \), and \( \bar{N} \) is the average number of galaxies per sector.

The least-squares solution for the coefficients \( A \) and \( B \) is given by

\[
A = 3(x_{15} - x_{75})/12n
\]

\[
B = (x_{15} + 2x_{45} - x_{75})/12n
\]

where

\[
\begin{align*}
x_{15} &= n_{15} - n_{105} + n_{195} - n_{285} \\
x_{45} &= n_{45} - n_{135} + n_{225} - n_{315} \\
x_{75} &= n_{75} - n_{165} + n_{255} - n_{345}
\end{align*}
\]

(A full derivation is found in Treanor 1958.)

\( A \) and \( B \) are statistically independent, and we may estimate their relative errors, \( E_a \) and \( E_b \). If we estimate the error in the number of galaxies per sector as \( \sqrt{N}/2 = (n/12)^{1/2} \), it is easily found that

\[
E_a = E_b = (2/n)^{1/2}
\]

Note that the errors are independent and equal.
Eq. (1) can obviously be written in the alternative form

\[ n_1 = \bar{n} C \cos \left[ 2(\theta_1 - \phi) \right] \quad , \]

where

\[ C = \left( A^2 + B^2 \right)^{1/2} \]

\[ \phi = \frac{1}{2} \tan^{-1} \left( B/A \right) \quad . \]

The ratio of \( C \) to \( E_a (= E_b = E) \) gives an indication of the statistical significance of the result. We will use all of these quantities in studying samples of the Coma cluster.

Finally, we should relate sector counts to ellipticity, a quantity that is normally defined by the shapes of equal-density contours. The reason for using sector counts, of course, is that they can be profitably analyzed even when the numbers are too ragged to allow clear contour lines to be drawn. Anticipating, however, that our results will still have sizable statistical uncertainties, we will use approximations that include ellipticity only to the first order.

The polar equation of an ellipse is

\[ r = u/(1 - c \cos 2\theta)^{1/2} \quad , \]

where \( u \) is a size parameter. Clearly \( r = u \) when \( \theta = 45^\circ \), and the semi-major and semi-minor axes are respectively

\[ a = r/(1 - c)^{1/2} \quad , \quad b = r/(1 + c)^{1/2} \quad . \]
To first order in $\epsilon$, the ellipticity $\epsilon = 1 - b/a$ is equal to $c$. It is also easily found from a Taylor expansion that to first order the azimuthal variation of density in a ring at constant $r$ is given by

$$n = \bar{n} \left[ 1 - \frac{1}{2} \left( \frac{d \ln n}{d \ln r} \right) c \cos 2\theta \right].$$  \hspace{1cm} (9)

The other quantity for which we will need an expression is the total number, in a sector, from the origin out to radius $r$. After a little manipulation we find that the azimuthal variation of this quantity is given by

$$N = \bar{N} \left[ 1 + \left( 1 - r^2 \bar{n}/2\bar{N} \right) c \cos 2\theta \right].$$  \hspace{1cm} (10)

For the radial variation of $\bar{n}$, Rood et al. (1972) noted that the surface density of galaxies in the Coma cluster is reasonably well approximated by

$$\bar{n} = \bar{n}_0/[1 + (r/r_c)^2],$$  \hspace{1cm} (11)

with $r_c = 6^{1/4}$. Noting that the radii that we shall study all have $r^2 > r_c^2$, we derive the approximations

$$- \frac{1}{2} \frac{d \ln \bar{n}}{d \ln r} = 1$$  \hspace{1cm} (12)

$$1 - r^2 \bar{n}/2\bar{N} = 1 - 1/[2 \ln (r/r_c)].$$  \hspace{1cm} (13)

These, with Eqs. (9) and (10), allow us to determine ellipticities from azimuthal variations in numbers. These correction factors are labeled $F$ in Table 2 (below); and we shall use primes to denote corrected values.
of A, B, and C, which should approximate the actual ellipticities.

II. SAMPLES OF THE COMA CLUSTER

For the present study we have been fortunate to examine three independently gathered samples of galaxies in the Coma cluster (Table 1). We formed an additional sample from the intersection of two of these samples, to allow study of luminosity distribution. The Abell (A) sample (shown as a scatter plot in Figure 1) contains all galaxies to Abell's visual magnitude ($m_A$) 18.0, within 75° of the cluster center; it will be referred to as the total sample. The Gregory (G) and Abell-intersect-Gregory (AG) samples will be termed "members" samples. The membership criterion is that the radial velocity of the individual galaxies, given by Gregory (1975) and by Gregory and Tifft (1976a), should lie between 4400 and 9300 km/sec. While there is always a non-zero probability that field objects might satisfy this criterion (Rood 1975), these limits are usually accepted as defining membership for galaxies within 3° of the center. In any case, possible inclusion of a non-member will have little effect on our results. Figure 2 is a scatter plot of the inner part of the G sample; our AG sample consists of the galaxies inside the 75° circle.

Additional tests were made to see if the ellipticity depended upon either magnitude or radial position. Galaxies were divided into groups of "bright" and "faint" magnitudes, the dividing magnitude and the magnitude system varying from sample to sample. The radial position dependence was tested by dividing the sample into the radial groups specified in Table 2. The largest sample was also divided into classes
that tested simultaneously both magnitude and radial position. In the case of sample A, we subtracted appropriate backgrounds (Rood et al. 1972). This increases the relative strength of the ellipticity in this sample.

We also examined the distribution in total luminosity as a function of sector position angle. Luminosity \( L \) from the Gregory-sample galaxies was taken as proportional to \( 10^{-0.4m_z} \) (using Zwicky's magnitudes \( m_z \)), while luminosity in the AB sample galaxies was taken as proportional to \( 10^{-0.4m_a} \). Since the two magnitude systems are not identical (see Abell 1977, pp. 322-323, for the relationship), the results are not absolutely comparable; but each test does give an indication of the relative variations in total luminosity from sector to sector.

Finally, we tested the radial velocities collected by Gregory (1975) to see if there is any systematic variation in average velocity with position angle.

III. RESULTS

The results of our investigation are shown in Table 2. It can be seen that galaxies in the Coma cluster show a strongly elliptical distribution. In Figure 3a-c, we show the ellipticity coefficients and errors for the most important of our results. On these graphs, the axes represent the \( A' \) and \( B' \) coefficients, and each ellipticity is plotted as a vector with a one-sigma error circle. For reference the position angle of NGC 4889 is also indicated. Note that \( \tan^{-1}(B'/A') \) is twice the position angle of the major axis of a sample and that the ellipticity is \( C' = (A'^2 + B'^2)^{1/2} \).
In some of the samples the number of galaxies per sector ($n_1$) varies as far as twice or half the mean value. The ellipticity test that we have described confirms that these variations are systematic.

Members-only samples, bright samples, and samples within 55' of the center showed the greatest ellipticity, with the greatest statistical significance. However, the ellipticity was still $>15\%$ in the weakest samples. Since the relative error is proportional to $n^{-1/2}$, the relative strength of the ellipticity seen in the members-only samples is somewhat offset by the increased uncertainty due to sampling errors, but the signal-to-noise ratio in these samples is still larger than in the complete but impure samples.

Certain of our results bear further amplification. The relatively low ellipticity for the entire A sample arises in part from the variation in the direction of principal axis from one radial group to another. This can be explained by noting two phenomena. First, NGC 4889 and 4874 lie along an axis at higher position angle than the major axis for any of our samples, and subclustering around these two giants influences the results in the inner ring. Second, the ellipticity of the 55' - 75' ring is oriented quite differently. These effects can be seen in Figure 3c.

To investigate the unusual results for the 55' - 75' ring, we examined plots of the positions of the Abell galaxies, with different magnitude ranges shown in different plots. In a sample complete to $m_a = 17.0$ (Fig. 4a) the central ellipse of the cluster is seen clearly against a field. Of the 519 objects in this sample, the adopted background densities suggest that approximately 85% should be cluster members.
In the fainter remainder of the sample (Figs. 4b, 4c) the shape of the cluster becomes more difficult to discern; furthermore, although 70% of the 858 objects can be expected to be members, the outer parts of the picture should be dominated by field, and the fainter galaxies are indeed clumped. Zwicky and Herzog's (1963) chart of the field including Coma, part of which we reproduce in Figure 5, shows several agglomerations that they call background clusters. The five that appear most prominently in our field are marked in Figure 4c. All of these are in the minor-axis quadrants of the 55' - 75' ring, with Zw 40 impinging also on the first quadrant along the major axis. The second quadrant, virtually empty of bright objects, is well-populated with fainter galaxies.

If the galaxies in the magnitude interval \([17.1, 17.8]\) (Fig. 4b) are analyzed for ellipticity, the result agrees closely with that found for the interval \([11.6, 17.0]\), both over the entire 75' radius and within a radius of 55'. The principal contribution to the anomalous orientation in the 55' - 75' ring is seen to come from the faintest galaxies \([17.9, 18.0]\), see Fig. 4c), which, by inspection, make up the largest part of the Zwicky agglomerations, and also of the field, in the second quadrant. Since the shift in principal axis is also seen weakly in the brightest Abell galaxies and in the members-only sample, we cannot rule out the possibility that some of this anomaly may be in the structure of the cluster itself; but the evidence points strongly to the clumpiness of the background as the source of the strange behavior of the ellipticity in the 55' - 75' ring. It is well known that the general field of galaxies is clumpy. A future paper will treat the effect of the background more fully.
Because of the behavior of the ellipticity in the 55'-75' shell we performed a more general test for clumpiness, examining the hypothesis that the ellipticity, a 2θ angular term, was only one component of a clumpy angular distribution of galaxies. We Fourier-analyzed the distribution of galaxies along the interval of position angle (0, 2π), using the sector counts. In nearly every case the 2θ term contained much more power than any of the four higher-order terms, most of which were not statistically significant anyway. Thus we conclude that ellipticity is the dominant structural feature of Coma, after the central concentration itself.

Another feature of our results is the apparent strength of the ellipticity in the brighter galaxies relative to the fainter ones. (This is most easily seen in the high ellipticities of the G sample.) Even if the background in the faintest set was underestimated by a factor of two, a very unlikely occurrence, the ellipticity of the fainter galaxies is still less than that in the brightest set. Since the weakening of the ellipticity at fainter magnitude shows up in all zones of Coma, it is unlikely that clumping of faint background galaxies, as noted above, could smear out the ellipticity present in the cluster itself for each radial set.

The distribution of luminosity, L, shows the same strong tendency towards ellipticity as do the counts of galaxies. All luminosity tests gave principal axes close to those of the corresponding tests of counts. That the ellipticity in luminosity distribution was even stronger than that of the counts is explained in part by the contributions of NGC 4889
and 4874, which lie close to the principal axis. Also we note that the brighter galaxies show stronger ellipticity than the fainter ones. On the other hand, the statistical uncertainties are larger in luminosity tests than in tests employing only counts, because the intrinsic relative Poisson error in total luminosity, $(IL^2)^{1/2}/L$ (King 1966) is usually greater than the Poisson error in counts alone. Results for the A and AG samples are shown in Figure 3a and given in Table 2.

Tests for elliptical variations in average radial velocity within a sector proved negative.

IV. DISCUSSION

Confirmation of the elliptic shape of clusters of galaxies is not surprising. One cause of ellipticity could be rotation of the whole cluster. Rood et al. (1972) tested a sample of Coma galaxies for systematic rotation by examining radial velocities along the directions of the major and minor axes; no effect of any statistical significance was found. Gregory (1975) tested a larger sample of Coma and again found no strong indication of rotation as a systematic tendency in the line-of-sight velocity across the cluster. Tifft and Gregory (1976) went farther from the center of Coma, discussing velocities of over 200 galaxies out to a radial distance of 6°. Their results suggest -- with marginal significance -- a systematic rotation, but around the major rather than the minor axis of the elliptical distribution that we have been discussing. We merely note their result in passing. First, their rotation occurs in a zone far outside the region with whose ellipticity we are concerned. Second, it involves galaxies so far from the center that their crossing
time is greater than the Hubble time, so that such a "rotation" can have little to do with the equilibrium shape of the cluster. In any case, we found no distribution that is elliptical in the sense that would fit the rotation described by Tifft and Gregory, except in the anomalous zone from 55 to 75 minutes; but this is not a zone in which they found rotation, and its ellipticity is in a direction different from that of both the interior and the exterior zones.

If the Coma cluster does not rotate, what is responsible for the elliptical shape? Gregory (1975) noted that the marginal rotation that he observed implied a rotational kinetic energy for the cluster that would be insignificant compared to the kinetic energy seen in the radial velocities. By contrast, King (1961) argued that, at least for globular clusters with ellipticities of 10 to 20%, the rotational kinetic energy should approach one-third of the total internal kinetic energy. Such a rotation in Coma seems completely ruled out by observation, so we conclude that the observed ellipticity is not caused by rotation.

However, there are studies that indicate that flattening or other asymmetries may occur in the absence of significant rotation. Aarseth (1969), incidentally to his study of rotating clusters, looked at statistical fluctuations. He performed a 100-body simulation of a non-rotating cluster to check for possible flattening. The RMS deviation about circular symmetry corresponded to 7% flattening. Aarseth suggested that the size of this effect was due to the small number of objects in his study. Since the Coma cluster has many more objects than Aarseth's model and since its flattening is several times greater, it seems very
unlikely that what we have observed is such a fluctuation about an otherwise symmetrical distribution.

Peebles (1970) modeled 300 objects in order to study the evolution of the Coma cluster with time. Some of his results (Fig. 1c in his paper) show asymmetrical features that would appear as ellipticity in our calculations. Since his figure showing ellipticity refers to a time when the cluster has completed only one "bounce", this asymmetry is probably related to a temporary stage of early evolution. The number of objects is small, however, and purely statistical fluctuations may play some role.

White (1976a, b) performed a simulation with over 700 objects, using a mass spectrum similar to that in Coma. He found that asymmetric distributions and subclustering arose during the course of cluster evolution, and they persisted even after many cluster collapse times. His results suggest that the ellipticity that we observe might be a long-persistence relic of initial anisotropies and/or subsequent subclustering. The observed degree of ellipticity in Coma is so striking, however, as to suggest that an even stronger cause may be operating.

Binney (1976, 1977) has discussed mechanisms that can create a strong ellipticity without rotation and that can even cause the ellipticity to increase. He shows (Binney 1976), in the context of galaxy formation, that an initially flattened system should, after violent relaxation, remain flattened. He later argues (Binney 1977) that in a flattened cluster of galaxies dynamical friction will lead toward greater flattening, which should manifest itself selectively among the most massive galaxies.
The orientation of the two Coma giants along the major-axis direction of the general distribution speaks in favor of such a suggestion, as does the generally higher ellipticity among the brightest galaxies.

It is interesting to contrast the three mechanisms just discussed. The fluctuations that we have noted in the computer runs of Aarseth and of Peebles are short-term transients, whereas White's suggestion is that anisotropies may die out on a much slower time scale than the crossing time. In conjunction with Binney's mechanisms, on the other hand, one must contemplate the possibility of a permanent ellipticity that is sustained by a permanent anisotropy in the velocities. Such a configuration might well be possible, but the relevant dynamical problems have yet to be investigated.

It would also be interesting to know whether the gas responsible for Coma's X-ray emission shows an ellipticity similar to that of the galaxy distribution. A flattening has indeed been observed in the X-ray emission from the Perseus cluster (Cash, Malina, and Wolff 1976), but in that case the X-rays may be associated with the peculiar chain of major galaxies. In any case, the spatial distribution of the Coma X-rays deserves further observation.

We have profited from discussions with Dr. James Binney. It is also a pleasure to thank Drs. George Abell and Laird Thompson, who made available unpublished surveys of the Coma cluster, and Mr. Robert Stevens for unusually competent assistance with the graphics. One of us (LS) acknowledges the support (through Dr. E. D. Commins) from the Lawrence Berkeley Laboratory, under contract W-Eng-74-05 with the U.S. Energy Research and Development Administration. The other (IRK) was partially supported by NSF Grant AST 76-00530.
### TABLE 1

Sample of the Coma Cluster Studied

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Number of Objects</th>
<th>Magnitude Limit</th>
<th>Radial Limit</th>
<th>Source and Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1377</td>
<td>( m_a = 18.0 )</td>
<td>75'</td>
<td>Unpublished. Referred to in Abell 1962, 1975, 1977. Background correction of 67 gal./sq. degree adopted. ( m_a ) is visual.</td>
</tr>
<tr>
<td>G</td>
<td>226</td>
<td>( m_z = 15.7 )</td>
<td>180'</td>
<td>Gregory (1975), Gregory and Tifft (1976a) photographic magnitudes and positions from Zwicky and Herzog (1963). Sample complete to ( m_z = 15.7, R = 180' ) and ( m_z = 14.9, R = 360' ). The outer galaxies were studied but are not included here. ( m_z ) is photographic, and ( m_z = 15.7 ) corresponds to ( m_a = 14.8 ).</td>
</tr>
<tr>
<td>AG</td>
<td>153</td>
<td>( m_a \sim 14.8 )</td>
<td>75'</td>
<td>Sample made from all objects in A with redshifts from G. Used for luminosity calculations. Limiting magnitude is approximate, since membership was determined by ( m_z ).</td>
</tr>
<tr>
<td></td>
<td>817</td>
<td>------</td>
<td>76'</td>
<td>Unpublished counts (to ( m_a \approx 17.5 )) of Laird Thompson (private comm.); used to check results of A sample.</td>
</tr>
<tr>
<td>-------</td>
<td>----</td>
<td>--------------</td>
<td>-----</td>
<td>--------------------</td>
</tr>
<tr>
<td>1377</td>
<td>87.3</td>
<td>0 - 75°</td>
<td>0.80</td>
<td>All</td>
</tr>
<tr>
<td>906</td>
<td>60.1</td>
<td>0 - 55</td>
<td>0.77</td>
<td>All</td>
</tr>
<tr>
<td>519</td>
<td>36.6</td>
<td>0 - 75</td>
<td>0.80</td>
<td>11.6 - 17.0</td>
</tr>
<tr>
<td>381</td>
<td>28.2</td>
<td>0 - 55</td>
<td>0.77</td>
<td>11.6 - 17.0</td>
</tr>
<tr>
<td>431</td>
<td>31.5</td>
<td>&lt; 30</td>
<td>0.68</td>
<td>All</td>
</tr>
<tr>
<td>106</td>
<td>8.7</td>
<td>&quot; &quot;</td>
<td>&lt; 15.5</td>
<td>-15</td>
</tr>
<tr>
<td>103</td>
<td>7.6</td>
<td>&quot; &quot;</td>
<td>15.6 - 17.0</td>
<td>-13</td>
</tr>
<tr>
<td>222</td>
<td>15.3</td>
<td>&quot; &quot;</td>
<td>&gt; 17.1</td>
<td>-12</td>
</tr>
<tr>
<td>475</td>
<td>29.2</td>
<td>30 - 55</td>
<td>1.0</td>
<td>All</td>
</tr>
<tr>
<td>81</td>
<td>6.4</td>
<td>&quot; &quot;</td>
<td>&lt; 15.5</td>
<td>-15</td>
</tr>
<tr>
<td>94</td>
<td>5.4</td>
<td>&quot; &quot;</td>
<td>15.6 - 17.0</td>
<td>-.49</td>
</tr>
<tr>
<td>303</td>
<td>17.6</td>
<td>&quot; &quot;</td>
<td>&gt; 17.1</td>
<td>-.30</td>
</tr>
</tbody>
</table>

Shown are the sample size N, corrected number per sector N₁, A', B', C' = (A² + B'²)¹/², mean error, phase angle and error in that angle. Recall that φ/2 = position angle. In the case of luminosity, the total in units of 10¹⁰ L₀ is given in the fifth column. The factor F (determined from Eq. (12) or (13) in the text) relates the ellipticity coefficients to the actual counts.
<table>
<thead>
<tr>
<th>N</th>
<th>N_1</th>
<th>Radial group</th>
<th>F</th>
<th>Magnitude</th>
<th>A'</th>
<th>B'</th>
<th>C'</th>
<th>E'</th>
<th>φ</th>
<th>Δφ</th>
<th>Label, if shown in Fig. 3</th>
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<tbody>
<tr>
<td>471</td>
<td>26.6</td>
<td>55 - 75</td>
<td>1.0</td>
<td>All</td>
<td>.32</td>
<td>-.09</td>
<td>.33</td>
<td>.10</td>
<td>344</td>
<td>16</td>
<td>A_75</td>
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<tr>
<td>53</td>
<td>4.0</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&lt; 15.5</td>
<td>.29</td>
<td>.25</td>
<td>.38</td>
<td>.21</td>
<td>41</td>
<td>29</td>
<td>A_75,B</td>
</tr>
<tr>
<td>85</td>
<td>4.4</td>
<td>&quot;</td>
<td>&quot;</td>
<td>15.6 - 17.0</td>
<td>.10</td>
<td>-.13</td>
<td>.25</td>
<td>.17</td>
<td>307</td>
<td>56</td>
<td>A_75,M</td>
</tr>
<tr>
<td>138</td>
<td>8.4</td>
<td>&quot;</td>
<td>&quot;</td>
<td>11.6 - 17.0</td>
<td>.19</td>
<td>.05</td>
<td>.20</td>
<td>.16</td>
<td>15</td>
<td>40</td>
<td>A_75,B+M</td>
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<td>18.4</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&gt; 17.1</td>
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<td>-.15</td>
<td>.41</td>
<td>.12</td>
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<td>19.1</td>
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<td>0.80</td>
<td>&lt; 15.5</td>
<td>-.21</td>
<td>.36</td>
<td>.43</td>
<td>.12</td>
<td>121</td>
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<tr>
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<td>0 - 75</td>
<td>0.80</td>
<td>15.6 - 17.0</td>
<td>-.30</td>
<td>.03</td>
<td>.30</td>
<td>.14</td>
<td>174</td>
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<td>A_M</td>
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<tr>
<td>858</td>
<td>51.3</td>
<td>0 - 75</td>
<td>0.80</td>
<td>17.1 - 18.0</td>
<td>-.05</td>
<td>.10</td>
<td>.11</td>
<td>.09</td>
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<tr>
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<td>20.8</td>
<td>0 - 75</td>
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<td>17.1 - 17.8</td>
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<td>.32</td>
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<td>.14</td>
<td>146</td>
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<tr>
<td>122</td>
<td>8.0</td>
<td>0 - 30</td>
<td>0.68</td>
<td>&quot;</td>
<td>-.66</td>
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<td>153</td>
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TABLE 2 (continued)

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### a. Luminosity

\((l_{10})\)

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\(A_L\)  

\(A_{CL}\)
REFERENCES


_____. 1976b, preprint.

FIG. 1. — Distribution of galaxies in the Abell sample. The spacing of the print grid is about 2.6" in the EW direction and 1.6" NS. Where more than one galaxy occupies a print cell, the number is printed. The
FIG. 2. — Distribution of galaxies in the Gregory sample, with non-members omitted.
FIG. 3. — Ellipticity components (as defined in the text) for Coma galaxies. Capital letters identify the sample and subscripts the subset or treatment; for details see last column of Table 2. Arrow corresponds to the direction of NGC 4889 from the center. Note that position angle of an ellipticity is $1/2 \tan^{-1} B'/A'$. Separate diagrams show (a) the A, G, and AG samples, (b) subsets of the G and AG samples, and (c) subsets
FIG. 4. — Distribution of Abell galaxies by magnitude group: (a) B and M ranges (11.6 - 17.0), (b) F range (17.1 - 17.8), (c) FF range (17.9 - 18.0). In the last part the Zwicky and Herzog agglomerations are marked.
Fig. 4b
FIG. 5. — Part of Field 160 of Zwicky and Herzog (1963), showing the background agglomerations (contours). The outer boundary is the limit of the Abell field, except at the right, where the Abell field would extend a few arc min beyond the edge of this Zwicky-Herzog field.

The shaded square is the cluster center, in which the symbols representing galaxies were too crowded to plot.
This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.