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MOYER MODEL APPROXIMATIONS FOR POINT AND EXTENDED BEAM LOSSES

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J.B. McCaslin, W.P. Swanson and R.H. Thomas

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MOYER MODEL APPROXIMATIONS
FOR POINT AND EXTENDED BEAM LOSSES

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### TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>ABSTRACT</th>
<th>v</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>2. Summary of Previous Work</td>
<td>3</td>
</tr>
<tr>
<td>3. Scope of This Paper</td>
<td>4</td>
</tr>
<tr>
<td>4. Calculations</td>
<td>5</td>
</tr>
<tr>
<td>4.1 Summary of the Moyer Model</td>
<td>5</td>
</tr>
<tr>
<td>4.2 Values of the Moyer Model Parameters</td>
<td>5</td>
</tr>
<tr>
<td>4.3 Point Source Calculations</td>
<td>6</td>
</tr>
<tr>
<td>4.3.1 Dose Equivalent at the Shield Surface</td>
<td>6</td>
</tr>
<tr>
<td>4.3.2 Location of the Maximum Dose Equivalent--Determination of the Value of $\vartheta_m$</td>
<td>7</td>
</tr>
<tr>
<td>4.3.3 Calculation of Maximum Dose Equivalent, $H_m$, on the Shield Surface</td>
<td>8</td>
</tr>
<tr>
<td>4.4 Extended Source Calculations</td>
<td>8</td>
</tr>
<tr>
<td>4.4.1 Infinite Uniform Line Sources</td>
<td>8</td>
</tr>
<tr>
<td>4.4.2 Finite Uniform Line Sources</td>
<td>9</td>
</tr>
<tr>
<td>4.5 Relation Between Point and Extended Uniform Sources</td>
<td>12</td>
</tr>
<tr>
<td>5. Conclusions</td>
<td>13</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>14</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>15</td>
</tr>
<tr>
<td>APPENDICES</td>
<td></td>
</tr>
<tr>
<td>A1 Definitions of Symbols Used</td>
<td>19</td>
</tr>
<tr>
<td>A2 Hewlett-Packard HP-97 Code for Calculating $F(\vartheta)$</td>
<td>21</td>
</tr>
<tr>
<td>A3 Hewlett-Packard HP-97 Code for Calculating $C(\lambda)$ and Numerical Approximations to the Function $C(\lambda)$</td>
<td>23</td>
</tr>
<tr>
<td>A4 Hewlett-Packard HP-97 Code for Calculating the Moyer Integral and Dose Equivalent for Extended Uniform Line Sources</td>
<td>24</td>
</tr>
<tr>
<td>A5 Hewlett-Packard HP-97 Code for Calculating Dose Equivalent from Point, Extended and Infinite Uniform Line Sources by Approximate Methods</td>
<td>27</td>
</tr>
<tr>
<td>List of Figures</td>
<td>30</td>
</tr>
<tr>
<td>Figures</td>
<td>32</td>
</tr>
</tbody>
</table>
"Then feed on thoughts, that voluntary move
Harmonious numbers"

From: Paradise Lost, Book iii, line 39
John Milton (1608-1676)
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ABSTRACT

The use of the empirical Moyer Model for the determination of transverse neutron shielding for high-energy proton accelerators is described and discussed. It is shown that an important advantage of the Moyer Model is the physical insight it offers towards understanding the complex interactions that comprise the shielding processes. Calculations for point-like and extended uniform beam loss distributions are discussed and their relationship to practical shielding conditions developed. The calculations required by the Model are readily performed on small programmable calculators and thus are widely accessible. Examples of program listings for practical calculations are given for a Hewlett-Packard HP-97 calculator.
1. INTRODUCTION

The development of the Moyer Model as an empirical method of determining transverse shield thicknesses for high-energy particle accelerators has been extensively described in the literature, to which the interested reader is referred (Mo61, Mo62, Pa73, Ri73, Ro69, St82).

Despite the development over the past decade of sophisticated high-energy radiation transport codes which may be applied to the detailed and accurate calculation of accelerator shielding (Ne80), Moyer Model calculations continue to be of great utility. As we shall show, since its first use in the early 1960s to calculate shielding for the improved Bevatron (Mo61, Mo62, We63) the Moyer Model has been used to design shielding for a large number of particle accelerators. A very recent example of its use is the design of the proposed Superconducting Super Collider (Th83, DOE84).

The reasons for this continuing interest in calculations utilizing the Moyer Model are threefold:

- The empirical Moyer Model is simple—employing only three parameters each of whose values are now well known (see Section 4). Calculations using the Model may be readily performed and yield physical insight into shielding that may be obscured when complex computer codes are used.

- The model is well adapted to the rapid, inexpensive estimation of accelerator and beam-line neutron shielding without recourse to complex computer codes. This is of great interest in industrially developing countries where large computers are of limited availability (Li81).

- Although under many conditions analogue Monte Carlo calculations are capable of great accuracy, they are themselves limited and may not permit direct calculation of shielding requirements for the occupational radiation intensities which exist at many operating high-intensity, high-energy accelerators e.g., typically $3 \times 10^{-22}$ Sv/proton. Analogue calculations such as HETC (Ar72) and FLUKA (Ra72) can extend down to about $3 \times 10^{-15}$ Sv/proton. Almost a factor of $10^{-4}$ may be obtained by particle splitting as is used, for example, in TRANKA (Ra74). Weighted Monte Carlo methods such as in CASIM (VaG75) may extend beyond FLUKA by a factor of $10^{-4}$ to $3 \times 10^{-19}$ Sv/proton. Beyond the intensities reached by these calculations extrapolation is achieved via the Moyer Model (St82). It is therefore important to understand the Model and optimize the choice of its parameters.

In order that Monte Carlo methods for shielding calculation may be successfully employed it is important that the user have both detailed technical knowledge of the specific computer codes used and physical insight into the radiation transport processes involved. Without this physical insight it is possible to obtain results which have no practical value.

The use of transport codes as "black-boxes" is dangerous and can often lead to false information. Booth, for example, offers the following simple example to show how confusing results may be obtained:
"Consider particles trying to penetrate a thick shield. If the shield is any good, very few of the particles will penetrate the shield. Thus, it is possible to simulate a huge number of particles without any of the particles penetrating the shield. If none of the simulated particles penetrate the shield, the sample mean and the sample variance will both be zero. However, even for the best shields the probability of penetration is some number $\epsilon > 0$. The sample mean is thus in error; at the same time the sample variance is indicating zero error." (Bo79).

It is therefore extremely useful to have available phenomenological shielding models which may be used as a rough check on the results of calculations made by more sophisticated techniques. The Moyer Model is one such model providing the physical insight useful for such comparisons and needed to fully understand the results of Monte Carlo calculations.
2. SUMMARY OF PREVIOUS WORK

Since the early work of Moyer in designing shielding for the Bevatron (Mo61, Mo62) his method has been used, in whole or in part, to estimate shielding for many accelerators including the 20 GeV Stanford Linear Accelerator (deS62), the CERN SPS (CERN64), the Fermilab Proton Synchrotron (LRL65, URA68), the Stanford Positron-Electron Project [PEP] (McC73, Th77), the LBL Experimental Superconducting Accelerator Ring [ESCAR] (McC76) and the Chinese 50 GeV Proton Synchrotron (AS80, Ch80). Most recently this method was used in preliminary studies for the Superconducting Super Collider (Th83, DOE84).

The use of the Moyer Model in these designs has led to substantial experimental and theoretical improvement in the understanding of accelerator shielding phenomena and of the Model itself (Pa73, Ri73, St82).

Routti and Thomas addressed the application of the Moyer Model to uniform line sources and introduced the concept of Moyer Integrals (Ro69). McCaslin and Thomas, in unpublished notes, related finite uniform beam loss to loss at a point, anticipating similar work by Sullivan (Th69, McC76, Su81).

The Moyer Model utilizes three constants which are obtained empirically—the source strength term, \( \kappa \), the angular distribution coefficient, \( \beta \), and the attenuation length, \( \lambda \).

During the design of the Stanford Positron Electron Project, de Staebler reviewed phenomenological shielding models and summarized them in a series of what he called "Moyer-like" equations convenient for rapid computation (deS77). In private communications, de Staebler drew attention to the conservative choice of the normalizing constant, \( \kappa \), in the work of Routti and Thomas. This conservatism was a consequence of the assumption that there were no gaps between the ring magnets. The experimental data were taken from Gilbert et al. (Gi68, Gi69).

In design studies for the Beijing 50 GeV Proton Synchrotron, Stevenson and Liu (St80a, St80b) supported the conclusions of de Staebler (deS77) and as a result of their work, revised Moyer Model parameters were evaluated (Li82, St82). More recently the variation of the parameter \( \kappa \) with primary proton energy has been investigated (the other two parameters \( \beta \) and \( \lambda \) are essentially invariant with energy—see Section 3) (Li84, Th84).

During the design of the shielding for PEP, McCaslin addressed the primary aspect of the de Staebler 1977 paper—that of simple computational techniques (deS77). Several programs were written for computers with limited memory. Examples of these programs are given in Appendices to this report.
3. SCOPE OF THIS PAPER

The design of high-energy accelerator shielding usually proceeds in two stages—first, an approximate calculation of shield thickness is made using semiphenomenological models for fairly simple geometries, and second, when accelerator parameters have been more closely defined, these simple calculations are verified by the use of more sophisticated numerical methods, usually involving Monte Carlo techniques to calculate electromagnetic and hadronic cascade phenomena in the shield (Ne80).

These numerical techniques are not necessarily more accurate than the empirical models in estimating the intensity of radiation fields outside shielding when the geometry is simple and the primary particle energy is in a region where good experimental data are available. Under these conditions both methods can predict radiation field intensities to within a factor of two or better. The numerical techniques are of greatest value in extrapolations to new energies or for calculations with complex geometries.

In the preliminary stages of shielding design, when simple but rapid methods suffice and are even to be preferred, it is necessary to idealize both the accelerator geometry and the pattern of beam loss in order to make the problem tractable.

It is common practice to assume two general types of beam loss which, although not necessarily realistic, serve to provide bounds to the shielding thickness:

- Operational error beam loss (e.g., magnet failure, collimator slip-page, etc.), generally assumed to be located at a point. This assumption may be taken as determining the upper bound to the shielding requirement.

- Randomly distributed beam loss resulting from normal operation which, when averaged over extended periods of time, may be assumed to be uniformly distributed along the beam path over a finite distance. This assumption may be taken as determining the lower bound to the shielding required.

Both the case of beam loss located at a point and of uniform beam loss along an infinite straight line are amenable to simple calculation. The case of uniform beam loss over a finite straight line is less tractable to analytical solution, and recourse to numerical techniques of integration is made.

The purpose of this paper is to explain the physical and geometrical aspects of the Moyer Model in such a way as to make the method more widely understood and useful to operational accelerator health physicists. The needed calculations are organized in such a way that they are within the capacity of many hand-held programmable calculators, without recourse to any supplementary material such as tables or graphs. Examples of specific programs written for the Hewlett-Packard HP-97 calculator are given (Appendices A2,A4,A5). The application of such programs to specific problems is demonstrated by the solution of several practical problems.
4. **CALCULATIONS**

4.1. **Summary of the Moyer Model**

In summary the Moyer Model expresses the dose equivalent $H(\theta)$, at some point, $P$, on the shield surface (Fig. 1), per proton interacting at a point on the beam axis, as:

$$H(\theta) = \kappa r^{-2} \exp(-\alpha \theta) \exp(-2 \csc \theta) , \quad \frac{\pi}{3} < \theta < \frac{2\pi}{3} \quad (1)$$

where:

- $\theta$ is the angle subtended between the beam axis and a line joining the point of interaction and the point, $P$, of interest on the shield surface (see Fig. 1).
- $\kappa$ is the number of attenuation lengths in the shield (given by $d/\lambda$ where $d$ is the transverse shield thickness and $\lambda$ the attenuation length of the shield material; see Fig. 1).
- $r$ is the distance from the point source to the point of interest on the shield surface (slant distance; see Fig. 1).

and the parameters $\kappa$, $\alpha$, and $\lambda$ have been determined empirically.

With appropriate choices of the three Moyer Model parameters the model has been found to give good estimates of the dose equivalent at the shield surface—within the angular range indicated in Equation (1).

4.2. **Values of the Moyer Model Parameters**

Stevenson and his colleagues have recently summarized the published data which enable the parameters $\kappa$, $\alpha$, and $\lambda$ to be calculated (St82, Li84, Th84). Experience has shown that both $\alpha$ and $\lambda$ may be considered to be independent of energy for $E > 5$ GeV, and the best values obtained from the literature according to these authors are:

$$\alpha = 2.3 \pm 0.2 \text{ radians}^{-1}$$

$$\lambda \text{ (in earth)} = 1170 \pm 20 \text{ kg} \cdot \text{m}^{-2}$$

The attenuation length in a material of mass number, $A$, is given by:

$$\lambda = \left(\frac{A}{A_E}\right)^{1/3} \lambda_E \quad (2)$$

where $A_E$ is the effective mass number for earth ($A_E = 20.4$ for a 95%:5% mixture by weight of SiO$_2$ and H$_2$O) and $\lambda_E$ is the attenuation length in earth.

The parameter $\kappa$ is a function of energy:
A recent review of published data extending up to 350 GeV by regression and analysis-of-variance techniques has suggested the variation of $X$ with proton energy is expressed by the equation:

$$X = H_0(E)$$  \hspace{1cm} (3)$$

with values of $m = 0.80 \pm 0.10$, $k = (2.84 \pm 0.14) \times 10^{-13}$ when $H_0(E)$ is in Sv·m$^2$ and $E$ in GeV (Th84).

The earlier work of Stevenson et al. (St82) had suggested that the Moyer Model Parameter, $X$, was directly proportional to energy and given by:

$$X = 1.60 \times 10^{-13} E \text{ Sv·m}^2$$  \hspace{1cm} (4a)$$

when $E$ is in GeV. This approximation is sufficiently accurate (±20 percent) in the energy range from 5 to 30 GeV. However, at higher energies Equation 4a becomes increasingly inaccurate, overestimating $X$ by a factor of two at 500 GeV and a factor of four at 20 TeV.

The discussion of this report is restricted to proton beam energies, $E$, above 5 GeV, where the above relationships hold. Tesch has recently studied the variation of $\lambda$ and $X$ for $0.05 < E < 1 \text{ GeV}$ (Te84).

4.3 Point Source Calculations

Although beam losses at high-energy particle accelerators are almost always of an extended nature, it is often the case that the region of high beam loss occurs over lengths small or comparable with the thickness of the accelerator shields and tunnel radius $R = (a + d)$ (see Fig. 1). Under such conditions the assumption of point loss may be used to determine the dose equivalent at the shield surface.

In calculating shield thickness it is often erroneously assumed that it is sufficient to determine the dose equivalent on the shield surface directly above the region of localized beam loss (i.e., $\theta = \pi/2$ radians). This section will explore the validity of this assumption and develop simple methods for the calculation of shielding which do not require it.

4.3.1. Dose Equivalent at the Shield Surface

Using the Moyer Model it is a simple matter to calculate the dose equivalent resulting on the shield surface at some point $P$ (Fig. 1) subtending an angle $\theta$ to the beam direction, by the substitution of appropriate values for the parameters $X$, $\beta$ and $\lambda$ into equation (1).

The dose equivalent at the shield surface directly above the point source ($\theta = \pi/2$) is given by:
where \( N \) is the number of protons stopped at the point. With \( \chi \) in \( \text{Sv} \cdot \text{m}^2 \), \( H(\pi/2) \) will be in \( \text{Sv} \). A calculation of \( H(\pi/2) \) is shown in Fig. 2 for a point loss of \( N = 1 \) proton at \( E = 10 \) GeV. Values are normalized to a transverse distance of \( R = 1 \) m. As these data are for a point source, scaling to a different distance may be done by the inverse-square law. Scaling to another energy is done by means of Equation (4).

However, the maximum dose equivalent, \( H_m \), occurs at a point downstream of the point source at angle \( \theta_m \); in general \( \theta_m < \pi/2 \). Figure 3 shows calculations of dose equivalent at the shield surface, normalized to that occurring at an angle of \( \pi/2 \) radians to the point source, as a function of shield thickness \( (1 < \ell < 20) \) and subtended angle, \( \theta \) \( (50^\circ < \theta < 120^\circ) \). The figure plots the function \( F(\theta) \) defined as:

\[
F(\theta) = \frac{H(\theta)}{H(\pi/2)} = \frac{\exp(-\alpha \theta) \exp[\alpha(1 - \csc \theta)]}{\exp(-\alpha \pi/2) \csc^2 \theta}
\]

Substituting \( \alpha = 2.3 \) radians\(^{-1} \), Equation (6) becomes:

\[
F(\theta) = 37.1 \sin^2 \theta \exp(-2.3 \theta) \exp[\alpha(1 - \csc \theta)] .
\]

[See Appendix A2 for details of a program to determine \( F(\theta) \) using an HP-97 calculator.]

The maximum value of dose equivalent, \( H_m \), is seen to occur at some angle, \( \theta_m \), in general less than \( \pi/2 \) radians. The thicker the shield the closer the value of \( \theta_m \) approaches \( \pi/2 \) radians (see Fig. 4); as seen in Fig. 2, the relative and absolute difference between \( H_m \) and \( H(\pi/2) \) diminishes with increasing shield thickness. But Figs. 2 and 3 show that, for thin shields, errors as much as a factor of two or more may be made by assuming that the maximum dose equivalent occurs at \( \theta = \pi/2 \) radians. For accurate calculations of shield thickness at regions of point loss it is therefore important to determine \( H_m \). \( H_m \) may be calculated when the corresponding value of \( \theta_m \) is known.

### 4.3.2. Location of Maximum Dose Equivalent—Determination of the Value of \( \theta_m \)

The value of \( \theta_m \) may be obtained by differentiating Equation (1) with respect to \( \theta \) and setting the result equal to zero, yielding:

\[
[\gamma \cot \theta_m \csc \theta_m - \beta + 2 \cot \theta_m] = 0 ,
\]

where use is made of the relationship \( r = R \csc \theta \) (Fig. 1). Figure 4 shows values of \( \theta_m \) as a function of shield thickness, \( \ell \), for a value of the angular distribution coefficient \( \beta = 2.3 \).
As was to be expected $\theta_m$ increases with $\lambda$ and approaches the value $\pi/2$ radians for very thick shields. Initially the rate of change of $\theta_m$ with $\lambda$ is quite rapid, but slows with increasing shield thickness. At $\lambda = 5$, $\theta_m$ has the value 1.3 radians ($72^\circ$), and at $\lambda = 10$ it has the value 1.4 radians ($80^\circ$). For practical high-energy accelerator shields $\theta_m$ will, as we have already inferred, be less than $\pi/2$ radians ($90^\circ$).

Figure 5 shows values of $\theta_m$ as a function of shield thickness for several values of beam spill length, $n$ ($n$ is defined in section 4.4.2).

4.3.3. Calculation of Maximum Dose Equivalent, $H_m$, on the Shield Surface

We define a function $C(\lambda)$ such that:

$$H_m = C(\lambda) H(\pi/2)$$  \hfill (8)

Substitution into Equation (5) and using $\beta = 2.3$ radians$^{-1}$ gives:

$$C(\lambda) = \exp[2.3(\pi/2 - \theta_m)] \exp[\lambda(1 - \csc \theta_m)] \sin^2 \theta_m$$  \hfill (9)

Appendix A3 shows that a good numerical approximation for $C(\lambda)$ is:

$$C(\lambda) = 2.20 \lambda^{-0.245}$$  \hfill (9a)

Combining Equations (5) and (9a) it follows that:

$$H_m = 0.0593 \frac{\kappa N \exp(-\lambda)}{R^2} \lambda^{-0.245}$$  \hfill (10)

with $\kappa$ in Sv·m$^2$ and the distance $R$ measured in meters, $H_m$ will be in Sv.

4.4. Extended Source Calculations

In practice, "point" sources do not generally occur in the operation of high-energy proton accelerators because the physical mechanisms resulting in beam loss usually spill beam over an extended region.

An approximation often useful in accelerator shield design is to assume that beam losses are uniform. This section discusses the calculation of dose equivalent for uniform beam loss both of infinite and finite extent.

4.4.1. Infinite Uniform Line Sources

Using the Moyer Model it is simply shown (Ro69) that for an infinite uniform line source of $S$ protons per meter the dose equivalent on the shield surface is given by:

$$H_\infty = \frac{\kappa S}{R} \int_0^\pi \exp(-\alpha \theta) \exp(-\lambda \csc \theta) \, d\theta$$  \hfill (11)
In contrast to the point-source geometry just described, Equation (11) shows that the dose equivalent outside the shield produced by an infinite uniform line source diminishes as the inverse distance.

The integral of Equation (11) has been designated by \( M(\varrho, \xi) \) and is known as a Moyer Integral. Tabulations of Moyer Integrals for arguments in the range \( 0 < \varrho < 10; 0 < \xi < 40 \) have been published. For the calculation of high-energy accelerator shielding the Moyer Integral \( M(2.3, \xi) \) is of particular interest. Appendix A4 gives a computer program for the calculation of this particular integral by Simpson's Rule using an HP-97 calculator.

Tesch has suggested that a suitably accurate approximation is given by:

\[
M(2.3, \xi) = 0.065 \exp(-1.09 \xi)
\]

valid for \( \xi = 2 - 15 \) (Te83). This expression is useful if tables of \( M(2.3, \xi) \) or means of calculating the integral are not available. If \( M(2.3, \xi) \) can be evaluated the dose equivalent on the shield surface \( H(\xi, R) \) is then given by:

\[
H_\infty(\xi, R) = \frac{\kappa S M(2.3, \xi)}{R} \quad \text{Sv}
\]

As an example:

with \( E = 10 \) GeV, and

\[
\kappa = 1.79 \times 10^{-12} \text{ Sv \cdot m}^2,
\]

then

\[
H_\infty(\xi, R) = \frac{1.79 \times 10^{-12} S M(2.3, \xi)}{R} \quad \text{Sv}
\]

when \( R \) is measured in meters and \( S \) in protons \( \text{m}^{-1} \).

With the same parameters and using the approximation of Tesch:

\[
H_\infty(\xi, R) = \frac{1.16 \times 10^{-13} S}{R} \exp(-1.09 \xi) \quad \text{Sv}
\]

Stevenson et al. suggest that the use of Equation (14a) or (14b) should predict values of dose equivalent rate to within a factor of about two (St82).

4.4.2. Finite Uniform Line Sources

Here we consider the more practical case of the dose equivalent produced at the shield surface by a finite, but uniform, beam loss. To quantify the region of beam loss it is convenient to define the parameter, \( n \), by the equation:
\[ n = \frac{L}{R} \quad (15) \]

where \( L \) = distance over which beam is uniformly distributed—"beam spill length" and \( R \) has previously been defined (see Fig. 6 and also Appendix A1).

The dose equivalent on the shield surface \( H(Z, L) \) per proton per meter lost along the beam axis is given by:

\[
H(Z, L) = \frac{kS}{R} \int_{a_1}^{a_2} \exp(-\theta) \exp(-\lambda \csc \theta) \, d\theta
\quad (16)
\]

where the angles \( a_1, a_2 \) limiting the integral are given by (see Fig. 6):

\[
a_1 = \tan^{-1} \left( \frac{R}{Z} \right) \quad (16a)
\]

\[
a_2 = \tan^{-1} \left[ \frac{R}{Z - L} \right] \quad (16b)
\]

The integral of Equation (16) differs from that in Equation (11) only in the limits of integration and may be evaluated by numerical methods (see Appendix A5).

When \( N \) protons are lost uniformly over length, \( L \),

\[
dN = S \frac{N}{L} \quad (17)
\]

and since \( n = L/R \) [Equation (15)], we may rewrite Equation (16) as:

\[
H(Z, L) = \frac{kN}{\pi R^2} \int_{a_1}^{a_2} \exp(-\theta) \exp(-\lambda \csc \theta) \, d\theta
\quad (18)
\]

Designating the integral of Equation (18) as a restricted Moyer Integral: \( M_{\alpha_1, \alpha_2}[^{\beta, \lambda}] \) we write:

\[
H(Z, L) = \frac{kN}{\pi R^2} M_{\alpha_1, \alpha_2}[^{\beta, \lambda}] \quad (18a)
\]

The calculation of dose equivalent on the shield surface from Equation (18a) may be carried out by numerical means, and Appendix A4 gives a suitable program for use with an HP-97 calculator.

Figure 7 shows the results of an illustrative calculation of \( H(Z, L) \) using Equation (18a). A family of curves is shown for various spill lengths, \( n = L/R \), and for the parameters:
$E = 10 \text{ GeV}$

$\mu = 1.79 \times 10^{-12} \text{ Sv m}^2$

$x = 5$

$R = 1 \text{ m}$

$0.002 < n < 4$

Distances along the Z axis are measured in units of $n = L/R$, and the origin of coordinates is at the start of beam spill. The parameter $\mu = 1.79 \times 10^{-12}$ Sv m$^2$, corresponding to 10 GeV, and $R = 1$ m, are chosen for convenience in normalization. As $N = 1$ is assumed for each spill length, the area under the curves in Fig. 6 is independent of $n$.

Figure 8 shows the behavior of the maximum dose equivalent, $H_m$, on the outside surface of the shield for a constant number of protons ($N = 1$ proton and $R = 1$ m) as a function of the beam spill length $n$. The limit for $n > 0$ at $4.7 \times 10^{-6}$ Sv m$^2$ per proton is indicated. At the other extreme, as one increases $n$ while holding $N$ fixed, $H_m$ approaches zero in a manner approximately proportional to the inverse spill length: $H_m \propto 1/n$. The approach to this limiting curve (hyperbola) is also indicated in Fig. 8. Values on the abscissa, $n_p$ and $n_{oo}$, show where the true behavior deviates from either limiting assumption by 10 percent for this distance. The shaded area, bounded above by the two limiting assumptions, indicates the amount of overestimation that can be made by assuming the minimum of either the point-source or hyperbolic approximation.

In Fig. 9, the family of curves shows the distribution of dose equivalent outside of the shield for the case where the beam loss rate is held constant at $S = 1$ proton per meter. The dose equivalent is calculated using Equation (16), using the same parameters as for Fig. 7. In Fig. 9 the total number of protons, and therefore the area under the curves, is proportional to the length of the spill, $L$.

It can be seen from Figs. 7 and 9 that, for short spill lengths, $n << 1$, the source is nearly point-like and substitution of the appropriate value of $N$ in Equation (10) will yield a good estimate of $H_m$. For values of $n > 1$ the source behaves more like an infinite uniform line source so that substitution of the appropriate value of $S$ into Equation (14a) will provide an acceptable estimate of $H_m$.

An attempt is made in Fig. 10 to further define the boundary between the point-like and extended line-like sources in terms of the spill length parameter, $n$. As in Fig. 8, we define $n_p$ as being that value of $n$ at which $H$ would be equal to 90 percent of that which would obtain if the same number of beam particles were deposited at a point. The reference distance of $R = 1$ m is assumed. In similar fashion, we define $n_{oo}$ as being that value of $n$ at which $H$ would be equal to 90 percent of that which would obtain if the source were an infinite uniform line source of the same loss per unit length, $S$.

The percentage overestimation resulting from use of either of the two limiting assumptions (Point source [constant $N$]) or (Infinite uniform line source [constant $S$]), is explicitly shown in Fig. 11. The percentage overestimation is defined as
where $H_m$ is the true maximum dose equivalent on the outside surface of the shield for the spill length in question, $n$, and $H_A$ is the dose equivalent estimated using either of the two assumptions. Using 10 percent as an arbitrary criterion, we define the limits of applicability for the point source [Equation (10)] and for the infinite uniform line source [Equation (14a) or (14b)] as follows:

For "short" spill lengths ($n < 0.6$), $H_m$ may be calculated using the "point-source" Equation (10);
For "long" spill lengths ($n > 1.4$), $H_m$ may be calculated using the infinite line source Equation (14a or 14b).

The error associated with these procedures amounts to an overestimate of $H_m$ which ranges from 0 to about 10 percent when $L = 5$. For the intermediate case $0.6 < n < 1.4$ the maximum dose rate must be calculated from Equation (18a) using the restricted Moyer integral. Figure 11 shows the amount of overestimation of $H_m$ by either the point source equation for extended sources (keeping $N$ constant) or by the infinite line source equation for finite line sources (keeping $S$ constant). Percentage errors are plotted as functions of $n$ for both of these assumptions.

4.5. Relation Between Point and Extended Uniform Sources

As we have suggested in Section 4.4, two simple assumptions are available to the accelerator shield designer if there exists some estimate of the total number of protons, $N$, lost to the accelerator. It may be assumed either that all the protons are lost at one location (point source) or that the beam loss is uniformly distributed along the accelerator structure of length, $L$.

The first assumption leads to an estimate of maximum dose equivalent for the number of protons lost, $N$, of:

$$H_m = \frac{K N \exp(-\beta \theta_m) \exp(-\lambda \csc \theta_m)}{R^2 \csc^2 \theta_m}$$  \hspace{1cm} (20)$$

where $\theta_m$ may be read from Fig. 4 and $K$ obtained from Equation (4). Section 4.3 discusses the calculation of $H_m$ in detail. Since "point-like" beam loss does not occur at high-energy proton accelerators this assumption will tend to overestimate the value of $H_m$ and hence the shield thickness.

The second assumption proceeds by substituting the value $N/L$ for $S$ into Equation (14a) or (14b); these equations assume an infinite uniform source. In Section 4.4.2 we showed that this was a reasonable procedure for values of spill length, $n > 1.4$.
These two assumptions clearly lead to different estimates of dose equivalent, \(H_m\) and \(H_\infty\). The relationship between \(H_m\) and \(H_\infty\) may be obtained by combining Equations (11) and (20):

\[
\frac{H_m}{H_\infty} = \frac{N}{SR} \frac{\exp(-\beta \theta_m) \exp(-\lambda \csc \theta_m)}{M(\alpha, \lambda) \csc^2 \theta_m} \quad (21)
\]

Substituting \(N = LS\) and \(n = L/R\) yields

\[
\frac{H_m}{H_\infty} = \frac{\sin^2 \theta_m \exp(-\beta \theta_m) \exp(-\lambda \csc \theta_m)}{M(\beta, \lambda)} \quad (22)
\]

For convenience in expressing this relation, we define a function \(B(\lambda)\), such that

\[
\frac{H_m}{H_\infty} = n \ B(\lambda) \quad (23)
\]

Figure 12 shows values of \(B(\lambda)\) calculated as a function of \(\lambda\). It is seen that \(B(\lambda)\) may be approximated by a second order polynomial (dashed curve) which fits the curve within \(\pm 5\) percent over the range \(1 < \lambda < 15:\)

\[
B(\lambda) = 0.44 + 0.12\lambda - 0.0028\lambda^2 \quad (24)
\]

Equation (24) facilitates the use of the Moyer Model for shielding calculations because \(H_m\) can be obtained from Equation (20) without too much difficulty, and the factor \(B(\lambda)\) can be evaluated from the polynomial [Equation (24)]. With these algorithms, Moyer Model calculations can be made with simple pocket calculators without having to solve the Moyer Integral directly.

5. Conclusions

The Moyer Model for high-energy neutron shielding has proved itself a very durable instrument since it was first developed in the early 1960's and has been utilized in the design of several important high-energy accelerator facilities. Its advantage over "sophisticated" methods lies in its simplicity; the algorithm contains only three parameters whose values are now well established over the energy range of at least 5 to 350 GeV. The figures presented and the sample codes for use with desk-top or hand-held calculators demonstrate the ease with which the Moyer Model can be applied to many shielding problems, some of which are beyond the present capability of well-established Monte-Carlo procedures. The procedure, which requires very modest computational power, is useful in its own right besides serving as a check on more elaborate, but less transparent, methods of shielding calculation.
ACKNOWLEDGMENTS

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LRL65 Lawrence Radiation Laboratory 200 BeV Study Report (2 vols.), University of California Lawrence Radiation Laboratory Internal Report, UCRL-16000 (June 1965).


Thomas, R.H. and McCaslin, J.B., "PEP Main Ring Shielding Design," Lawrence Berkeley Laboratory Internal Note HP 63 (February 1977).


Appendices
APPENDIX A1. Definitions of Symbols Used

Figures 1 and 6 of the text show the accelerator and beam loss geometries considered in this paper. The following symbols are used throughout:

- $Z$ distance along the proton beam, measured from the point at which beam loss starts
- $a$ radius of accelerator tunnel
- $d$ transverse shield thickness
- $\lambda$ attenuation length of shield material
- $\zeta$ number of attenuation lengths in shield in transverse direction
- $R$ distance normal to the proton beam axis to the shield outer surface
- $r$ distance from point source to point of interest on shield surface (slant distance)
- $L$ beam spill length
- $n$ beam spill length in units of $R$
- $\theta$ angle subtended between beam axis and line joining source element and point of interest on shield outer surface
- $H$ dose equivalent
- $H_m$ maximum dose equivalent on shield outer surface
- $\theta_m$ angle $\theta$ at which $H_m$ occurs
- $H_\infty$ dose equivalent due to infinite uniform line source
- $H_L$ dose equivalent due to extended uniform line source
- $S = \frac{dN}{dZ}$ number of protons lost per unit length along beam axis
- $N$ total number of protons lost
- $\rho$ density of shield material (mass per unit volume)
- $E$ primary proton energy
The following relationships automatically follow from the definitions given above:

\[ R = (a + d) \]

\[ \lambda = d / \lambda \]

\[ n = \frac{L}{(a + d)} = \frac{L}{R} \]

\[ r = (a + d) \csc \theta = R \csc \theta \]

\[ r^2 = R^2 + z^2 \]

\[ dZ = -(a + d) \csc^2 \theta \theta = -R \csc^2 \theta \theta \]

\[ N = L \frac{dN}{dZ} = nR \frac{dN}{dZ} = nRS \]
APPENDIX A2. Hewlett-Packard HP-97 Code for Calculating $F(\theta)$

This program, written for the Hewlett-Packard HP-97 calculator, determines values of $F(\theta)$, where

$$F(\theta) = \frac{H(\theta)}{H(\pi/2)} = \frac{\exp(-\theta) \exp[\lambda(1 - \csc \theta)]}{\exp(-\lambda \pi/2) \csc^2 \theta}$$  \hspace{1cm} (6)\

in increments of 5° for given values of shielding thickness in units of $\lambda$, assuming a point radiation source.

TO RUN THE PROGRAM: Store in the indicated registers the following:

Register A  $\theta_{\text{min}}$(deg), the smallest angle of concern;
B  $\theta_{\text{max}}$(deg), the largest angle of concern;
D  $\lambda = 2.3$, the angular distribution parameter;
E  $\lambda$, the number of attenuation lengths.

Press A. The program will stop when the angular range from $\theta_{\text{min}}$ to $\theta_{\text{max}}$, in steps of 5°, has been processed. New values of $\lambda$ can then be stored in Reg E, and the process is repeated by pressing A.

PRINTOUT:

Line 1  \( g(\theta) = (\sin^2 \theta) \exp(-2.3 \theta) \exp(-\lambda \csc \theta) \)

Line 2  \( F(\theta) = g(\theta)/[0.2697 \exp(-\lambda)] \)

Line 3  \( \theta \), in degrees.

*Equation numbering is as in text.*
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Legend:
- *LBLA: Label A
- *LBL2: Label 2
- *LBL3: Label 3
- *LBL4: Label 4
- RCLD: Recall from Data Register
- RCL: Recall
- STO: Store
- STO1: Store register 1
- STO2: Store register 2
- STO3: Store register 3
- RCLl: Recall from register l
- X^2: Square
- X-35: Subtract 35
- X-35 01: Subtract 35 from register 1
- X=Y?: Equal to Y?
- GT04: Greater than or equal to 4
- GT03: Greater than or equal to 3
- GT02: Greater than or equal to 2
- GT01: Greater than or equal to 1
- PRTX: Print
- R/S: Return from subroutine
APPENDIX A3. Hewlett-Packard HP-97 Code for Calculating $C(\lambda)$ and Numerical Approximations to the Function $C(\lambda)$

This program, written for the Hewlett-Packard HP-97 calculator, evaluates $C(\lambda) = \frac{M}{H(\pi/2)}$, i.e., the maximum dose equivalent at the shield outer surface for a point source, divided by the value at 90°. Values of the angle, $\theta_m$, at which the maximum dose equivalent is observed at the shield outer surface, assuming a point source, have been determined from Equation (7) and plotted in Fig. 3*:

$$\lambda \cot \theta_m \csc \theta_m - \beta + 2 \cot \theta_m = 0 \quad (7)$$

These values of $\theta_m$, when substituted in Equation (9) shown below, yield values of $C(\lambda)$ as a function of $\lambda$. These are shown as the solid line in Fig. A3-1.

$$C(\lambda) = \exp[2.3(\pi/2 - \theta_m)] \exp[\lambda(1 - \csc \theta_m)] \sin^2 \theta_m \quad (9)$$

Simple approximations to $\theta_m$ and $C(\lambda)$ are given below. The angle $\theta_m$ can be approximated to better than ±5 percent from $\lambda = 1$ to $\lambda = 15$ by the simple expression

$$\theta_m = 55.9 + 9.8 \ln(\lambda) \text{ (degrees)} \quad (A3-1)$$

and $C(\lambda)$ can be approximated by the following expression:

$$C(\lambda) = 2.2 \lambda^{-0.245} \quad (9a)$$

This is shown as the dashed line in Fig. A3-1. Although greater accuracy is not required, the following polynomial fits $C(\lambda)$ very well as shown by the dots in Fig. A3-1:

$$C(\lambda) = 2.652 - 0.589\lambda + 0.116\lambda^2 - 0.0124\lambda^3$$
$$+ 6.588 \times 10^{-4}\lambda^4 - 1.378 \times 10^{-5}\lambda^5 \quad (A3-2)$$

*Equation and figure numbering are as in text.
APPENDIX A4. Hewlett-Packard HP-97 Code for Calculating the Moyer Integral and Dose Equivalent for Extended Uniform Line Sources

This program uses Simpson's Rule to calculate the Moyer Integral for extended, finite and infinite, uniform line sources. It also calculates the dose-equivalent on the shield outer surface for unit beam loss rate [Equation (16)] and for unit beam loss [Equation (18a)] i.e., \( S = 1 \) proton per meter, and \( N = 1 \) proton lost along \( L \):*

\[
H = \frac{\pi E}{R} \frac{M_{\alpha_1,\alpha_2}^{B,\lambda}}{a_1,a_2} \quad \text{and} \quad (16)
\]

\[
H = \frac{\pi E}{n R^2} \frac{M_{\alpha_1,\alpha_2}^{B,\lambda}}{a_1,a_2} \quad . \quad (18a)
\]

To use the program:

(1) Store the following in the indicated registers:

- Register \( E \) is the number of attenuation lengths along \( R \);
  - 1 \( R \) (meters);
  - 2 \( L \) (meters);
  - 3 \( E \) proton beam energy (GeV);
  - 4 \( Z \) (>0, meters), the distance along the shield outer wall at which the dose equivalent is determined. \( Z = 0 \) coincides with the start of beam spill.

(2) Press B. This step performs the geometry routines which store the integration limits in registers A and B. This is followed by the Simpson's Rule integration to determine the Moyer integral.

(3) Press C. This calculates the loss per unit length case \( (S = 1 \text{ proton m}^{-1}) \) from Equation (16).

(4) Press D. This calculates the unit loss case \( (N = 1 \text{ proton}) \) from Equation (18a).

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EXAMPLES:

Example 1. Calculate the Moyer integral, $M(2.3, \lambda)$, for an infinite line source. Approximate the infinite line source by a 10 km line ($L = 10,000 \text{ m}$) with the point of measurement at 5 km ($Z = 5000 \text{ m}$). Let $\lambda = 5, R = 1 \text{ m}$ and $E = 10 \text{ GeV}$. Press B. Read $M(2.3, 5) = 2.63 \times 10^{-4}$.

Example 2. Calculate the dose equivalent under the above conditions for a proton beam energy of 25 GeV when the loss rate is 1 proton per meter. Press C. Read $9.82 \times 10^{-16} \text{ Sv}$.

Example 3. Repeat the 2nd example for a loss of 1 proton along $L$. Press D. Read $9.82 \times 10^{-20} \text{ Sv}$.

Example 4. Calculate the restricted Moyer integral $M_{\alpha_1, \alpha_2} [\beta, \lambda]$ at $Z = 2\text{ m}$ for a spill length, $L$, of 4 meters where $R = 1 \text{ m}$ ($n = 4$). Let $\lambda = 5$. Press B. Read $M_{\alpha_1, \alpha_2} [\beta, \lambda] = 2.63 \times 10^{-4}$.

Example 5. Calculate the dose equivalent under the above conditions when $E = 25 \text{ GeV}$ for a loss rate of 1 proton per meter. Press C. Read $H_L = 9.81 \times 10^{-16} \text{ Sv}$.

Example 6. Repeat Example 5 for a loss of 1 proton along $L$. Press D. Read $H_L = 2.42 \times 10^{-16} \text{ Sv}$.
APPENDIX A5. Hewlett-Packard HP-97 Code for Calculating Dose Equivalent from Point, Extended and Infinite Uniform Line Sources by Approximate Methods

This program, written for the Hewlett-Packard HP-97 calculator, solves the following equations. First, assuming a point source:

1. \( H(\pi/2) = \pi R^{-2} \exp(-2.3\pi/2) \exp(-\varepsilon) \) (Sv per proton); (A5-1)

2. \( H(\theta) = \pi R^{-2} \sin^2 \theta \exp(-2.3 \theta) \exp(-\varepsilon \csc \theta) \) (Sv per proton); and (A5-2)

3. \( H(\theta_m) = C(\varepsilon) H(\pi/2) \) (Sv per proton), where \( C(\varepsilon) = 2.2 \varepsilon^{-0.245} \). (A5-3)

\( C(\varepsilon) = 2.2 \varepsilon - 0.245 \). The expression \( \varepsilon_m = (55.9 + \ln \varepsilon) \) (degrees) is used to estimate \( \varepsilon_m \), and \( Z(\varepsilon_m) = (R/\tan \varepsilon_m) \) (meters) is used to define the distance along the outer surface of the shield at which the maximum dose equivalent, \( H(\varepsilon_m) \), is observed relative to the position at 90° to the loss point. Then, assuming an extended uniform line source of length \( L \):

4. \( H_L = \frac{S L H(\varepsilon_m)}{B(\varepsilon_m)} = \frac{R H(\varepsilon_m)}{B(\varepsilon)} \) (Sv per proton per meter). (A5-4)

Here, \( H(\varepsilon_m) \) is multiplied by \( S L \) so that the number of protons lost is the same for \( H(\varepsilon_m) \) as for \( H_L \). \( S \) is assumed in this program always to be equal to one proton per meter along \( L \). The function, \( B(\varepsilon) \), is approximated by

\[ B(\varepsilon) = 0.44 + 0.12 \varepsilon - 0.0028 \varepsilon^2 \]  \hspace{2cm} (A5-5)

This calculation should be restricted to values of \( n > n_L \), where \( n_L \) is dependent on \( \varepsilon \). For values of \( n \) in the intermediate region, the restricted Moyer integral solution should be used (Appendix A4).

To Use the Program

Store in the following registers:

Register D \( \phi \) (degrees), any angle of interest
E \( \varepsilon = d/\lambda; \)
1 \( R(m); \)
2 \( L(m); \)
3 \( E, \) proton beam energy (GeV);
4 \( Z(m). \)

Press A to calculate \( H(\pi/2); \)
B \( H(\phi); \)
C \( H(\varepsilon_m); \)
D \( H_L. \)
Examples:

1. Calculate $H(\pi/2)$, $H(\theta_m)$ and $H(60^\circ)$ for the following parameters:

   $\lambda = 5$ (dimensionless);
   $R = 2$ m;
   $E = 10$ GeV.

   Find $\theta_m$ and $Z(\theta_m)$. Also find $H_L$ when $L = 8$ m and $Z = 6$ m:

   \begin{align*}
   H(\pi/2) &= 8.147964468 \times 10^{-17} \text{ Sv/proton} \\
   H(60^\circ) &= 9.399262676 \times 10^{-17} \text{ Sv/proton} \\
   H(\theta_m) &= 1.208437395 \times 10^{-16} \text{ Sv/proton} \\
   Z(\theta_m) &= 0.663 \text{ meters} \\
   \theta_m &= 71.672 \text{ degrees} \\
   H_L &= 2.491623494 \times 10^{-16} \text{ Sv per proton per meter.}
   \end{align*}

2. Calculate the dose equivalent for an "infinite" uniform source of $L = 10,000$ m.

   Let $R = 1$ m, $\lambda = 5$, and the point of measurement be halfway along the spill, $Z = 5000$ m. The proton beam energy is 25 GeV.

   $H_L = 1.037204566 \times 10^{-15} \text{ Sv per proton per meter.}$
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LIST OF FIGURES

Fig. 1. Diagram showing the accelerator geometry assumed in Moyer Model calculations and defining the symbols used. P(\(\theta\)) indicates a point on the shield outer surface at angle \(\theta\) from point loss and H(\(\theta\)) is the corresponding dose equivalent. Subscript m indicates maximum H. (a) longitudinal section; (b) cross section.

Fig. 2. Dose equivalent at \(\theta = \pi/2\), H(\(\pi/2\)), and maximum dose equivalent, H_m, on the shield outer surface, as functions of shield thickness (\(\lambda = d/\lambda\)). A point loss of N = 1 proton at E = 10 GeV is assumed, and normalization is to a transverse distance of R = 1 m.

Fig. 3. Dose equivalent on the shield outer surface as a function of angle with respect to beam direction (point source loss is assumed) relative to the same quantity at \(\pi/2\) (90°). Parameter \(\lambda\) is shield thickness in units of the attenuation length, \(\lambda\).

Fig. 4. Angle \(\theta_m\), at which maximum dose equivalent, H_m, on the shield outer surface occurs, as a function of shield thickness, \(\lambda = d/\lambda\). Point source loss is assumed.

Fig. 5. Angle \(\theta_m\), at which maximum dose equivalent, H_m, occurs on the shield outer surface as a function of shield thickness, \(\lambda\), for various spill lengths, \(\eta = L/R\). For extended sources, \(\theta_m\) is defined such that vertex is on beam axis where beam loss begins.

Fig. 6. Definition of the angles \(\alpha_1\) and \(\alpha_2\).

Fig. 7. Variation of dose equivalent on the shield outer surface as a function of position along the beam direction, for different spill lengths, \(\eta = L/R\). Origin is at start of beam spill. Same beam loss is assumed (N = 1 proton) for each curve. Parameters are: E = 10 GeV; R = a + d = 1 m; \(\lambda = d/\lambda = 5\); 0.002 \(<\eta < 4\).

Fig. 8. Maximum dose equivalent on the shield outer surface, H_m, for a line source of length \(\eta = L/R\), for one proton lost (N = 1) and \(\lambda = 5\), as a function of \(\eta\). The limit for \(\eta \rightarrow 0\) (point source) is indicated. The dashed curve is a hyperbola which approaches the true behavior for large \(\eta\). Values on abscissa \(\eta_p\) and \(\eta_x\) show where true H_m deviates from either limiting assumption by 10 percent. Shaded region shows amount of overestimation that can be made by assuming the minimum of either the point source or hyperbolic approximation.

Fig. 9. Same as Fig. 7 except that the beam loss per unit length is set constant (i.e., S = 1 proton per meter). In this case, area under each curve is proportional to beam spill length.
Fig. 10. Maximum dose equivalent, $H_m$, on the shield outer surface for a finite uniform beam loss of 1 proton/m over a spill length $n = L/R$, as a function of $n$. Curve is calculated with the parameters shown using restricted Moyer Integrals. The rising straight line is the asymptote obtained by assuming all beam loss occurs at a point. The horizontal dashed line is the limit for an infinite uniform line source. Intermediate region shows where actual prediction deviates from either the point-source or infinite-line-source approximations by more than 10 percent when $l = 5$. Shaded region indicates amount of overestimation that can be made by assuming the minimum of either the point source ($N = S \times L$) or infinite uniform line source assumptions.

Fig. 11. Percent overestimation of true maximum dose equivalent, $H_m$, on the shield outer surface, as a function of beam spill length $n = L/R$ (for $l = 5$), if either the infinite-line source or point-source approximation is assumed instead. Values $n_P$, $n_\infty$ indicate where 10 percent overestimate occurs. Curves are based on the data of Fig. 9.

Fig. 12. Function $B(\varphi) = n^{-1} H_m/H_\infty$, which relates the actual maximum dose equivalent, $H_m$, to that given by an infinitely extended uniform source having the same beam loss per unit length $S = N/L$. Solid curve: calculation; dashed curve: polynomial fit to calculated values.

Fig. A3-1. Function $C(\varphi) = H_m/H(\pi/2)$, which relates the maximum dose equivalent on the shield outer surface to that at $90^\circ$, is shown as a solid curve, the power law as a dashed curve, and the 5th order polynomial, practically indistinguishable from $C(\varphi)$, is shown by the points.
Fig. 1
Point source

\( E = 10 \text{ GeV} \)
\( N = 1 \text{ proton} \)
Point source

$F(\theta) \equiv H(\theta)/H(\pi/2)$

Fig. 3
Fig. 4

Point source

Angle of maximum dose equivalent

$\theta_m$ (degrees)

Shield thickness ($\ell$)

XBL 8411.6348
Fig. 5

Angle at which maximum occurs

$\theta_m$ (measured from start of spill) (deg)

Spill length parameter $\eta = L/R$

$\eta = 0.01$

$\eta = 0.5$

$\eta = 1$

$\eta = 2$

Shield thickness ($\ell$)
Finite uniform line source

Fig. 6  XBL 8411-6355
Fig. 7

Distance from start of beam loss (Z/R)

Dose equivalent at 1m (Sv)

- Start of beam loss

$N = 1$ proton

$\ell = 5$
Fig. 8

Maximum dose equivalent $H_m$ at 1 m (Sv)

Point source limit

Finite line source

Hyperbolic limit

$N = 1$ proton
$l = 5$

Spill length ($\eta$)

$5 \times 10^{-16}$
Infinite line source $H \propto S$

Point source $H(\theta_m) \propto N$

$S = 1$ proton m$^{-1}$
$\lambda = 5$

Region of validity of infinite uniform line source

Point source region

Intermediate region

Fig. 10
100 \times \left[ \frac{H_A - H_m}{H_m} \right]

Point source assumption (constant N)

Infinite line source assumption (constant S)

Fig. 11
\[ B(\ell) \equiv \frac{1}{\eta} \frac{H_m}{H_\infty} \]

\[ B(\ell) \approx 0.44 + 0.12\ell - 0.0028\ell^2 \]

\begin{figure}
\centering
\includegraphics{figure12}
\caption{Fig. 12}
\end{figure}

XBL 8411-6353
\[ C(\ell) \equiv \frac{H_m}{H(\pi/2)} \]

\[ 2.2 \ell^{-0.245} \]

- Fifth order polynomial

Fig. A-3.1
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