Simple heuristic or knowledge-based inference?
Model comparison of binary choice inference

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Abstract

We investigated the processes of making inference in binary choice tasks. We proposed statistical models for two simple heuristics and two knowledge-based inferences, and compared how well these models could explain the observed patterns of inferences. It was found that the best model for explaining choice patterns varied depending on the inference problem. In particular, results suggested that participants used a simple heuristic when they had difficulty in retrieving clues pertaining to the inference problem. In contrast, when participants could retrieve enough clues pertaining to inference problems, they made inferences based on these clues.

Keywords: simple heuristics; knowledge-based inferences; binary choice inferences; model comparison

Introduction

In the last decade, a highly controversial topic in research on judgment and decision-making has been the recognition heuristic (Goldstein & Gigerenzer, 2002). When the recognition heuristic is applied to a binary choice problem, the inference rule is described as follows:

“If one of two objects is recognized and the other is not, then infer that the recognized object has the higher value with respect to the criterion.”

(Goldstein and Gigerenzer, 2002, p.76)

For example, consider the following question, “Which city has the larger population, Tokyo or Chiba?” In this problem, the recognition heuristic predicts that when a person recognizes Tokyo but not Chiba, the person will infer that Tokyo has the larger population.

Many researchers have discussed the recognition heuristic from theoretical, empirical, and rational perspectives. For example, in the journal Judgment and Decision Making, a special issue on the recognition heuristic was published in 2010 and 2011, which included 24 papers in three volumes.

In particular, researchers have been interested in whether the recognition heuristic is actually used in making inferences. Some researchers have claimed that people often make inferences based on the recognition heuristic in binary choice situations, in which people recognize one of the objects in a pair1 (Goldstein & Gigerenzer, 2002; Pachur, Bröder, & Marewski, 2008; Pachur & Hertwig, 2006; Reimer & katsikopoulos, 2004; Snook & Cullen, 2006; Volz et al., 2006). In contrast, others have argued that although people make inferences using the recognition heuristic in some R-U pairs, people often make inferences using knowledge pertaining to the inference problem when such knowledge is available (Bröder & Eichler 2006; Hilbig, Pohl, & Bröder, 2009; Glöckner & Bröder, 2011; Newell & Fernandez, 2006; Newell & Shanks, 2004; Oesouonthornwattana & Shanks, 2010; Oppenheimer, 2003; Pohl, 2006; Richter & Späth, 2006).

The recognition heuristic can be applied only to R-U pairs. Thus, although many studies have accumulated evidence for both the arguments, they have focused on the processing of inference in R-U pairs. Then, how do people make inferences when they recognize both objects of the pair? Previous studies have assumed that in the case of R-R pairs, inferences are made by using knowledge (e.g., Goldstein & Gigerenzer, 2002; Hilbig, 2010). As described above, many studies have suggested that when people have knowledge in addition to mere recognition, inferences are affected by the knowledge. However, specific models of knowledge-based inferences have not been proposed to date.

Furthermore, people may make inferences using a simple heuristic such as the recognition heuristic, even in the case of R-R pairs. For example, Hertwig, Herzog, Schoolder, and Reimer (2008) proposed a fluency heuristic, which is an inference strategy using the retrieval fluency of objects, and Honda, Abe, Matsuka, and Yamagishi (2011) proposed a familiarity heuristic, which is the inference strategy using the familiarity of objects.

The goal of the present study was to examine binary choice inferences in not only R-U pairs, but also in R-R pairs. Brighton and Gigerenzer (2009) argued that when examining the validity of the recognition heuristic, alternative models should be proposed and compared. Accordingly, we approached the above issue by comparing models. In particular, we examined binary choice of population inferences, by comparing the validity of models representing a simple heuristic and knowledge-based inference.

Models of binary choice inference

For each pair of cities, the two cities can be ordered by their actual populations, and we name them accordingly. For example, if cities X and Y are presented, and the city X has a larger population than city Y, we call city X the “larger city,” and city Y the “smaller city.”

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1 Hereafter, depending on the recognition of objects in a pair, we use following abbreviations: R-U (Recognized-Unrecognized), R-R (Recognized-Recognized), and U-U (Unrecognized-Unrecognized).
Simple heuristic

We examined inferences in not only R-U pairs, but also in R-R pairs. Simple heuristics that can be applied to both R-U and R-R pairs are appropriate for this purpose. Hence, we used the fluency heuristic and the familiarity heuristic for modeling simple heuristics.

Fluency heuristic. This model is based on Hertwig et al. (2008). They have defined the model as follows: If two objects, a and b, are recognized, and one of the two is fluently retrieved, then infer that this object has the higher value with respect to the criterion (p. 1192). Based on this definition, the response pattern in pair i in the binary choice task is represented by the following variable, \( \text{Flu}_i \):

\[
\text{Flu}_i = \frac{\text{Flu}_{Li} - \text{Flu}_{Si}}{\text{Flu}_{Li} + \text{Flu}_{Si}}
\]

where \( \text{Flu}_{Li} \) and \( \text{Flu}_{Si} \) represent the fluency for the larger and smaller cities in pair i.

Familiarity heuristic. This model is based on Honda et al. (2011). It assumes that when there is a difference in familiarity between two cities, people will infer that the more familiar city has the larger population. Response patterns in pair i in the binary choice task are assumed to be represented by the following variable, \( \text{Fam}_i \):

\[
\text{Fam}_i = \frac{\text{Fam}_{Li} - \text{Fam}_{Si}}{\text{Fam}_{Li} + \text{Fam}_{Si}}
\]

where \( \text{Fam}_{Li} \) and \( \text{Fam}_{Si} \) represent the familiarity for the larger and smaller cities in pair i.

Models: Knowledge-based inference

Some researchers have pointed out that people use available knowledge in binary choice inferences. Although it might be difficult to clarify the specific processes of knowledge-based inferences, previous findings have been straightforward: When people have knowledge pertaining to population size, that knowledge directly affects the inference processes. For example, when one knows that a city has a professional soccer team, implying a larger population, that city is more likely to be selected in a binary choice. Contrarily, when one knows that a city is rapidly aging, implying a declining population, it is unlikely to be chosen.

We assumed that people are clued to retrieve information when making inferences about the population of cities, as to whether the cities are large, or small. Based on this assumption, we modeled two knowledge-based inferences, a Z-score model and a Decision by Sampling model.

Z-score model (ZM). This model assumes that people have correct knowledge of absolute population sizes of cities, and that the availability of knowledge depends on their familiarity with cities. In each list, we calculated the z-scores for 15 cities (See Table 1). Using the z-scores, the response pattern in pair i is represented by the following variable, \( \text{ZM}_i \):

\[
\text{ZM}_i = \frac{\text{Fami}_{Li}z_{Li} - \text{Fami}_{Si}z_{Si}}{\alpha(z_{max} - z_{min})}
\]

where \( z_{Li} \) and \( z_{Si} \) denote z-scores for the larger and smaller cities in the pair i, and \( z_{max} \) and \( z_{min} \) denote maximum and minimum z-scores in the list. In this equation, \( \alpha \) is a positive constant determining the range of this variable. The most important feature of \( \text{ZM}_i \) is that when people are familiar with a small city (i.e., the city with a negative z-score), that city is unlikely to be chosen. This feature differs from the familiarity heuristic, which assumes that familiarity leads to choice.

Decision by Sampling (DbS). As noted above, ZM assumes that people have correct knowledge about population sizes. However, this assumption may be inappropriate. Hilbig, Pohl, and Bröder (2009) suggested that participants have knowledge about the relative ranks of a criterion value within a given set. Thus, we have proposed another model of knowledge-based inference, Decision by Sampling (DbS), which was originally proposed by Stewart, Chater, and Brown (2006).

In this model, the subjective value of an object is determined by relative rank. In the present study, we assumed that knowledge about population sizes is determined by the relative rank in the list. The rank of each city in the list is calculated using following equation:

\[
r = \frac{R - 1}{14} - 0.5
\]

where \( R \) is the rank of the population in a list in an ascending order (the specific value of \( R \) for each city is shown in Table 1). The value, \( r \), ranges from -0.5 to 0.5 depending on population size. Using \( r \), the response pattern in pair i is represented by following variable, \( \text{DbS}_i \):

\[
\text{DbS}_i = \frac{\text{fam}_{Li}r_{Li} - \text{fam}_{Si}r_{Si}}{\beta}
\]

where \( r_{Li} \) and \( r_{Si} \) denote \( r \) for the larger and smaller cities in pair i, respectively. Value \( \beta \) is a positive constant determining the range of this variable. The feature of this variable is analogous to \( \text{ZM}_i \). Thus, when people are familiar with a small city, it is unlikely to be chosen in binary choice inferences.

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\(^2\) In the present study, fluency is operationally defined using elapsed time for city recognition. See the Results and Discussion.

\(^3\) We set the minimum values of \( \text{Fam}_{Li} \) and \( \text{Fam}_{Si} \) with zero. See the Results and Discussion section.
 Experiment

 Method

 **Participants.** Japanese undergraduates from Wako University, Keio University, and Toho University (n = 79; 26 men and 53 women) participated in this experiment. They were given course credits for participation.

 **Tasks and Materials.** We conducted three tasks, a binary choice task of population inference, measurement of familiarity, and a recognition task.

 In the binary choice task, participants were presented with two Japanese city names and were asked to choose the city that they thought had the larger population.

 In this task, we used Lists A and B (Table 1) that were used in Honda et al. (2011). Honda et al. (2011) suggested that these two lists differ in availability of knowledge pertaining to populations, and participants made inferences using different strategies. Inference patterns for List A could be well explained by the familiarity heuristic. In contrast, patterns of population inference using List B could be well explained by differences in the actual populations, indicating that participants had a good knowledge about the population sizes of cities on List B.

 For the recognition test, participants were asked whether they knew the 30 cities that were presented in the binary choice task. For the measurement of familiarity, participants were asked how well they knew the 30 cities presented in the binary choice task.

 **Procedure.** The three tasks were individually conducted using a computer, and they were presented in the following order: binary choice task, recognition task, and measurement of familiarity.

 In the binary choice task, when participants pressed the key named, “Next,” the focal point “*” was presented for 2000 ms on a computer screen. Then, two city names were presented on the screen. Participants responded to the question by pressing one of two keys assigned to make a choice. Participants were instructed to respond as quickly and correctly as possible. Half of the participants first received 105 pairs from List A, then 105 pairs from List B. The other participants were presented the lists in the opposite order. The presentation order of the listed cities was randomized.

 In the recognition task, by pressing the key named, “Next,” the focus point “*” was presented for 2000 ms, then a single city name was presented. Participants responded to the question by pressing one of two keys named, “Recognized,” or “Unrecognized.” The time that elapsed between the presentation of city name and the participants’ key-press was recorded. Participants were instructed to respond as quickly and correctly as possible.

 For the measurement of familiarity, pressing the key named, “Next,” presented the participants with a city name. They were asked to rate their familiarity with the city using a 100-point scale shown on the screen. The scale ranged between (not know at all) on the far left to (know a lot) on the far right.

 The above three tasks took 45-60 minutes to complete.

 **Results and Discussion**

 In the following analysis, we operationally define familiarity with cities based on participants’ responses during the measurement of the familiarity task. Unrecognized cities were assigned a zero. Retrieval fluency for a city was operationally defined, based on the elapsed time recorded in the recognition task, as in Hertwig et al. (2008). It was assumed that the more quick the recognition response was, the more fluent was a participant’s retrieval of an object.

 The constant values, \( \alpha \) and \( \beta \), were set at 100. The range of \( ZM_i \) and \( DbS_i \) are \(-1 < ZM_i \leq 1 \) and \(-0.5 \leq DbS_i \leq 1 \).


<table>
<thead>
<tr>
<th>List</th>
<th>Population</th>
<th>Z-score</th>
<th>DbS (r)</th>
<th>List</th>
<th>Population</th>
<th>Z-score</th>
<th>DbS (r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kawasaki-shi</td>
<td>479,486</td>
<td>2.364</td>
<td>0.500</td>
<td>Yokohama-shi</td>
<td>3,544,104</td>
<td>2.608</td>
<td>0.500</td>
</tr>
<tr>
<td>Machida-shi</td>
<td>405,142</td>
<td>1.588</td>
<td>0.429</td>
<td>Osaka-shi</td>
<td>2,506,456</td>
<td>1.351</td>
<td>0.429</td>
</tr>
<tr>
<td>Kohriyama-shi</td>
<td>334,756</td>
<td>0.853</td>
<td>0.357</td>
<td>Nagoya-shi</td>
<td>2,145,208</td>
<td>0.913</td>
<td>0.357</td>
</tr>
<tr>
<td>Takasaki-shi</td>
<td>317,686</td>
<td>0.675</td>
<td>0.286</td>
<td>Sapporo-shi</td>
<td>1,869,180</td>
<td>0.579</td>
<td>0.286</td>
</tr>
<tr>
<td>Tsu-shi</td>
<td>283,167</td>
<td>0.315</td>
<td>0.214</td>
<td>Kobe-shi</td>
<td>1,498,805</td>
<td>0.130</td>
<td>0.214</td>
</tr>
<tr>
<td>Sasebo-shi</td>
<td>260,348</td>
<td>0.077</td>
<td>0.143</td>
<td>Kyoto-shi</td>
<td>1,392,746</td>
<td>0.002</td>
<td>0.143</td>
</tr>
<tr>
<td>Hachinohe-shi</td>
<td>248,776</td>
<td>-0.044</td>
<td>0.071</td>
<td>Fukuoka-shi</td>
<td>1,352,221</td>
<td>-0.047</td>
<td>0.071</td>
</tr>
<tr>
<td>Matsumoto-shi</td>
<td>223,472</td>
<td>-0.308</td>
<td>0.000</td>
<td>Hiroshima-shi</td>
<td>1,141,304</td>
<td>-0.303</td>
<td>0.000</td>
</tr>
<tr>
<td>Hitachi-shi</td>
<td>201,607</td>
<td>-0.536</td>
<td>-0.071</td>
<td>Sendai-shi</td>
<td>998,402</td>
<td>-0.476</td>
<td>-0.071</td>
</tr>
<tr>
<td>Yamaguchi-shi</td>
<td>187,539</td>
<td>-0.683</td>
<td>-0.143</td>
<td>Chiba-shi</td>
<td>905,199</td>
<td>-0.589</td>
<td>-0.143</td>
</tr>
<tr>
<td>Takaoka-shi</td>
<td>182,408</td>
<td>-0.736</td>
<td>-0.214</td>
<td>Niigata-shi</td>
<td>804,873</td>
<td>-0.710</td>
<td>-0.214</td>
</tr>
<tr>
<td>Imabari-shi</td>
<td>176,966</td>
<td>-0.793</td>
<td>-0.286</td>
<td>Hamamatsu-shi</td>
<td>786,776</td>
<td>-0.732</td>
<td>-0.286</td>
</tr>
<tr>
<td>Miyakonojo-shi</td>
<td>174,473</td>
<td>-0.819</td>
<td>-0.357</td>
<td>Kumamoto-shi</td>
<td>662,599</td>
<td>-0.883</td>
<td>-0.357</td>
</tr>
<tr>
<td>Ogaki-shi</td>
<td>159,661</td>
<td>-0.974</td>
<td>-0.429</td>
<td>Okayama-shi</td>
<td>659,561</td>
<td>-0.886</td>
<td>-0.429</td>
</tr>
<tr>
<td>Ashikaga-shi</td>
<td>159,040</td>
<td>-0.980</td>
<td>-0.500</td>
<td>Kagoshima-shi</td>
<td>601,675</td>
<td>-0.957</td>
<td>-0.500</td>
</tr>
</tbody>
</table>
Analysis of aggregated data

Similarity among models. Although the four models were based on different assumptions about the processes of inference, predictions of binary choice might be analogous. Thus, we analyzed the similarity among the four models.

For 105 pairs on Lists A and B respectively, we calculated $Flu_i$, $Fam_i$, $ZM_i$, and $DbS_i$ using mean familiarity or fluency for each city. Then we analyzed similarities among the four models in terms of correlation.

Table 2 shows the correlation coefficients among the four models. It was found that two heuristic models were analogous to each other. So were two knowledge-based inference models. Similarity between the heuristic and knowledge-based inference models varied depending on the list. With List A, heuristic models did not show a similarity with knowledge-based inferences. However, heuristic models showed a moderate similarity with the knowledge-based inference models in List B.

Predictions of choice patterns. Next, we examined the relationship between variables of the four models and choice patterns. For choice patterns, we used proportion of correct choice (i.e., rate of the larger city choice in a pair). For 105 pairs from Lists A and B respectively, we calculated the proportion of correct choice, and then examined correlations between the proportions and $Flu_i$, $Fam_i$, $ZM_i$, and $DbS_i$.

Figure 1 shows this relationship. As to the results of List A (the upper four figures), the two heuristic models predicted the choice patterns better than the knowledge-based inference models. In particular, a strong relationship between $Fam_i$ and choice patterns was observed. Hence, these results suggest that the familiarity heuristic explains inference processes in List A.

The results for List B showed a different picture. Of the knowledge-based inference models, $DbS_i$ showed a stronger relationship with the choice patterns than the others. Although $Fam_i$ also showed a relationship with choice patterns comparable with $DbS_i$, $DbS$ will be a more appropriate model than the familiarity heuristic to explain choice patterns in List B. List B consists of well-known cities, and therefore, it is quite likely that participants could retrieve many clues pertaining to population size (Honda, et al., 2011). As a result, participants could have knowledge about population size such as relative rank, which is assumed in $DbS$.

<table>
<thead>
<tr>
<th></th>
<th>List A</th>
<th></th>
<th>List B</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$Flu_i$</td>
<td>0.628</td>
<td>0.070</td>
<td>0.059</td>
<td>0.622</td>
</tr>
<tr>
<td>$Fam_i$</td>
<td>-</td>
<td>-0.113</td>
<td>-0.017</td>
<td>-</td>
</tr>
<tr>
<td>$ZM_i$</td>
<td>-</td>
<td>-</td>
<td>0.857</td>
<td>-</td>
</tr>
<tr>
<td>$DbS_i$</td>
<td>-</td>
<td>-</td>
<td>0.384</td>
<td>0.597</td>
</tr>
</tbody>
</table>

Figure 1. Relationship between choice pattern and variables of the four models. The upper four figures demonstrate results for List A, and the lower for List B.
Taken together, it was found that choice patterns were explained by different models depending on which list was used. In List A, heuristic models explained choice patterns better than knowledge-based inference models. In contrast, in List B, where participants were predicted to easily access many clues pertaining to the population size, choice patterns were explained by knowledge-based inference models better than heuristic models.

**Analysis of individual data**

**Similarity among models.** As in the aggregated analysis, we first analyzed the similarity of correlations among the four models. We calculated correlation coefficients for each participant, and examined the distribution of the coefficients.

Table 3 provides medians of correlation coefficients for the six pairs. In List A, results were analogous to those in the aggregated data analysis. However, the observed similarities were much stronger than with the aggregated data analysis. For List B, similarity between two knowledge-based inference models was observed as in the aggregated data analysis. However, similarities were not observed between the fluency heuristic and familiarity heuristic. Moreover, similarity was not observed between the two heuristics and two knowledge-based inferences.

Thus, we found that there was no similarity between the two models at the individual level, even when it was observed with the aggregated data.

**Predictions of choice patterns.** Next, we examined choice patterns using individual data. Specifically, we adopted a model-based approach using a multilevel-logistic regression analysis (Gelman & Hill, 2007). We compared the four models that predicted the choice patterns in terms of model fitting.

The four models, fluency heuristic, familiarity heuristic, ZM, and DbS are represented as follows:

\[
\begin{align*}
\log \frac{P_{\text{Flu}_i}}{1-P_{\text{Flu}_i}} &= a Flu_i + b \\
\log \frac{P_{\text{Fam}_i}}{1-P_{\text{Fam}_i}} &= a Fam_i + b \\
\log \frac{P_{\text{ZM}_i}}{1-P_{\text{ZM}_i}} &= a ZM_i + b \\
\log \frac{P_{\text{DbS}_i}}{1-P_{\text{DbS}_i}} &= a DbS_i + b
\end{align*}
\]

where \( P_{\text{Flu}_i} \) denotes the choice rate for the larger city in pair \( i \). Values \( a \) and \( b \) denote free parameters for weight and intercept, respectively. For Lists A and B, these four models were regressed, based on individual data from 79 participants. Then we assessed the goodness of fit of the models using log-likelihood values. Table 3 shows the result of this analysis. For List A, the familiarity heuristic resulted in the best fit among the four models. For List B, DbS was a better fit than the other models.

Hence, findings based on individual data were analogous to those based on aggregated data. It was found that participants changed their inference strategies, depending on the list. Choice patterns with List A were well explained by the familiarity heuristic, and those with List B were well explained by DbS.

**General Discussion**

In the present study, we examined the processes of inference regarding binary choices by model comparisons. We proposed statistical models for simple heuristic and knowledge-based inferences. It was found that the familiarity heuristic well explained the inference patterns for List A, and that DbS well explained those for List B. These findings suggest that people use different strategies depending on the situation.

The most important difference between Lists A and B is how easily people can retrieve clues relevant to population size. Given that List B consisted of well-known cities (Hon-

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4 In the following analyses, we excluded U-U pairs.
da et al., 2011), participants could easily retrieve clues pertaining to population size. For larger cities, participants could retrieve many clues suggesting that the population size would be large.Similarly, participants could retrieve many clues suggesting smaller population sizes for small cities. As a result, inference patterns could be well explained by knowledge-based inference models. Note that DbS explained the inference patterns better than ZM. This finding suggests that participants could have a good sense of population size in relative level. This is consistent with the findings of Hilbig, Pohl, and Bröder (2009).

On the other hand, participants might not have retrieved clues relevant to population size, and then used a simple inference strategy, such as the familiarity heuristic. In other words, a simple heuristic is the likely strategy when people have difficulty in retrieving clues relevant to inferences. This finding is consistent with the framework for understanding heuristics proposed by Shah and Oppenheimer (2008).

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**References**


