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Systematic Design for Roadway Interfaces with Application to Automated Highways

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SYSTEMATIC DESIGN FOR ROADWAY INTERFACES
WITH APPLICATION TO AUTOMATED HIGHWAYS

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ABSTRACT

This report provides interim results on the design of interfaces between automated highways and conventional street systems. The purpose here is to identify the strategic issues in interface design, and to provide preliminary analysis on just one of these issues (separation between highway entrance and exits). Future research will explore a full set of strategic issues in greater depth. The central concept explored in this report is how to design a roadway system that comprises multiple layers, some of which are designed for the purpose of accessibility, and others of which are designed more for the purpose of speed and capacity. With multiple layers, efficient functioning of the roadway system depends on creating effective roadway interfaces. The interface between an automated highway system and conventional roadways presents new challenges for interface designed, which are explored in this paper.
EXECUTIVE SUMMARY

Roadway systems provide the infrastructure for rubber-tire vehicles to efficiently travel between trip origins and destinations. By providing a smooth and obstruction-free travel surface, vehicles can move at high velocity, with low risk of damage. By providing traffic control devices, signage and structures, vehicles can also move at large volumes with a high level of safety. Taken as a whole, the roadway/vehicle system provides a mechanism for the movement of people and goods from place to place, with access to most trip origins and destinations, and with the flexibility for travel at almost any time of the day, week, month or year.

The economics of roadways, and their variability in demand, favor construction of multi-layered and inter-connected networks. Different network layers are designed to different standards and to perform somewhat different functions, though all provide the common function of mobility for a reasonably homogeneous class of vehicles. Yet interfaces have been constructed to provide a smooth transition between network layers, with little delay and inconvenience to travelers.

Recent research on vehicular automation presents new challenges for interface design. Currently, vehicles do not change their fundamental mode of operation when they move between network layers, and interfaces permit most (or all) vehicle types to move from one layer to another without restriction. With automation, vehicles may need to transition between human and computer control at the interface. Furthermore, certain roadways may be restricted to vehicles that are capable of automatic control.

One of the strategic issues in interface design – interchange separation -- is examined in this paper. Access, and usage, of the AHS (or highway) depend on this
spacing, along with the speed of the highway and the orientation of the highway relative to local streets. Slower highway speeds (relative to street speeds) cause vehicles to travel longer distances to reach the highway, and cause more vehicles to bypass the highway completely. A consequence is increased traffic on streets, especially on those that are more horizontally oriented relative to the highway.

Future research will examine highway orientation and interchange separation in greater depth through computer simulations of more detailed scenarios. In addition, models will be developed for determination of capacity requirements for the interface, lateral streets, critical intersections and transitional streets. Future research will explore specific case studies, including visits to sites for potential AHS roadways.
1. INTRODUCTION

Roadway systems provide the infrastructure for rubber-tire vehicles to efficiently travel between trip origins and destinations. By providing a smooth and obstruction-free travel surface, vehicles can move at high velocity, with low risk of damage. By providing traffic control devices, signage and structures, vehicles can also move at large volumes with a high level of safety. Taken as a whole, the roadway/vehicle system provides a mechanism for the movement of people and goods from place to place, with access to most trip origins and destinations, and with the flexibility for travel at almost any time of the day, week, month or year.

Accessibility and flexibility are primary advantages of roadway/vehicular systems. Disadvantages include their susceptibility to crowding and congestion, their harmful effects on the environment (e.g., emissions and noise) and their large space requirements. With respect to the latter point, safe vehicle spacing virtually mandates that most of the area used by freeway lanes remains unoccupied for most of the time (i.e., the space between vehicles exceeds the space occupied by vehicles). Except under congested conditions, freeway occupancy (percentage of time that a section of roadway is covered by a vehicle) rarely exceeds 10%; the figure is much smaller for local roadways.

The figures for vehicle occupancy are low for two reasons: (1) drivers are incapable of driving safely at high speed with short separation, and (2) the demand for most roadways is intermittent, time varying and, lastly, small relative to their capacity. As a point of comparison, the length of the United States’ roadway system exceeds 4

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1 The occupancy is even lower when one considers that most passenger cars carry only one or two people, utilizing, perhaps, just 10% of the space occupied by the vehicle. Occupancy is lower still when
million lane-miles, a distance sufficient to accommodate in excess of 1.4 billion automobiles, or about 7 times the number of vehicles owned in the country. On average, less than 5% of these vehicles are on the roadway at any given time, making the average vehicular occupancy well below 1%, a striking figure in light of the congestion problems facing many urban areas. The figure becomes even smaller if the lateral occupancy is factored in (i.e., the width of vehicles are far less than the width of the right-of-way occupied by a roadway).

The low demand for most roadways is a direct consequence of their accessibility. By providing connections to virtually all addresses, and by permitting dispersion of these addresses, it is impossible to accumulate high levels of demand on all roads. In fact the US Department of Transportation classifies more than 2/3 of roadway mileage as “local”, with the majority of the remainder falling in the classifications of rural collector or rural arterial. Only 6% of roadway mileage is classified as urban collector, urban arterial or interstate (the types of roadways that are most prone to congestion). Thus, most roadway miles are constructed for the purpose of accessibility, and not for the purpose of serving traffic volumes.

The economics of roadways, and their variability in demand, favor construction of multi-layered and inter-connected networks. Different network layers are designed to different standards and to perform somewhat different functions, though all provide the common function of mobility for a reasonably homogeneous class of vehicles. Because they accommodate less traffic, local roadways may have different surfaces and widths than collectors. An arterial may have more lanes than a collector, along with additional

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considering that roadway right-of-ways are much wider than the widths of the vehicles that they serve. All factors considered, event the busiest roadways are sparsely populated by people.
traffic control devices. And an interstate will have barriers and bridges to separate traffic. These design characteristics produce different attributes for each roadway layer, attributes that include (1) design capacity, (2) design speed, (3) weight limitation on vehicles, and (4) ability to access/egress local addresses. In this way a roadway can be designed to serve its expected demand for an appropriate cost.

Most roadway trips cannot be completed without traveling through more than one roadway layer. The juncture between a pair of layers constitutes a roadway interface. Roadway interfaces are designed to enable merging and diverging of traffic flows in a safe and efficient manner. An interface can range in complexity from a simple uncontrolled intersection to a fully connected highway interchange, equipped with surveillance and control devices. In all cases, an interface permits vehicles to diverge from the traffic stream in one layer and merge into the traffic stream of another, while preventing conflicts and collisions with crossing traffic.

From a strategic perspective, some of the important issues in the design of multi-layered roadway networks include:

- Density (i.e., roadway separation) for each network layer
- Geometric orientation of each network layer (e.g., grid, radial, etc.)
- Design attributes for each network layer (e.g., speed, capacity)
- Design attributes for network interfaces
- Density, frequency and provision of interfaces between each pair of network layers
Recent research on vehicular automation presents additional challenges for network design. Currently, vehicles do not change their fundamental mode of operation when they move between network layers, and interfaces permit most (or all) vehicle types to move from one layer to another without restriction. With automation, vehicles may need to transition between human and computer control at the interface. Furthermore, certain roadways may be restricted to vehicles that are capable of automatic control. These issues motivate the research in this paper.

The following sections represent an interim report on the design of interfaces between automated highways and conventional street systems. The purpose here is to identify the strategic issues in interface design, and to provide preliminary analysis on just one of these issues (separation between highway entrance and exits). Future research will explore the full set of strategic issues in greater depth. The concept of “roadway layers” is used throughout the paper to represent the functions performed by different types of roadways.

Section 2 reviews research on automated highway system (AHS) entrances and exits, with emphasis on capacity analysis and system design. Section 3 describes roadway interfaces in general, and Section 4 describes issues for AHS/street interfaces in particular. Section 5 describes alternative designs for AHS/street interfaces. In Section 6, interchange separations are analyzed. Lastly, Section 7 summarizes the paper and presents future research.
2. RELATED RESEARCH ON AUTOMATED HIGHWAYS

This section summarizes research on topics related to the design of interfaces between automated highway systems and conventional roadways. A good introduction and review on conventional highway/road interchanges can be found in TRB (1994), which describes geometrics, placement of traffic control devices and measures of performance. There is also a fairly large literature on driver/vehicle behavior around interchanges, such as acceleration and gap acceptance (e.g., Kou and Machemehl, 1997; Michaels and Frazier, 1989; Polus et al., 1985). The aggregate behavior of the traffic stream around merge points has also been studied with respect to weaving behavior (e.g., Wang et al., 1993; Moskowitz and Newman, 1963; Cassidy and May, 1991; Cassidy et al., 1989).

Because Automated Highway Systems would operate under computer control (with fewer random disturbances and less variation from vehicle to vehicle), the existing conventional models are unlikely to represent AHS behavior. Hall et al. (1996) and Hall and Li (1998) developed a series of models for analyzing queueing delays at highway entrances. These represent random arrivals of vehicles, and the delays that result from randomness under a range of operational concepts (e.g., spacing rules, mixed vehicles classes, etc.). Analytical models were also created to upper-bound throughput with mixed heavy-duty and light-duty vehicles. Ran et al. (1998) and Leight (1997) created and applied a model for simulating the entrance of automated vehicles through a dedicated on-ramp. The model utilizes car-following and platoon formation models to simulate vehicle movements at a fairly microscopic level, and is intended for application to platoon based operating strategies.
Relatively little has been written about the AHS exiting process. Ran et al (1997a,b) provide a conceptual analysis of street interface issues, including an intermodal strategy in which automobiles are parked in garages in the vicinity of an AHS, and occupants are then shuttled to their destinations in transit vehicles. The authors also propose a strategy in which through street traffic (i.e., traffic that does not enter or exit the highway) is diverted to parallel streets to accommodate the entering and exiting highway traffic. At the other end of the spectrum, Sachs and Varaiya (1996) and Varaiya (1995) specify a system for controlling vehicle maneuvers during entry and exit, which allows for transitions between manual and automated lanes to occur through defined gates. The work does not provide evaluative results, and does not compare alternative configurations for the street interface.

Castillo et al (1997) developed analytical flow and queue models for both entrance and exit processes. They argue that one of the challenges for AHS is to store queues that could spill back from exits onto the mainline. Because AHS operate under much higher density than conventional highways, these queues can spill back over a much greater distance than they do on conventional highways. The authors also model capacity losses that occur when vehicle spacing is altered to accommodate entrance and exit requirements. Other reports were written under the Precursor Systems Analysis program, including Obrien et al (1995), and Youngblood et al (1995). These works are rather preliminary, though they are useful in analyzing check-id-check-out facilities as well as acceleration/deceleration profiles.

At more of a system-wide level, Hall et al (1997) developed an analytical model to study capacity within a corridor, with arterials running parallel to an automated
highway. This model represents trip lengths and delay as a function of highway spacing, ramp spacing, and capacity concentration. The model is most useful in estimating the relationship between trip diversions and highway speeds over the width of an entire corridor. It also allows analysis of flow rates on streets as a function of their distance from highway entrances and exits. Hall (1995) also analyzes alternative AHS deployments, comparing dense networks of lower capacity AHS to sparser networks of higher capacity AHS. Cost-effectiveness analysis of alternative deployment strategies can be found in Hall (1996).
3. INTERFACE COMPONENTS

The ease by which vehicles (and their occupants) can transfer from one network layer to another is, perhaps, the single most important factor favoring roadway construction. Roadway interfaces enable people to travel from origin to destination without leaving their vehicle, and with minimal delay and inconvenience at interfaces. The same cannot usually be said for other forms of transportation, including those that travel on rails, or in water or air.

A transportation network interface comprises four elements, which we refer to as the infrastructure interface, vehicular interface, operational interface and managerial interface. The following describes each.

**Infrastructure Interface**

The infrastructure interface represents the physical intersection or interchange that joins roadways in different layers. It includes the roadways, ramps and turn-lanes that permit the vehicle to travel between one roadway and another. Infrastructure interfaces naturally vary in complexity, from simple uncontrolled intersections to elaborate grade-separated interchanges with ramps that connect all possible directions of travel. They also include the associated equipment (sensors, signals, etc.) mounted along the intersection or interchange.

Functionally, interchanges are designed to fulfill several objectives. First and foremost is to provide safety, especially for vehicles following intersecting paths. Second is to provide smooth flow, especially for merging and diverging traffic. Last is to provide speed, so that vehicles can change direction of travel with minimal delay turning from one roadway to another. Interchanges, in which roadway elements are constructed at
multiple elevations and control devices regulate traffic, can provide a higher level of functionality, but they obviously only make sense when traffic volumes are large. Hence, the design of infrastructure interfaces depends on the attributes of the roadway layers that are being connected.

**Vehicular Interface**

The vehicular interface represents changes in vehicle functionality that occur when a vehicle transitions between roadway layers. The obvious objective for a vehicle interface is either to eliminate changes, or to make any required changes as simple as possible for drivers and passengers. Fortunately, vehicles and roadways have been designed around the world to permit a high level of vehicular interoperability, thus obviating the need for functional changes at interfaces. A notable exception is right-hand and left-hand driving. But this isn’t even a drastic limitation, both because vehicles can be driven off their designed roadway and because left-hand roadway networks are reasonably isolated from right-hand roadway networks. Another example of a change in functionality is the transition from 2-wheel to 4-wheel drive when some all-terrain vehicles move from paved to unpaved surfaces.

By comparison, railway networks pose much greater challenges for vehicular interoperability. First, to operate on a set of tracks, vehicles must be designed for a specific gauge (wheel separation; a much more stringent requirement than for rubber-tire vehicles). Because gauges are not identical around the world, railway cars cannot freely move across all country boundaries, or sometimes between railroads within the same country. Second, some locomotives operate from electrified wire or track, and cannot
freely move to non-electrified tracks, or even tracks that are electrified by a different method. Third, cars and locomotives are sometimes designed to track-specific conditions, dictated by overhead clearances, tunnel widths, track speeds and track curvature.

Railway interoperability is sometimes addressed with dual-mode vehicles, such as locomotives that can operate either from electrified wire or from diesel, or wheel sets (also called “trucks”) that can be adjusted to multiple track gauges. Interoperability can also sometimes be accommodated through exchange of vehicle parts. However, interfaces still require passengers, containers or freight to be transferred between vehicles on one network to vehicles on another before continuing on their journey.

**Operational Interface**

The operational interface represents changes in vehicle operation and control as they transition between roadway layers. The interface may produce a change in vehicle speed and direction, a change in how the vehicle interacts with roadside traffic control devices and surrounding vehicles, or a change in operations that are specific to the interface (such as a ramp meter). Driver attention may also change at the interface, as different types of hazards are encountered on different roadway layers, and because different speeds demand different levels of attention.

It is unrealistic to expect no change in vehicle operation at an interface, as roadway layers are designed for specific speeds and volumes. Instead, the interface should be designed to maintain an appropriate level of driver attention through the transition, and to provide consistent modes of operation for merging and diverging traffic streams. For instance, vehicles entering a traffic stream should be moving with a speed
and separation that is consistent with vehicles already in the traffic stream, and they should be alert for appropriate hazards.

**Managerial Interface**

The managerial interface represents changes in ownership, as well as strategic and tactical oversight, that occur at the boundaries between network layers. The managerial interface is in part jurisdictional, as local agencies are typically responsible for streets, and state agencies are typically responsible for highways. The managerial interface affects such issues as highway maintenance, highway investment, and policies for sharing information and coordinating operation. It also carries over to enforcement of traffic laws, issuance of licenses and permits, and clearance of incidents, for which jurisdictional boundaries may differ.

Returning to railways as a point of comparison, the managerial interface can pose an obstacle to interoperability. Ownership arrangements place limits on vehicle movements, as cars and locomotives (rarely) cannot move from one railroad to another without following prescribed interline agreements. Fairly elaborate policies have been created for routing cars across the tracks of other railroads, return of cars to the originating railroad and pricing for car movements. These sorts of agreements are not needed for roadways because roadway ownership has been decoupled from vehicle ownership. The vehicle owner manages the vehicle through the entire trip, and the infrastructure owner manages its own infrastructure while permitting the movement of all qualified vehicles through the roadway. Nevertheless, managerial interfaces are still
important, especially with respect to accommodating and controlling vehicle flows in areas of high demand.
4. INTERFACE ISSUES FOR AUTOMATED VEHICLES

Simple interfaces enable the roadway system to serve three objectives: accessibility to all addresses, speed on longer trips, and capacity to accommodate large volumes of traffic on fast roads. As automation is introduced in roadway vehicles, to further enhance capacity as well as safety, it will be highly desirable to retain mobility for moving between roadway layers. Yet automation presents special challenges for all aspects of the network interface. These issues are summarized here.

Infrastructure Interface

As currently conceived, automated travel would be restricted to specially designed roads that are isolated from manually controlled vehicles, as well as from potential obstacles. This concept creates an additional network layer, as well as concomitant interfaces with conventional roadways. An automated roadway interface may also require additional infrastructure elements for vehicle and driver inspection and for regulating vehicle flows.

The infrastructure interface should also provide for a buffer to accommodate changes in flow patterns that occur as vehicles transition between networks. Flows on an automated highway may be somewhat bursty, in correspondence to the passage of platoons. Flows on connecting streets may be even more bursty, in correspondence to signal cycles at adjacent intersections. The interface should provide sufficient buffer to accommodate increases and decreases in queues that occur within these cycles.

The infrastructure interface must also provide sufficient capacity to disperse traffic that is exiting from the automated highway and to collect traffic entering the
automated highway. To circumvent local congestion on arterials and streets, designs may be needed to connect the automated highway to multiple streets at a single interchange.

**Operational Interface**

Vehicles entering an automated roadway must transition from manual to automated control (and the opposite upon exit). Once they enter the automated roadway, control may also become somewhat more centralized, as vehicles are instructed to travel at a particular speed or within a particular lane. Upon entry, the sensing requirements will also change. Manually operated vehicles rely in entirety on human sensing of the surrounding environment, which is decentralized by nature. Automatically controlled vehicles would depend on a mixture of vehicle-mounted sensors, roadway-mounted sensors and communication to and from vehicles.

The operational interface should also regulate traffic flows in a way that prevents queues from spilling back from one network layer to another in the event that buffer space is exhausted. This may include altering metering rates at highway entrances to prevent spillback to connecting streets, or regulating traffic signals near exits to prevent spillback onto the highway and blockage of highway flows. It may also be possible to divert traffic to alternative interchanges in the event that a first choice becomes overloaded.

**Vehicular Interface**

Automated vehicles would likely be designed for dual-mode operation, thus providing capability for both manual and automated control. It is a technical matter to
design the vehicle so that it can safely transition from one operating mode to another without incurring delay, and to provide necessary inspections to determine whether the driver is fit to resume control upon exit.

Managerial Interface

Personnel employed to maintain and operate an automated highway would need skills that are atypical of highway departments, especially in the areas of computer hardware, software and communication. Automated highways might also be financed through different mechanisms than conventional roadways. For these reasons, it may be preferable to separate their management, and ownership, from ordinary highway and street departments. Even if the automated highway is owned by a state highway agency, it is likely to be organized as a separate operating unit. In either case, an additional managerial interface is created.
5. DESIGN ALTERNATIVES FOR AUTOMATED HIGHWAY INTERFACES

The interface between the AHS and other layers of the roadway system can be defined along several dimensions, which we place in the decision hierarchy: (1) automation concept, (2) roadway layering, (3) interface concept, (4) interchange separation and placement, (5) buffer sizing, and (6) flow control. These are described here, along with the interface between the AHS and surrounding land uses, which affects design decisions.

Automation Concept

The highway/street interface depends on the automation concept, which is defined by fundamental design decisions, such as control hierarchy (e.g., which decisions are made locally, at roadside or centrally), sensing and communication capabilities, permissible variations in vehicle design (e.g., size, weight and performance standards), car-following methods and standards (e.g., platooned versus free-agent control, and separation as a function of velocity), mixing of automated and non-automated vehicles, and separation of automated vehicles from potential hazards.

Roadway Layers

The AHS can be viewed as a distinct roadway layer that is interfaced to other layers at defined locations, through defined methods. AHS can be constructed independently of conventional highways, or designed to coexist in some manner (e.g., share right-of-way, operate on adjacent lanes; or possibly even operate within existing roadway layers). The AHS can be constructed to interface solely to a street layer, solely
to a highway layer, or to some combination of highways and streets. If the interface is through streets, then the AHS is functionally segregated from conventional highways, though they may still share right-of-way. If the interface is through highways, then the entrance process entail two steps, first entering the conventional highway and second entering the AHS from the conventional highway.

**Capacity**

The capacity of the street/highway system as a whole depends on the capacities of the individual roadway layers combined with the capacity of the interfaces that join the layers. No matter how the roadway system is layered, capacities should be balanced at interfaces, so that receiving roads can accommodate traffic leaving another layer, and vice versa. This depends on a rather complex interaction between several elements, listed below:

- **Interface Capacity**: represents the localized capacity of the interface that connects conventional roadways to the AHS, and vice versa.
- **Aggregate Lateral Capacity**: represents the combined capacity of streets used to accommodate traffic that crosses the highway, enters the highway or exits from the highway.
- **Critical Intersection Capacity**: represents the capacity of intersections located in the vicinity of interfaces, which are likely to experience the largest traffic flows.
- **Transitional Capacity**: represents the capacity of intersections and parallel roadways to properly position traffic for access from AHS to end destination, or from origin to AHS.
Parking Capacity: represents the capacity of parking facilities to accommodate traffic flows traveling to and from the automated highway.

As indicated by these elements, traffic flows are not evenly distributed across the street system. Any of the elements (or the AHS itself) can prove to be the system bottleneck that limits capacity of the roadway system.

Interface Concept

The interface concept defines the actions that occur at the interface, along with infrastructure and vehicular characteristics that enable these actions. Likely actions include:

- Inspection of vehicle for fitness to enter new roadway layer
- Inspection of driver for fitness to enter new roadway layer
- Regulation of vehicle release based on desired flow characteristics
- Regulation of vehicle speed for safe vehicle and roadway following, and safe merge and diverge from traffic streams.

The interface concept is derivative from the automation concept and the roadway layering, in that actions must be taken at the interface to prepare vehicles and drivers for operation on the AHS itself (and vehicles/drivers must be returned to their prior state upon exit). If, for instance, vehicles travel in platoons on the AHS, then the entrance must help vehicles form platoons, or prepare them to form platoons upon entry. These
steps can only be accomplished through controlling the release of vehicles and insuring that vehicles and drivers are fit to join platoons.

**Interchange Separation and Placement**

Interchange separation affects the performance of both the AHS and streets. Larger separations force vehicles to travel longer distances on streets to access entrances, and to reach destinations after exiting from the AHS, thus adding to traffic levels on streets. They also cause traffic to be concentrated in a smaller number of locations, creating congestion on the streets surrounding exits and entrances.

Large separations are, however, potentially beneficial to AHS operation, by reducing the occurrence of weaving (lane changes in vicinity of exits and entrances), as well as capacity losses as traffic streams merge and de-merge. Highways with larger separations also tend to have longer trip lengths, which can also be beneficial to capacity as weaving may be reduced.

**Buffer Sizing**

Queueing can occur at network interfaces, due to the merging of traffic streams and, potentially, the inspection of vehicles for readiness to enter a new traffic stream. The buffer is the maximum number of vehicles that can queue without causing a disturbance in upstream traffic flows. Buffers may be needed both at entrances to an AHS (e.g., to prevent disruption of street traffic) and exits from an AHS (e.g., to prevent disruption of AHS traffic).
Buffers are created through the construction of physical facilities to accommodate queued vehicles. They require investment at the time the AHS is built, both in terms of construction expenditures and in the acquisition and allocation of space. Buffer sizes can be physically constrained by surrounding land-uses, especially in areas where buildings are located in the immediate vicinity of highways. Buffer sizes can be defined both in terms of the amount of space that they occupy and the number of vehicles they can accommodate. The latter depends on the sizing of vehicles and their spacing while in queues, which in turn depends on the speed of vehicles while they are in queue.

**Flow Control**

Buffers can be regulated to prevent overflows, and the subsequent disruption of traffic. Regulation entails dynamic control of the rates at which vehicles enter and exit the queue as a function of the state of the queue. The release rate from an entrance queue depends on the passage of vehicles on the AHS, along with the allocation of capacity on the AHS for downstream entrances. When a buffer fills, it may be possible to reallocate capacity that is reserved for downstream, thus allowing vehicles to be released at a larger rate from the buffer. It might also be possible to reduce the upstream rate, providing more unused capacity at the entrance to accommodate additional vehicles. In addition, platoon size and composition might be changed to increase release rate or, if different vehicle classes wait in different queues, release rates can be increased in the queue that is in danger of overflow. Lastly, signal timing at nearby intersections can be used to control the rates at which vehicles enter the buffer, especially with respect to cyclic variations in queue sizes.
With respect to AHS exits, the release rate is dependent on traffic flows, and signal patterns on streets. By controlling upstream signals, capacity can be reallocated from local street traffic to the vehicles that are exiting from the AHS. By controlling downstream signals, additional capacity can be added for traffic exiting from the AHS.

Release rates can also be controlled at entrance queues with the objective of optimizing vehicle flows on the AHS itself. This can be used to prevent the formation of traffic congestion and queues on the AHS, or to optimize other AHS performance measures. Flow control can also moderate anticipated problems at downstream exits by holding back traffic at upstream entrances, or diverting traffic to alternate ramps when congestion materializes.

**Interactions with Surrounding Land Uses**

Surrounding land uses constrain AHS design in two ways: (1) they may limit the amount of space available to construct the AHS, and (2) once the AHS is in operation, they may limit the modes of operation. Though the mainline portion of an AHS is envisioned to be space efficient, land requirements are significantly greater at interfaces, due to the requirements of inspection, buffering, acceleration/deceleration, and extra ramps needed to inter-connect various roadways.

In operation, AHS will almost certainly impose externalities on surrounding land-uses, in such forms as noise, emissions and dust. These impacts are little different from conventional highways, but they do point to the need for creating buffer zones around the highway, along with mitigating the impacts with devices such as sound walls.
EFFECTS OF STREET ORIENTATION

This section provides preliminary analysis on the issues of interchange separation. This decision has the potential to affect both the performance of the AHS, and the performance of the street system that accommodates local traffic. Here the issue of travel distances on the street system is examined. Other design decisions will be evaluated in future research. In addition, interchange separation will be evaluated in greater depth in future research.

Hall (1997) examined the performance of roadway systems consisting of a series of parallel highways and a grid of identically oriented local streets. Local street mileage, highway mileage and congestion were evaluated as a function of highway separation, interchange spacing and various trip characteristics. In reality, street systems frequently have a different orientation than the highways, perhaps rotated by some angle as in Figure 1. Other roadway configurations also exist in real cities, but our focus here is on evaluating the effects of the rotation, as the issue if prevalent in many roadway systems.

We begin by considering a simple system comprising a single highway, along with a dense network of homogeneous streets. The spacing between entrance and exit ramps is assumed to be very small, allowing highway access from any point on the street system without backtracking. Let:

\[ a = \text{local street speed, as a proportion of speed on highways} \]
\[ x = \text{vertical distance from trip origin to highway} \]
\[ y = \text{vertical distance from highway to trip destination} \]
(y is positive if destination is below highway, negative otherwise)
\[ z = \text{horizontal distance between trip origin and trip destination} \]

\[ \text{(z is positive if destination is to the right of origin, negative otherwise)} \]
\[ 8 = \text{rotation angle for highway relative to streets.} \]

Without loss in generality, 8 is assumed to be less than or equal to 45° in the following analysis (route lengths exhibit a cyclic pattern over 45° intervals). We assume that the highway is no slower than streets and therefore \( a \leq 1 \).

For any trip, a traveler has the option to travel to his destination entirely by street, or alternatively use the highway for some portion of the trip. If the highway is used, the traveler must also select a place to enter the highway and a place to exit from the highway (streets are always used for highway access and egress). We shall assume that the traveler makes these choices with the objective of minimizing travel time.

Travel by street is rectilinear. If a trip is entirely by street, the total travel time is defined by the sum of the horizontal and vertical distances (Figure 1):
\[ T_s = |z| + |x-y+z\tan(\theta)| \]

Highway trips, by contrast, comprise three segments: access via streets, travel by highway, and egress via streets. Access and egress can occur along either horizontal streets or vertical streets (Figure 2). With \( \theta \leq 45^\circ \), vertical access always minimizes street distance, and is therefore preferred for most origin/destination pairs. Nevertheless, horizontal access can still be optimal when it sufficiently reduces travel time in the highway portion of the trip to compensate for the added street travel (especially when 8 and \( a \) are large). Vertical streets are optimal for highway access, independent of origin.
and destination location, when the following condition holds (and are sometimes optimal when the condition does not hold; Figure 2):

Vertical Access Always Optimal if \( a < \cos(\theta) - \sin(\theta), \theta < 45^\circ \) \hspace{1cm} (2)

As an illustration, Figure 3 plots the “breakeven angle” (value of \( \theta \) for which Eq. 2 is an equality) as a function of \( a \). When the highway is very fast (small \( a \)), vertical (i.e., shortest distance to highway) access is always optimal for most rotation angles; for slower highways, horizontal access is utilized some of the time, except when the rotation angle is very small (streets and highway have nearly the same orientation).

**Route Choice**

Whether it is optimal to use the highway for a trip depends on the relative distance between the origin and destination, along with their positions relative to the highway. When \( a < \cos(\theta) - \sin(\theta) \) and \( \theta < 45^\circ \), access/egress is in the vertical direction, resulting in a travel time of

Vertical Access Travel Time: \( T_v = |x| + |y| + |z - \alpha \cos(\theta)| \) \hspace{1cm} (3)

Allowance for both horizontal or vertical access complicates the travel time calculation as it presents additional routing options. For a fixed origin with the location shown, Figure 4 divides the travel region into sections. Each section represents a set of potential destination locations, and each section defines a unique travel time equation (provided in Table 1). It should be noted that horizontal streets are only used for highway access when the destination falls in Section A or H, and horizontal streets are only used for highway egress when the destination falls in Section A or F. Even in these cases, horizontal streets are only used when \( \theta \) is sufficiently small to satisfy Eq. 1. Outside of
these sections, vertical street travel always equals |x|+|y|, and highway distance always equals |z|/\cos(\theta).

**Direct and Highway Regions**

It is not difficult to derive regions for which travel is entirely by streets (called the *street region*), and regions for which travel is in part by highway (called the *highway region*). In both cases, “region” refers to the destination’s location relative to a fixed origin.

We again, without loss of generality, limit analysis to $\theta \leq 45^\circ$. As illustrated in Figure 5, two distinct region shapes are possible. The cases are defined entirely by $a$ and $\theta$, and do not depend on the distance from the highway to the origin. The cases are evaluated in the following sections.

**Case 1: $a < \cos(\theta) - \sin(\theta)$**

In this case the street region falls entirely on the origin’s side of the highway, meaning that all destinations on the opposite side of the highway are reached in part by highway. The region’s boundaries are defined by three vertices:

- Point on the highway having the same horizontal coordinate as the origin.
- Two points having identical vertical coordinate as the origin, with horizontal coordinates displaced from the origin by:

  \[-2x/[1+\tan(\theta) - \alpha/\cos(\theta)], 2x/[1-\tan(\theta) - \alpha/\cos(\theta)] \} = \{b_1,b_2\} \]  

  \[ (4) \]
It should be observed that the right-hand boundary \((b_2)\) is greater or equal in magnitude than the left-hand boundary \(|b_1|\), due to the upward tilt in the highway’s orientation in that direction. For \(\theta = 0\), symmetry exists and the boundaries have identical magnitude equaling \(2x/(1-a)\). In the limit as \(a\) approaches \(\cos(\theta) - \sin(\theta)\), \(b_2\) increases without bound, and \(b_1\) approaches \(x/\tan(\theta)\) (situated exactly on the highway).

As a measure of the attractiveness of street routes, the width of the vertical portion of the street region \((|b_1| + b_2)\) is plotted in Figure 6 as a ratio to \(x\), illustrating these points.

**Case 2: \(a > \cos(\theta) - \sin(\theta)\)**

In this case, as shown in Figure 5b, the street region occupies two entire quadrants of the plane, plus additional sections defined by the tilt in the highway. Thus even destinations that are very far away, or on the opposite side of the highway, are better served entirely by streets, as the added circuity in highway access is too large to justify the available travel time savings.

**Comparing Sections**

Returning to the sections in Figure 4, Table 2 summarizes the cases where the highway provides the shortest time route. For sections A, C, F and H, the highway is always utilized for a portion of the trip, keeping to the assumption that \(a \leq 1\). The highway is used in Section B if \(a \leq \cos(\theta) - \sin(\theta)\). In sections D, E, G and I, the
highway is used if the destination falls outside the street region of Case 1 (it is never used for Case 2), as indicated by the equations provided.

**Ramp Placement**

In reality, highway access and egress can only occur at distinct points defined by highway ramps. The spacing between these ramps affects travel time and route choice, as infrequent ramps necessitate more street mileage and greater trip circuity, making highway paths less desirable. In the limit, as the spacing between ramps approaches zero, travel time and optimal routes are identical to those in the prior sections. But when ramps are infrequent, travelers will be affected in the following ways:

1) Travelers will enter/exit the highway at different locations
2) The average distance traveled on streets to/from highway ramps will increase.
3) More travelers will find it advantageous to complete their trip entirely on streets.

Without loss in generality, suppose that a highway passes through the point \((0,0)\).

Further suppose that highway ramps are sequentially number from 1 to n, with coordinates \((0,0),(x_2,y_2), \ldots, (x_n,y_n)\) (Figure 7). Lastly, let \((v_x,v_y)\) represent the coordinates of the vertical projection of the origin onto the highway (Figure 6) and \((l_k,h_y)\) represent coordinates of the horizontal projection.
Theorem: An optimal (i.e., shortest time) path can be found from the street-only path along with highway paths that include the following four alternative ramps for entering the highway:

A. \( \max \{(x_n, y_n) \mid y_n \leq h, \} \), designated as \((x_A, y_A)\)

B. \( \min \{(x_n, y_n) \mid y_n \geq h, \} \), designated as \((x_B, y_B)\)

C. \( \max \{(x_n, y_n) \mid x_n \leq v_y, \} \), designated as \((x_C, y_C)\)

D. \( \min \{(x_n, y_n) \mid x_n \geq v_y, \} \), designated as \((x_D, y_D)\)

We call these the set of candidate ramps for highway entry.

**Proof.** Consider three possible contradictions to the theorem: (1) The highway is accessed at a ramp with lower number than ramp \(A\), (2) The highway is accessed at a ramp with number between ramp \(B\) and ramp \(C\), and (3) The highway is accessed at a ramp with number higher than ramp \(D\). (No ramps exist between \(A\) and \(B\) or \(C\) and \(D\), so these three cases are exhaustive.)

As shown in Figure 8, Case (1) is clearly non-optimal, as it substitutes street travel for a shorter route by the faster highway. Similarly, Case (3) is also non-optimal, as it also substitutes a longer street route for a shorter highway route.

For case (2), consider the example in Figure 9. Suppose that the freeway can be entered at any point between ramps \(B\) and \(C\), and that the position of the entrance point is designated \(\{x_B + m, y_B + m \tan(\theta)\}\) (\(m\) is the horizontal separation between ramp \(B\) and the intermediate point). Potentially, this entrance point could be used to travel to the left on the highway or to the right. However, it is clearly non-optimal to travel to the right, as...
the path would entail backtracking (i.e., greater street mileage than alternative paths without a commensurate reduction in highway mileage).

If travel occurs to the left of the intermediate entrance point, the exit can be between B and the intermediate point, or to the left of B. Suppose first that the destination falls between B and the intermediate entrance point. In this case the entrance point will still be non-optimal. When \( a > \cos(\theta) - \sin(\theta) \), any point on the highway between B and C falls in the street region, making a street only route preferable. When \( a < \cos(\theta) - \sin(\theta) \), any point on the highway between B and C can be reached in shorter (or equal) time by accessing the highway at ramp C. Thus, in either case an alternative path is no worse than using the intermediate point.

Finally, consider a highway exit to the left of point B, again for case (2). Then total travel time is the following function of m:

\[
T(m) = k + (x_C-x_B-m) + m\tan(\theta) + \alpha m/\cos(\theta)
\]  

(6)

Where \( k \) is a constant representing the time traveled from ramp B to the destination. Eq. 6 can be optimized by taking the derivative of \( T(m) \) with respect to m:

\[
\frac{dT(m)}{dm} = -1 + \tan(\theta) + \alpha/\cos(\theta)
\]  

(7)

The derivative is a constant, meaning that \( T(m) \) is optimized at an extreme point, either ramp B or ramp C. Hence, no ramp located between B and C can be preferred to the better of B or C, when the exit ramp is to the left of ramp B. This completes case (2), thus proving that no entrance ramp can be better than the best of A, B, C or D.

The value of the derivative in Eq. 7 also determines whether ramp C is preferred to ramp D, or vice versa:
Ramp C is Preferred to D if \( a > \cos(\theta) - \sin(\theta) \) \hspace{2cm} (8a)

Ramp D is Preferred to C if \( a < \cos(\theta) - \sin(\theta) \) \hspace{2cm} (8b)

The similarity of Eq. 8 to the cases defined by Equation 2. Specifically, when \( a \) is large (i.e., freeway is relatively slow) or when \( \theta \) is large (highway orientation has large angular displacement relative to the street system), then entry at ramp B is preferred for destinations to the left of B, increasing travel on horizontal streets with a reduction in total trip length. For smaller values of \( a \) and \( \theta \), ramp C is preferred for destinations to the left of B, with an increase in trip length, and the benefit of increasing the proportion of miles traveled by highway.

Though this section has addressed trip origins only, symmetry dictates that the same rules apply to trip destinations. Hence, there are at most 4 entrance ramps to consider and at most 4 exit ramps, producing no more than 16 distinct highway routes. When the origin and destination are close to the highway, the number of candidate entrance ramps and exit ramps reduces to two each, producing no more than 4 distinct highway routes (even less if origin and destination are close to each other).

**Selecting Among the Four Candidate Ramps**

The optimal entrance ramp among the four candidates (A,B,C and D) depends on the ultimate destination, along with \( a \) and \( \theta \). From the calculations in the prior section, it is relatively simple to construct “drawing regions”, representing the set of origin locations that would utilize each ramp. These regions depend on the ultimate destination.
Examples are shown in Figure 10, representing the cases \( a < \cos(\theta) - \sin(\theta) \) and \( a > \cos(\theta) - \sin(\theta) \), and for destinations to the far left of the origin. As noted in earlier sections, street-only routes are also preferred for some nearby destinations.

For the case \( a < \cos(\theta) - \sin(\theta) \), the drawing region is oriented vertically relative to the highway, and street travel is predominantly in the vertical direction. For the latter, the drawing region takes an L shape, with vertical travel dominant for origins above the highway and horizontal travel dominant for origins below the highway.

Travel time is identical for all points on the boundary (iso-time) line with respect to a pair of adjacent ramps. The iso-time line crosses the highway at a point satisfying:

\[
m(1 + \tan(\theta)) = (l-m)(1 + \tan(\theta)) + \alpha l / \cos(\theta)
\]

where \( l \) is the horizontal separation between adjacent ramps. The left-side of Eq. 9 represents travel time via the left ramp and the right-side represents travel time via the right ramp (minus a constant on both sides, representing travel time beyond the left ramp). Equation 9 can be reduced to:

\[
m/l = (1/2)(1 + \alpha/[\cos(\theta)+\sin(\theta)])
\]

It can be noted that for very fast highways (\( \alpha \) close to zero), the iso-time line intersects the highway midway between the terminals. In another extreme, when \( \Theta = 45'' \) and \( a = 1 \), the intersection is moved to the right at \( m/l = .853 \). And in still another extreme, when \( \Theta = 0'' \) and \( a = 1 \), the intersection moves all the way to \( m/l = 1 \). It can be further concluded that when the highway is rotated (\( \Theta > 0'' \)), it is still used when its speed is no faster than city streets, because route length can be shortened by using the highway.

When the highway is not rotated, it offers no advantage over streets if \( a = 1 \).
A circuity penalty is easily calculated as a function of the separation between adjacent ramps. This penalty represents the added travel time, relative to the alternative of continuous entry points along the highway. The penalty is naturally a linear function of the ramp spacing. The worst-case penalty occurs along the iso-time line at the point of intersection with the highway. Figure 11 shows that the worst-case penalty (represented as a ratio to the ramp spacing) increases as \( \theta \) increases, though at declining rate; the penalty decreases as \( a \) increases. Thus, frequent ramp spacing is most important for large orientation angles and fast highway speeds.

The iso-time line as a whole consists of a diagonal segment (45° angle), and horizontal and vertical segments, which terminate at the rectangle enclosing the pair of adjacent terminals. The pattern is similar for trip destinations to the right of the origin. It should also be noted that symmetry exists for orientation angles greater than 45°. That is, the patterns are identical to those shown, with the exception that horizontal and vertical axes are exchanged (e.g., an orientation of 50° relative to the horizontal axis is equivalent to a 40° orientation relative to the vertical axis).
The economics of roadways, and their variability in demand, favor construction of multi-layered and inter-connected networks. Different network layers are designed to different standards and to perform somewhat different functions, though all provide the common function of mobility for a reasonably homogeneous class of vehicles. Yet interfaces have been constructed to provide a smooth transition between network layers, with little delay and inconvenience to travelers.

Roadway interfaces consist of four components: (1) infrastructure (i.e., physical) interfaces, (2) vehicular interface, (3) operational interface, and (4) managerial interface. Automated vehicles present special challenges for all four components. Unlike conventional vehicles, they must undergo a fundamental change in their mode of operation at the interface. Special infrastructure facilities will be needed to support this mode change. Changes in vehicle operation must also occur under automation. Lastly, the personnel requirements for managing an automated highway are quite different than conventional roadways, likely necessitating a different managerial structure.

The interface design can be described along seven dimensions, which form a type of hierarchy: (1) automation concept, (2) roadway layering, (3) capacity, (4) interface concept, (5) interchange separation and placement, (6) buffer sizing, and (7) flow control. The automation concept, roadway layering, and design capacity impose requirements on the interface, which must be satisfied to ensure that vehicles are properly prepared before entering the highway. These in turn affect the requirement for interchange separation, along with the provision of buffers to accommodate flow fluctuations as well as real-time strategies for controlling flows entering and leaving the AHS.
One of the strategic issues in interface design – interchange separation -- was examined in this paper. Access, and usage, of the AHS (or highway) depend on this spacing, along with the speed of the highway and the orientation of the highway relative to local streets. Slower highway speeds (relative to street speeds) cause vehicles to travel longer distances to reach the highway, and cause more vehicles to bypass the highway completely. A consequence is increased traffic on streets, especially on those that are more horizontally oriented relative to the highway.

Future research will examine highway orientation and interchange separation in greater depth through computer simulations of more detailed scenarios. In addition, models will be developed for determination of capacity requirements for the interface, lateral streets, critical intersections and transitional streets. Future research will explore specific case studies, including visits to sites for potential AHS roadways.
7. REFERENCES


Transportation Research Board (1994). Interchange operations on the local street side: state of the art. Transportation Research Circular Number 430, Washington, D.C.


Table 1. Distance Calculations by Trip Segment for Trips that Use Highway

<table>
<thead>
<tr>
<th>Section (Figure 4)</th>
<th>Street Horizontal</th>
<th>Street Vertical</th>
<th>Highway</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: $\alpha \leq \cos(\theta) - \sin(\theta)$</td>
<td>$0$</td>
<td>$x+</td>
<td>y</td>
</tr>
<tr>
<td>A: $\alpha &gt; \cos(\theta) - \sin(\theta)(x+</td>
<td>y</td>
<td>)/\tan(\theta)$</td>
<td>$0$</td>
</tr>
<tr>
<td>F: $\alpha \leq \cos(\theta) - \sin(\theta)$</td>
<td>$0$</td>
<td>$x+y$</td>
<td>$z/\cos(\theta)$</td>
</tr>
<tr>
<td>F: $\alpha &gt; \cos(\theta) - \sin(\theta)y/\tan(\theta)$</td>
<td>$x$</td>
<td>$z/\cos(\theta)-y/\sin(\theta)$</td>
<td></td>
</tr>
<tr>
<td>H: $\alpha \leq \cos(\theta) - \sin(\theta)$</td>
<td>$0$</td>
<td>$x+y$</td>
<td>$</td>
</tr>
<tr>
<td>H: $\alpha &gt; \cos(\theta) - \sin(\theta)x/\tan(\theta)$</td>
<td>$y$</td>
<td>$</td>
<td>z</td>
</tr>
<tr>
<td>B,C,D,E,G,I</td>
<td>$0$</td>
<td>$</td>
<td>x</td>
</tr>
</tbody>
</table>

Table 2. Conditions When Highway is Utilized on Fastest Path

<table>
<thead>
<tr>
<th>Section</th>
<th>Highway Utilized</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Always</td>
</tr>
<tr>
<td>B</td>
<td>$\alpha &lt; \cos(\theta) - \sin(\theta)$</td>
</tr>
<tr>
<td>C</td>
<td>Always</td>
</tr>
<tr>
<td>D</td>
<td>$(x/</td>
</tr>
<tr>
<td>E</td>
<td>$(y/</td>
</tr>
<tr>
<td>F</td>
<td>Always</td>
</tr>
<tr>
<td>G</td>
<td>$(x/z) &lt; .5[1-\tan(\theta)-\alpha/\cos(\theta)]$</td>
</tr>
<tr>
<td>H</td>
<td>Always</td>
</tr>
<tr>
<td>I</td>
<td>$(y/z) &lt; .5[1+\tan(\theta)-\alpha/\cos(\theta)]$</td>
</tr>
</tbody>
</table>
Figure 1. Highway Rotation Relative to Street Systems
Figure 2. Alternative Highway Access Paths
Figure 3. Breakeven Angle for Vertical/Horizontal Access
Figure 4. Destination Sections That Define Route Lengths
Figure 5. Street and Highway Regions, $\alpha \leq \cos(\theta) - \sin(\theta)$
Figure 6. Street and Highway Regions, $\alpha \geq c \theta - \sin(\theta)$
Figure 7. Width of Street Region/\(x\); \(a \leq \cos(\theta) - \sin(\theta)\)
Figure 8. Location of Highway Ramps
Figure 9. Entrance Ramps Beyond A are Inferior
Figure 10. Entrance Ramps Between B and C are Inferior
Vertical Drawing Region

$$m/l = .5(1 + \alpha/([\cos(\theta)+\sin(\theta)])$$

Figure 11. Drawing Region for Ramp, $\alpha \leq \cos(\theta)-\sin(\theta)$
Figure 12. Drawing Region for Ramp, $\alpha \geq \cos(\theta) - \sin(\theta)$
Figure 13. Maximum Time Penalty Due to Ramp Spacing