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CONCERNING ANOMALOUS CONVERSION COEFFICIENTS OF DIPOLE TRANSITIONS

Sven G. Nilsson and John O. Rasmussen

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ABSTRACT

General aspects of the problem of anomalous nuclear-structure-dependent contributions to the internal-conversion process are considered with the qualitative conclusion that the most likely cases for observation of anomalies will be in highly retarded electric or magnetic dipole transitions. Formulas for an elementary theory of anomalous internal conversion for El transitions are given. Selection rules for the relevant nuclear matrix elements are given in the quantum numbers appropriate to spheroidally deformed nuclei (K, N, n_z, L, Σ). Similar selection rules for ML transitions are given on the basis of the anomalous operators previously derived by Church and Weneser.

The experimental data on dipole conversion coefficients of retarded transitions for odd-mass spheroidal nuclei are surveyed. It is noted that where retardation is ascribable to K forbiddenness (up to retardation from the single-proton rate by a factor of 10^9) no detectable anomalies are found, but where transitions are allowed by K-selection rules detectable conversion-coefficient anomalies may generally be found at retardations greater than 10^5 to 10^6 and are not found at lesser retardation. There are some exceptions to this general rule, though. From the present meager data the utility of selection rules in the asymptotic quantum numbers, N, n_z, Λ, and Σ, for anomalous-conversion matrix elements is open to question, although their utility in qualitatively explaining retardation of the radiative transitions is very evident.

The simple El theory is applied in an attempt to quantitatively explain the very anomalous 85-kev transition in Pa^{231}. Values of the two parameters in the simple theoretical expressions can be found to explain all three L-
subshell conversion coefficients. The magnitude of one parameter, the nuclear matrix element $\langle r^3 Y_1 \rangle$, is consistent with estimates from the single-particle model. However, the magnitude required of the other parameter is such as to suggest that there are important shortcomings in the theory.
CONCERNING ANOMALOUS CONVERSION COEFFICIENTS OF 
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INTRODUCTION 

The internal conversion process whereby a bound orbital electron is 
ejected during a nuclear electromagnetic transition generally occurs in parallel 
with photon emission. The ratio of conversion-electron ejection to photon 
emission is defined as the conversion coefficient, $\alpha$, with $\alpha_K$, $\alpha_{LI}$, etc., refer-
ring to conversion of K, L, or other electrons alone. Comparison of experimental 
absolute conversion coefficients or relative conversion coefficients (K/L ratios, 
L- or M-subshell ratios) with theoretical values constitutes the most generally 
useful means of determining gamma-transition multipolarities. 

The overwhelming contribution to the normal internal-conversion process 
comes from regions outside the nuclear volume. The original calculations by 
Rose et al.\textsuperscript{1} assuming a point nucleus represent therefore a good approximation 
in most cases, as the probability of the electrons penetrating the nucleus is 
small even for the heaviest nuclei. However, later calculations, by Sliv 
and Band\textsuperscript{2}, show some conversion coefficients (particularly ML) to be quite 
seriously affected when, instead of the point-nucleus model, they assume a nucleus 
of finite size but with all nuclear currents restricted to the surface. This 
correction is essentially a correction corresponding to improved electron wave 
functions. The intranuclear effects of the electron penetrating the nucleus 
are accounted for only in an average way by the model of Sliv et al.\textsuperscript{2} restrict-
ing the currents to the surface of the nucleus. The correction is, however,

\textsuperscript{*}This work was done under the auspices of the U. S. Atomic Energy Commission.  
\textsuperscript{**}On leave from University of Lund, Lund, Sweden.
of real importance, and recent experimental evidence on ML conversion coefficients agrees better with the latter theoretical values.\textsuperscript{3}

Church and Weneser\textsuperscript{4} have further suggested that anomalous, model-dependent conversion coefficients may occur for retarded ML transitions if one takes into account the distribution of currents throughout the nuclear volume. They have considered contributions to the internal conversion arising from integrals over the electron density within the nuclear volume, and they have shown that terms of this intranuclear contribution may obey certain selection rules in various approximate nuclear quantum numbers, which selection rules may be different or less restrictive than the selection rules governing both photon emission and the ordinary (electron outside the nucleus) internal-conversion contributions. Thus, if a transition is forbidden by the ordinary selection rules and is highly retarded but an intranuclear contribution to internal conversion is allowed, the conversion coefficient may be anomalous. There is an experimental case of an anomalous ML conversion coefficient in Ta\textsuperscript{181}, which we shall refer to later.

It has been known for some time that L-subshell conversion ratios for the 60-kev transition to ground in Np\textsuperscript{237} were not in agreement with theoretical values.\textsuperscript{5} More recently, evidence has been collected for other EL transitions in the heavy region. (cf. Asaro, Stephens, Hollander and Perlman\textsuperscript{6}) Some transitions exhibit $L_{I,II,III}$ conversion coefficients in agreement with the theoretical values of Rose, while others (notably the 85-kev transition to ground in Pa\textsuperscript{231}) exhibit anomalously large $L_I$ and $L_{II}$ conversion coefficients. Many of the low-energy EL transitions of odd-A spheroidal nuclei have rates measurable by fast-coincidence techniques ($t \geq 10^{-9}$ sec), and are thus greatly retarded from single-proton lifetime formula estimates.

From simple qualitative considerations one might suspect that intranuclear contributions to the internal conversion may be responsible for the anomalous EL conversion coefficients for $s_{1/2}$ and $p_{1/2}$ electrons. In order for such special contributions to be at all competitive, a necessary condition is that neither the initial nor final electron wave function be vanishingly small within the nuclear volume. Terms involving both initial and final electron states with $j = 1/2$ (i.e., $s_{1/2}$ or $p_{1/2}$) would seem more likely to
give anomalous intranuclear contributions than would terms involving \( j_1 \) or \( j_f > 1/2 \).

Table I lists the continuum states available for internal conversion from various bound states for different multipolarities.

Table I

<table>
<thead>
<tr>
<th>Multipolarity</th>
<th>bound state</th>
<th>El</th>
<th>M1</th>
<th>E2</th>
<th>M2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_{1/2} )</td>
<td>( p_{1/2}, p_{3/2} )</td>
<td>( s_{1/2}, d_{3/2} )</td>
<td>( d_{3/2}, d_{5/2} )</td>
<td>( p_{3/2}, f_{5/2} )</td>
<td></td>
</tr>
<tr>
<td>( p_{1/2} )</td>
<td>( s_{1/2} , d_{3/2} )</td>
<td>( p_{1/2}, p_{3/2} )</td>
<td>( p_{3/2}, f_{5/2} )</td>
<td>( d_{3/2}, d_{5/2} )</td>
<td></td>
</tr>
<tr>
<td>( p_{3/2} )</td>
<td>( s_{1/2} , d_{3/2}, d_{5/2} )</td>
<td>( p_{1/2}, p_{3/2}, f_{5/2} )</td>
<td>( p_{1/2}, p_{3/2}, f_{5/2}, f_{7/2} )</td>
<td>( s_{1/2}, d_{3/2}, d_{5/2}, f_{7/2} )</td>
<td></td>
</tr>
</tbody>
</table>

The combinations where anomalous intranuclear terms might have the best chance of being significant are those with \( j_1 = j_f = 1/2 \), namely, El and M1 conversion of \( s_{1/2} (K,L) \) and \( p_{1/2} (L_{II}) \) electrons.

In the sections following there is given an elementary theory of the anomalous electric dipole conversion coefficients and an examination of its implications. Comparison with experimental data is made, and the strong points and shortcomings of the theory discussed.

THEORY OF THE ANOMALOUS TERMS IN THE EL CONVERSION PROCESS

The probability for the ejection of an electron by the process of internal conversion is proportional to the matrix element \( |U_{fi}|^2 \) (in a perturbation approximation). We limit ourselves here to internal conversion accompanying El gamma radiation and write,

\[
U_{fi}(El) \sim \sum_M \int_0^\infty d\tau_e \psi_f^* O_e(h_1) \psi_i \int_0^\infty d\tau_n \phi_f^* O_n(j_1) \phi_i + \int_0^\infty d\tau_e \psi_f^* \int_0^\infty d\tau_n \phi_f^* O_n(h_1) \phi_i \int_0^\infty d\tau_e \psi_f^* O_e(j_1) \psi_i
\]

(1)
Here $\psi$ and $\phi$, respectively, are the electron and nucleon wave functions. The integration over the nucleon coordinates $\int_0^1 dr_n$ implies a complete angular integration over the angles of the nucleon position but an integration in the radial coordinates $r_n$ only out to the radius of the electron $r_e$. The first term thus accounts for the case when the electron is outside the nucleon. The second term represents the reversed situation in which the electron is inside the nucleon radius.

The nuclear operator is

$$0_n(j_1) = iW \frac{\partial}{\partial r} \left[ r j_1(Wr) \right] Y_{1-M},$$

(2)

where $j_1(Wr)$ is the regular spherical Bessel function and $W$ the energy of the gamma ray (for a more complete account of the derivations leading up to Eq. (13) see Ref. 8.) The quantity $0_n(j_1)$ is rather independent of the assumed interaction of the nucleon with the transverse photon field provided (a) that the assumed interaction is linear in the electromagnetic field, (b) that it is gauge-invariant, (c) that the long-wave length limit is approached (i.e., $Wr_n << 1$).

The electron operator is

$$0_e(j_1) = \left[-iW \frac{\partial}{\partial r} (rj_1) + \alpha r W^2 r j_1 \right] Y_{1-M},$$

(3)

where $j_1$ as before is a function of $Wr$. The matrix $\alpha$ is defined as

$$\alpha_r = \frac{(01)^2 r}{10}.$$

This expression (3) is derived on the basis that the interaction of the electron with the transverse photon field is

$$H_e = -e \Theta_e \cdot A(r_e),$$

(4)

where the matrix $\Theta = (01)^2$. 

The quantities $0_n(h_1)$ and $0_e(h_1)$ are obtained from Eq. (2) and (3) by everywhere replacing $j_1$ by $h_1$, the Hankel function of the first kind, corresponding to an outgoing wave.
Equation (1) may be conveniently rewritten as

\[ U_f(El) = \sum_M \left\{ \int_0^\infty d\tau_n \phi_f^* O_n(h_1) \phi_1 \int_0^\infty d\tau_e \psi_e^* O_e(j_1) \psi_1 \right. \\
+ \int_0^\infty d\tau_n \phi_f^* O_n(h_1) \phi_1 \int_0^n d\tau_e \psi_e^* O_e(j_1) \psi_1 \\
- \int_0^\infty d\tau_n \phi_f^* O_n(j_1) \phi_1 \int_0^n d\tau_e \psi_e^* O_e(h_1) \psi_1 \right\}. \]  

(5)

The first term in Eq. (5) now corresponds to the "point nucleus" case, i.e., it is the only surviving term if we let the nucleus shrink to a point. The other two terms represent "finite size" corrections to this limiting situation.

As the angular functions are identical in the corresponding operators for all three terms, it is easily shown that the "partial" conversion coefficient corresponding to ejection of a bound electron in state \( K' \) into the free state \( K \) is

\[ \alpha_{K'K}(El) = \delta_{K'K} |1 + \lambda|^2, \]

(6)

where \( \delta_{K'K} \) is the partial conversion coefficient corresponding to a point nucleus, as far as nuclear matrix elements are concerned (and thus corresponds to the case in which only the first term in (5) is retained). Calculating \( \delta_{K'K} \), one should, strictly speaking, use electron wave functions adjusted for the finite extension of the central charge. In the qualitative considerations employed in the following, values of either Sliv or Rose are sufficiently accurate, even though neither strictly corresponds to our definition of \( \delta_{K'K} \).

The term \( \lambda \) of Eq. (6) is defined as

\[ \lambda = \frac{\langle I || O_n(h_1) S(r,j_1) - O_n(j_1) S(r,h_1) || I' \rangle}{\langle I || O_n(j_1) || I' \rangle S(\infty,h_L)}, \]

(7)

where

\[ S(r,j_1) = \int_0^r 2^2 d\rho \left\{ W^2 r j_1 (f_k g_{K'} + g_k f_{K'}) + \left[ \frac{\partial}{\partial r} (r j_1) \right] (f_{K'} g_{K'} + g_{K'} f_{K'}) \right\}. \]

(8)
In Eq. (8) it is assumed that the expansion of $j_1(\omega r)$ is employed, with only the leading term retained. Under this condition, $\omega r \ll 1$, the second of the terms in Eq. (8) is dominant. In Eq. (7) the "double-bar" matrix element (reduced matrix element) is employed in the usual definition. (The quantities $I'$ and $I$ in the bra- and ket-vectors of Eq. (7) really denote all quantum numbers necessary to represent initial and final state, apart from the spacial projection $m_1$ of the total angular momentum).

The expression $S(r,h_1)$ is obtained from Eq. (6) by employing $h_1$ instead of $j_1$. Furthermore $S(\omega,h_1)$ corresponds to $S(r,h_1)_{r \to \infty}$.

Finally the total conversion coefficient $\alpha_{\chi'}$ (i.e., respectively $\alpha_{K}$, $\alpha_{L_I}$, $\alpha_{L_{II}}$, etc.) is defined as

$$\alpha_{\chi'} = \sum_{\chi} \alpha_{\chi \chi'}.$$

The electron wave functions $f_{\chi}$ and $g_{\chi}$ ("small" and "large" components of the Dirac electron wave functions) have now to be estimated.

In the interior of the nucleus one may assume an electrostatic potential corresponding to a homogeneous charge distribution,\textsuperscript{10,11}

$$v(r) = \frac{e^2}{4\pi \epsilon_0 R} \left[ 3 - \left( \frac{r}{R} \right)^2 \right],$$

where $R$ is the nuclear radius. In this potential, which is finite at the origin, one may find series expansions in $r$ of $f_{\chi}$ and $g_{\chi}$. The amplitude of the leading term is determined by matching at the nuclear boundary with the external solution. The leading terms are only weakly dependent on the particular shape of the interior potential assumed.

For $L_I$ and $L_{II}$ conversion one would expect the main contribution to the structure-dependent terms to originate from transitions $\chi' = -1$ to $\chi = 1$ (i.e., $s_{1/2} \rightarrow p_{1/2}$) and $\chi' = 1$ to $\chi = -1$ (i.e., $p_{1/2} \rightarrow s_{1/2}$), respectively. (The wave functions corresponding to these states have the largest amplitudes at the nuclear surface). Indeed, these are the leading contributions, although they are considerably weakened owing to a particular cancellation, discussed in the following paragraphs.
We employ the following internal expansions, treating as an example the case \( s_{1/2} \rightarrow p_{1/2} \):

\[
\begin{align*}
  f'_{1/2} &= f_{1/2} \left( \frac{r}{R} \right) + \ldots = -\frac{1}{3} \left[ E - v'(0) - 1 \right] \frac{\partial}{\partial x}, r + \ldots, \\
  g'_{1/2} &= g_{1/2} + \ldots, \\
  f'_{1} &= f_{1} + \ldots, \\
  g'_{1} &= g_{1} \left( \frac{r}{R} \right) + \ldots = \frac{1}{3} \left[ E - v(0) + 1 \right] \frac{\partial}{\partial x}, r + \ldots,
\end{align*}
\] (11a, 11b, 11c, 11d)

where use has been made of the Dirac equation to determine the relation between \( f_X \) and \( g_X \).

Retaining terms to leading order only and neglecting terms of order \( \hbar \), (valid for transition energies much less than the electron rest mass), we obtain by substitution into Eq. (8) the expression

\[
  f_{\xi'} g_{\xi'} + g_{\xi} f_{\xi'} \approx r \left[ \frac{f_{\xi'} g_{\xi'}}{f_{\xi} g_{\xi'}} \right] \left[ 1 + \frac{v'(0) - v(0)}{2} \right] + \ldots, \tag{12}
\]

where units \( m = \hbar = c = 1 \) are assumed throughout. Thus \( R \) is the nuclear radius in units of the electron Compton wave length, and \( v'(0) \) and \( v(0) \) are the initial and final values of the electrostatic potential at the origin. The two terms on the left side of (12) are opposite in sign and very nearly of equal magnitude unless there is a large change in effective potential at the origin during the transition. Assuming, e.g., a homogeneous charge distribution over a spheroid of constant volume but varying eccentricity, the potential at the origin depends on the eccentricity parameter \( \delta \) (excess of major axis over minor axis) as

\[
  v(0) = \frac{q_f(0)}{2} \left( 1 - \frac{h}{9} \delta^2 \right). \tag{13}
\]

If thus the nucleonic transition associated with the conversion process changes the equilibrium deformation from \( \delta = 0.3 \) to \( \delta = 0.2 \), \( v(0) \) changes by approximately 2\%, i.e., by \( \sim 1 \) Mev in the heavy-element region. It is quite possible that other effects may also tend to lift this cancellation.
Employing the expansions of Eqs. (11a-d) one arrives at the following simplified expression for the corrected conversion coefficient

$$\alpha_{\text{k}k'} = \frac{1}{\omega + \imath 2 \varepsilon} \left| 1 - \frac{\mathcal{C}_{\text{k}k'}}{\mathcal{M}_{\text{k}k'}} \right|^2$$  \hspace{1cm} (14)

Here $\alpha_{\text{k}k'}$ is the normal-point nucleus partial-conversion coefficient defined previously; $\varepsilon$ is the phase of the integral $S(\omega, \varepsilon)$, which enters into the expression for $\alpha_{\text{k}k'}$ as the absolute value squared. These phases have not been published, but we estimate, for low-energy transitions, $\varepsilon = \pm \frac{\pi}{2}$ (i.e., $S(\omega, \varepsilon)$ almost purely imaginary) for $E1$ as well as for $M1$ conversion. The phase problem for the $M1$ case has been previously discussed by Church and Weneser.\textsuperscript{4} Here $\omega$ is the transition energy in units of $mc^2$; $\mathcal{C}_{\text{k}k'}$ is a factor depending on the change of electrostatic potential at the origin brought about during the transition; $\mathcal{x}$ is a real quantity, the ratio of two nuclear matrix elements,

$$\mathcal{x} = \frac{\langle \mathcal{I}|r^3|\mathcal{I}\rangle}{\langle \mathcal{I}|r|\mathcal{I}\rangle},$$  \hspace{1cm} (15)

where $r$ is expressed in units of $\sqrt{\frac{\mathcal{r}}{M\mathcal{w}}}$ appropriate to the nucleon wave functions of Nilsson,\textsuperscript{12} where the basic energy $\mathcal{w}$ of the nuclear oscillator potentials employed in Ref. 12 is given as $80 A^{-1/3}$ in units of $mc^2$. The correction term in Eq. (14) can thus be expected to be almost purely real for low-energy transitions. Furthermore, for nuclei in the heavy-element region and for gamma-ray energies less than $\sim 100$ kev the estimates of $M_{\text{k}k'}$ of Table II may be employed. The accuracy of the values of $M_{\text{k}k'}$ depend on the accuracy with which $\mathcal{g}$ and $\mathcal{g}$ may be estimated.

Values of these latter quantities may be obtained for bound states from the diagrams of Brysk and Rose\textsuperscript{13} based on calculations that allow for screening and the finite size extension of the central nuclear charge. The tables of Reitz\textsuperscript{14} have supplied values of the free-electron wave functions. A more detailed discussion on this point is found in Ref. 8.

It is clear that the calculated $M_{\text{k}k'}$ of Table II are not very exact but should be sufficiently accurate for semiquantitative estimates of the conversion anomalies.
Table II

<table>
<thead>
<tr>
<th>Shell</th>
<th>Initial</th>
<th>Final</th>
<th>$M_{\chi \chi'} \times 10^6$</th>
<th>$C_{\chi \chi'}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>$1s_{1/2}$</td>
<td>$p_{1/2}$</td>
<td>-5.3</td>
<td>$1 + \frac{v'(0) - v(0)}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$p_{3/2}$</td>
<td>3.9</td>
<td>1</td>
</tr>
<tr>
<td>L_I</td>
<td>$2s_{1/2}$</td>
<td>$p_{1/2}$</td>
<td>-2.1</td>
<td>$1 + \frac{v'(0) - v(0)}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$p_{3/2}$</td>
<td>1.6</td>
<td>1</td>
</tr>
<tr>
<td>L_{II}</td>
<td>$2p_{1/2}$</td>
<td>$s_{1/2}$</td>
<td>-1.9</td>
<td>$1 + \frac{v(0) - v'(0)}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d_{3/2}$</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>L_{III}</td>
<td>$2p_{3/2}$</td>
<td>$s_{1/2}$</td>
<td>1.4</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d_{3/2}$*</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$d_{5/2}$*</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

*In these cases the leading anomalous conversion operator is of the type $r^5 Y_1$; the coefficient corresponding to $M_{\chi \chi'}$ is, however, so small that the anomalous contributions to these terms may be safely neglected.

Let us examine the implications of the simple theory, as expressed in Eqs. (14) and (15) and in Table II. The conversion coefficients for K, L_I, and L_{II} (i.e., $s_{1/2}$ and $p_{1/2}$) electrons in this formulation are functions of two parameters—the matrix element ratio $\chi$, defined in Eq. (15), and the correction factor $C_{\chi \chi'}$, which depends on the electrostatic potential change. However, the L_{III} conversion coefficient is essentially a function of $\chi$ only. For a typical heavy-element case ($Z = 91$, $W = 0.17$) an $\chi$ value of about 400 to 600 should give rise to a second term in Eq. (13) of order 0.1, causing the partial-conversion coefficient for $p_{3/2} \rightarrow s_{1/2}$ transitions to increase or decrease by 20%, depending on the relative signs of $\chi$ and $e^{15}$. For $v'(0) = v(0)$ the theory predicts anomalies in the L_I and L_{II} subshells only slightly greater than that in the L_{III} subshell.
The correction factor $C_{X'X}$ theoretically will usually differ for $s_{1/2}$ and $p_{1/2}$ electrons. As $v'(0) - v(0)$ is increased from zero, $C_{X'X}$ increases from unity for $s_{1/2}$ and decreases from unity for $p_{1/2}$, and in the limit of very large potential change $C_{X'X}$ will be nearly of equal magnitude but of opposite sign for $s_{1/2}$ and $p_{1/2}$ electrons. In the next section we attempt some comparison with experiment.

The nuclei in which the anomalous cases occur lie in the region of spheroidal nuclei. Thus, one may expect selection rules in $K$ and to a lesser extent in $N$, $n_z$, $\Lambda$, and $\Sigma$ to be applicable to transition-matrix elements. The $K$-selection rules as applied to beta and gamma transitions have been frequently discussed elsewhere. The $N$, $n_z$, $\Lambda$, $\Sigma$ selection rules have also been applied successfully to beta\textsuperscript{15,16} and gamma\textsuperscript{16,17,18,19} transitions previously, although they are not generally as restrictive as the $K$-selection rules.

As has been pointed out, the $N$ quantum number should properly not be called an asymptotic quantum number as it is not dependent on the assumption of a very large deformation.\textsuperscript{20} The evidence from Ref. 20 and from the studies by Hoffman and Dropesky\textsuperscript{21} on the $K$-capture of Pu\textsuperscript{237} to Np\textsuperscript{237} may suggest that a breaking of the selection rule in $N$ is associated with a quantitatively somewhat greater hindrance than in $n_z$ and $\Lambda$.

Let us now consider the selection rules in $K$ and in $N$, $n_z$, $\Lambda$, and $\Sigma$ for the matrix elements ($r^3Y_1$) giving rise to anomalous El conversion contributions.

If the matrix element of $rY_1$ is weakened by $K$-forbiddenness ($\Delta K \neq 0$), then this is also the case for $r^3Y_1$, which has the same $K$-selection rule. However, the severe nucleonic selection rules that hinder El transitions, to some extent accounted for by the asymptotic selection rules in $N$, $n_z$, $\Lambda$, and $\Sigma$, are relaxed for $r^3Y_1$.

Tables III and IV list the appropriate selection rules. For $\Delta K = -1$ the selection rules are obtained by changing signs in the $\Delta \Lambda$ and $\Delta \Sigma$ columns.
## Table III

Selection rules for radiative El transitions \( (\Sigma r^3 Y_{lm}) \)

<table>
<thead>
<tr>
<th>( \Delta K )</th>
<th>Operator</th>
<th>( \Delta N )</th>
<th>( \Delta n_z )</th>
<th>( \Delta \Lambda )</th>
<th>( \Delta \Sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( x+iy )</td>
<td>±1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>( z )</td>
<td>+1</td>
<td>+1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

## Table IV

Selection rules for anomalous El conversion \( (\Sigma r^3 Y_{lm}) \)

<table>
<thead>
<tr>
<th>( \Delta K )</th>
<th>Operator</th>
<th>( \Delta N )</th>
<th>( \Delta n_z )</th>
<th>( \Delta \Lambda )</th>
<th>( \Delta \Sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>((x+iy)(x^2+y^2))</td>
<td>±1,±3</td>
<td>0</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>((x+iy):z^2)</td>
<td>±1</td>
<td>0</td>
<td>+2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>((x+iy):z^2)</td>
<td>+1,±3</td>
<td>+2</td>
<td>-1,±3</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>(z(x^2+y^2))</td>
<td>±1,±3</td>
<td>+1</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(z^3)</td>
<td>+1</td>
<td>+1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+3</td>
<td>+3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1</td>
<td>-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-3</td>
<td>-3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
NUCLEAR-STRUCTURE CORRECTIONS FOR ML CONVERSION COEFFICIENTS

The case of magnetic dipole K-shell ICC is treated in Ref. 4 by Church and Weneser. For the purpose of the survey of empirical data below we rewrite the final formulae of that reference. To preserve the analogy with the EL case, Eq. (14), we express r in the units of nuclear dimensions \( \sqrt{\frac{m}{M \omega}} \) employed in Eq. (14). We obtain for the K-shell partial ICC (denoted \( \beta_{K,K'} \), and leading to the free \( s_{1/2} \) state, i.e., \( \kappa' = -1, \kappa = 1 \)).

\[
\beta_{-1,-1} = \beta_{-1,-1}^0 (1 + x N_{-1,-1})^2
\]

where

\[
x = \frac{\langle r^2 (\frac{\sigma}{2} + 2 \mu g) - \mu r (\sigma \cdot r) \rangle}{\langle \| \frac{\sigma}{2} + \mu g \| \rangle},
\]

and where the value of the constant \( N_{-1,-1} \) may be obtained from Eq. (6) of Ref. 4. Furthermore, Church and Weneser, on the basis of available ML partial ICC's, \( \beta_{K,K'}^0 \), rewrite the correction in terms of the total ICC. In the notation of the present paper (adopted for the use of the nucleonic wave functions of Ref. 12). Their result may be rewritten

\[
\beta_K \approx \beta_K^0 [1 + x N_K]^2.
\]

The step between Eqs. (16) and (18) (or Eqs. (6) and (7) in Ref. 4) appears to invoke the approximating assumption of \( N_K \ll 1 \).

The constant \( N_K \) in Eq. (18) is given as

\[
N_K = C(Z,W) \cdot R^{-2}.
\]

The energy involved in the transition is denoted \( \bar{W} \) as before \(( \kappa \text{ in Ref. 4})\). Values of the constant \( C(Z,W) \) are tabulated in Ref. 4. The nuclear radius \( R \) is to be expressed in the units above \(( \text{for } A \approx 230, R \approx 3)\). For \( \bar{W} \leq mc^2 \) the factors \( N_K \) take on the values listed in Table V; for more accuracy Table I of Ref. 4 should be used for \( C(Z,W) \). It should be noted that if Sliv's values are used for \( \beta_K^0 \) and \( \beta_{-1,-1}^0 \) it is appropriate to replace \( x \) by \( (x-R^2) \), as pointed out in Ref. 4. The same holds true for the EL case. As large values of \( x \) are required for the transition to be detected as anomalous, this correction term to \( x \) is negligible, however.
Table V

<table>
<thead>
<tr>
<th>Z</th>
<th>70</th>
<th>85</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_K$</td>
<td>2 to $3(-3)$</td>
<td>4$(-3)$</td>
<td>7 to $8(-3)$</td>
</tr>
</tbody>
</table>

The numbers in parentheses denote powers of 10.

It is thus found that the quantity $N_K$, characterizing the anomalous corrections to the $M_l$ ICC, is in general much larger than the corresponding quantity for $E_l$ transitions $M_{KK'} W^{-3/2} \delta K^{1/2}$ (see Eq. (14); the correction there is, however, expressed in terms of the partial ICC). For example, in the experimentally interesting case of the 84-kev $E_l$ transition in $^{231}$Pa, the latter quantity corresponding to the partial ICC $\alpha_{L_1}$ of $L_{1/2}$ conversion (connecting the bound electron state $2s_{1/2}$ with the free state $p_{1/2}$) equals $\sim 2 \times 10^{-4}$. (For the purely hypothetical case of $K$ conversion of the same energy we would have twice this value.) If the theoretically undetermined factor $\psi_{KK'}$ were of the order 1, the nuclear-structure deviations in ICC would be expected to be observed in $E_l$ first for transitions that were 100 to 1000 times as hindered as $M_l$ transitions showing anomalous conversion. However, there seems to be some experimental indication that $\psi_{KK'}$ indeed is of order 10, in which case the difference between $E_l$ and $M_l$ in this respect is less important.

Church and Weneser gave three categories of hindered $M_l$ transitions in which anomalies might be observable, and to them may be added the fourth category of transitions in strongly deformed nuclei, transitions for which there is hindrance in $N$, $n_z$, $\Lambda$, or $\Sigma$.

In Table VI we give the "asymptotic" selection rules for $M_l$ radiation and in Table VII the selection rules for the anomalous $M_l$ conversion operator. As with Tables III and IV, the selection rules are only for $\Delta K = +1$ and 0, and the corresponding rules for $\Delta K = -1$ are obtained by changing signs in the $\Delta \Lambda$ and $\Delta \Sigma$ columns.
**Table VI**

Selection rules for radiative ML transitions

\[ \sum_m \hat{e}_m \cdot (L + \mu \cdot g) \]

<table>
<thead>
<tr>
<th>( \Delta K )</th>
<th>Operator</th>
<th>( \Delta N )</th>
<th>( \Delta n_z )</th>
<th>( \Delta \Lambda )</th>
<th>( \Delta \Sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \sigma_+ )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>( L_+ )</td>
<td>0</td>
<td>±1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>( \sigma_z )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>( L_z )</td>
<td>0</td>
<td>±2</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table VII**

Selection rules for anomalous ML conversion contribution

\[ \sum_m \hat{e}_m \cdot [(L + \mu \cdot g)r^2 - \mu \cdot r \cdot (g \cdot \mu)] \]

<table>
<thead>
<tr>
<th>( \Delta K )</th>
<th>Operator</th>
<th>( \Delta N )</th>
<th>( \Delta n_z )</th>
<th>( \Delta \Lambda )</th>
<th>( \Delta \Sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( z^2 \sigma_+ )</td>
<td>( {+2, +2 } )</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>( {0, 0 } )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( {-2, -2 } )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x^2 + y^2 )</td>
<td>( \sigma_+ )</td>
<td>0,±2</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( z^2 L_+ )</td>
<td>( {+2, +1, +3 } )</td>
<td>1</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( {0, ±1 } )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( {-2, -1, -3 } )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x^2 + y^2 )</td>
<td>( L_+ )</td>
<td>0,±2</td>
<td>±1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( x + iy )</td>
<td>( z \sigma_+ )</td>
<td>0,±2</td>
<td>0</td>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>( x + iy )</td>
<td>( z \sigma_z )</td>
<td>0</td>
<td>2</td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>( x^2 + y^2 )</td>
<td>( L_z )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( x^2 + y^2 )</td>
<td>( \sigma_z )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
QUALITATIVE SYSTEMATIZATION OF CONVERSION-COEFFICIENT ANOMALIES

The occurrence in the 60-kev El transition of \( \text{Np}^{237} \) of L-subshell ICC's in disagreement with Rose's theoretical values has been pointed out by Hollander et al.\(^5\) L-subshell conversion coefficients have been studied for El transitions in neighboring isotopes, and some are found to be anomalous (notably 85-kev El in \( \text{Pa}^{231} \)), whereas others are normal. The experimental evidence in the heavy region is detailed in a forthcoming paper by Asaro, Stephens, Hollander, and Perlman.\(^6\) Vartapetian\(^{22,23}\) has reviewed lifetime and conversion-coefficient data for M1 and El transitions and has made the general observation for the heavy element El's that the more delayed transitions usually exhibit conversion coefficients higher than theoretical for El and requiring more M2 admixture than is reasonable for explanation in many cases. Vartapetian suggests that the anomalies may be due to nuclear-structure effects not treated by the Rose or Sliv theoretical calculations.

We now wish to make a brief survey of experimental conversion-coefficient data for El and M1 transitions in the principal two regions of spheroidal nuclear deformation. It is beyond the scope of this paper to analyze exhaustively the experimental evidence. We confine our cases to those in which a lifetime or limit is known and exclude from consideration those cases for which only a single conversion coefficient is known and in which there is no independent evidence bearing on possible quadrupole admixture.
The simple theoretical treatment exhibited in the first section of this paper predicts (a) that significant anomalies should appear only in transitions highly hindered from the single-particle transition value, (b) that when such hindrance is attributable to violation of the $K$-selection rule, anomalies should not be appreciable, and (c) that anomalies should be favored for transitions in which the anomalous operator is allowed by "asymptotic" selection rules. The relevant experimental cases are summarized briefly in Tables VII through XI in inverse order of retardation from the single-particle transition rate. The cases are discussed individually in the Appendix. Separate tables are given for $K$-forbidden and $K$-allowed cases. Altogether there are four certain and four probable anomalous $E1$ cases and one certain $M1$ case.

Concerning the first of the three general theoretical predictions enumerated above we see, indeed, that all the clear cases of anomalies occur for transitions retarded from single-particle rates by factors of $1.5 \times 10^4$ or more.

Concerning the second, we see that except for the exceedingly retarded $^{180m}Hf$ case, normal conversion coefficients are found in $K$-forbidden cases even though the retardation may be as large as $10^5$ or more.

Concerning the third theoretical prediction, that asymptotic quantum-number selection rules for the anomalous operator are valid, there is some uncertainty. At the outset it should be borne in mind that violation of selection rules in $n_{\Sigma}L,\Sigma$ is found to result, on the average, in retardation of only one order of magnitude in beta decay. It has been suggested that cases such as those in Tables VIII and X owe their retardation to violation of these selection rules, although the especially high retardation of $E1$ transition associated with the removal of almost all the oscillator strength to the giant-resonance region of excitations seems to indicate some higher-order cancellation of matrix elements, in addition.

(Violation of the selection rules in $\Sigma$ and $\Lambda$ in $E1$ transitions is (for the normal cases $\Delta N = 1$) also associated with a violation $n_{\Sigma}$. One may possibly make the distinction, however, as to the order of forbiddenness in $n_{\Sigma}$ alone.) What the theory of anomalous ICC would lead us to seek is separate "threshold" values of retardation above which transitions would show anomalies (of greater
Table VIII

Survey of El transitions not K-forbidden

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Gamma energy (keV)</th>
<th>Retardation factor</th>
<th>Probable state assignments</th>
<th>Asymptotic classification for</th>
<th>Radiative trans. op.</th>
<th>Anomalous ICC op.</th>
<th>Conversion-coefficient observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Np²³⁷</td>
<td>267</td>
<td>5.5 x 10⁸</td>
<td>I Kα[Nn²/₂]</td>
<td>αₗ probably high by factor of 10.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pa²³¹***</td>
<td>84</td>
<td>3 x 10⁶</td>
<td>5  3  2-[642]→3  5  2-[521] h(?) u(?)</td>
<td>α₉ high by 20.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lu¹⁷⁷</td>
<td>146</td>
<td>5 x 10⁶</td>
<td>9  2  2-[514]→7  7  2-[404] h h</td>
<td>α₉ normal.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lu¹⁷⁵</td>
<td>282</td>
<td>1.4 x 10⁶</td>
<td>9  2  2-[514]→7  7  2-[404] h h</td>
<td>α₉ not measurable.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pu²³⁹</td>
<td>106</td>
<td>5 x 10⁵</td>
<td>7  7  2-[743]→5  5  2-[622] h u</td>
<td>α₉ high by 1.6±0.2.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Np²³⁷</td>
<td>60</td>
<td>2.8 x 10⁵</td>
<td>5  5  2-[523]→3  5  2-[642] h h</td>
<td>α₉ high by 3.8.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Np²³⁷</td>
<td>26</td>
<td>3 x 10⁵</td>
<td>5  5  2-[523]→7  5  2-[642] h h</td>
<td>α₉ normal.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pm¹⁶⁹</td>
<td>63</td>
<td>8 x 10⁴</td>
<td>7  7  2-[523]→7  7  2-[404] h h</td>
<td>α₉ normal.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ac²²⁷</td>
<td>27</td>
<td>6 x 10⁴</td>
<td>State assignments unknown.</td>
<td>α₉ normal but low.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The table contains atomic and nuclear data for various elements, including their gamma energy, retardation factor, probable state assignments, and asymptotic classification for radiative and anomalous transitions. The conversion-coefficient observations are noted, with some transitions indicated as high or low.
### Table VIII (cont'd.)

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Gamma energy (keV)</th>
<th>Retardation factor</th>
<th>Probable state assignments</th>
<th>Asymptotic classification for radiative trans. op.</th>
<th>Anomalous ICC&lt;sub&gt;3&lt;/sub&gt; op.</th>
<th>Conversion-coefficient observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pa&lt;sup&gt;231&lt;/sup&gt;</td>
<td>26</td>
<td>~5 x 10&lt;sup&gt;4&lt;/sup&gt;</td>
<td>5/2&lt;sup&gt;5&lt;/sup&gt; + [642] → 5/2&lt;sup&gt;2&lt;/sup&gt; + [521]</td>
<td>h(?)</td>
<td>u(?)</td>
<td>α&lt;sub&gt;M&lt;/sub&gt;-subshell ratios normal.</td>
</tr>
<tr>
<td>Pa&lt;sup&gt;234m&lt;/sup&gt;</td>
<td>29</td>
<td>1.5 x 10&lt;sup&gt;4&lt;/sup&gt;</td>
<td>? Note: odd-odd nucleus</td>
<td></td>
<td></td>
<td>α&lt;sub&gt;L&lt;/sub&gt; probably high (?)</td>
</tr>
<tr>
<td>Hf&lt;sup&gt;177&lt;/sup&gt;</td>
<td>208</td>
<td>&lt;3 x 10&lt;sup&gt;4&lt;/sup&gt;</td>
<td>9/2&lt;sup&gt;9&lt;/sup&gt; + [624] → 7/2&lt;sup&gt;7&lt;/sup&gt; + [514]</td>
<td>h</td>
<td>h</td>
<td>α&lt;sub&gt;K&lt;/sub&gt; normal.</td>
</tr>
<tr>
<td>Am&lt;sup&gt;243&lt;/sup&gt;</td>
<td>85</td>
<td>&lt;2 x 10&lt;sup&gt;4&lt;/sup&gt;</td>
<td>5/2&lt;sup&gt;5&lt;/sup&gt; + [642] → 5/2&lt;sup&gt;2&lt;/sup&gt; + [523]</td>
<td>h</td>
<td>h</td>
<td>L-subshell ratio and α&lt;sub&gt;total&lt;/sub&gt; normal.</td>
</tr>
<tr>
<td>Eu&lt;sup&gt;153&lt;/sup&gt;</td>
<td>98</td>
<td>&lt;9 x 10&lt;sup&gt;3&lt;/sup&gt;</td>
<td>5/2&lt;sup&gt;5&lt;/sup&gt; + [532] → 5/2&lt;sup&gt;2&lt;/sup&gt; + [413]</td>
<td>h</td>
<td>h</td>
<td>α&lt;sub&gt;K&lt;/sub&gt; normal</td>
</tr>
<tr>
<td>W&lt;sup&gt;182&lt;/sup&gt;</td>
<td>152</td>
<td>?</td>
<td>? Note: even-even nucleus</td>
<td></td>
<td></td>
<td>α&lt;sub&gt;K&lt;/sub&gt; low, α&lt;sub&gt;L&lt;/sub&gt; high.</td>
</tr>
</tbody>
</table>

* h = hindered.
** u = unhindered.
*** Question marks indicate that the orbital assignments seem somewhat less certain than in the other cases listed.
### Table IX

Survey of K-forbidden El transitions

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Gamma energy (kev)</th>
<th>Retardation factor</th>
<th>ΔK</th>
<th>Conversion-coefficient observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hf$^{180m}$</td>
<td>57.6</td>
<td>$10^{15}$</td>
<td>8 or 9</td>
<td>L-subshell pattern anomalous with L$^1$ too high</td>
</tr>
<tr>
<td>Hf$^{178m}$</td>
<td>88.8</td>
<td>$2 \times 10^{14}$</td>
<td>8 or 9</td>
<td>Total conversion coeff. normal for El within 20%</td>
</tr>
<tr>
<td>Pu$^{239}$</td>
<td>316</td>
<td>$9.4 \times 10^8$</td>
<td>3</td>
<td>$\alpha_K$ normal</td>
</tr>
<tr>
<td>Re$^{183}$</td>
<td>382</td>
<td>$2 \times 10^6$</td>
<td>2</td>
<td>$\alpha_K$ normal</td>
</tr>
</tbody>
</table>

### Table X

Survey of Ml transitions not K-forbidden

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Gamma energy (kev)</th>
<th>Retardation factor</th>
<th>Probable state assignments</th>
<th>Asymptotic classification for rad. anomalous Conversion coefficient trans. ICC op. observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ta$^{181}$</td>
<td>482</td>
<td>$2.6 \times 10^6$</td>
<td>$5 \frac{5}{2} ^+[402] \rightarrow 7 \frac{7}{2} ^+[404]$</td>
<td>h</td>
</tr>
<tr>
<td>Np$^{237}$</td>
<td>208</td>
<td>$1.3 \times 10^4$</td>
<td>$3 \frac{3}{2} ^-[521] \rightarrow 5 \frac{5}{2} ^-[523]$</td>
<td>h</td>
</tr>
<tr>
<td>Eu$^{153}$</td>
<td>102</td>
<td>$5 \times 10^2$</td>
<td>$5 \frac{5}{2} ^+[411] \rightarrow 7 \frac{7}{2} ^+[413]$</td>
<td>h</td>
</tr>
</tbody>
</table>

### Table XI

Survey of K-forbidden Ml transitions

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Gamma energy (kev)</th>
<th>Retardation factor</th>
<th>ΔK</th>
<th>Conversion-coefficient observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tm$^{169}$</td>
<td>178</td>
<td>$5 \times 10^5$</td>
<td>3</td>
<td>$\alpha_K$ normal (or slightly high)</td>
</tr>
<tr>
<td></td>
<td>199</td>
<td>$5 \times 10^5$</td>
<td>3</td>
<td>$\alpha_K$ normal</td>
</tr>
<tr>
<td>Pu$^{239}$</td>
<td>277.9</td>
<td>$6.0 \times 10^4$</td>
<td>2</td>
<td>$\alpha_K$ normal</td>
</tr>
<tr>
<td></td>
<td>228.2</td>
<td>$4.7 \times 10^4$</td>
<td>2</td>
<td>$\alpha_K$ normal</td>
</tr>
<tr>
<td></td>
<td>209.7</td>
<td>$9 \times 10^3$</td>
<td>2</td>
<td>$\alpha_K$ normal (or low)</td>
</tr>
</tbody>
</table>
than, say, 50%): one threshold for transitions with unhindered anomalous operators and another higher threshold for transitions hindered in this operator. There is also a theoretical dependence of the anomalous ICC on atomic number. Numerical values exhibiting this dependence for M1 have been given by Church and Weneser, and a similar dependence is expected for E1. (Our Table II gives only values of $M_{1\lambda}$ for $Z = 91$.)

Among the E1 cases in Table VIII all examples of transitions unhindered in $r^3Y_1$ are anomalous with the probable exception of the 26-kev in Pa$^{231}$, so we may presume the threshold retardation is about $10^5$. It is somewhat surprising that the factor by which these examples are anomalous does not vary more, in view of the variation in retardation factor from $5 \times 10^5$ to $6 \times 10^8$. Perhaps in some cases of high retardation there is a change of configuration involving nucleons other than the odd one, and such rearrangement would decrease the anomalous-conversion matrix element classed as unhindered. Of the Table VIII heavy-element cases hindered in $r^3Y_1$, the 60- and 26-kev Np$^{237}$ transitions at $3 \times 10^5$ retardation show anomalies while in Am$^{243}$ at a retardation $< 2 \times 10^4$ (with presumably the same proton states as the Np$^{237}$ 60-kev) the ICC's are normal. In the rare earth region the E1 cases of Table VIII are all hindered in $r^3Y_1$, and just one of them, Lu$^{177}$ at $5 \times 10^6$ retardation, shows a possible anomaly. The other four cases with retardation ranging down from $1.4 \times 10^6$ to $< 9 \times 10^3$ are all normal. The threshold for anomalies in these "h" cases seems around $10^5$ in the heavy region and $10^6$ in the rare earth region. There is no evident difference in threshold for the cases hindered in $r^3Y_1$, and the unhindered cases. Probably the scatter in magnitudes of matrix elements is greater than the average separation of the "h" and "u" groups, but more experimental cases will be needed to establish the point.

All three M1 isomers of Table X are unhindered in the anomalous operator, and they indicate a threshold retardation somewhere between $2 \times 10^4$ and $10^6$. The cases designated "normal" show $\alpha_K$ values somewhat lower than the Sliv theoretical values, but the unretarded M1 transitions probably generally exhibit such slightly lower values according to the analysis of Wapstra and Nijgh.
MORE DETAILED COMPARISONS OF THEORY AND EXPERIMENT

The simple theory of Church and Weneser for anomalous M1 conversion and the corresponding theory for El given in this paper, together with considerations of selection rules in the quantum numbers of deformed nuclei, have provided a basis for some systematization of the occurrence of anomalous conversion coefficients. It is next of interest to see if the simple theories are also capable of quantitative explanation of the anomalies.

In order to make really quantitative comparisons for El conversion it would be necessary to have theoretical partial-conversion coefficients (i.e., how much of El conversion of $s_{1/2}$ goes to $p_{1/2}$ and how much to $p_{3/2}$) and phases for the normal-conversion matrix elements, and these quantities have not been published. Nevertheless, in a particular case of exceedingly large deviations from the normal values of the ICC's we are relatively independent of a knowledge of these partial values. The experimental $L_1$, $L_{11}$, and $L_{111}$ coefficients for the 85-kev transition in Pa$^{231}$ are 1.32, 0.84, and 0.047, respectively, and Rose's theoretical values are 0.063, 0.042, and 0.037.

As is readily evident, the experimental ratios cannot be explained solely by M2 admixture* ($\alpha_{M2 \ \text{theo.}} = 96, 8.5, 29.2$) but some M2 admixture cannot at present be excluded.

One might attempt to test the theory (cf. Eqs. (14) and (15) and Table II) by examining first any anomaly of the $L_{111}$ subshell conversion, which should depend only on $x$, the ratio of nuclear matrix elements. This procedure involves several difficulties: first, there is experimental uncertainty of at least 50% in the value; second, the relative partial-conversion coefficients to $s_{1/2}$, $d_{3/2}$, and $d_{5/2}$ final states are not known to us; third, it cannot be excluded that the entire small increase of experimental $L_{111}$-conversion coefficient over theory could be due to a small M2 admixture (0.03%). This admixture corresponds to an M2 half life of $4 \times 10^{-4}$ sec. which is 10 times as long as the single-proton estimate, but still probably not long enough to be consistent with the fact that the M2 transition between the single-particle states assigned is classified as hindered. Depending upon which fraction of the total normal

* Provided M2 is not anomalous.
\[ L_{III} \] internal conversion goes to final state \( s_{1/2} \) (with which latter transition almost all the anomalous conversion is associated), the upper limit on \( x \) may be put between \( 5 \times 10^{-3} \) (0\%) and \( 2 \times 10^{-4} \) (100\%). (If we assume the fraction to be, e.g., \( 1/2 \), the upper limit on \( x \) from the experimental \( L_{III} \) ICC may be put at \( 1.5 \times 10^{-4} \).) This upper limit corresponds to the hypothetical case (somewhat improbable in view of other empirical \( L_{III} \) cases) that the correction term is almost twice as large as the normal term but enters with opposite sign.

The experimental \( L_I \) and \( L_{II} \) ICC's depend both on the structure parameter \( x \) and on \( C_{XX'} \). Because they are an order of magnitude (\( \approx 20 \)) larger than the normal values, the analysis is rather independent of a knowledge of the partial ICC's \( \alpha_{XX'} \). We may rewrite Eq. (14) in the form

\[ \alpha'_{XX'} = \sum_{XX'} \left| \frac{\alpha_{XX'}}{M_{XX'}} - ie^{-15} W^{-3/2} M_{XX'} C_{XX'} x \right|^2. \]  

(20)

It is then apparent that in these particular cases \( \alpha'_{XX'} \) may be neglected, and from the empirical values of \( \alpha_{L_I} \) and \( \alpha_{L_{II}} \) for the Pa\(^{231}\) case considered, we obtain the relations

\[ |x|^2 (0.37 + 0.63 C_{-1-1}^2) \approx 8.2 \times 10^8, \]  

(21)

\[ |x|^2 C_{-11}^2 \approx 10 \times 10^8. \]  

(22)

From the analysis of the \( L_{III} \) conversion we had

\[ |x|^2 < 4.0 \times 10^8. \]  

(23)

In the most elementary form of the theory [Eq. (14) and Table II] \( C_{XX'} \) would be unity, corresponding to no change in electrostatic potential at the center of the nucleus and to a correspondingly high degree of cancellation of the two terms on the left of Eq. (12). With the uncertainty of our values of \( M_{XX'} \) with both \( C_{XX'} \) and, in addition, the uncertainty introduced by neglecting the contribution from the normal terms in Eq. (20), we cannot entirely exclude a solution \( x \approx 3 \times 10^{-4} \) and \( C_{XX'} = 1 \) to Eqs. (21) and (22). This \( x \) is somewhat larger than what is allowed by the inequality (23). The upper limit of this inequality corresponds in turn to a somewhat improbable
case that the anomalous amplitude for $L_{III}$ conversion enters with twice the size of the normal contribution, and with the opposite sign. The limit is furthermore lowered if part of the $L_{III}$ conversion is due to M2 radiation.

Furthermore, we have calculated the single-particle value of the matrix element $< r_{11}^3 >$ for the state assignments of Table VIII, using the wave functions of Ref. 12 in the so-called asymptotic approximation representing an approximate solution to the potential of Ref. 12 in the limit of large deformations. Using the empirical value on $| < r_{11}^3 > |$ from the gamma lifetime, we obtain $| x | \approx 11.00$. The asymptotic approximation in particular and any single-particle wave function in general obtained from a simple model is more likely to overestimate than underestimate the value of $< r_{11}^3 >$. However, a possible enhancement of $x$ might result from a collective octupole deformation of the nucleus. An enhancement by a factor of 20 seems, however, excessive.

It seems more probable that the true physical situation is more nearly represented by a solution of relations (21)-(23) with an $x$ of the order of the estimated single-particle value and large factors $C_{K'K}$. On a naive basis one may insert the expressions of Table II for $C_{K'K}$ and solve Eqs. (21) and (22) in terms of $x$ and the quantity $[v'(0) - v(0)]$, i.e., the change of depth of the electrostatic potential. (One may notice that this quantity enters with a different sign in $C_{1-1}$ and $C_{-11}$.) The quantity $[v'(0) - v(0)]$ is then given by the ratio between Eqs. (21) and (22), i.e., $[v'(0) - v(0)]$ is related essentially to the ratio $L_{I} / L_{II}$. Of the two solutions to the new relation so obtained, one corresponds to a very small value of $[v'(0) - v(0)]$ and requires the large $x$ value already discussed and evaluated as improbable for other reasons. The other solution corresponding to a large value of $| v'(0) - v(0) |$, of the order of 10 Mev or more, is very sensitive to the exact ratio of the right sides of Eqs. (21) and (22). In view of the fact that the normal conversion amplitudes are neglected in comparison with the anomalous ones (the latter being only five times as large) and furthermore in view of the uncertainty of the estimate of $M_{K'K}$, the numbers on the right-hand sides of Eqs. (21) and (22) cannot be considered very accurate. The solution corresponding to $| v'(0) - v(0) | >> 1$ is, however (in contrast to the other solution), very sensitive to the value of the ratio discussed. We can then mainly conclude that $C_{K'K}$ seems to be of order 10, and $x$ of the order of the single-
particle estimate or somewhat larger.

In summary, we can assign values to the two parameters \( r^3 Y_1 \) and \( v'(0) - v(0) \) to give a consistent explanation of the three L-subshell coefficients in the \( \text{Pa}^{231} \) case, and such that the actual value of the matrix element \( r^3 Y_1 \) is not inconsistent with reasonable single-particle values. However, the magnitude of \( v'(0) - v(0) \) required seems quite excessive. This shortcoming of the elementary theory clearly calls for refinements and consideration of effects in addition to the change in the electrostatic potential, which would tend to lift the cancellation in Eq. (12). The anomalous internal-conversion interaction takes place wholly within the nuclear volume (i.e., at short distances), and it would not be surprising if vacuum polarization or higher-order radiative corrections were significant. It is believed, for example, that such corrections are significant in calculation of x-ray fine-structure energy levels.\(^{24}\) Another effect that might tend to remove the \( s_{1/2} \leftrightarrow p_{1/2} \) cancellation in the "unperturbed" electron wave functions would result from a large change in the state of magnetization throughout the nuclear volume.

The probably anomalously converted 29-kev transition in \( \text{Pa}^{234} \) discussed by Vartapetian\(^ {22}\) (where possibly the same orbitals are associated with the transition as in \( \text{Pa}^{231} \)) presents a difficulty of a quantitative kind for this elementary theory, as this transition is only 1/100 as hindered as the 84-kev transition in \( \text{Pa}^{231} \). In view of possible experimental difficulties in conversion coefficient measurements at such low energies, further work on the ICC and subshell ratios is important.

It is interesting to note the pattern of L-subshell anomalies for the El cases of Tablé VIII where such information is known (the 85-kev transition of \( \text{Pa}^{231} \), the 60-kev and 26-kev transitions of \( \text{Np}^{237} \), and the 106-kev of \( \text{Pu}^{239} \)). In no case is the L\( _{III} \) subshell definitely anomalous, although it may be about 30\% too high in \( \text{Pa}^{231} \). (This may be interpreted as strong support for the argument of \( C_{\text{KK'}} \gg 1 \)). In \( \text{Pa}^{231} \), the most striking case, the L\( _I \) and L\( _{II} \) subshells are equally enhanced by a factor of 20. In \( \text{Np}^{237} \) and \( \text{Pu}^{239} \) the enhancement is more modest, and is greater in the L\( _{II} \) subshell than in the L\( _I \).

One may speculate on a possible effect on the K/L\( _I \) ratio associated with the existence of large anomalous matrix elements but more directly dependent on the phase of the normal-conversion amplitude. The K-as well as the
L\textsubscript{1}\textsuperscript{-}conversion involves initial s\textsubscript{1/2} states; the initial states, however, are characterized by different radial quantum numbers. The sign of S (\omega|h\textsubscript{L}\rangle) (cf. Eq. (8)) determines the phase of the "point charge" amplitude \sqrt{\alpha_{KK'}}. As we have not performed the calculation of S(\omega|h\textsubscript{L}\rangle) we have no way of quantitatively estimating the effect; it seems, however, conceivable that the phase of this quantity may be greatly different for the K-conversion and the L\textsubscript{1}-conversion. It is thus possible that in anomalous conversion the K and L\textsubscript{1} ICC's could deviate in different directions. This effect may be thought of as a possible explanation of the anomalous K/L ratio reported for an El transition in \textsuperscript{182}W (see discussion Appendix).

One might attempt a quantitative comparison also for some of the M\textsubscript{l} transitions. The case that lends itself most readily for such a comparison is the 480-kev M\textsubscript{l} transition in Ta\textsuperscript{181}. The M\textsubscript{l} K-conversion coefficient obtained on the basis of the measured total ICC and angular correlation data, as discussed in the Appendix, is \approx 0.5 as compared with Sliv's value \alpha_{K}(M\textsubscript{l}) = 0.06. Using Eq. (18) to fit the observed ICC, one may roughly estimate |x| \textbf{expt} = 1000 - 2000.

Actually the approximation involved in the step between Eqs. (16) and (18) (i.e., between Eqs. (6) and (7) of Ref. 4) seems to require that x (or \lambda) not be too large. This introduces an additional error in the estimate of the order

\[ \sqrt{\frac{\beta_{1-1} + \beta_{2-1}}{\beta_{1-1}}} \]

for large x values. Equation (18) thus underestimates x.

A semitheoretical estimate of x is obtained by using the observed partial M\textsubscript{l} lifetime to determine the absolute value of the normal M\textsubscript{l} matrix element and the wave functions of Ref. 12 in the "asymptotic approximation" to determine the anomalous internal-conversion matrix element. This estimate of x gives

\[ |x|^{\text{theo}} \approx 5000. \]

It is not disturbing to find the theoretical anomalous conversion matrix element as much as a factor of five too large. Indeed, most single
particle theoretical matrix elements for gamma or beta transitions overestimate the experimental transition rates unless collective effects are important.

It is interesting to compare the Ta\(^{181}\) 480-kev case just treated with the case of the 208-kev Ml transition in Np\(^{237}\). Both transitions are hindered for the radiative operator in the "asymptotic" selection rules and unhindered in the same rules for the anomalous internal-conversion operator, but they differ in their experimental hindrance factors. This latter transition is hindered by a factor \(\approx 10^4\), compared with \(\approx 10^6\) for the Ta\(^{181}\) transition considered.

The experimental value of \(\alpha_k\) is 2.3, compared to Sliv's theoretical value of 2.4. This may be used to put a limit on \(x^{\text{expt}}\). With a deviation between theoretical and experimental \(\alpha_k\) of, say, 10%, we have

\[ |x|^{\text{expt}} < 10 \]

(excluding the unlikely case that the anomalous contribution is of twice the magnitude but opposite in sign to the normal term.) The semitheoretical value of \(x\) obtained by taking the absolute value of the "normal" matrix element from the gamma lifetime and calculating the anomalous matrix element from the wave function of Ref. 12 in the "asymptotic approximation" is

\[ |x|^{\text{theo}} \approx 50 \]

Again the single-particle estimate of \(x\) seems too high, but as discussed with the Ta\(^{181}\) case, this overestimation is not particularly disturbing.

As a summary of the El and Ml cases treated quantitatively one may state that the anomalous terms introduced by Church and Weneser seem to account semi-quantitatively for the anomalous Ml case encountered and on the whole to be consistent with the cases of hindered Ml transitions in which no anomaly is found. As for the quantitative comparison of the anomalous El internal-conversion operator, the experimental effect seems somewhat greater than theoretically expected, and the large values of the correction factor \(C_{\alpha_k}\) obtained by the attempted fitting of experimental data may probably more appropriately be considered an indication of higher-order effects, neglected in the treatment presented here.
EFFECTS ON ANGULAR DISTRIBUTION

A few words may be in order regarding the effects of ICC anomalies on angular-distribution experiments involving conversion electrons. Church and Weneser have already discussed the ML transitions. For El transitions the $s_{1/2}$ electrons (K,L) convert into $p_{1/2}$ and $p_{3/2}$ continuum states, and contributions to anisotropy arise from the $p_{3/2}$ part and--more importantly--from a $p_{1/2}$-$p_{3/2}$ interference. In the normal case $p_{1/2}$ and $p_{3/2}$ occur in comparable amount, whereas it is clear empirically from anomalous subshell ratios that the $p_{1/2}$ final state is generally most affected and $p_{3/2}$ very little. In a greatly enhanced K conversion, as is likely for the 267-kev transition in Np$^{237}$, the anisotropy in a conversion-line angular correlation would be depressed from normal, and the sign of the anisotropy might or might not be reversed from normal, depending on the relative phases of the normal and anomalous conversion to $p_{1/2}$ states. Angular-correlation experiments on conversion electrons could thus yield unique information on these relative phases.

ACKNOWLEDGMENTS

We are grateful to Prof. I. Perlman for his encouragement and to Drs. A. Bohr, K. Alder, B. Stech, and A. Winther for discussing with us certain aspects of the work. We wish also to thank Drs. F. Asaro, J. M. Hollander, and F. S. Stephens, Jr., and also Drs. G. T. Ewans and T. W. Knowles for supplying us with experimental data in advance of publication. We have furthermore profited from correspondence with Dr. G. Källen and Dr. P. O. Olsson.
APPENDIX

The cases in Table VIII in the heavy-element region are thoroughly discussed in the paper of Asaro et al., but other cases in Tables VIII-XI that need special discussion are covered briefly here.

Table VIII Cases (K-Allowed E1)

Lu$^{177}$, 146 kev

The decay scheme of Yb$^{177}$ to Lu$^{177}$ has been studied by several groups in recent years. A prominent feature of the photon spectrum is the 146-kev gamma ray depopulating a state measured as having $1.2 \times 10^{-7}$ sec half life. The conversion coefficient was reported by de Waard as $\alpha_K = 0.63 \pm 0.08$ and $K/LM \approx 3.5$. On this basis he designated the transition as 10% M2 - 90% E1, and Vartapetian points out that this admixture requires M2 radiation of 1.3 times the single-particle rate. From this observation one might suspect the ICC for E1 to be slightly high. However, Mize et al. re-measured $\alpha_K$ for the 146-kev gamma and set an upper limit $\alpha_K < 0.4$ and, using de Waard's K/LM ratio, they set a limit $M2/E1 < 0.4$. With this limit the M2 comparative lifetime is quite similar to that in the analogous transitions of 396 kev and 282 kev in neighboring Lu$^{175}$. For the 282-kev transition the angular correlation measurements fix the M2/E1 ratio and lead to the conclusion that $\alpha_K$ for the E1 is probably normal. We must conclude from the present uncertain data that the 146-kev transition in Lu$^{177}$ probably has a normal E1 conversion coefficient. Experimental work to resolve the disagreement on $\alpha_K$ and to establish L-subshell conversion ratios would be valuable.

Lu$^{175}$, 282 kev

Vartapetian has measured the lifetime of the 396-kev level from which this E1 transition originates as $(3.4 \pm 0.3) \times 10^{-9}$ sec. Several studies on Yb$^{175}$ have helped to establish the decay scheme, and a theoretical interpretation of some features has been given by Chase and Wilets.

From the angular correlation work of Åkerlind et al. on the 282-113 cascade it is possible to determine the E1/M2 ratio of the 282-kev independently of its conversion coefficient. Assuming de Waard's value of $E2/M1 = 0.3$ for the 113-kev gamma (recent work of Hatch et al. gives $E2/M1$
0.25, in reasonable agreement), one finds that the anisotropy measured by Åkerlind et al. supports 2% M2 – 98% E1 for the 282-kev transition. With this mixing ratio and Sliv's theoretical K-conversion coefficients (for E1, 0.0205, and for M2, 0.67) we calculate a theoretical normal ICC of 0.0334. Hatch et al. give an experimental $\alpha_K$ of 0.030 and Mize et al. $^{28}$ give 0.038, so we conclude that the E1 ICC here is normal. Vartapetian $^{22}$ has pointed out that the 2% M2 admixture requires an M2 transition rate half that of the single particle formula.

$^{169}$Tm, 63 kev

The Yb$^{169}$ electron capture decay has been studied in several recent investigations. $^{27,34,36,37}$ Some features of the decay have been discussed also by Mottelson and Nilsson. $^{38}$

The 63-kev transition proceeds from a level at 380 kev according to the decay scheme of Mihelich et al. $^{37}$ and they measured a lifetime of 4.5 x 10$^{-8}$ sec. for the state. They determined an L-conversion coefficient of 0.19 ± 0.04 and Hatch et al. $^{34}$ give 0.15 for the same quantity. The corresponding Rose theoretical value for E1 is 0.15. Mihelich et al. measure relative L subshell ratios of 2.3/0.8/1.0, consistent with the theoretical ratios 2.2/0.8/1.0. We conclude that this transition has normal conversion coefficients.

$^{177}$Hf, 208 kev

The beta decay of Lu$^{177}$ to Hf$^{177}$ has received considerable study in recent years, $^{22,39-41}$ and the ground state spin of Hf$^{177}$ has been determined as 7/2 by Speck and Jenkins. $^{42}$ Some information has also come from study of electron capture of Ta$^{177}$, $^{43,44}$ and from Coulomb excitation. $^{45}$

There appears to be a level at 321 kev populated both in beta decay and in electron capture. It decays by 321-kev and 208-kev gamma rays. For our study of the ICC problem the latter transition is of the greater interest, since there is information on the E1-M2 mixing from angular-correlation data. $^{22,40}$ The interpretation of the angular correlation requires independent knowledge of the ML-E2 ratio in the 113-kev cascade transition. From L-subshell ratios $^{39,43}$ one would calculate a small ML admixture of 2 to 3%. The experimental anisotropy is consistent with ML/E2 = 0.03±.005 and a pure
El transition; this interpretation is proposed by Ofer.\textsuperscript{41} However, the angular-correlation experiment does not fix the limit on M2 admixture in the 208-kev very well. Two percent M2 in the 208-kev is consistent with 2.8\% or 3.2\% M1/E2 possibilities in the 113-kev.

The experimental $\alpha_K$ of the 208-kev is 0.044,\textsuperscript{39} to be compared with Sliv's theoretical $\alpha_K(\text{El}) = 0.0446$ and $\alpha_K(\text{M2}) = 2.05$. We conclude in Table VIII that $\alpha_K$ is normal, although we cannot exclude the remote possibility that $\alpha_K$ is anomalously small and that there is M2 admixture.

A limit on the 321-kev state lifetime was set by Vartapetian\textsuperscript{22,23} as $t_{1/2} < 4 \times 10^{-10} \text{sec}$.

Eu\textsuperscript{153}, 98 kev

The level system of Eu\textsuperscript{153} has been studied by Coulomb excitation\textsuperscript{46} and by beta decay\textsuperscript{47-52} of Sm\textsuperscript{153} and electron capture\textsuperscript{53-58} of Gd\textsuperscript{153}. Among the transitions is a 98-kev El transition to ground with a lifetime of < 10\textsuperscript{-9} sec. Marty and Vergnes\textsuperscript{56} report $\alpha_K = 0.3 \pm 0.1$, and Church\textsuperscript{59} has given the $\alpha_K$ relative to that of the 103-kev M1 transition discussed later in this appendix. From his ratio we calculate $\alpha_K \approx 0.17$. The Sliv and Band theoretical $\alpha_K$ is 0.23. Hence, we conclude that $\alpha_K$ is normal within experimental uncertainty.

The initial- and final-state assignments are $5/2, 5/2 - [532]$ and $5/2, 5/2 + [413]$, respectively.\textsuperscript{19}

W\textsuperscript{182}, 152 kev

Wapstra and Nijgh\textsuperscript{3} have recently renormalized and discussed conversion coefficients in W\textsuperscript{182} and W\textsuperscript{183} from measurements by Murray et al.,\textsuperscript{60} and they list conversion-coefficient comparisons for a few El transitions in W\textsuperscript{182}. The $\alpha_K$ of the 152-kev transition from $3, 2^- (I_\pi = 2)$ to $2, 2^+$ is less than half the Rose theoretical values (which differ very little from Sliv and Band values, in this case). The 222-kev transition from $4, 4^- \rightarrow 3, 2^+$ exhibits a normal $\alpha_K$ for El. It is interesting to note that the anomalous case is not K-forbidden, while the normal case is K-forbidden. With only one determination of these conversion coefficients we might maintain some reservation about labeling the 152-kev transition as anomalous. However, recent work of Gallagher on Re\textsuperscript{182} decay to W\textsuperscript{182} has clearly shown that there is an anomaly
in the K/L\textsubscript{1} ratio, the experimental value being 2.8 and Rose's theoretical being 8.3. This K/L\textsubscript{1} measurement and the \(\alpha\textsubscript{K}\) measurement together indicate an \(\alpha\textsubscript{L\textsubscript{1}}\) higher than normal. In the main text of this paper we have discussed the possibility that the El anomaly might give a constructive contribution to L\textsubscript{1} (or K) conversion and a destructive contribution to K (or L\textsubscript{1}). Gallagher also finds probable L\textsubscript{1}/K anomalies in other El transitions of W\textsuperscript{182}, but there is considerably more experimental uncertainty with them than with the 152-kev transition.

We do not have a lifetime determination for the 152-kev transition, but we may estimate the order of its retardation from that of the analogous 67-kev transition (2,2\(\rightarrow\)2,2\(+\)). The 2,2\(-\) state has a half life of \(1.03 \times 10^{-9}\) sec according to measurements of Sunyar. From this measurement and relative gamma intensity measurements we calculate a retardation factor of \(4.5 \times 10^{-3}\) for the 67-kev El.

It seems surprising that El anomalies should set in at such low retardation, but W\textsuperscript{182} is an even-even nucleus and is not really to be compared closely with odd-mass nuclei. Further experimental studies in this and similar even-even nuclei are certainly desirable.

### Table IX Cases (K-Forbidden El)

<table>
<thead>
<tr>
<th>Hf\textsuperscript{180m}</th>
<th>57.6 kev</th>
</tr>
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</table>

This unusual 5.5-hr El isomeric transition is the slowest El transition known; it is slower by a factor of \(10^{15}\) than the single-particle formula predicts. The values L\textsubscript{I}:L\textsubscript{II}:L\textsubscript{III} \(\approx 5:0.5:1\) and \(\alpha\textsubscript{L} \approx 0.4\) have been reported, from which one determines approximately that \(\alpha\textsubscript{L\textsubscript{I}}, \alpha\textsubscript{L\textsubscript{II}},\) and \(\alpha\textsubscript{L\textsubscript{III}}\) are 0.31, 0.03, and 0.06 compared with theoretical values of 0.11, 0.051, and 0.058 for the three L subshells for El. (Corresponding values for M2 are 67, 6.1, and 21). Clearly it is not proper to invoke M2 admixture, since \(\alpha\textsubscript{L\textsubscript{III}}\) agrees for El. The threefold enhancement of \(\alpha\textsubscript{L\textsubscript{II}}\) and possible decrease of \(\alpha\textsubscript{L\textsubscript{II}}\) is to be ascribed to anomalous El conversion contributions. The great retardation has been attributed to a high order of K-forbiddenness, \(\Delta K = 8\) or 9. The appearance of anomalous internal conversion according to this interpretation indicates, however, that for the small components of the wave function by which the transition may proceed the anomalous matrix elements may greatly exceed those for the radiative transition.
This El transition\(^{43,67,68}\) seems analogous to the case of Hf\(^{180m}\) just discussed. With a lifetime of \(3\) sec, the retardation factor is \(2 \times 10^{14}\). It is reported that the total conversion coefficient is \(0.5_{43}\) compared to the theoretical value of \(0.46\). Here again the retardation is presumably due mainly to K-forbiddenness, \(\Delta K = 8\) or \(9\).

\(\text{Pu}^{239}, 316\text{ kev and } 334\text{ kev}\)

These weak El transitions have been seen arising from the \(1.93 \times 10^{-6}\) sec level at \(391.8\) kev.\(^{69,70}\) From the branching ratios for transitions from this level we calculate that the 316-kev transition is retarded by a factor of \(9.4 \times 10^8\) and the 334 by \(8.4 \times 10^8\).

Ewan and Knowles\(^{70}\) measured the K-conversion coefficients of these gammas and found them in agreement with theoretical El values of Sliv or Rose, which are not very different from each other in this case.

The great slowness of these transitions is largely to be attributed to K-forbiddenness, since \(K_i = 7/2\) (or possibly \(5/2\)) and \(K_f = 1/2\).\(^{69,19}\)

\(\text{Re}^{183}, 382\text{ kev}\)

Newton and Shirley\(^{71}\) have found a 382-kev El transition in \(\text{Re}^{183}\) following the decay of Os\(^{183}\). The state at \(496\) kev from which this transition originates was measured to have a half life of \(8 \times 10^{-9}\) sec. According to Newton's interpretation the initial state is \(9/2 - (K = 9/2)\) and the final state is \(7/2 + (K = 5/2)\), being the first excited state of the ground rotational band. Thus, the transition is K-forbidden.

The experimental K-conversion coefficients is \((1.0 \pm 0.1) \times 10^{-2}\), to be compared with Sliv's theoretical value of \(1.12 \times 10^{-2}\), and thus seems normal.)

\begin{table}[h]
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\caption{Cases (K-Allowed Ml)}
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\hline
\text{Ta}^{181}, 482\text{ kev}\
\hline
The levels and transitions of Ta\(^{181}\) have received much study through Coulomb excitation and beta decay of Hf\(^{181}\). We shall not attempt to list references or give a detailed discussion, since the data have been thoroughly reviewed recently by Debrunner et al.\(^{72}\) Suffice it to say here that the Ml-E2
\end{tabular}
\end{table}
mixing ratio of the 482-kev transition has been carefully determined by gamma-gamma and electron-gamma angular correlations as $98\% E2 + 2\% M1$. Its half life is $1.06 \times 10^{-8}$ sec. Several determinations of $\alpha_K$ are listed by Debrunner et al., and they choose $\alpha_K = 0.026$ as a best value. Using Sliv's theoretical conversion coefficients one finds a significant discrepancy, which we choose to interpret as the $M1$ conversion coefficient being a factor of about 10 larger than normal and the unhindered $E2$ ICC's being normal. The experimental uncertainty on this enhancement factor is considerable, but there is clearly some enhancement outside limits of experimental error.

The initial- and final-state assignments $\frac{5}{2}, \frac{5}{2} + [402] \rightarrow \frac{7}{2}, \frac{7}{2} + [404]$ are supported by spin determinations as discussed by Debrunner et al., and the orbital assignments are based on a variety of evidence to be reviewed in a forthcoming publication. The radiative $M1$ transition is allowed by $K$-selection rules but hindered in "asymptotic" quantum-number rules (see Table VI), probably explaining at least partially its great retardation ($2.6 \times 10^6$). The anomalous $M1$ conversion operator is unhindered ($\Delta n_z = 0, \Delta \lambda = 2, \Delta \Sigma = -1$) (see Table VII), a favorable situation for appearance of the ICC anomaly.

$^{237}Np$, 208 kev

The 208-kev $M1$ transition is a prominent feature of the beta decay of $^{237}U$ (cf. Rasmussen, Canavan, and Hollander and references listed therein). By L-subshell conversion ratios the $E2/M1$ mixing ratio is 0.5% or less. Within experimental error the $\alpha_K$ of 2.3 agrees with the Sliv theoretical $M1$ value of 2.4. The half life of the state at 267 kev from which the 208-kev transition originates was measured by Bunker, Mize, and Starner as $5.4 \times 10^{-9}$ sec.

The reasons supporting the state assignments associated with this transition exhibited in Table X are detailed in Ref. 73. With respect to the asymptotic quantum-number selection rules, the situation is the same as in $^{181}Ta$ just discussed: $\Delta n_z = 0, \Delta \lambda = 2, \Delta \Sigma = -1$. Hence, the radiative operator is hindered and the anomalous operator is unhindered.
The reader is referred to the paragraph on the 98-kev E1 transition in Eu\(^{153}\) for references on the Eu\(^{153}\) decay scheme. The 103-kev M1 transition is thought to proceed to ground from a state with half life \(4 \times 10^{-9}\) sec, as measured by McGowan,\(^{48}\) by Graham and Walker,\(^{49}\) and by Vergnes and Marty.\(^{51}\) Its K-conversion coefficient has been variously measured as 1.19 (Dubey et al.,\(^{52}\) 1.2 (Marty\(^{51}\)), 1.1 (McGowan\(^{48}\)), and 1.2 (Cohen et al.\(^{54}\)). Bhattacharjee and Raman\(^{57}\) reported \(\alpha_K = 0.67\), and Bisi et al.\(^{58}\) reported 0.68, but these two last-mentioned studies apparently failed to take into account the presence of the 98-kev E1 transition, which would not have been resolved from the 102-kev in the photon spectrum. From the L-subshell conversion ratios of the 102-kev transition it is established\(^{75}\) that there is less than 5% E2 admixture. Sliv's theoretical K-conversion coefficients for this case are 1.48 for M1 and 1.10 for E2. The experimental ICC seems significantly lower than the theoretical by about 20%. However, we have classified the conversion coefficient as normal in Table X, since the analysis by Wapstra and Nijgh\(^3\) on a number of M1 transitions, most of which were not of the retarded nature considered here, showed their conversion coefficients to be systematically still somewhat lower than the Sliv theoretical values.

### Table XI Cases (K-Forbidden M1)

**Tm\(^{169}\), 178 and 199 kev**

In the first section of this Appendix, where the 63-kev E1 transition in Tm\(^{169}\) was discussed, we listed the references to experimental work elucidating the Tm\(^{169}\) level system. In Tm\(^{169}\) there is a level at 316.2 kev with a half life of \(6.4 \times 10^{-7}\) sec. The level is assigned \(7/2, 7/2 + [404]\),\(^{19}\) and it decays by K-forbidden (\(\Delta K = -3\)) transitions to states of the ground rotational band, \(7/2 +\) and \(5/2 +\), \(K = 1/2 [411]\). The analysis by Mihelich et al.\(^{37}\) determines the M1-E2 mixing ratios of the 178- and 199-kev transitions on the basis of L-subshell conversion coefficients, and they give the value \(5 \times 10^5\) as factors of retardation for the M1 components of both transitions.

Using the Mihelich et al. mixing ratios and Sliv ICC values, we obtain the theoretical \(\alpha_K\) for the 178-kev transition of 0.49, while Mihelich et al. find experimentally 0.49 ± .10 and Hatch et al.\(^{34}\) find 0.51. For the 199-kev transition, the corresponding theoretical value is 0.35 and the
experimental values $0.45 \pm 0.09^{37}$ and $0.40^{34}$. Thus, we conclude that the ICC's are normal.

Pu$^{239}$, 277.9, 228.2, and 209.7 keV

There are three prominent M1 transitions seen following beta decay of Np$^{239}$ or Am$^{239}$ and following alpha decay of Cm$^{243}$. These transitions of 277.9, 228.2, and 209.7 keV arise from a level at 285.8 keV with half life $1.1 \times 10^{-9}$ sec. The experimental work relevant to this level scheme has been reviewed by Perlman and Rasmussen, and we refer to this review work for the original references. The level at 285.8 keV has been assigned $5/2$, $5/2 + [622]$, and the three M1 transitions go to excited states of the ground rotational band of spins $3/2$, $5/2$, and $7/2$. The ground state of this latter band has been assigned $K = 1/2 +$ and the orbital $[631]$. Hence, the retardation of the M1's may be largely ascribed to K-forbiddenness ($\Delta K = 2$). The best determinations of conversion coefficients of these three transitions are probably those of Ewan and Knowles. Limits of < 10% E2 admixture were set on the basis of L-subshell conversion coefficients, and Table XII lists their experimental $\alpha_K$ values and Sliv theoretical values for pure M1 and for the upper limit of 10% E2 admixture. We conclude that these $\alpha_K$ values are probably normal although the 210-keV conversion coefficient shows a discrepancy with the theoretical values by twice the probable error.

<table>
<thead>
<tr>
<th>Gamma energy (kev)</th>
<th>$\alpha_K$ (Exptl.)</th>
<th>$\alpha_K$ (Theoretical)</th>
<th>Retardation factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Pure M1</td>
<td>90% M1–10% E2</td>
</tr>
<tr>
<td>278</td>
<td>1.16±.12</td>
<td>1.18</td>
<td>1.07</td>
</tr>
<tr>
<td>228</td>
<td>1.60±.16</td>
<td>2.04</td>
<td>1.85</td>
</tr>
<tr>
<td>210</td>
<td>1.76±.30</td>
<td>2.58</td>
<td>2.33</td>
</tr>
</tbody>
</table>
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