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Superdeformed Band Relationships, Mass-190 Region

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A new region of superdeformed (SD) bands in the mercury nuclei has been extended to the thallium and lead isotopes. A surprising number of excited, as well as yrast, bands have been found, and they have very interesting properties. So far 25 such bands have been observed, including six in one nucleus, $^{194}$Tl. They show some startling features. More than half have transition energies equivalent within a few keV to transitions in other SD bands in the region. And in these examples the relative alignments with respect to an optimum reference SD band is most often integral with the value 1$\hbar$, whether comparing an odd- or an even-mass nucleus with the reference. We believe the pseudo-spin formalism gives clues to explaining these features, but not the fact that changes in orbital alignment, in deformation, in pairing, and in mass seem to cancel almost perfectly although individually they may cause larger variations than are observed. Perhaps some new basic physics is involved, or at least some new symmetries.

With the great increase in information on superdeformed (SD) bands during the past year, we have come to realize that they have some startling properties. I shall tell you a little about some recent work we have done (the we is a combined LBL-LLNL group and their visitors), and then some speculation to try to explain some of the results.

It has been suggested\textsuperscript{1-7) that another region of SD bands, of rotational cascades in a second strongly deformed minimum in the nuclear potential energy surface, would occur around the atomic number $Z = 80$. The first example of such a band was found\textsuperscript{8) one-and-a-half years ago by the Argonne group in $^{191}$Hg. Since then more than two dozen discrete SD bands have been found in the mass-190 region. The first figure shows the spectrum of the SD band in $^{192}$Hg; it was discovered shortly after $^{191}$Hg independently but simultaneously by the Argonne group\textsuperscript{9) and by our combined LBL-
This spectrum shows features common to most of the SD bands in the mass-150 and mass-190 regions, namely, an increasing intensity with decrease in spin as the band is fed until about midway in the cascade where the intensity becomes essentially constant, and finally an abrupt decay out of the band in one or two transitions. The maximum intensity of the band is of the order of 2% of the channel intensity, and this is one of the strongest bands observed in this region; others are weaker by factors as large as five. Angular correlation measurements on the stronger transitions compared to those for known stretched electric quadrupole transitions in the ground band indicate that they are also stretched quadrupole transitions. Finally, no connecting transitions could be identified between the SD and the yrast ground band, just as for the mass-150 examples and all the other cases in this region. But coincident transitions in the ground band of $^{192}$Hg and the excitation function permitted the assignment of this SD band to that nucleus.

There are also differences from the bands in the mass-150 region. In the latter, the dynamic moments of inertia, $J(2) = dI/d\omega$, generally decrease with increasing transition energy, $E_T$, or rotational frequency, $\omega = dE/dI = E_T/2$, but there is a great deal of individuality in the observed discrete SD bands, and some of them even show the opposite trend. Clearly this behavior depends upon the nature of the individual orbitals being occupied. In the mass-190 region, the transitions form good rotational cascades, but the dynamic moments of inertia of all bands tend to increase with spin or $\omega$, Fig. 2. They are remarkably similar, and this suggests that the increase is due to alignment of a relatively large number of high-$j$ orbitals, or more generally, to a gradual alignment of
several orbitals and to a decrease in pairing, weak as it may be.

Now let me show you one of the more startling features of some of these SD bands.

Fig. 2. Map of dynamic moments of inertia, $J^{(2)}$, vs. $\omega$ for the 25 known SD bands in the mass-190 region at this time.

Fig. 3. Differences in transition energies between the excited SD bands in $^{194}$Hg and the SD band in $^{192}$Hg.
Figure 3 shows the difference in transition energies\textsuperscript{11}) between the excited SD bands in $^{194}$Hg and the yrast band in $^{192}$Hg. For $\omega > 0.2$ MeV the transition energies are almost exactly equivalent to about 1 keV (By equivalent is meant that the energies are either the same, or for the other member of a signature-partner pair with vanishingly small splitting, they are the average of two consecutive transitions in the other band). If one uses the macroscopic scaling of $A^{5/3}$ for the moment of inertia, these energies should differ by 1.7\%, or 10 keV for a 600 keV transition; this is nearly an order of magnitude more than the observed difference. And more than half of the SD bands in Hg, Tl, Pb show such an equivalence in transition energies. Other cases may not be so accurately similar (some are), but they are all within a few keV. For example, consider the six SD bands we have found in the one nucleus, $^{194}$Tl. They form three signature-partner pairs\textsuperscript{12)}, Fig. 4, and have transition energies that are equivalent to a few keV in the sense mentioned above. This equivalence has also been seen in the mass-150 region in two cases\textsuperscript{13)}, but seems

\begin{figure}[h]
    \centering
    \includegraphics[width=\textwidth]{Fig_4.png}
    \caption{Triple-coincidence spectra of the six SD bands in $^{194}$Tl; each is the sum of a number of double gates in that band.}
\end{figure}
to be a general phenomenon in the heavier mass range, and it cannot be an accident. So the first big puzzle is; why do so many SD bands show such an equivalence in energies?

But there are more. Because the spins of these SD rotational cascades go down to much lower values than in the mass-150 region, they can be determined. This comes about because the transition energy of a rigid, axially symmetric rotor is proportional to its spin \[ E_T = (4\langle I \rangle + 2)\hbar^2 / I \] where \( \langle I \rangle \) is the intermediate spin of a transition and so changes most rapidly at low spin. Nuclei are not rigid rotors, but the SD bands do follow this pattern quite well. Our Livermore\(^{14} \) and University of California, Davis\(^{15} \) colleagues have shown, in somewhat different developments, that one can express the dynamic moment of inertia, \( J(2) \), as a function of \( \omega^2 \),

\[
J(2)/\hbar^2 = 2\alpha + 4\beta\omega^2 + \cdots \quad (1)
\]

and then evaluate the parameters \( \alpha \) and \( \beta \) from the experimental values of \( J(2) \) at low spin. Integration of (1) gives the average or intermediate spin of the transition \( \langle I \rangle + 1 \rightarrow \langle I \rangle - 1 \),

\[
\langle I \rangle + 1/2 = 2\alpha\omega + 4/3\beta\omega^3 + \cdots \quad (2)
\]
as long as there is no discontinuous change in alignment in the extrapolated range from zero to the lowest spins observed. Thus no initial alignment or sudden change, such as a sharp backbend, may take place, but a gradual crossing with a large mixing matrix element is alright. This is an important caveat, but seems not to be a problem at the low spins found in the mass-190 region (6-12\( \hbar \)), as the values determined so far are less than 0.2\( \hbar \) away from integer values for even-mass nuclei and less than 0.3\( \hbar \) from half-integral values for odd-mass examples (which tend to be of higher spin); irregular alignments might not be expected to behave so well. Also, the values found are compatible with the average entrance spins into the ground bands wherever those have been observed in coincidence with the SD bands.

Using the spins so determined, we can plot the intermediate spins of the transitions against the rotational frequency \( \omega \). Figure 5 shows such a plot for the SD band in \(^{192}\text{Hg}\) and for the two excited signature-partner bands in \(^{194}\text{Hg}\). First of all, note that the two curves are quite parallel for \( \omega > 0.2 \text{ MeV} \). This says that the three bands have the same values of \( J(2) \) for transitions above 400 keV; they have equivalent energies. The vertical difference between the two curves is the difference in alignment between the bands, or if
Fig. 5. Average spin vs. frequency $\omega$ for the two excited signature-partner bands in $^{194}$Hg and the yrast band in $^{192}$Hg.

we pick that of $^{192}$Hg as the reference (an arbitrary choice), it is the alignment relative to that reference. This relative alignment is plotted $^{11}$ in Fig. 6 against the rotational frequency. It tells us that at low transition energies (low spins) the excited $^{194}$Hg bands show an increase in alignment relative to the yrast SD band in $^{192}$Hg. But above $\omega = 0.2$ MeV, the difference is constant at $1.00 \pm 0.04\%$. Since alignments are not quantized, there is no obvious reason for this value of unity, but the equivalence of the transition energies,

Fig. 6. Difference in spin, or relative alignment, vs. $\omega$ for the two excited bands of $^{194}$Hg compared to yrast band in $^{192}$Hg.

Fig. 7. Alignments vs. frequency $\omega$ for the nine bands listed relative to the yrast band in $^{192}$Hg.
the puzzle already mentioned, does require that the relative alignments of these doubly-even nuclei be integral; they could be 0, 1, 2, 3, etc. units.

The next figure shows a plot against \( \omega \) of the alignments relative to \(^{192}\text{Hg}\) of nine SD bands from this mass region. For all but \(^{194}\text{Pb}\) there is an initial rise indicating a more rapid increase in alignment with rotational frequency than for the reference band, and then the relative alignments do reach near-integral values; one is \(~0\), two are \(~2\), and six are \(~4\). As mentioned above, the feature that over a range of rotational frequencies these values are nearly integral is a reflection of the fact that their transition energies are nearly equivalent to those of \(^{192}\text{Hg}\). But that a majority are \(4\) is a second puzzle, and a third is that this is true even for those of the plotted curves that are odd nuclei, whose expected "natural" alignment with respect to a doubly-even reference would be half-integral\(^{16}\). This result is completely unexpected and perhaps even more difficult to explain than the transition-energy equivalence.

About half of the mass-190 SD bands do not seem to be closely related to \(^{192}\text{Hg}\) and the other half of the bands. For example, Fig. 8 shows the alignments of the SD bands in \(^{193}\text{TI}\) (Ref. 17), \(^{194}\text{TI}\) (Ref. 12), \(^{195}\text{TI}\) (Ref. 18) against the band in \(^{192}\text{Hg}\). They become roughly horizontal but are not grouped as nicely as in the previous figure. This is not too surprising, as the properties of the SD bands must depend upon the orbitals occupied,
particularly the high-j ones which contribute greatly to the moments of inertia, and the added proton in thallium probably goes into an additional $i_{13/2}$ orbital. But it also means that the optimal reference band can be different for different groups of nuclei. This is shown in Fig. 9 where the alignment of the six bands in $^{194}$Tl and two bands in $^{195}$Tl are shown referred to the appropriate one of the two signature-partner bands in $^{193}$Tl, thus putting the extra proton into the reference. Now the bands show near-integral relative alignments, indicating that the 113th and 114th neutrons are "better" spectators than the 81st proton. Each of two pairs in $^{194}$Tl has alignment one against its respective $^{193}$Tl reference band, and one pair has alignment zero against one of the $^{193}$Tl bands. This suggests that there might be still another pair of bands with alignment zero against the other $^{193}$Tl band. These are being looked for but have not been found yet, either because they are too weak or they do not in fact exist.

Finally we come to possible explanations for the puzzles presented. We do not know the answers, but we can speculate. Consider the features that many of the nuclei have equivalent energies and that many of the alignments relative to a common reference appear to be quantized with the value $\hbar$. Think of adding two particles to a reference core, for example two neutrons to $^{192}$Hg to form the excited SD bands in $^{194}$Hg. The only scenario we have thought of involves the alignment of the intrinsic spins of the two neutrons while the orbital angular momenta remain coupled to the deformation, to the nuclear symmetry axis. The tendency for such behavior has been shown to be what is expected in the pseudo-spin formalism. This formalism treats the natural-parity orbitals in a shell, excluding the opposite-parity orbitals intruding from the shell above, and ignoring the highest-j normal-parity orbitals which similarly are pushed down into the shell below. For example, for the neutron numbers we are interested in, 108-118, the natural-parity orbitals in shell $N=5$, namely, $h_{9/2}$, $f_{7/2}$, $f_{5/2}$, $p_{3/2}$, $p_{1/2}$ are rather close-spaced and form a pseudo-oscillator shell with $\bar{N}=4$ and $g_{9/2}$, $g_{7/2}$, $d_{5/2}$, $d_{3/2}$, $f_{1/2}$ orbitals, as shown in Fig. 10. The feature of interest to us is that the pseudo spin-orbit splitting is quite small, of order ten times smaller than in the Nilsson scheme, so that the spin-orbit doublets with $\lambda \pm 1/2$ are quite close, and this separation remains small, a few hundred keV, over the range of deformation given in the figure. This means that the
Coriolis interaction, with increasing rotational frequency, can mix the doublet states to give an increasing alignment of the pseudo intrinsic spins, yielding a maximum quantized alignment of $\hbar$ when the pair of particles are fully aligned if there is no additional orbital alignment. When fully aligned, the intrinsic spins cannot contribute to the moment of inertia and thus to the transition energies, and so give the equivalent energies observed. All the natural-parity (pseudo-spin) orbitals can generate such quantized alignments of $\hbar$, and this is the only way we have been able to think of such a process.

However, in suggesting this possible answer to the first two puzzles, we meet a fourth: why do other effects not disturb the equivalence in energies and the unit value for the relative alignment? There are several which should contribute larger differences than are seen in the energies by as much as an order of magnitude. These include orbital alignment, deformation, mass change, and pairing. There are reasons to believe that the effects of the first two do tend to cancel. For the full harmonic oscillator one gets the rigid-body moment of inertia at equilibrium at zero spin, and this does not change as the spin increases, even though the deformation decreases. This will not happen exactly for a realistic potential, but the tendency appears to exist. There is some evidence for this even
with normally deformed rotational bands. Figure 11 shows the behavior of the moment of inertia and of the deformation as represented by the reduced transition probability, the $B(E2)$, with increasing spin in the normally deformed yrast band of $^{158}$Er (Ref. 22). Above the first backbend, the moment of inertia stays almost constant (actually increases slightly) while the $B(E2)/B(E2)_{\text{rot}}$ decreases by about an order of magnitude.

The mass dependence referred to above is the $A^{5/3}$ scale factor that even the harmonic oscillator moment of inertia should follow. However, it is true only on the average. In a range of nuclei making up a real shell, the variation is much smaller, particularly at higher spin where pairing and stretching effects are smaller. Both the $l\cdot s$ term and the flattening of the bottom of the potential well (Woods-Saxon potential or $l^2$ term in the Nilsson one) lower in energy the high-$l$ orbits that give the largest contributions to the moment of inertia. Thus extra moment will be produced at the beginning and less than average at the end of a shell, systematically reducing the mass dependence in the shell (both for normal and SD bands).

The effects of pairing must also be considered. Changes (decrease) in the pairing correlations almost certainly are involved in the gradual, smooth increase in moment of inertia observed for all the SD bands in the mass-190 region. So there is pairing, but it
must be rather weak, and this is both because of the moderately high spins involved and because the particle numbers are around those of deformed magic numbers. Possibly pairing vibrations rather than static pairing is involved, and I do not know if the effects of changes in that have been studied theoretically, but it would appear that the effects of a decrease in the residual pairing on the moment of inertia must be rather uniform for the SD bands in the nuclei studied so far.

The last puzzle on the list is why alignments of $\frac{1}{2}l'$ are so common for odd-mass nuclei when $\frac{1}{2}l'$ is to be expected for a single particle. This suggests that a pair of pseudo-spin aligned particles are also involved to give the alignment, and that the odd nucleon (possibly in an excited or hole state) is in a strongly deformation-coupled zero-alignment orbital such as a high-$\Omega$ intruder level. But why this, rather than the simple alignment of the odd particle? Maybe the pair of pseudo-spin aligned particles (whose orbital motion is time reversed) can scatter into other empty pseudo-spin orbitals (remember that all the normal-parity levels can form them) and so contribute to a triplet pairing which lowers the energy of the aligned pair in competition with the system of the odd particle alone. Although not familiar in nuclear systems, triplet pairing does exist in superfluid $^3$He. To form the bosonic system necessary for the superfluid condensate, two $^3$He atoms pair off in the triplet state\(^{23}\), each contributing $\frac{1}{2}l'$ of spin. The singlet state, corresponding to the Cooper pair of electrons in superconductivity or the usual singlet pairing of nuclei, has not been seen yet, but theory predicts it should exist in dilute solutions of superfluid $^4$He. The catch is that this should occur at temperatures well below those to which liquid helium can be cooled at present, although there is hope to achieve such temperatures in the future. It is also possible that triplet pairing of neutrons occurs in neutron stars and pulsars\(^{24}\). If true, the formation of such pairs with their intrinsic spins aligned to $\frac{1}{2}l'$ might resemble the situation which we have been discussing.

It is fun and interesting to speculate, but it should be remembered that at present it is just that. However, it seems clear that the study of superdeformed bands will tax our ingenuity in developing new experimental techniques and theoretical approaches to solve the problems described and ones we have not yet uncovered. It will surely provide us with new views on nuclear structure, and perhaps there are more surprises ahead.
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