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ON THE POSSIBILITY OF DISCOVERING THE ASYMPTOTIC HADRON SPECTRUM THROUGH ULTRA-RELATIVISTIC NUCLEAR COLLISIONS

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February 1979

ABSTRACT

The asymptotic form of the hadronic mass spectrum, beyond the region where individual resonances can be identified, is a fundamental property of matter, interesting in both particle physics and cosmology. We investigate the possibility of discovering its form through the properties of hot hadronic matter such as might be produced in ultra-relativistic nuclear collisions. We find that at collision energies in the tens of GeV per nucleon in the c.m., a sensitivity of certain measurements to the underlying hadronic spectrum can be expected.
I. INTRODUCTION

The enormous number of hadronic resonances that have been discovered during the last few years,\(^1\) and in particular the growth in their number with increasing mass, raises very interesting questions that have significance at both the particle and cosmological level. Is the number of resonances finite or infinite? In either case, what is their mass spectrum? (i.e., how does the density of hadronic states vary with their mass?) The situation as it is known up to the present is depicted in Fig. 1, where the number of hadronic states per unit (pion) mass interval is plotted as a function of mass. The spectrum increases, essentially exponentially up to masses of about 14 \(m_\pi\) \((m_\pi = \text{pion mass} \approx 140 \text{ MeV, used as a convenient unit of mass})\). Thereafter there is an apparent cutoff, with only a few resonances of higher mass known. However, in the region of the cutoff, the average width of the resonances is of the order of several hundred MeV, while the "known" density of states in the cutoff region is of the order of several hundred states per pion mass interval. Therefore the experimental problem of resolving individual resonances at masses of the order of 10 \(m_\pi\) or more becomes extremely severe, not to mention the low production rates. The cutoff is therefore probably more apparent than real.

Is it interesting to know the form of the hadronic spectrum in the region where it has become impossible to distinguish individual resonances? The answer appears to be emphatically yes. For example, in particle physics the form of the spectrum of hadronic states at high mass will provide an asymptotic constraint on the theory of hadronic structure that may prove useful either in distinguishing between theories, or in fixing constants within a theory. On the cosmological scale, while the thermal
history of the universe can be traced backward in time to when the temperature was about 1 MeV with reasonable confidence, the unknown hadronic spectrum must have played a crucial role in determining the temperature and composition of the universe in its earliest instants when the energy density was enormous. Likewise the temperature of black holes in the late stage of their evolution, and the energy released in their ultimate explosion, depends on the unknown hadronic spectrum. In both cases these assertions follow from the partition of energy between kinetic energy and particle creation. The interesting dependence of the temperature and composition of matter at very high energy density on the form of the hadronic spectrum will become vivid later in the paper.

At this point it is appropriate to indicate the range of theoretical expectations concerning the asymptotic region of the hadronic mass spectrum. Currently the most popular theory of hadronic structure involves the hypothesis of colored quarks that interact through massless colored vector bosons (quantum chromodynamics). Baryons consist of three valence quarks and a sea of quark-antiquark pairs, while mesons consist of a valence quark and antiquark (generally of different flavors) and a sea of pairs. The hadronic resonances, in this picture, consist of the radial and angular momentum excitation modes of the quarks. By considering a naive idealization of the quark structure of hadrons, characterized only by the valence quarks contained in a rigid bag, we estimate a lower bound on the spectrum. A calculation of the phase space of the internal degrees of freedom of such a structure of mass $m$ yields a power law for the asymptotic density of states at that mass, namely, $m^5$ for baryons and $m^2$ for mesons. The extrapolation from the "known" density of states at $10 m_n$ of such a power law is shown in Fig. 1. This is a lower bound provided by the quark
Fig. 1. The hadronic spectrum is plotted as the density of states as a function of mass of the state. The resonances with reasonably certain determination of spin and isospin as of 1976 are represented by the histogram. Only the baryons and mesons are plotted, not the antibaryons. The curves are explained in the text.
hypothesis. Any relaxation of the rigidity of the hadron size, or the participation of the sea of quark-antiquark pairs in the excitation will increase the density of states. In addition, note that the apparent confinement of quarks may imply an infinity of hadronic states, and a possibly exponential spectrum. The bootstrap theory, a modern version of which appears to imply a quark substructure, provides another estimate of the hadronic spectrum in the asymptotic region. The bootstrap hypothesis can be implemented using the "known" particles and antiparticles as input, as was done by Hamer and Frautschi. The spectrum that they generate from a given group of low mass particles and antiparticles is represented by the dotted histogram in Fig. 2. This spectrum is exponential, in agreement with the thermodynamic formulation of the bootstrap hypothesis due to Hagedorn. In Fig. 1, we plot the Hagedorn spectrum (labelled Hagedorn)

\[ \rho(m) = \frac{c}{m^3} e^{m/T_0} \]

with the constants adjusted to agree with the "known" spectrum as to normalization at 10 m and the slope as determined by the bootstrap iteration shown in Fig. 2.

The two curves labelled "rigid quark bag" and "Hagedorn" provide the theoretically expected range on the hadronic spectrum in the high mass region. The rigid bag is presumably a rather rigorous lower bound, while the slope of the exponential curve is related to the known spectrum in a reasonable way. As already noted, although the exponential spectrum is known to follow from the bootstrap hypothesis, it may also be consistent with the quark hypothesis.

Now the full enormity of the problem of determining the asymptotic spectrum of the hadrons can be stated. In the vicinity of the cutoff of known hadrons, the number of resonances per unit (pion) mass interval
Fig. 2. The bootstrap iteration (dots) based on the particles and antiparticles represented by the dense cross-hatched histogram (adapted from Ref. 6). The dashed line is an exponential Hagedorn spectrum adjusted to pass through the bootstrap iteration.
expected on the basis of the exponential spectrum is $\sim 10^4$. On the basis of the lower bound this number is merely deferred to a higher mass. If a new resonance were discovered every day it would take a hundred years to determine this one point on the curve. Obviously it is impractical to determine the spectrum by an actual measurement of individual resonances as has been done in the past. The sheer numbers prevent this, even were it not for the already mentioned problem of resolving overlapping resonances.

We have already stressed the relevance of knowing the form of the hadronic spectrum. Its relevance in the contexts mentioned does not rest on the possibility (or impossibility) of resolving and discovering the properties of individual resonances, but only on the density of states as a function of mass. Fortunately, it may be possible to discover the form of the spectrum without the need to discover each resonance of which it is composed. For example, the specific heat of a material object depends on its composition. The composition of matter at very high energy density depends in turn on the number, types, and masses of particles or resonances that can be created at that energy density, that is, on the hadronic spectrum, whether or not the individual resonances are or ever will be known.

Therefore, properties of enormously hot matter, if they could be measured, would reveal the underlying form of the hadronic spectrum. This paper is devoted to studying whether ultra-relativistic collisions between nuclei might be used to reveal this fundamental property of matter. The thought is that enormous energy densities may thus be achieved, however fleetingly, and that the way in which the material disassembles or decays may retain a dependence on the conditions during the dense instants.
We investigate this possibility with the goal of learning what observables are most sensitive to the underlying hadronic spectrum, how high an energy one must attain to achieve a reasonable sensitivity, and in particular, whether the energy lies within the range of existing or conceivable accelerators.

II. PERSPECTIVE

A complete dynamical description of a nuclear collision at energies sufficient to create particles is obviously out of the question. It is presumably a field theoretic problem involving a Lagrangian describing all of the elementary fields and their interactions. Although there is a good candidate for the Lagrangian, that of quantum chromodynamics, its solution in the context of nuclear collisions cannot be envisioned in the near future. On the other hand, while a many-body system poses enormous problems if a microscopic account is sought, certain bulk properties can be inferred without solving the detailed equations of motion. This is the approach that we explore. We conceptualize the collision as progressing through three stages. We shall say nothing about the first stage, in which the high-energy primary collisions between nucleons are taking place, other than to assume that it leads through a succession of collisions to the stopping of one nucleus by the other. Statistical models have been applied to hadron-hadron collisions. We shall apply a statistical thermodynamic description to the second stage of the nuclear collision, after a sequence of hadron-hadron collisions have brought the system to an assumed state of equilibrium. Our view is that the hard parton collisions that seem to play an important part in the high-energy hadron-hadron
collisions may not be amenable to a statistical theory, whereas the
singular behavior of the parton collisions become irrelevant if equilibrium
is achieved in a high energy nuclear collision. Factors that would tend
to bring about equilibrium are: 1) an enormous phase space is opened up
by particle production; 2) the strong interaction of pions and nucleons
especially in the resonance region and the corresponding high velocity
of the pions (near c) provides a fast mechanism for thermalization;
3) thermalization may require only 3 or 4 collisions anyway; 4) the
finite size of a nuclear complex inhibits the disassembly of its interior;
5) particle creation increases the density. In the third stage the
collision complex, called a nuclear fireball, disassembles. During this
stage, the fastest particles escape first while the whole fireball
expands and the constituents eventually cease to interact. Thereafter
resonances decay freely and only stable particles survive to be seen in
the counters. This work goes considerably beyond our initial exploration
published elsewhere, especially as concerns the composition of the
fireball, a description of the third expansion stage of the reaction,
and the identification of the signals that are the most sensitive to the
hadronic spectrum.

III. THERMODYNAMICS OF HADRONIC MATTER

For the five reasons stated earlier, we assume that a state of
quasi-equilibrium is achieved in a very high energy nuclear collision.
Here we set down the thermodynamic equations for a relativistic gas of
hadrons. These are known for a free gas. We need to clarify why such
equations make sense for an interacting gas of hadrons. In his thermo-
dynamic formulation of the bootstrap theory of hadrons, Hagedorn invoked
an old statistical mechanical technique that allows one to (at least
c spuriously) systematically shift the effect of the interactions between
particles to the density of states in phase space. The technique rests on the observation that the (non-relativistic) wave function of an interacting pair of particles contained within a cavity of radius $R$, is on the boundary

$$\psi \sim \sin(kR - \ell \pi/2 + \delta_\ell) = 0.$$ 

From this we learn that the density of states is

$$\frac{dn_\ell}{dk} = \frac{1}{\pi} \left( R + \frac{d}{dk} \delta_\ell \right).$$

This shows how the phase space $R/\pi$ of free particles is modified by interactions that are characterized by phase shifts $\delta_\ell$. A generalization in relativistic statistical mechanics has been demonstrated more recently. Hereafter we regard the density of hadron states, $\rho(m)$, to represent the combined "bare" spectrum and the contribution to phase space of the interaction between particles or resonances in the "bare" spectrum. We stress that in the region of overlapping resonances, where it is impractical to identify all of the resonances individually, it is this combined density which is the most interesting object anyway. It determines the available phase space, and for the astrophysical implications of the hadronic spectrum, that is what is needed.

The Fermi and Bose distribution functions for a relativistic ideal gas of particles of mass $m$ and statistical weight $g = (2J+1)(2I+1)$ at temperature $T$ and occupying a volume $V$ are (units are $\hbar = c = k_{\text{Boltzman}} = 1$):

$$f(p,T) d^3p = \frac{gV}{2\pi^2} \frac{p^2 dp}{\exp(E_p - \mu)/T + 1}, \quad \left( \begin{array}{c} F \\ B \end{array} \right)$$

(1)
where

\[ E_p = \sqrt{p^2 + m^2} \]

The chemical potential is denoted by \( \mu \).

We want to describe a gas of baryons and mesons distributed in mass according to unknown functions \( \rho_\alpha(m) \). Here \( \alpha \) labels the families of particles, ordinary and strange baryons and mesons, some of which are known and listed in Table I. The number of particles and energy residing in the family \( \alpha \) is given by the following convenient expressions which converge rapidly when \( (E_p - \mu)/T \gg 1 \).

\[
N_\alpha = \frac{VT}{2\pi^2} \int_{m_\alpha}^{\infty} dm \rho_\alpha(m) \, m^2 \sum_{n=1}^{\infty} \frac{(\pi)^{n+1}}{n} \, K_n \left( \frac{nm}{T} \right) \, e^{n\mu_\alpha/T} \tag{2}
\]

\[
E_\alpha = \frac{VT}{2\pi^2} \int_{m_\alpha}^{\infty} dm \rho_\alpha(m) \, m^3 \sum_{n=1}^{\infty} \frac{(\pi)^{n+1}}{n} \left[ K_1 \left( \frac{nm}{T} \right) + \frac{3T}{nm} K_2 \left( \frac{nm}{T} \right) \right] \, e^{n\mu_\alpha/T} \tag{3}
\]

where \( \mu_\alpha \) is the chemical potential for the family \( \alpha \), \( m_\alpha \) is the threshold (i.e. lowest mass in the family \( \alpha \)) and \( K \) is a Kelvin (modified Bessel) function.

Chemical equilibrium implies certain relations among the chemical potentials which are dictated by the possible reactions. These relations are derived in Appendix I. The result is that all chemical potentials, \( \mu_\alpha \), can be expressed in terms of two others, corresponding to the additive conserved quantum numbers, one for (non-strange) baryons, \( \mu_B \), and one for strange mesons, \( \mu_S \). Antiparticles have chemical potentials of opposite sign to the particles. The two independent chemical potentials
are determined at given temperature $T$ and volume $V$ by the conservation of net baryon charge and strangeness. Regarding the equation for $N_\alpha$ as a function of $\mu_\alpha$, the number of antiparticles is given by

$$\bar{N}_\alpha = N_\alpha(-\mu_\alpha) \quad .$$

So denoting the baryon charge of family $\alpha$ (indicated in Table 1) by $B_\alpha$, the net baryon charge is:

$$A = \sum_\alpha \left[ N_\alpha(\mu_\alpha) - N_\alpha(-\mu_\alpha) \right] B_\alpha \equiv B - \bar{B} \quad ,$$

where $A$ is the total number of initial nucleons involved in the collision. The net strangeness is similarly:

$$0 = \sum_\alpha \left[ N_\alpha(\mu_\alpha) - N_\alpha(-\mu_\alpha) \right] S_\alpha \equiv S - \bar{S} \quad .$$

If we specialize to symmetric collisions between $N=Z$ nuclei, then the above two conservation conditions also imply conservation of electric charge, because $Q = (B + S)/2$.\textsuperscript{14}

Equations (5) and (6) determine the two independent chemical potentials $\mu_B$ and $\mu_S$ at given temperature and volume. The populations can be calculated from (2) and (4) and the total energy conservation condition reads

$$AE = \sum_\alpha \left( E_\alpha + \bar{E}_\alpha \right) = \sum_\alpha \left( E_\alpha(\mu_\alpha) + E(-\mu_\alpha) \right)$$

where $E$ is used to denote the c.m. energy per nucleon in the colliding nuclei (including rest mass).

However, the initial conditions are a little harder to solve because $V$ is not an independent variable but depends instead on the energy.
Consider a central symmetric collision in the center of mass frame between two nuclei of atomic number $A/2$. Each nucleus is Lorentz contracted by the factor $1/\gamma = m/E$, where $m$ is the nucleon mass and $E$ is the energy per nucleon (including rest mass). Assume that the collision is perfectly inelastic: that the nuclei are stopped by each other. Then the largest possible volume in which all nucleons are contained just after the nuclei have stopped each other is the Lorentz-contracted volume originally occupied by one of them.\(^\text{15}\) Thus, the initial baryon density of the fireball is at least

$$\rho_{\text{initial}} \geq 2\gamma \rho_0 = \frac{2E\rho_0}{m} = \frac{1}{v} \quad (8)$$

where $\rho_0$ is the normal nucleon density. Hence the initial volume $V = Av$ multiplying (2) and (3) depends on the energy. However, numerically it is more efficient to treat the energy as a dependent variable since it is $\mu$ and $T$ that are embedded in integrals (2) and (3). So we define the energy density and baryon density by

$$\varepsilon_\alpha = \frac{E_\alpha}{V}, \quad \text{and} \quad \nu_\alpha = \frac{N_\alpha B_\alpha}{V} \quad (9)$$

which according to (2) and (3) are functions of $T$ and $\mu$. Then the baryon conservation and energy equations read

$$\Delta A = V \Sigma (\nu_\alpha - \bar{\nu}_\alpha) \equiv V (\nu - \bar{\nu})$$

$$\Delta E = V \Sigma (\varepsilon_\alpha + \bar{\varepsilon}_\alpha) \equiv V (\varepsilon + \bar{\varepsilon}) = A \frac{m}{2E\rho_0} (\varepsilon + \bar{\varepsilon}).$$  

So,

$$E = \sqrt{\frac{m(\varepsilon + \bar{\varepsilon})}{2\rho_0}}, \quad V = A \sqrt{\frac{m}{2\rho_0 (\varepsilon + \bar{\varepsilon})}} \quad (10)$$

Hence the baryon conservation equation can be written

$$\nu - \bar{\nu} = \sqrt{\frac{2\rho_0 (\varepsilon + \bar{\varepsilon})}{m}} \quad (11)$$
This equation, together with (6), are transcendental equations which for fixed $T$ can be solved for the two independent chemical potentials $\mu_B$ and $\mu_S$. The corresponding energy per nucleon is then given by (10).

IV. EXAMPLES OF HADRONIC SPECTRA

We shall study the properties and decay of hot nuclear matter under three possible assumptions concerning the underlying hadronic mass spectrum. They are illustrated in Fig. 1 and detailed below. For brevity we shall sometimes refer to the results for different spectra as different worlds.

1. The Known Hadrons.

As one extreme case, suppose that all of the hadrons have already been discovered, as of the last edition of the Particle Data Tables. Their density as a function of mass is shown in Fig. 1 aside from isolated recent discoveries. They can be arranged according to statistics and strangeness into seven families. For each family, a few of the lowest mass multiplets and the average mass of the multiplet is shown in Table I, together with the total multiplicity of states known. This amounts to $\sim 531$ which together with antiparticles comprises the some 1000 hadronic particles and resonances known to date. We include all of them by using the average mass and width, and the multiplicity for each multiplet.

2. Exponential (Hagedorn) Spectrum.

As mentioned in the introduction, the bootstrap theory implies an exponential spectrum, as may also the quark hypothesis. This spectrum lies at the opposite extreme to the "known" spectrum since it rises
TABLE I. The families of light mass multiplets, their average masses in MeV, and their baryon and strangeness quantum numbers (B, S). Total multiplicity including the unlisted multiplets is indicated in the bottom row for each family.

<table>
<thead>
<tr>
<th>Family (B,S)</th>
<th>Π (0,0)</th>
<th>K (0,1)</th>
<th>η (1,0)</th>
<th>Λ (1,-1)</th>
<th>Σ (1,-1)</th>
<th>Ξ (1,-2)</th>
<th>Ω (1,-3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>η(138)</td>
<td>K(495)</td>
<td>N(940)</td>
<td>1116</td>
<td>1193</td>
<td>1318</td>
<td>1672</td>
</tr>
<tr>
<td></td>
<td>η(549)</td>
<td>K*(892)</td>
<td>N*1430</td>
<td>1405</td>
<td>1385</td>
<td>1533</td>
<td></td>
</tr>
<tr>
<td>Multiplets</td>
<td>ρ(773)</td>
<td>K*(1421)</td>
<td>N*1520</td>
<td>1519</td>
<td>1670</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ω(783)</td>
<td>N*1515</td>
<td></td>
<td>1670</td>
<td>1745</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>η (958)</td>
<td>Δ(1232)</td>
<td></td>
<td>1690</td>
<td>1773</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total multiplicity</td>
<td>103</td>
<td>18</td>
<td>248</td>
<td>38</td>
<td>108</td>
<td>12</td>
<td>4 = 531</td>
</tr>
</tbody>
</table>
exponentially and is unbounded. We test the consequences of the exponential spectrum by using the Hagedorn form in the region $m_\Pi > 12 m_\pi$ for the $\Pi$ and $N$ families

$$\rho_\alpha(m) = C_\alpha \left( \frac{T_o}{m} \right)^m e^{m/T_o}, \quad m > 12 m_\pi. \quad (12)$$

Below $m = 12 m_\pi$, we use the "known" spectrum. The slope, $T_o$, we fix for all families by adjusting it to the bootstrap iteration of Hamer and Frantschi based on the known particles and antiparticles (Fig. 2). This gives

$$T_o = 0.958 \ m_\pi, \quad 140 \ MeV.$$ 

The constants $C_\alpha$ for each family are fixed as the average multiplicity in five pion mass intervals around $10 m_\pi$. For ordinary mesons, $\Pi$, and baryons, $N$, this number is about equal

$$C_\Pi = C_N = 0.56/\text{pion mass}.$$ 

In the calculations reported in Sections V and VI, only these two families are assigned continuum spectra, but in the final sections all families have continuum exponential spectra. To obtain the continua for strange hadrons, one has to somehow find their contributions to the level density; the experimental situation is less well defined since fewer strange multiplets are known. We choose to normalize by a simple quark counting argument; the resulting coefficient and mass thresholds are listed in Table II.

3. Rigid Quark Bag

Intermediate in multiplicity is the spectrum corresponding to the
### TABLE II. Normalization constants and mass thresholds for continua.

<table>
<thead>
<tr>
<th>Family</th>
<th>N</th>
<th>Π</th>
<th>K</th>
<th>Λ</th>
<th>Σ</th>
<th>Ξ</th>
<th>Ω</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold normalization</td>
<td>$C_N$</td>
<td>$C_\Pi$</td>
<td>$(1/2)C_\Pi$</td>
<td>$(3/4)C_N$</td>
<td>$(3/4)C_N$</td>
<td>$(1/2)C_N$</td>
<td>$(1/4)C_N$</td>
</tr>
<tr>
<td></td>
<td>1680</td>
<td>1680</td>
<td>1520</td>
<td>2200</td>
<td>2300</td>
<td>2600</td>
<td>3000</td>
</tr>
<tr>
<td></td>
<td>(2130)*</td>
<td>(1630)*</td>
<td>(1770)*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*The values in parenthesis represent the mass at which the last particle of this family is known plus 100 MeV. From this mass to threshold an interpolation in degeneracy was used.*
rigid quark bag described in the Introduction. It is, however, the lower bound provided by confined quark models. The form is\[\rho_\alpha(m) = C_\alpha \left(\frac{m}{m_\pi}\right)^n , \quad m_\pi > 12 \ m_\pi , \quad (13)\]

\[n = \begin{cases} 
2 & \text{for mesons} \\
5 & \text{for baryons} 
\end{cases} \]

\[C_\pi = 0.16/\text{pion mass} , \quad C_N = 10^{-3} C_\pi \]

Again we normalize this spectrum in the same way as for the Hagedorn spectrum. The normalizations \(C_\alpha\) for the other families, and the thresholds are listed in Table II. Below \(m = 12 \ m_\pi\) we use the "known" spectrum of all particles.

V. THE TEMPERATURE AND COMPOSITION OF THE INITIAL FIREBALL

Neither the temperature nor composition of the initial fireball are observables because any conceivable experiment must look at the products of the collision after the fireball has disassembled. Nonetheless, it is interesting to look at the calculated populations because they are the starting point of the subsequent expansion or decay of the fireball. They also give us a glimpse of what the temperature and composition of the universe might have been at the beginning of time. However, because of the time scales involved, we do not have to consider photons and leptons in equilibrium with the hadrons, which is different from the cosmological problem.

The first indication that our calculation can give of differences in hadronic matter constructed from the three assumed hadronic spectra of Section IV is registered in the temperature to which the matter would be heated for a given energy input. Since we assume for simplicity that
the nuclear collisions are perfectly inelastic, then the c.m. collision energy including rest mass is the total fireball energy. The temperature as a function of energy is shown in Fig. 3. For the case of the "known" spectrum, the temperature of matter is by far highest at energies greater than several GeV. Because energy goes into making additional resonances in the quark bag spectrum that were not present in the "known" spectrum, the temperature is lower at corresponding energy. For the exponential spectrum, as first discovered and emphasized by Hagedorn, the temperature is limited to a maximum value corresponding to the constant $T_0$ in the spectrum. $T_0$ is not determined by the theory but deduced by a comparison with data, as outlined in Section IV, and turns out to have a value very close to the pion mass.

This limiting temperature of matter, if composed of hadrons having an exponential spectrum (12) is a truly remarkable property. No matter how much energy is injected the temperature cannot be raised. We can see the mathematical nature of the limit by referring to Eq. (3). Inserting the Hagedorn spectrum into (3), which causes the integral to be dominated by high $m$, we can use the limit of the Kelvin function, valid for $m \gg T$,

$$K(x) = x^{-\frac{3}{2}} e^{-x}, \quad x \to \infty$$

to find

$$E \propto \int_{m_0}^{\infty} \frac{dm}{m^3} \exp \left(-\left(\frac{T_0 - T}{T_0 T}\right) m\right)$$

$$= \left(\frac{T_0 T}{T_0 - T}\right)^{\frac{1}{2}} \int_{x_0}^{\infty} \frac{dx}{x^\frac{3}{2}} e^{-x}.$$

The integral is finite, and the result shows that $E$ is finite so long
Fig. 3. For the three hadronic spectra considered, the temperature of hot hadronic matter as a function of energy per nucleon is plotted. Assuming a colliding beam central collision between identical N=Z nuclei in which the nuclei are stopped by each other, this is the c.m. collision energy per nucleon including the rest mass.
as $T < T_0$; but for $T > T_0$, the energy diverges, showing that infinite energy would be required to raise the temperature to $T_0$.

To display the composition of the fireball it is necessary to make some arbitrary groupings because there are so many discrete resonances (Table I), not to mention the continua. Therefore, each family is divided into light particles comprising the lightest five, and heavy particles comprising all the rest, including continuum resonances in the case of the exponential and quark bag spectra. Thus for example, in Fig. 4, the ordinary (non-strange) mesons are represented by two curves $\pi_<$ and $\pi_>$. There are no heavy kaons but there are anti-kaons. In Fig. 4 the curves marked K that asymptotically approach each other represent these populations, with the anti-kaons below the kaons.

These figures show remarkable differences in the initial composition of the fireball, depending on the underlying hadronic spectrum. For both the known and the rigid quark bag spectrum, the heavy baryons and anti-baryons dominate the composition at very high energy, with heavy mesons being the next most populous group. In the case of the quark bag, this happens at rather low energy (on the particle creation scale). The composition at 1 GeV is of course all nucleon (the original particles of the colliding nuclei), but the light baryons and anti-baryons become less populous than the heavy baryons and anti-baryons above 10 GeV in the bag model.

The composition corresponding to the exponential Hagedorn spectrum is remarkably different from the other two. The light meson population rises to a maximum of about 10% and then declines. The heavy baryon populations rise sharply above 3 GeV. Above this energy the fireball is composed almost entirely of heavy baryons! In both the "known" and
Fig. 4. The fireball populations corresponding to the three assumptions of the underlying hadronic spectrum. The light and heavy members of the family of ordinary mesons ($\Pi$), strange mesons ($K$), and ordinary baryons ($N$), are plotted as a function of collision energy. *Light* refers to the first five multiplets (if that many) of each family, and are denoted by $<$. *Heavy* refers to all others, including the continuum where applicable, and are denoted by $>$. Antiparticles, in the case of the "known" spectrum, approach the particle populations at high energy and can be identified in the figure by this property. In contrast, for the exponential spectrum, the heavy baryons dominate. Refer to Table I for some of the members of the families.
"rigid bag" worlds all particle/anti-particle populations approach each other at high energy (with anti-particles slightly less numerous). In the "exponential" world all anti-particles and mesons have vanishing populations at asymptotic energies. It is a world that is dominated by heavy baryons at high energy density.

This remarkable property of the exponential spectrum, can easily be understood, both intuitively and mathematically. At high energy density the phase space associated with heavy particles becomes enormous because of the exponential spectrum, and as already noted, far exceeds the phase space associated with kinetic motion (related to temperature). Since however, baryon conservation is enforced, at any given energy the largest phase space is achieved by committing the energy to the conserved particle type, to the exclusion of all others. Hence at very high energy density such as might be produced in high-energy nuclear collisions, or such as what existed in the earliest instants of the universe, or in the late stages of black holes, matter is dominated by heavy baryons if the spectrum of hadrons is exponential, and if no phase change to quark matter can occur.

This structure emerges from the mathematics through the following observation. The integral in the expression (2) for the populations, in the case of the exponential spectrum, tends to

\[
N \propto \left( \frac{T_0 - T}{T T_0} \right)^{1/2} \int_{x_0}^{\infty} \frac{dx}{x^{3/2}} e^{-x}
\]

aside from the factor \( e^{\mu/T} \) which is present for conserved particles. Hence the baryon conservation condition takes on the form

\[
\left( \frac{\mu/T}{e - e^{-\mu/T}} \right) \left( \frac{T_0 - T}{T T_0} \right)^{1/2} > 0
\]
It can be satisfied when $T \to T_0$ only by $\mu$ becoming an appropriate infinity. Now note that baryons have the factor $e^{\mu/T}$, antibaryons $e^{-\mu/T}$ and mesons the factor 1, multiplying the integral like $N$ above, which itself tends to zero. This proves the stated result.

VI. ISOERGIC EQUILIBRIUM EXPANSION OF THE FIREBALL

We have seen that the initial conditions in the nuclear fireball are very sensitive to the underlying hadronic spectrum at sufficiently high energy density. However, the only possible observations that can be made occur after it has expanded and come apart. This is the third stage in our conceptualization of the collision process. The simplest assumption that we can make is to assume, as in cosmology, that the expansion occurs through a sequence of equilibrated states. At some point during the expansion, the density will fall below a critical value where the interactions cease to maintain equilibrium. This is referred to as the freezeout density. Thereafter relative populations do not change except by decay of isolated resonances.

The above assumption will provide some insight, but it is oversimplified because there is nothing to prevent some of the fast outward moving particles from leaving the equilibrated region of the fireball before it has expanded to the freeze-out density. Therefore, there are two indistinguishable components to the particles that reach the counters—those that escape from the fireball during the expansion and prior to freezeout, and those that remain in thermal contact until the freezeout. This is the scenario that we shall model in Section VII in order to
calculate the spectra of stable particles. However, as a first orientation we consider the simpler isoergic expansion in which no prefreezeout radiation of particles occurs.

The freezeout density, $\rho_F$, below which thermal contact ceases, must be somewhat less than normal nuclear density, $\rho_N \approx 0.17 \text{ fm}^{-3}$, but is not less than the density corresponding to each particle having a sphere of radius equal to a pion wavelength, $\rho_\lambda \approx 0.085 \text{ fm}^{-3}$.

$$\rho_N > \rho_F > \rho_\lambda$$

We shall plot our results for the thermal expansion stage as a function of $1/\rho$ where $\rho$ is the hadron density and we measure it in units of normal nuclear density $\rho_N$. On such a scale the freezeout is expected between 1 and 2.

The fall in temperature of a 20 GeV/nucleon fireball as it expands, and the corresponding evolution of the particle populations for the two extreme worlds, are shown in Figs. 5 and 6. Although the initial temperatures are quite different at this energy, as shown in Fig. 3, the known and rigid bag worlds cool during the expansion and are virtually equal over the scale shown in Fig. 5. This contrasts with the exponential (Hagedorn) world where the temperature remains essentially constant until the fireball has expanded to roughly normal nuclear density. Thereafter all three worlds cool in much the same way. At the same time, the particle populations, initially very different, become more similar as the density falls. This is so because as the energy density decreases, only the lower mass particles, common to all three worlds, can remain populated. Therefore at some sufficiently low density the three worlds must become indistinguishable if all of the material remains in the
Fig. 5. The temperature of the fireball as it expands with constant energy equal to 20 GeV/baryon. The ordinate is the reciprocal of the total hadron density in units of the density of normal nuclei (0.17 fm$^{-3}$). On this scale, 2 corresponds to a density such that each hadron has a share of the volume corresponding to a sphere with radius equal to the pion Compton wavelength 1.4 fm.
Fig. 6. Composition of a 20 GeV/baryon fireball as it expands, plotted as a function of reciprocal total hadron density measured in units of normal nuclear density (0.17 fm⁻³). Notation as in Fig. 4. Antiparticles indicated by bar.
expanding fireball and stays in thermal contact.

For a 20 GeV/nucleon fireball, the density at which the three worlds become indistinguishable is about one-half normal density, that is to say, at about the expected freezeout density. Therefore, under these conditions, 20 GeV/nucleon would be too low an energy to distinguish between the worlds. One could contemplate a much higher energy collision. However as seen in Fig. 6, the populations in the high density phase of the expansion are very different in the two extreme worlds. This strongly suggests that particle radiation from the early, high temperature, stage will be very different depending on the underlying hadron spectrum, and in particular, that these differences will be reflected most in the high energy tails of the spectra of stable particles. Thus we are motivated to develop a more realistic scenario for the expansion stage that takes account of the pre-freezeout radiation.

VII. QUASI-DYNAMICAL EXPANSION

The hint gained in the preceding study, that the pre-freezeout radiation from the three worlds will be different because of the differing populations in the early, high-temperature phase of the expansion, provides the motive for attempting to follow the time development of the expansion. The assumption of hydrodynamics has been made for single or several component models of hadron-hadron collisions and therefore provides a dynamical description. Here, however, we deal with a system containing many species which would make a detailed hydrodynamical model very difficult to implement. Instead we develop an expansion model which incorporates the main physical processes that would govern the expansion, namely the
populations in the fireball, their velocity distributions, the mean free path, and the mean resonance lifetime.

We shall assume that at any instant the particles that lie within a mean free path of the surface of the fireball, and are directed outward, will move into vacuum. Those that are unstable will decay within a resonance mean life into lighter stable and unstable particles, so that in the immediate vicinity of the surface the density remains high. Therefore we take this to define an instantaneous new surface and we assume that a new quasi-equilibrium state is established within three concentric zones in the outward moving material. Meanwhile, those of the original outward moving particles that are stable and moving faster than the unstable ones that established the position of the new surface escape to the vacuum. Their quantum numbers and energy are subtracted from those defining the state of the new quasi-equilibrated fireball. These steps are iterated until the density has dropped to the critical density or the fireball contains negligible energy and conserved quantum numbers in resonance states, whichever comes sooner. At that point the remaining particles move freely to the vacuum. More details are given in Appendix 2.

Of course the expansion does not occur isotropically in the center of mass because of the initial Lorentz-contracted shape of the fireball. It is clear from the geometry that the shape of the fireball will evolve from the oblate spheroidal shape to a sphere. This evolution is indicated in Fig. 7.

We exhibit in Figs. 8 to 10 the energy spectra of some of the stable particles produced in a central collision of equal mass nuclei with 10 GeV kinetic energy per nucleon. The particles are emitted almost
isotropically so we show the spectra at only one angle. Results for a collision of two mass number 4 and two mass number 200 nuclei, are compared in the center of mass (colliding beams).

All spectra possess the common feature that they are concave upward. This corresponds to the emission of particles from a cooling object, the high-energy tails arising predominantly from emission from the early high-temperature stage, which is the stage during which the three worlds are most markedly different. The spectra are, so to speak, a convolution of the Fermi or Bose distributions over the history of the fireball. Asymptotically their slopes would characterize the initial temperature of the fireball. However, in the real world we recognize that there will be contamination far out in the tail from unthermalized primary nucleon-nucleon collisions.

Comparison of the spectra for the baryon number 8 and 400 collisions reveals the interplay of the various factors that control the decay of the fireball. These are the underlying hadron mass spectrum, the mean free path, mean resonance lifetime, and the size of the fireball — or more precisely, the ratio of (emitting) surface to volume.

From Section V we learned that the richer the underlying hadron spectrum is in high mass particles, the lower will be the particle density at any given energy density. This implies on the one hand that the fireball consists of a few ponderous heavy particles, but with a longer mean free path because of the low particle density, than in the case of, say, the "known" spectrum where the particles are lighter and faster but more dense and therefore have shorter mean free path.

In the higher energy part of the spectrum of all stable particles
Fig. 7. Typical evolution of a nuclear fireball. This particular case used an exponential spectrum and $B = 100, 10\text{ GeV/nucleon c.m. kinetic energy (roughly a symmetric }^{40}\text{Ca collision at } b = 0).$
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$T_{CM} = 10. \text{ GeV/NUCLEON}$

Fig. 8. For a symmetrical head-on collision at 10 GeV/nucleon of two equal mass nuclei, the spectrum of emitted pions at 90° in the c.m. frame is shown corresponding to the three underlying hadronic spectra. Results for heavy and light nuclei are shown.
Fig. 9. Spectrum of kaons. See caption to Fig. 8.
Fig. 10. Nucleon spectrum. See caption to Fig. 8.
the exponential world is more sensitive to the mass of the colliding nuclei than the other two worlds. As became evident in the earlier sections, the energy is converted to massive baryons for the exponential world and according to Fig. 5, the temperature is rather highly saturated in this case. This implies that the relatively larger ratio of volume to surface achieved for the collision of larger nuclei provides a longer lived reservoir of energy which is contained mainly in the high mass resonances. This results in a roughly constant temperature source.

The advantage of having the mass as a variable by which to alter the interplay of the factors controlling the fireball disassembly is shown in Figs. 11 and 12. The ratio of high-energy emitted pions and kaons to nucleons is plotted as a function of the mass of the colliding nuclei. Each of the three worlds has its own signature by comparison of the behavior of these two ratios as a function of the mass. For example, both ratios are rather constant for the known world, while only one is a strong function for the bag world. This illustrates a signature of the underlying spectrum of hadrons that is easier to read than the spectra of emitted particles themselves.

Figures 13 and 14 show the ratio of antibaryons to baryons and strange baryons to nucleons as a function of the energy of the emitted particles for a collision of mass 4 and 200 nuclei. Again, a unique signature can be read through a comparison of such ratios. Only for the known world does the number of antibaryons and strange baryons approach the number of baryons. This happens because at the collision energy of 10 GeV per nucleon, the masses in the known spectrum appear rather degenerate. In such a situation there might be a significant production of light hypernuclei.
Fig. 11. Ratio of emitted pions to nucleons at 1.6 GeV kinetic energy as a function of total nucleon number in a symmetric collision of equal mass nuclei at 10 GeV/nucleon in c.m. The ratio behaves differently depending on the assumed hadronic spectrum.
Particle ratio at $90^\circ$, 1.6 GeV kinetic energy

Fig. 12. Ratio of emitted pions to kaons at 1.6 GeV kinetic energy as a function of total nucleon number in a symmetric collision of equal mass nuclei at 10 GeV/nucleon in c.m. The ratio behaves differently depending on the assumed hadronic spectrum.
Fig. 13. The ratio of emitted antibaryons to baryons as a function of their kinetic energy for the three underlying hadronic spectra corresponding to a central symmetric collision of two nuclei at 10 GeV kinetic energy per nucleon in the c.m. Two cases are shown for heavy nuclei and one for light.
Fig. 14. The ratio of emitted strange baryons to nucleons as a function of their kinetic energy for three underlying hadronic spectra corresponding to a central symmetric collision of two nuclei at 10 GeV kinetic energy per nucleon in the c.m. Two cases are shown for heavy nuclei and one for light.
VIII. QUARK PHASE

One possibility that we have not mentioned is that quark matter might be formed in the early high-density stage of the fireball. Several qualitative statements can be made in such an eventuality, although the detailed mechanism of recondensation into hadrons is currently beyond our power and is one of the severest problems (infrared) in QCD.

We have emphasized the importance of the pre-freezeout radiation, especially of those particles that are emitted in the early high-temperature stage of the reaction. If a transition to a quark phase occurred, during that phase there would be little or no radiation. On the other hand, the system is presumably not bound, but rather highly unstable. It will therefore expand and at some point must recondense into hadrons, either at once or with a hadron halo surrounding a quark matter core. The temperature of an asymptotically free quark phase at given energy density must be very different and higher than the hadron phase. A hadron halo in contact with it might therefore emit high-energy hadrons, at the same time cooling the core. Without a plausible model for this evolution, we cannot guess the spectrum characterizing it. If the recondensation occurs all at once, then we expect the disassembly to proceed from that point in the approximate way described in Section VII. The spectra would be modified in having the contribution of the early radiation removed. Again nothing quantitative can be said in the absence of a plausible scenario.
Summary

The asymptotic form of the hadronic spectrum has interesting implications in both particle physics and cosmology. We discussed why it is impossible to determine it in the traditional way of discovering the individual resonances. We then drew attention to the fact that, in principle, the macroscopic properties of matter at very high energy density must depend on the hadronic spectrum. Very high-energy nuclear collisions offer the only possibility of producing matter at very high energy density in the laboratory. The purpose of our research was to assess whether there are likely to be signals from such collisions that are sensitive to the underlying hadronic spectrum, and whether they can be expected at energies that can be achieved in existant or conceivable accelerators. Recognizing that a dynamical theory of such collisions is beyond reach at the present stage, we take the statistical mechanical approach which is applied, not at the first stage of primary hadron-hadron collisions, but after an assumed sequence of collisions has stopped the colliding nuclei. While our assumptions may be crude, they are applied equally to three possible forms of the hadronic spectrum. Although single particle inclusive spectra do have some sensitivity to the hadron spectrum, we place greater reliance on the measurement of ratios of produced particles as a function of energy and of the mass number of colliding nuclei, because depending on the hadronic spectrum, this brings into play in different ways the factors that govern the disassembly of the nuclear fireball.

The signals appear to be sufficiently large that it seems likely that they will survive a refinement of the collision dynamics. Therefore
it should be possible to distinguish between the three extreme examples of hadronic spectra that we considered, even within the limitations of the present scheme. The center of mass energies are in the tens of GeV/nucleon region. While these are high energies, they are within the reach of present technology. We consider that the prognosis for discovering the asymptotic hadron spectrum is good.
APPENDIX 1

Consider the additive-conserved quantum numbers: electric charge Q, baryon number B, and strangeness S. The Gell-Mann-Nishijima relations for an isospin averaged $Q = A/2$ system yields $Q = \frac{B+S}{2}$ (e.g., for the $\pi$, one has $\pi^+, \pi^0, \pi^-, \langle Q \rangle = 0$. $B_\pi = 0$, $S_\pi = 0$, so $(0+0)/2 = 0$; for the $N$: $p^+, n^0, \langle Q \rangle = 1/2$, $B_N = 1$, $S_N = 0$, $1/2 = (1+0)/2$; for the $K$: $K^+, K^0, \langle Q \rangle = 1/2$, $B_K = 0$, $S_K = 1$, $1/2 = (0+1)/2$; for the $\Sigma$: $\Sigma^+, \Sigma^-, \Sigma^0, \langle Q \rangle = 0$, $B_\Sigma = 1$, $S_\Sigma = -1$, $0 = (1-1)/2$, etc.) Thus if one conserves B and S, Q is automatically conserved on the average.

Consider some typical reactions in chemical equilibrium (grand canonical ensemble):

\[
\begin{align*}
\text{NN} & \rightarrow \text{NN}\pi \Rightarrow \mu_\pi = 0 \\
\text{NN} & \rightarrow \text{NN}(\overline{\text{NN}}) \Rightarrow \mu_N = -\mu_N \\
\text{NN} & \rightarrow \text{NK}\Lambda \Rightarrow \mu_K + \mu_\Lambda = \mu_N \\
\text{NN} & \rightarrow \text{NK}\Sigma \Rightarrow \mu_K + \mu_\Sigma = \mu_N \\
\text{NN} & \rightarrow \text{N}\overline{\text{KK}} \Rightarrow \mu_N = \mu_\Xi + 2\mu_K \\
\text{NN} & \rightarrow \text{N}\Xi\overline{\text{KK}} \Rightarrow \mu_N = \mu_\Omega + 3\mu_K
\end{align*}
\]

Let

\[
\begin{align*}
\mu_K & \equiv \mu_S \\
\mu_N & \equiv \mu_B
\end{align*}
\]

Then a consistent solution to the relations between the chemical
potentials is:

\[ \mu_\pi = 0 \]

\[ \mu_N = \mu_B \]

\[ \mu_K = \mu_S \]

\[ \mu_\Lambda = \mu_\Sigma = \mu_B - \mu_S \]

\[ \mu_\Xi = \mu_B - 2\mu_S \]

\[ \mu_\Omega = \mu_B - 3\mu_S \]

or in general, if \( \mu_\alpha \) is a chemical potential for additive quantum number \( \alpha \), and \( q_{\alpha i} \) is the amount of \( \alpha \) possessed by particle \( i \),

\[ \mu_i = \sum_\alpha q_{\alpha i} \mu_\alpha . \]
APPENDIX 2

As a model of the initial state of the system, we have assumed a homogeneous distribution of quantum numbers and energy throughout a volume of given shape. We shall discuss here the specific details of the model we have used for the evolution of this initial state, that is, its disassembly. Physically, three processes come into play, perhaps simultaneously: radiation, expansion, and decay. Let us clearly define the terms for the purpose of this discussion.

Radiation. Particles which have escaped from the system and which carry their quantum numbers and 4-momenta to the detectors.

Expansion. The growth in volume of the initial homogeneous distribution caused by the lack of any restraining vessel; this also implies fractionation (gradients) since different particles travel at different speeds and have different interactions.

Decay. Resonances decay strongly into the stable hadrons, which are \( \pi, K, \Lambda, \Sigma, \Xi, \Omega \). Only stable hadrons reach the detectors. We shall now explain how each of these processes are handled.

1. Radiation. The only particles which were allowed to escape were those stable hadrons within a mean free path of the surface, moving faster than the surface, and directed outwards. We assumed that one-half of the particles were directed outwards, and the mean free path, \( \ell \), was universal for all hadrons, depending only on a universal temperature-dependent cross section (to mock up energy dependence in the cross section) and the local hadron density (particles and antiparticles)

\[
\ell \approx \frac{1}{\rho \sigma}
\]
\[ \sigma[\text{fm}^2] = \frac{670}{T} + 3.3 \quad \text{T in MeV}. \]

Thus, for \( T = 100 \text{ MeV} \), \( \sigma = 10 \text{ fm}^2 = 100 \text{ mb} \), which is qualitatively correct.

We wish to emphasize that except for the hadronic level density, all parameters were kept fixed in these calculations. Resonances were not allowed to escape (see the further two sections).

2. Expansion. The details of the expansion scenario are as follows:

Let us define the average thermal speed \( \langle \beta \rangle = \langle p/E \rangle \); we approximate this as \( \langle p \rangle / \langle E \rangle \). If the expectation value is over all the particles in the volume, then we have, very roughly, the speed with which the linear dimensions of the volume as a whole is attempting to grow, and specifically, the outward growth. Thus, if the original ellipsoid (Lorentz-contracted sphere) has axes \( R_\perp \) and \( R_\parallel \), and \( \Delta t \) is the time step of the expansion,

\[ R_\perp(\Delta t) = R_\perp + \langle \beta \rangle \Delta t \quad , \quad R_\parallel(\Delta t) = R_\parallel + \langle \beta \rangle \Delta t. \]

We pick \( \Delta t = \tau = 10^{-23} \text{ sec} \). This expansion continues until \( R_\parallel(t) = R_\perp(t) \), at which time a sphere is assumed and \( \mathcal{R}(t+\Delta t) = \mathcal{R}(t) + \langle \beta \rangle \Delta t \), where throughout \( \langle \beta \rangle \) is a function of the composition and energy, which change with time (see below).

However, this would be the expansion scenario without zones; to include zones (e.g., gradients) we proceed as follows. We divide the particles into groups by mass range, since the lower the mass the higher the speed. In fact, we use the three mass ranges \( m_\pi, m_K + m_\pi, \) and \( > m_\Delta \); the calculation is not very sensitive to the precise demarcations as long as the \( \pi \) is split off. Call these mass ranges \( L, M, H \) (for light, medium, heavy); then empirically, \( \langle \beta \rangle_L \approx 0.9 \text{ c}, \langle \beta \rangle_M \approx 0.4 \text{ c}, \)
$<\beta>_H \approx 0.1 \, c$, and it seems reasonable that up to mean free path considerations, a gas of such a markedly different speed composition, freely expanding, would develop gradients. In the initial stage, we find for each constituent species the number of particles in each of the three $<\beta>$ ranges, $c > <\beta>_L > <\beta>_M > 0$ and within a mean free path of the surface, and compute for each within these speed ranges the $<\beta>$, and then use this $<\beta>$ to define the new volume by

$$R_1(\Delta t) = R_1 + \beta \Delta t \quad \text{etc.}$$

and

$$V_i = (4\pi/3) R_i R^2 - V_{i+1} \quad (i = 1, 2, 3 = L, M, H),$$

$$V_4 = 0 \quad .$$

Into this new volume are placed the quantum numbers brought by the particles and equilibrium is attempted. If the attempted equilibrium hadron density is greater than the freezeout density (that is, the hadron density where all hadrons are, on the average, one pion Compton wavelength separated), we permit the volume to equilibrate; if not, all stable particles which were going into it are allowed to reach the detectors and all unstables stay behind. This process is continued permitting particles to migrate from zone to zone, and those which attempt to migrate from the outermost zone outward are allowed to escape to the detectors. The equilibration by zones also simulates final state interactions, since it effectively forces the particles to share 4-momentum and quantum numbers, which is a similar effect to the rescattering afforded by a final state interaction. This is also a reason to retain a fixed freezeout density irrespective of the local hadronic mean free path, as the latter determines the stable radiation
and volume growth, but the former via thermodynamics determines resonance decay (see below). The process stops when the density in the remaining zone drops low enough or the resonances are an insignificant fraction of the remaining energy and quantum numbers. Fortunately, this occurs empirically at about the same point. For simplicity, recondensation into nuclei is neglected.

3. Decay. We used in the rest frame of a resonance, a mean resonance lifetime $\tau \approx 10^{-23}$ sec, and we make one other assumption -- the decay of a resonance can be described by thermodynamics. One may justify this for our purposes because, even if the decay is binary, a statistical sample will still appear statistically distributed throughout microcanonical phase space, and we extend this to grand canonical phase space, which in this case greatly simplifies the calculation and incurs negligible errors as long as one stays away from kinematic boundaries. Hence, our aim is to calculate single-particle inclusive spectra.
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