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Component procurement strategies in decentralized assembly systems under supply uncertainty

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ABSTRACT

In this article we analyze the interactions among the assembler and two component suppliers in their procurement decisions under a Vendor-Managed Inventory (VMI) contract. Under the VMI contract, the assembler first offers a unit price for each component and will pay component suppliers only for the amounts used to meet the actual demand. The two independent component suppliers then decide on the production quantities of their individual components before the actual demand is realized. We assume that one of the component suppliers has uncertainty in the supply process, in which the actual number of components available for assembly is equal to a random fraction of the production quantity. Under the assembly structure, both component suppliers need to take into account the underlying supply uncertainty in deciding their individual production quantities, as both components are required for the assembly of the final product. We first analyze the special case under deterministic demand and then extend our analysis to the general case under stochastic demand. We derive the optimal component prices offered by the assembler and the corresponding equilibrium production quantities of the component suppliers.

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1. Introduction

Sourcing components from a complex global supplier network can lead to a high degree of uncertainty in the supply process. Various supply chain glitches such as unexpected supply disruptions, insufficient supplier capacity, or transportation delays across borders can cause unexpected shortfalls in the required components and halt the assembly of the final products. At the same time, long procurement lead times in a global supply network make it difficult and expensive to deal with such component shortfalls using emergency orders. Consequently, firms need to effectively manage supply uncertainty in their component procurement decisions to avoid such potential component shortfalls. This is especially critical for products with rapidly changing technology or short life cycle such as electronic products, as it is expensive to keep safety stock of components due to high obsolescence costs.

In this article we analyze the interactions among the assembler and component suppliers in their procurement decisions under a Vendor-Managed Inventory (VMI) contract. Specifically, we assume that the assembler needs to procure two required components from two independent suppliers to assemble the final product. Under the VMI contract, the assembler first offers a unit price for each component and will pay component suppliers only for the amounts used to meet the actual demand. Based on the unit component prices offered by the assembler, the two independent component suppliers then decide on the production quantities of their individual components and ship the components to the assembler before the actual demand is realized. Here, we assume that the production and/or transportation lead times are long, so there is no opportunity for a second shipment after the actual demand is realized. Excess components, if any, will be salvaged. The VMI contract allows the assembler to transfer the associated risk of inventory mismatching to the component suppliers.

We assume that one of the two component suppliers has uncertainty in the supply process, in which the actual amount of components available for assembly is equal to a random fraction of the production quantity. As such, this component supplier needs to take into account the underlying supply uncertainty in deciding his production quantity. At the same time, the other component supplier will also need to consider this supply uncertainty in deciding her production quantity, as both components are required for the assembly of the final product. Thus, supply uncertainty of one component can affect the production and procurement decisions of all other required components under the assembly structure.

The objective of this article is to understand how supply quantity uncertainty (referred to simply as *supply uncertainty* hereafter) can affect the production/procurement decisions of the assembler and the component suppliers in a decentralized assembly system under the VMI contract. We first analyze the case under deterministic demand, so that we can isolate the impact of supply uncertainty on the interactions among the assembler and the component suppliers. We derive the optimal component prices offered by the assembler and the corresponding equilibrium production quantities of the component suppliers. We then extend our analysis to the general case under stochastic demand.

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For the deterministic demand case, we first provide a (deterministic) threshold product price below which it is not optimal for the assembler to assemble the product and show that this threshold price critically depends on the ratio of the unit production cost of the two components. We then derive the optimal component prices offered by the assembler in the VMI contract and the corresponding equilibrium production quantities of the components. Our result shows that the optimal component prices largely depend on the ratio of the unit production cost of the two components and the product price. Furthermore, the equilibrium production quantity of the component with supply uncertainty depends on the unit component price offered, unit production cost, and supply reliability distribution, whereas the equilibrium production of the other component, interestingly, is always equal to the deterministic demand as long as the unit component price offered is above a certain level.

For the stochastic demand case, we characterize an additional (stochastic) threshold price for the product. When the product price is below the deterministic threshold price, it is not optimal for the assembler to assemble the product. When the product price is between the deterministic and stochastic threshold prices, it is optimal for the assembler to target the production to meet only the minimum demand level. In this case, the optimal component price offered by the assembler and the corresponding equilibrium production quantities are given by the results for the deterministic demand case with demand being equal to this minimum demand level. Only when the product price is above the stochastic threshold price is it then profitable for the assembler to target production above the minimum demand level. We further provide a Lagrangian method to solve for the optimal component prices offered by the assembler and the corresponding equilibrium production quantities of the two components. We then conduct a set of numerical experiments to illustrate the impact of supply uncertainty, demand uncertainty, and product price on the optimal decisions of the assembler and component suppliers in the system.

One important result from our analysis is the identification of two threshold prices, for which the optimal procurement decisions critically depend on whether the product price is above or below these two threshold prices. Interestingly, we also show that supply uncertainty greatly affects these two threshold prices, whereas demand uncertainty has no impact on these two threshold prices. On the other hand, both supply and demand uncertainty affect the equilibrium production quantities of the suppliers when the product price is above the stochastic threshold price. This behavior is similar to that observed for a centralized assembly system studied by Pan and So (2010).

The rest of the article is organized as follows. Section 2 provides a literature review of relevant research. Section 3 provides our problem formulation and the basic notation of our model. In Section 4 we analyze the optimal component pricing by the assembler and the corresponding equilibrium production decisions by the component suppliers under deterministic demand. In Section 5 we extend our analysis to the general case under stochastic demand. We also provide some numerical results to illustrate our analysis and gain some managerial insights. We summarize the results of our article in Section 6. All mathematical proofs are provided in the Appendix.

2. Literature review

Our article is concerned with the optimal procurement strategies under supply uncertainty. Recently, there is increasing interest in this research area, as firms need to effectively manage supply uncertainty in today's complex global supply chains; see, for instance, the recent works of Tomlin (2006), Dada *et al.* (2007), Serel (2008), Burke *et al.* (2009), Federgruen and Yang (2009), Tang *et al.* (2014), and Xu and Lu (2013). However, none of the above papers consider component procurement strategies under an assembly structure. Under the assembly structure, it is important to coordinate the joint component procurement decisions, as all components need to be available for the final assembly. We refer to Song and Zipkin (2003) for a recent review on research works that study the coordination of component-ordering decisions and inventory policies in evaluating the performance of assemble-to-order systems. However, most research papers discussed in this review do not consider supply uncertainty of components.

Our article specifically deals with supply quantity uncertainty, which is closely related to research works on random yields; see Yano and Lee (1995) for a comprehensive review on earlier research papers in this area. Therefore, our article is closely related to the stream of research that analyzes the component inventory policies for assembly systems with random yields. Yao (1988) examines the inventory decision of components in an assembly system with random yields. Singh *et al.* (1990) present some models that explicitly take into account random yield losses of individual components in a semiconductor manufacturing environment. Gerchak *et al.* (1994) analyze a single-period lot sizing decision model for an assembly system with random yields. Gurnani *et al.* (2000) study the supply management of a two-component assembly system with stochastic demands and random yields. Pan and So (2010) study a centralized assemble-to-order system, in which one of the components faces supply quantity uncertainty and demand is assumed to be price-dependent. They analyze how the supply quantity uncertainty of one component can affect the product pricing and ordering quantities of all components under the assembly structure. However, none of the above papers considers pricing and production decisions in a decentralized setting.

Some recent research has studied the coordination of component procurement decisions in decentralized assembly systems, including Wang and Gerchak (2003), Bernstein and DeCroix (2006), Bernstein *et al.* (2007), and Fang *et al.* (2008). However, none of these papers consider supply uncertainty. To our knowledge, Gurnani and Gerchak (2007) is the earliest work that examines the coordination of component production decisions in a decentralized assembly system with supply quantity uncertainty. They consider the setting where the assembler and suppliers choose their ordering and production quantities based on their own cost/reward structures. They analyze the conditions when system coordination can be achieved and derive the optimal contract parameters. Yan *et al.* (2010) later extend Gurnani and Gerchak's model to allow for the case of asymmetric suppliers. Guler and Bilgic (2009) also consider a decentralized assembly system with uncertain yields and demands and study the contracting issues in the coordination of the supply chain. In contrast, our article studies a decentralized system, in which the assembler and the component suppliers use a VMI contract to coordinate their procurement and production decisions.

3. Problem formulation

Consider a contract assembler who is anticipating a one-time future order for an assemble-to-order customized product at some fixed price p. The specific order quantity, denoted by D, is uncertain. The assembly of the product requires one unit of two major components. The two components, denoted by 1 and 2, are produced by two independent suppliers, referred to as supplier 1 and supplier 2, respectively. The supply process of component 1 is subject to uncertainty, in which the actual available quantity is equal to ϵQ if Q is the production quantity, where ϵ represents the supply reliability factor with support [0, 1]. This supply uncertainty can be due to various potential glitches such as unexpected supply disruptions, random yield of the production process, or spoilage or theft during transportation or at the assembler's warehouse. On the other hand, the supply process of component 2 is assumed to be perfectly reliable.

The assembler uses a VMI contract to coordinate the component procurement process of the two suppliers. Under this VMI contract, the assembler specifies a unit price w_i paid to supplier i for each sold component i. Based on the assembler's unit component prices offered, the two suppliers decide the individual production quantities of their components and deliver the components to the assembler's warehouse ready for product assembly. The sequence of events are as follows:

1. The assembler specifies the unit prices (w_1, w_2) offered to the two component suppliers.
2. The two suppliers choose their individual production quantities (Q_1, Q_2).
3. ϵQ_1 units of component 1 and Q_2 units of component 2 are available for assembly.
4. Demand D is realized.
5. The assembler assembles and delivers $\min(\epsilon Q_1, Q_2, D)$ units of the customized product.
6. The assembler pays each individual suppliers based on the components sold.

We use the following notation throughout this article.

p = unit product price;
c_i = unit production cost of component i;
w_i = unit component price paid to supplier i for each sold component i;
Q_i = the production quantity of component i;
ϵ = supply reliability factor of component 1;
$g(.)$ = density function of ϵ;
$G(.)$ = distribution function of ϵ;
μ_ϵ = mean of the supply reliability distribution $G(.)$;
$f(.)$ = density function of demand D; and
$F(.)$ = distribution function of demand D.

We also define $\bar{G}(.) = 1 - G(.)$ and $\bar{F}(.) = 1 - F(.)$. To simplify our technical exposition, we assume that for $0 \le l < u \le 1$, the density function $g(.)$ has positive support on $[l, u]$, with $g(x) = 0$ for all $x < l$ and $x > u$. Furthermore, $g(x)$ is differentiable for $x \in [l, u]$.

We assume that there is no shortage penalty and no salvage value for excess components. However, adding such costs in our model will not change our analysis and insights in any qualitative manner. Also, we assume that both suppliers will choose to produce the components when their expected profits are equal to zero. For instance, this happens where the unit component production cost c_i already includes some minimum acceptable profit for supplier i.

We formulate the above decision process as a Stackelberg game played by the assembler against the two component suppliers. The assembler acts as the Stackelberg leader by first choosing the unit component prices (w_1, w_2) offered to the suppliers. The two component suppliers act as the followers by simultaneously choosing their individual component production quantities (Q_1, Q_2). For any given pricing scheme chosen by the assembler, the expected profit of each supplier depends on his/her own production quantity as well as the production quantity of the other supplier. Thus, for any given (w_1, w_2), the decision problem of the two suppliers constitutes a Nash game. We refer to the Nash game between the two suppliers as the supplier problem and the decision problem for the assembler in the Stackelberg game as the assembler problem. We assume that all parameters and distribution functions in the model are known to the assembler and both suppliers.

4. Deterministic demand

We first analyze the situation where the one-time future demand D is deterministic. This allows us to isolate the impact of supply uncertainty to better understand the interactions among the assembler and two suppliers.

4.1. *The suppliers' problem*

Consider some fixed pricing scheme (w_1, w_2). Let (Q_1^*, Q_2^*) be any Nash equilibrium production quantities. Since there is no uncertainty in the supply of component 2, it is never optimal for supplier 2 to produce more than D units of component 2 or more than the production quantity of component 1, Q_1. Thus, (Q_1^*, Q_2^*) must satisfy the condition that $Q_2^* \le \min(D, Q_1^*)$. For any given $Q_2 \le D$, the expected profit of supplier 1 for any Q_1, with $Q_1 \ge Q_2$, is given by

$$\pi_1(Q_1|Q_2) = w_1 E[\min(\epsilon Q_1, Q_2, D)] - c_1 Q_1 = w_1 \left[\int_0^{\frac{Q_2}{Q_1}} t Q_1 g(t) dt + \int_{\frac{Q_2}{Q_1}}^1 Q_2 g(t) dt \right] - c_1 Q_1. \quad (1)$$

For any given Q_1, the expected profit of supplier 2 for any Q_2, with $Q_2 \le \min(Q_1, D)$, is given by

$$\pi_2(Q_2|Q_1) = w_2 E[\min(\epsilon Q_1, Q_2, D)] - c_2 Q_2 = w_2 \left[\int_0^{\frac{Q_2}{Q_1}} t Q_1 g(t) dt + \int_{\frac{Q_2}{Q_1}}^1 Q_2 g(t) dt \right] - c_2 Q_2. \quad (2)$$

Observe that one can interpret c_1/μ_ϵ as the expected production cost for having one unit of component 1 available for assembly. For any fixed $w_1 \geq c_1/\mu_\epsilon$, define k as

$$\int_0^k tg(t)dt = \frac{c_1}{w_1}, \tag{3}$$

and define

$$\hat{w}_2 = \frac{c_2}{1 - (\int_0^k G(t)dt)/k}. \tag{4}$$

Observe that $0 < k \leq 1$. Intuitively, supplier 2 needs to inflate her effective unit component cost to take into account the possibility that supplier 1 does not produce sufficient number of matching components due to supply uncertainty. In particular, supplier 2 will multiply her unit component cost c_2 by an inflation factor that depends on the distribution function of the supply reliability factor ϵ as given in Equation (4). We can prove the following result.

Proposition 1.

(i) If $w_1 < c_1/\mu_\epsilon$, then $(Q_1^*, Q_2^*) = (0, 0)$.

(ii) If $w_1 \geq c_1/\mu_\epsilon$ and $w_2 \geq \hat{w}_2$, then $(Q_1^*, Q_2^*) = (D/k, D)$, where k is defined in Equation (3). Otherwise, $(Q_1^*, Q_2^*) = (0, 0)$.

Proposition 1(i) proves the intuitive result that the assembler must offer at least a unit price of c_1/μ_ϵ to suppler 1 to produce component 1. Proposition 1(ii) further provides the minimum prices (w_1, w_2) that the assembler needs to offer to the two component suppliers below which the suppliers will not produce any component. We refer to these two minimum prices as the threshold component prices. The threshold component price for supplier 1 is equal to c_1/μ_ϵ, which is simply the effective cost of producing one unit of component 1 available for assembly. On the other hand, the threshold component price for supplier 2 is equal to $\hat{w}_2$ as defined in Equation (4), which depends on c_1, c_2, w_1, and the supplier reliability distribution $G(.)$.

Proposition 1(ii) further shows that the equilibrium production quantity $Q_1^* = D/k$ depends on c_1, w_1, and the supplier reliability distribution $G(.)$, whereas the equilibrium production quantity $Q_2^* = D$ is independent of any other parameters in our model. This is a rather interesting result, which suggests that supplier 2 manages the risk due to supply uncertainty by adding a price premium to her component rather than by adjusting her production quantity. In particular, the assembler needs to offer a higher unit price $\hat{w}_2$ than his unit production cost c_2 for supplier 2 to produce her components. Each unit of component now earns an expected profit of $(\hat{w}_2 - c_2)$ for supplier 2 to offset the potential loss due to insufficient amount of (matching) component 1, resulting in the expected profit of supplier 2 being zero. Also, as long as it is profitable for supplier 2 to produce, supplier 2 will produce up to the deterministic demand D.

4.2. *The assembler's problem*

The assembler, as the Stackelberg leader, chooses the optimal unit component prices (w_1^*, w_2^*) offered to the suppliers to maximize his own expected profit. Observe from Proposition 1 that the equilibrium production quantity Q_2^* is always equal to D as long as $w_2 \geq \hat{w}_2$. Therefore, it is clear that the assembler will simply select $w_2^* = \hat{w}_2$. For any fixed $w_1 \geq c_1/\mu_\epsilon$ and $w_2 = \hat{w}_2$, Proposition 1 shows that $Q_1^* = D/k$ and $Q_2^* = D$, and the expected profit of the assembler can be expressed as

$$\begin{aligned}\pi_a(w_1) &= (p - w_1 - \hat{w}_2)E[\min(\epsilon Q_1^*, Q_2^*, D)] \\ &= (p - w_1 - \hat{w}_2)D\left[1 - \frac{\int_0^k G(t)dt}{k}\right],\end{aligned} \tag{5}$$

where both $\hat{w}_2$ and k depend on w_1 as given in Equation (4) and Equation (3), respectively.

Observe that $(p - w_1 - \hat{w}_2)$ represents the unit profit margin for the assembler. Define

$$\hat{p} = \min\left\{w_1 + \hat{w}_2 : w_1 \geq \frac{c_1}{\mu_\epsilon}, \pi_a(w_1) \geq 0\right\}. \tag{6}$$

We refer to $\hat{p}$ as the deterministic threshold price, such that $\hat{p}$ represents the minimum product price below which it is not profitable for the assembler to assemble the product. Clearly, $\hat{p} \geq (c_1/\mu_\epsilon) + c_2$ since $\hat{w}_2 \geq c_2$. We illustrate our results in Fig. 1. In particular, the optimal (w_1^*, w_2^*) can be found along the line $\hat{w}_2$ when $p > \hat{p}$.

We next analyze how the different model parameters affect the deterministic threshold price $\hat{p}$. In order to derive some useful analytical results that allow us to better understand the impact of supply uncertainty, we require some specific properties on the distribution function for the supply reliability factor ϵ. In particular, we define the following technical assumption.

Assumption 1: $[xg'(x) + 3g(x)] \int_0^x tg(t)dt - 3x^2 g(x)^2 < 0$ for all $x \in [l, u]$.

Proposition 2. *Suppose that Assumption 1 holds:*

(i) If $c_2/c_1 \leq ug(u)/\mu_\epsilon$, then $\hat{p} = (c_1/\mu_\epsilon) + (c_2 u/\mu_\epsilon)$.

(ii) If $(c_2/c_1) > ug(u)/\mu_\epsilon$, then $\hat{p} = \frac{c_1}{\int_0^{k^*} tg(t)dt} + \frac{c_2}{1-(\int_0^{k^*} G(t)dt)/k^*}$, where k^* is given by $\frac{k^* g(k^*)(k^* - \int_0^{k^*} G(t)dt)^2}{(\int_0^{k^*} tg(t)dt)^3} = \frac{c_2}{c_1}$.

Proposition 2(i) shows that when the ratio between the two unit component costs, c_2/c_1, is low, the deterministic threshold price $\hat{p}$ is given by $(c_1/\mu_\epsilon) + (c_2/\mu_\epsilon/u)$ at which c_1/μ_ϵ is the unit

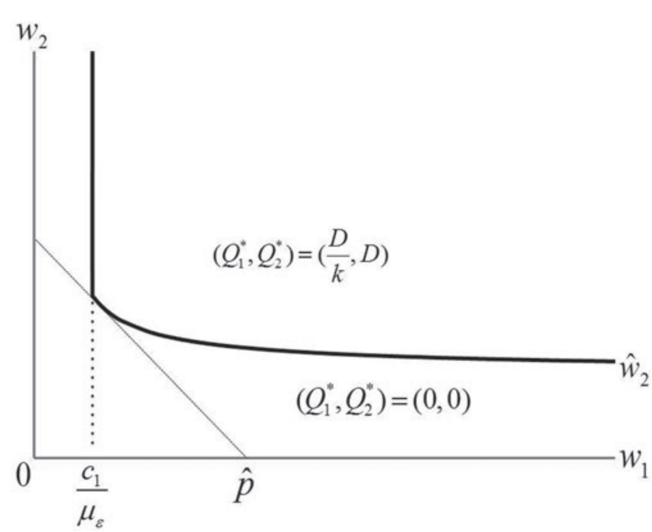

Figure 1. Summary for the deterministic demand case.

component price offered to supplier 1 and $c_2/(\mu_\epsilon/u)$ is the unit component price offered to supplier 2. In this case, no price premium above the effective unit production cost of component 1 is needed for supplier 1, whereas a price premium above the unit production cost of component 2, c_2, is required to induce supplier 2 to produce the component. Proposition 2(ii) further shows that when the ratio c_2/c_1 is high, a price premium above the corresponding effective unit production cost is required to induce both suppliers to produce the components. In this case, $k^* < u$, so $w_1 > c_1/\mu_\epsilon$ and $\hat{w}_2 < c_2/(\mu_\epsilon/u)$. In other words, the assembler now needs to offer a price premium above the effective unit cost c_1/μ_ϵ for supplier 1, while lowering the price premium for supplier 2.

Proposition 2 shows that the deterministic threshold price depends critically on the ratio c_2/c_1. We can explain this result as follows. There are two ways to induce supplier 2 to produce the components: (i) by inducing supplier 1 to produce more units or (ii) by offering a higher unit price for component 2. When the ratio c_2/c_1 is low, the assembler can simply offer a higher unit component price to supplier 2. However, when component 2 becomes more expensive relative to component 1, it is cheaper for the assembler to induce supplier 1 to produce a larger amount of component 1 by offering supplier 1 a price premium. By doing so, the assembler can reduce the price premium offered to supplier 2 accordingly.

Let $\hat{p} - ((c_1/\mu_\epsilon) + c_2)$ be the price premium above the combined effective unit cost of the two components for the assembler to assemble the product. We refer to this quantity as the assembler's price premium. It is clear from Proposition 2(i) that the assembler's price premium is increasing in c_2 but is independent of c_1 when $c_2/c_1 \le ug(u)/\mu_\epsilon$. Also, it is straightforward to derive from Proposition 2(ii) and the Envelope Theorem that the assembler's price premium is strictly increasing in c_1 and c_2 when $c_2/c_1 > ug(u)/\mu_\epsilon$.

Assume that the product price p is above the deterministic threshold price $\hat{p}$ given in Proposition 2 so that the assembler would offer sufficiently high unit component prices to the suppliers to produce the two components. We now analyze the optimal contract parameters (w_1^*, w_2^*) to maximize the assembler's expected profit. Using Equation (4), we can rewrite the assembler's expected profit function given in Equation (5) as

$$\pi_a(w_1) = (p - w_1)\left(1 - \frac{\int_0^k G(t)dt}{k}\right)D - c_2D. \tag{7}$$

Note that k depends on w_1 as given in Equation (3). Differentiate $\pi_a(w_1)$ in Equation (7) with respect to w_1 and obtain

$$\pi_a'(w_1) = -\left(1 - \frac{\int_0^k G(t)dt}{k}\right)D + (p - w_1)\frac{c_1\int_0^k tg(t)dt}{w_1^2k^3g(k)}D,$$

which, using Equation (3), can be rewritten as

$$\pi_a'(w_1) = -\left(1 - \frac{\int_0^k G(t)dt}{k}\right)D + (p - w_1)\frac{(\int_0^k tg(t)dt)^3}{c_1k^3g(k)}D. \tag{8}$$

We can establish the following result.

Proposition 3. *Suppose that Assumption 1 holds and $p > \hat{p}$ as given in Proposition 2:*

(i) If $c_2/c_1 \le ug(u)/\mu_\epsilon$ and $p \le (c_1/\mu_\epsilon) + (c_1u^2g(u)/\mu_\epsilon^2)$, then $w_1^* = c_1/\mu_\epsilon$ and $w_2^* = c_2u/\mu_\epsilon$.

(ii) If $c_2/c_1 > ug(u)/\mu_\epsilon$ or $p > (c_1/\mu_\epsilon) + (c_1u^2g(u)/\mu_\epsilon^2)$, then w_1^* is uniquely solved by $\pi_a'(w_1^*) = 0$ and $w_2^* = \hat{w}_2$ as defined in Equation (4) accordingly.

Observe that the optimal prices (w_1^*, w_2^*) given in Proposition 3(ii) have the property that $w_1^* > c_1/\mu_\epsilon$ and $w_2^* < c_2u/\mu_\epsilon$. Proposition 3(i) shows that when the ratio between the two unit component costs, c_2/c_1, is low and the product price p is low, the assembler should simply offer the threshold component price c_1/μ_ϵ to supplier 1 but offer a unit component price of c_2u/μ_ϵ to supplier 2 to offset the risk for supplier 2 to produce the component 2 due to the supply uncertainty of component 1. However, when either the ratio c_2/c_1 is high or the product price p is high, the assembler needs to offer a higher price than the threshold component price so that supplier 1 would produce a larger amount of component 1. When the ratio c_2/c_1 is high, it would be cheaper for the assembler to entice supplier 1 to produce a larger amount of component 1 by offering a higher unit component price to supplier 1. When the product price p is high, it would also be beneficial for the assembler to entice supplier 1 to produce a larger amount of component 1 due to the higher profit margin. In either case, the risk of having an insufficient amount of component 1 is reduced with a higher production quantity of component 1 and thus the assembler can reduce the unit component price offered to supplier 2 accordingly.

The results in Propositions 2 and 3 require that Assumption 1 holds. It could be difficult to verify Assumption 1 directly for specific supply reliability distributions. However, we can show that the following generalized uniform distributions and truncated exponential distributions satisfy Assumption 1.

Proposition 4.

(i) Suppose that the distribution function of ϵ is given by

$$G(t) = \begin{cases} 0, & t \le l \\ (\frac{t-l}{u-l})^y, & l < t < u, \\ 1, & t \ge u \end{cases} \tag{9}$$

where $0 \le l < u \le 1$ and $y > 0$. For $l = 0$ or $y \ge (3\sqrt{5} - 5)/10 \approx 0.17$, Assumption 1 holds.

(ii) Suppose that the supply reliability distribution of ϵ is given by

$$G(t) = \begin{cases} 0, & t \le l \\ \frac{1-e^{-\lambda(t-l)}}{1-e^{-\lambda(u-l)}}, & l < t < u \\ 1, & t \ge u, \end{cases} \tag{10}$$

where $\lambda > 0$ and $0 \le l < u \le 1$. Then, Assumption 1 holds.

It is straightforward to verify that the distribution function $G(t)$ as defined in Equation (9) is concave in t for $0 < y < 1$ and is convex in t for $y > 1$. Thus, this class of generalized uniform distributions is useful as an approximation for more general supply reliability distribution functions by choosing the appropriate parameter values of l, u, and y.

When the supply reliability density function $g(.)$ fails to satisfy Assumption 1, we can use a numerical search to find the deterministic threshold price $\hat{p}$ given in Equation (6). In this case, we can also conduct a numerical search to determine the optimal w_1^* by finding all solutions to the first-order condition given in Equation (8). If there exists no solution to $\pi_a'(w_1) = 0$ for $w_1 > c_1/\mu_\epsilon$, then $w_1^* = c_1/\mu_\epsilon$ and $w_2^* = c_2 u/\mu_\epsilon$. Otherwise, we can evaluate the profit function $\pi_a(w_1)$ given in Equation (7) at all of those solution points together with the boundary solution $w_1 = c_1/\mu_\epsilon$ to determine the optimal w_1^*. Then, $w_2^* = \hat{w}_2$ as defined in Equation (4) accordingly. We conducted a set of numerical experiments in which $g(t)$ follows a beta distribution, which does not satisfy Assumption 1. We found that the results given in Propositions 2 and 3 remain valid in all of our numerical experiments. Therefore, the results in Propositions 2 and 3 are robust with respect to different distributional assumptions.

4.3. *Comparison with the centralized system*

We next compare the results here with those for the centralized system in which the assembler decides the order quantity and purchases the components from the two suppliers at unit costs c_1 and c_2 as studied in Pan and So (2010). Let $\bar{Q}_1^*$ and $\bar{Q}_2^*$ represent the optimal order quantities in the centralized system. Also, let $\bar{p}$ denote the threshold price in the centralized system, below which it is not profitable for the assembler to purchase the components and assemble the product in the centralized system. We have the following results:

Proposition 5.
(i) $Q_1^* < \bar{Q}_1^*$ *and* $Q_2^* = \bar{Q}_2^* = D$.
(ii) $\hat{p} \geq \bar{p}$.

Proposition 5(i) shows that the equilibrium production quantity of the supplier with supply uncertainty in the decentralized system is less than that of the corresponding optimal order quantity by the assembler in the centralized system, whereas the equilibrium production quantity of the supplier with no supply uncertainty in the decentralized system remains the same as the corresponding optimal order quantity by the assembler in the centralized system. Proposition 5(ii) also shows that the deterministic threshold price in the decentralized system is always larger than the corresponding threshold price in the centralized system. This result seems intuitive, as a higher price premium is required to induce both suppliers to produce the components in the decentralized system.

5. Stochastic demand

We now consider the general case where demand D is stochastic with density function $f(.)$ and distribution function $F(.)$. Assume that $f(.)$ is continuous and differentiable. Also, let $L \geq 0$ denote the minimum demand such that $F(x) = 0$ for any $x < L$ and $F(x) > 0$ for any $x \geq L$.

5.1. *The suppliers' problem*

Consider some fixed pricing scheme (w_1, w_2). For any given Q_2, the expected profit of supplier 1 for any Q_1 is given by

$$\pi_1(Q_1|Q_2) = w_1 E[\min(\epsilon Q_1, Q_2, x)] - c_1 Q_1, \tag{11}$$

where

$$E[\min(\epsilon Q_1, Q_2, x)] = w_1 \left[\int_0^{\frac{Q_2}{Q_1}} \int_0^{tQ_1} x f(x) dx g(t) dt + \int_0^{\frac{Q_2}{Q_1}} \int_{tQ_1}^{\infty} tQ_1 f(x) dx g(t) dt + \int_{\frac{Q_2}{Q_1}}^1 \int_0^{Q_2} x f(x) dx g(t) dt + \int_{\frac{Q_2}{Q_1}}^1 \int_{Q_2}^{\infty} Q_2 f(x) dx g(t) dt \right]$$

for $Q_1 \geq Q_2$, and $E[\min(\epsilon Q_1, Q_2, x)] = E[\min(\epsilon Q_1, x)]$ for $Q_1 < Q_2$ as $\epsilon Q_1 \leq Q_1$. Similarly, for any given Q_1, the expected profit of supplier 2 for any Q_2 is given by

$$\pi_2(Q_2|Q_1) = w_2 E[\min(\epsilon Q_1, Q_2, x)] - c_2 Q_2.$$

It is clear that $\pi_2(Q_2|Q_1) < \pi_2(Q_1|Q_1)$ for any $Q_2 > Q_1$; i.e., it is never optimal for supplier 2 to produce more than the production quantity of component 1. Thus, for any Nash equilibrium production quantities (Q_1^*, Q_2^*), if they exist, we must have $Q_2^* \leq Q_1^*$.

For $Q_2 \leq Q_1$, we can differentiate the above expected profit function of the two suppliers with respect to Q_1 and Q_2 and obtain the following two first-order conditions:

$$\frac{\partial \pi_1(Q_1|Q_2)}{\partial Q_1} = w_1 \int_0^{\frac{Q_2}{Q_1}} [1 - F(tQ_1)] t g(t) dt - c_1 \tag{12}$$

and

$$\frac{\partial \pi_2(Q_2|Q_1)}{\partial Q_2} = w_2 \left[1 - G\left(\frac{Q_2}{Q_1} \right) \right] [1 - F(Q_2)] - c_2. \tag{13}$$

Then, we can easily obtain the analogous result of Proposition 1 that the threshold component price for supplier 1 is equal to c_1/μ_ϵ, which is the same as in the deterministic demand case.

Proposition 6. *If* $w_1 < c_1/\mu_\epsilon$, *then* $(Q_1^*, Q_2^*) = (0, 0)$.
For any fixed $w_1 \geq c_1/\mu_\epsilon$, *define*

$$\tilde{w}_2 = \frac{c_2}{1 - G(k)}, \tag{14}$$

where k is given in Equation (3). (Note that $\tilde{w}_2 \equiv \infty$ *if* $w_1 = c_1/\mu_\epsilon$.*) Let* (Q_1^*, Q_2^*) *denote the Nash equilibrium production quantities. The next result provides two threshold component prices for supplier 2 and characterizes* (Q_1^*, Q_2^*).

Proposition 7. *Suppose that* $w_1 \geq c_1/\mu_\epsilon$.
(i) If $w_2 < \hat{w}_2$, *then* $(Q_1^*, Q_2^*) = (0, 0)$.
(ii) If $\hat{w}_2 \leq w_2 \leq \tilde{w}_2$, *then* $(Q_1^*, Q_2^*) = (L/k, L)$.
(iii) If $w_2 > \tilde{w}_2$, *then* (Q_1^*, Q_2^*) *is solved by the two first-order conditions* $\partial \pi_1(Q_1^*|Q_2^*)/\partial Q_1 = 0$ *and* $\partial \pi_1 (Q_2^*|Q_1^*)/\partial Q_2 = 0$ *as given in Equations (12) and (13).*

Proposition 7 provides two threshold component prices for supplier 2. First, both suppliers will not produce any component if the unit price w_2 is below the threshold $\hat{w}_2$, which is the same threshold as in the deterministic demand case. When w_2 is between $\hat{w}_2$ and $\tilde{w}_2$, the two suppliers will only produce to meet the guaranteed minimum demand level L, with $Q_1^* = L/k$ and $Q_2^* = L$. Only when $w_2 > \tilde{w}_2$ are both suppliers willing to produce additional components above the minimum demand level L, where the equilibrium production quantity (Q_1^*, Q_2^*) can be uniquely determined by the two first-order conditions $\partial\pi_1(Q_1^*|Q_2^*)/\partial Q_1 = 0$ and $\partial\pi_1(Q_2^*|Q_1^*)/\partial Q_2 = 0$ as given in Equation (12) and Equation (13). Using the Implicit Function Theorem, it is straightforward to show that Q_1^* and Q_2^* are strictly increasing in w_1 and w_2.

We can consider $\hat{w}_2$ as the minimum threshold price for component 2 below which supplier 2 will not produce any component and $\tilde{w}_2$ as the risk-taking threshold price for component 2 above which supplier 2 is willing to produce components above the minimum demand level L. Thus, we can interpret $(\tilde{w}_2 - \hat{w}_2)$ as the risk premium for supplier 2 for taking production risk due to the underlying demand uncertainty. Interestingly, this risk premium depends only on the supply uncertainty function but is independent of the underlying demand distribution. In other words, our result shows that demand uncertainty has no impact on the threshold component price for both suppliers or the risk premium for supplier 2.

5.2. *The assembler's problem*

For any given (w_1, w_2), the equilibrium production quantity (Q_1^*, Q_2^*) is given by Proposition 7 and the corresponding expected profit of the assembler is given by

$$\begin{aligned}\pi_a(w_1, w_2) &= (p - w_1 - w_2)E[\min(\epsilon Q_1^*, Q_2^*, D)]\\ &= (p - w_1 - w_2)\Bigg[\int_0^{\frac{Q_2^*}{Q_1^*}}\int_0^{tQ_1^*} x f(x)dx g(t)dt\\ &\quad + \int_0^{\frac{Q_2^*}{Q_1^*}}\int_{tQ_1^*}^{\infty} tQ_1^* f(x)dx g(t)dt\\ &\quad + \int_{\frac{Q_2^*}{Q_1^*}}^{1}\int_0^{Q_2^*} x f(x)dx g(t)dt\\ &\quad + \int_{\frac{Q_2^*}{Q_1^*}}^{1}\int_{Q_2^*}^{\infty} Q_2^* f(x)dx g(t)dt\Bigg]. \qquad (15)\end{aligned}$$

The objective of the assembler's problem is to determine the optimal unit component prices (w_1^*, w_2^*) to be offered to the two suppliers to maximize the expected profit $\pi_a(w_1, w_2)$ given in Equation (15).

In view of the results in Proposition 7, we define an additional (stochastic) threshold price:

$$\tilde{p} = \min\left\{w_1 + \tilde{w}_2 : w_1 \geq \frac{c_1}{\mu_\epsilon},\ \pi_a(w_1, \tilde{w}_2) \geq 0\right\}. \qquad (16)$$

This stochastic threshold price $\tilde{p}$ represents the minimum product price below which it is not profitable for the assembler to offer any VMI contract to the two suppliers to assemble products above the minimum demand quantity L. The next result shows how to determine the value of $\tilde{p}$ for a general class of supply reliability function that satisfies the following technical assumption:

Assumption 2: $x\bar{G}(x)^2/[\int_0^x tg(t)dt]^2$ is strictly decreasing in x for all $x \in [l, u]$.

Proposition 8. *Suppose that Assumption 2 holds. Then*

$$\tilde{p} = \frac{c_1}{\int_0^{k^*} tg(t)dt} + \frac{c_2}{1 - G(k^*)}, \qquad (17)$$

where k^* given by $k^*\bar{G}(k^*)^2/[\int_0^{k^*} tg(t)dt]^2 = c_2/c_1$. Furthermore, $\tilde{p} > \hat{p}$.

We note that when the supply reliability density function $g(.)$ fails to satisfy Assumption 2, there could exist multiple k^* that satisfy $k^*\bar{G}(k^*)^2/[\int_0^{k^*} tg(t)dt]^2 = c_2/c_1$. In this case, we need to evaluate the value of $\tilde{p}$ as given in Equation (17) for each of these k^* to determine the minimum $\tilde{p}$. In general, it could be difficult to verify Assumption 2 directly for specific supply reliability distributions. However, we can show the following result.

Proposition 9. *The class of generalized uniform distributions given in Equation (9) and the truncated exponential distributions given in Equation (10) satisfy Assumption 2.*

We can summarize our results as follows, which is illustrated in Fig. 2. The optimal component prices offered by the assembler and the corresponding equilibrium production quantities of the components depend on the product price p. First, when the product price is below the deterministic threshold price ($p < \hat{p}$), the assembler will not offer any VMI contract to the component suppliers and, consequently, no product will be assembled. As discussed earlier, this deterministic threshold price $\hat{p}$ depends on the unit costs of the components c_i and the supply reliability function $G(.)$ but is independent of the underlying demand distribution. In other words, supply uncertainty affects the deterministic threshold price, but demand uncertainty has no impact.

Second, when the product price is between the deterministic and stochastic threshold prices ($\hat{p} \leq p \leq \tilde{p}$), the assembler will only be willing to assemble products to meet the minimum demand quantity L and to offer a VMI contract to the two component suppliers accordingly. The optimal component prices will be given by the results for the deterministic case with $D = L$ (i.e., Proposition 3) and the corresponding equilibrium production quantities (Q_1^*, Q_2^*) are given by $(L/k, L)$ (i.e., Proposition 1). Again, the stochastic threshold price $\tilde{p}$ depends on the unit costs of the components c_i and the supply reliability function $G(.)$ but is independent of the underlying demand distribution. In this case, supply uncertainty affects the deterministic and stochastic threshold prices, the optimal component prices,

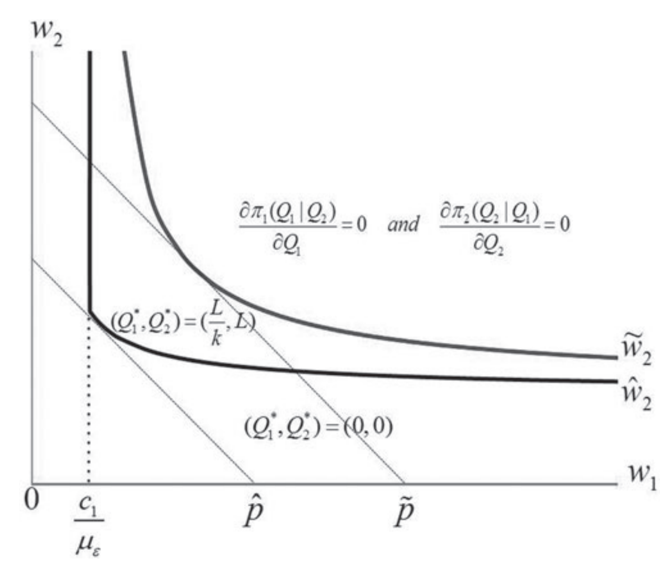

Figure 2. Summary for the stochastic demand case.

and equilibrium production quantities. On the other hand, the specific demand distribution has no impact on the threshold prices, and the equilibrium production quantities depend only on the minimum demand quantity L.

Finally, when the product price is above the stochastic threshold price ($p > \tilde{p}$), it is profitable for the assembler to assemble products above the minimum demand quantity L and to offer a VMI contract to the two component suppliers accordingly. To solve the assembler problem in this case, we need to consider two possible scenarios: (i) the assembler targets only the minimal demand level L and (ii) the assembler targets to produce above level L. The optimal solution is then given by the scenario that provides the higher expected profit for the assembler.

For scenario (i), we can solve the assembler's problem using the results for the deterministic case with $D = L$ as before. For scenario (ii), we provide the following procedure to solve the assembler's problem where the optimal component prices offered by the assembler and the corresponding production quantities of the two component suppliers will be affected by both the underlying supply and demand uncertainties. Specifically, we determine the optimal component prices (w_1^*, w_2^*) that maximize the assembler's profit function $\pi_a(w_1, w_2)$ given in Equation (15), while the corresponding equilibrium production quantities of the two component suppliers (Q_1^*, Q_2^*) are given by the two first-order conditions $\partial\pi_1(Q_1^*|Q_2^*)/\partial Q_1 = 0$ and $\partial\pi_1(Q_2^*|Q_1^*)/\partial Q_2 = 0$ as given in Equation (12) and Equation (13) with $w_1 = w_1^*$ and $w_2 = w_2^*$.

The next result shows that we can solve the assembler's problem for scenario (ii) by the solution to the first-order conditions of the following Lagrangian function:

$$
\begin{aligned}
L(Q_1, Q_2, w_1, w_2, u_1, u_2) &= (p - w_1 - w_2) \\
&\left[\int_0^{\frac{Q_2}{Q_1}} \int_0^{tQ_1} x f(x)dxg(t)dt + \int_0^{\frac{Q_2}{Q_1}} \int_{tQ_1}^{\infty} tQ_1 f(x)dxg(t)dt \right. \\
&\left. + \int_{\frac{Q_2}{Q_1}}^1 \int_0^{Q_2} x f(x)dxg(t)dt + \int_{\frac{Q_2}{Q_1}}^1 \int_{Q_2}^{\infty} Q_2 f(x)dxg(t)dt \right] \\
&-\lambda_1\left[w_1 \int_0^{\frac{Q2}{Q1}} \bar{F}(tQ_1)tg(t)dt - c_1\right] \\
&-\lambda_2\left[w_2\bar{F}(Q_2)\bar{G}(\frac{Q_2}{Q_1}) - c_2\right],
\end{aligned}
\tag{18}
$$

where λ_1 and λ_2 represent the Lagrangian multipliers for the two first-order conditions given in Equation (12) and Equation (13).

Proposition 10. *The optimal component prices (w_1^*, w_2^*) are given by the solution to the following set of first-order conditions:*

$$
\begin{aligned}
&(p - w_1^* - w_2^*) \int_0^{\frac{Q2}{Q1}} \bar{F}(tQ_1)tg(t)dt + \lambda_1 w_1^* \\
&\times \left[\frac{Q_2^2}{Q_1^3}\bar{F}(Q_2)g\left(\frac{Q_2}{Q_1}\right) + \int_0^{\frac{Q_2}{Q_1}} f(tQ_1)t^2g(t)dt\right] \\
&- \lambda_2 w_2^* \frac{Q_2}{Q_1^2} g\left(\frac{Q_2}{Q_1}\right)\bar{F}(Q_2) = 0,
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
&(p - w_1^* - w_2^*)\bar{F}(Q_2)\bar{G}\left(\frac{Q_2}{Q_1}\right) - \lambda_1 w_1^* \frac{Q_2}{Q_1^2}\bar{F}(Q_2)g\left(\frac{Q_2}{Q_1}\right) \\
&+ \lambda_2 w_2^*\left[\bar{G}\left(\frac{Q_2}{Q_1}\right) f(Q_2) + g\left(\frac{Q_2}{Q_1}\right)\frac{\bar{F}(Q_2)}{Q_1}\right] = 0,
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
&-\left[\int_0^{\frac{Q_2}{Q_1}} \int_0^{tQ_1} x f(x)dxg(t)dt \right. \\
&+ \int_0^{\frac{Q_2}{Q_1}} \int_{tQ_1}^{\infty} tQ_1 f(x)dxg(t)dt + \int_{\frac{Q_2}{Q_1}}^1 \int_0^{Q_2} x f(x)dxg(t)dt \\
&\left. + \int_{\frac{Q_2}{Q_1}}^1 \int_{Q_2}^{\infty} Q_2 f(x)dxg(t)dt \right] - \lambda_1 \int_0^{\frac{Q2}{Q1}} \bar{F}(tQ_1)tg(t)dt = 0,
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
&-\left[\int_0^{\frac{Q_2}{Q_1}} \int_0^{tQ_1} x f(x)dxg(t)dt + \int_0^{\frac{Q_2}{Q_1}} \int_{tQ_1}^{\infty} tQ_1 f(x)dxg(t)dt \right. \\
&\left. + \int_{\frac{Q_2}{Q_1}}^1 \int_0^{Q_2} x f(x)dxg(t)dt + \int_{\frac{Q_2}{Q_1}}^1 \int_{Q_2}^{\infty} Q_2 f(x)dxg(t)dt \right] \\
&- \lambda_2\bar{F}(Q_2)\bar{G}\left(\frac{Q_2}{Q_1}\right) = 0,
\end{aligned}
\tag{22}
$$

$$
w_1^* \int_0^{\frac{Q2}{Q1}} \bar{F}(tQ_1)tg(t)dt - c_1 = 0, \tag{23}
$$

$$
w_2^*\bar{F}(Q_2)\bar{G}\left(\frac{Q_2}{Q_1}\right) - c_2 = 0. \tag{24}
$$

Proposition 10 provides an efficient approach to solve for the optimal solution of the assembler problem for scenario (ii). Using the analytical results derived in this section, we conducted a comprehensive set of numerical experiments to gain additional insights as how supply uncertainty, demand uncertainty, and product price can affect the optimal component prices offered by the assembler in the VMI contract as well as the corresponding equilibrium production quantities of the two suppliers.

5.3. *Numerical study*

We performed a set of numerical experiments to generate additional insights from our analysis. We used both the uniform and beta distributions for the supply reliability distribution $G(.)$. For our base cases, we assume ϵ to be uniformly distributed on $[0.75 - b, 0.75 + b]$, where $0 \le b \le 0.25$. We note that the mean and variance of the beta distributions, $B(\alpha, \beta)$, are given by $\alpha/(\alpha + \beta)$ and $\alpha\beta/((\alpha + \beta)^2(\alpha + \beta + 1))$, respectively. For the base cases where ϵ follows a beta distribution, we set $\alpha = 3\beta$ such that $E(\epsilon) = 0.75$ and varied the values of β for different degrees of supply variability.

We also used both the uniform and beta distributions for the demand distribution $F(.)$. In particular, we ran a set of numerical experiments where D is assumed to be uniformly distributed on $[100 - a, 100 + a]$, where $0 \le a \le 100$, such that the minimum demand $L = a$ with $E(D) = 100$. We then ran a parallel set of experiments with $D = a + 2(100 - a)X$ where $0 \le a \le 100$ and X follows a beta distribution $B(2, 2)$ such that $L = a$ with

Table 1. Impact of supply uncertainty—Example 1.

Parameters: $p = 8$, $c_1 = c_2 = 1$, $\epsilon = Beta(3\beta, \beta)$, $D = 200 \times Beta(2, 2)$							
β	w_1^*	w_2^*	Q_1^*	Q_2^*	π_1^*	π_2^*	π_a^*
6	3.09	2.70	69.2	53.2	76.8	74.6	104.8
5	3.09	2.71	69.0	53.2	76.6	74.3	103.5
4	3.10	2.72	68.6	53.2	76.1	73.9	101.6
3	3.11	2.74	67.9	53.1	75.5	73.2	99.0
2	3.13	2.77	66.6	53.0	74.3	72.0	94.6
1	3.15	2.85	63.1	52.5	71.2	69.1	85.6

Table 3. Impact of supply uncertainty—Example 3.

Parameters: $p = 20$, $c_1 = c_2 = 1$, $\epsilon = Beta(9\beta, \beta)$, $D = 200 \times Beta(2, 2)$							
β	w_1^*	w_2^*	Q_1^*	Q_2^*	π_1^*	π_2^*	π_a^*
6	3.81	3.63	101.0	91.4	187.6	182.8	950.1
5	3.82	3.63	101.0	91.6	187.6	182.8	949.0
4	3.82	3.63	101.1	91.7	187.7	182.8	947.5
3	3.83	3.64	101.1	92.0	187.8	182.9	945.2
2	3.84	3.66	101.0	92.4	187.9	182.9	941.3
1	3.85	3.69	100.5	93.2	187.7	182.6	931.9

$E(D) = 100$. In our numerical experiments, we set $c_1 = c_2 = 1$ and varied the values of p.

We next present a sample of our numerical results that demonstrate the analytical results derived in earlier sections and illustrate some basic insights gained from our numerical experiments. These insights are valid in all our numerical experiments and thus appear to be robust with respect to the specific distributional assumptions on the supply reliability factor ϵ and the stochastic demand D.

5.3.1. *Impact of supply uncertainty*

We provide three numerical examples to illustrate the impact of supply uncertainty, and the specific parameter values used in each example are included in Tables 1 to 3. In particular, we assume that the supply reliability factor ϵ follows a beta distribution, $B(\alpha, \beta)$, and we varied the value of the shape parameter β in each example. Therefore, a lower value of β corresponds to a higher degree of supply uncertainty in all three examples.

First, observe from Tables 1 to 3 that the optimal unit component price offered to supplier 1 (w_1^*) always increases as supply uncertainty increases (i.e., as β decreases). However, the change in the optimal unit component price offered to supplier 2 (w_2^*) is not monotone. For example, Table 2 shows that w_2^* first increases and then decreases as supply uncertainty increases. We can explain this behavior as follows. As supply uncertainty increases, the assembler needs to entice both suppliers to produce sufficient amount of components by offering higher component prices to either supplier 1 and/or supplier 2. Since supplier 1 faces supply uncertainty, the magnitude of component price increase for supplier 1 needs to be more significant than that for supplier 2. Consequently, we observe that w_1^* always increases as supply uncertainty increases. In Example 2, the resulting increase in the production quantity Q_1^* is sufficient to entice supplier 2 to increase her production quantity Q_2^*. In fact, the assembler can actually reduce the unit component price offered to supplier 2 in this case.

Table 2. Impact of supply uncertainty - Example 2.

Parameters: $p = 200$, $c_1 = c_2 = 1$, $\epsilon = Beta(3\beta, \beta)$, $D = 200 \times Beta(2, 2)$							
β	w_1^*	w_2^*	Q_1^*	Q_2^*	π_1^*	π_2^*	π_a^*
6	10.46	8.70	186.8	140.1	791.7	674.1	16915
5	10.55	8.71	187.6	140.5	797.7	673.0	16887
4	10.67	8.71	188.7	141.0	806.5	671.1	16847
3	10.87	8.71	190.3	141.8	820.4	667.9	16780
2	11.23	8.70	192.9	142.8	845.8	661.3	16650
1	12.16	8.67	198.0	144.5	906.8	643.7	16284

Second, observe from Tables 1 to 3 that the changes in the equilibrium production quantities, Q_1^* and Q_2^*, can increase or decrease as supply uncertainty increases. There are two factors that affect these equilibrium production quantities. On one hand, the equilibrium production quantities would decrease as supply uncertainty increases, as both suppliers would be less willing to produce components due to the increased supply risk. On the other hand, as discussed above, the assembler would always increase the unit component price offered to supplier 1 as supply uncertainty increases to entice supplier 1 to produce more components, which, in turn, could indirectly entice supplier 2 to increase her production quantity. These two factors counteract each other and thus the effect of supply uncertainty on the equilibrium production quantities is not necessarily monotone.

Furthermore, observe that as the supply uncertainty increases, the expected profit of supplier 1 can either increase or decrease. In Example 1 given in Table 1, the increase in component price offered to supplier 1 is not sufficient to offset the increased risk due to a higher supply uncertainty. As a result, the production quantity and expect profit of supplier 1 both decrease. In Example 2 given in Table 2 where the product price is much higher, the assembler is now willing to offer a much higher component price to supplier 1 to counter a higher supply uncertainty. Consequently, the production quantity and expected profit of supplier 1 both increase in this case. Table 3 further provides an example where the expected profit of either supplier 1 or 2 is not even monotone. However, the expected profit of the assembler always decreases as supply uncertainty increases, as he needs to offer higher component prices to either supplier 1 and/or supplier 2.

Finally, we note that the equilibrium production quantities would always satisfy the following relationship: $Q_1^* = Q_2^*/\mu_\epsilon$ when there is no supply uncertainty in the system, which implies that Q_1^* and Q_2^* (and thus the expected profit π_1^* and π_2^*) would always behave in the same manner when there is a change in any of the model parameters. However, these relationships become much more complicated in the presence of supply uncertainty. For instance, Q_2^* increases while Q_1^* is not monotone, as supply uncertainty increases for Example 3 given in Table 3. Similarly, π_1^* increases while π_2^* decreases, as supply uncertainty increases for Example 2 given in Table 2.

5.3.2. *Impact of demand uncertainty*

We provide two numerical examples to illustrate the impact of demand uncertainty, and the specific parameter values used in each example are included in Tables 4 and 5. In particular, we

Table 4. Impact of demand uncertainty—Example 1.

Parameters: $p = 10$, $c_1 = c_2 = 1$, $\epsilon = Beta(3, 1)$, $D = a + 2(100 - a) \times Beta(2, 2)$							
a	w_1^*	w_2^*	Q_1^*	Q_2^*	π_1^*	π_2^*	π_a^*
100	1.67	1.27	105.8	100	26.0	0	557.1
60	1.67	1.27	63.5	60	15.6	0	334.3
48	1.67	1.27	50.8	48	12.5	0	267.4
47	3.07	2.70	89.4	73.7	102.9	95.3	264.1
40	3.12	2.76	87.2	71.9	102.2	95.6	249.6
20	3.27	2.92	82.0	67.8	100.5	95.3	212.4
0	3.42	3.07	78.3	64.8	97.9	93.5	180.5

assume that the supply reliability factor ϵ follows a beta distribution $B(3, 1)$, and the demand $D = a + 2(100 - a)X$ where $0 \leq a \leq 100$ and X follows a beta distribution $B(2, 2)$ such that $L = a$ with $E(D) = 100$ in both examples. Therefore, a smaller value of a corresponds to a higher degree of demand uncertainty in both examples.

Table 4 shows that the optimal unit component price offered to the two suppliers remain the same when $48 \leq a \leq 100$. In this case, both suppliers are only willing to produce to meet the minimal demand level $L = a$. This corresponds to the situation as depicted in Proposition 7(ii), whereas the optimal w_1^* and w_2^* are given by the results of Proposition 3(ii) with deterministic demand $D = a$. Accordingly, the equilibrium production quantities (Q_1^*, Q_2^*) are equal to $(L/k, L)$. Also, observe from Table 4 that the expected profits of supplier 1 and the assembler both decrease as the value of a decreases for $48 \leq a \leq 100$. This is due to the fact that the minimal demand level $(L = a)$ decreases as a decreases, hurting the expected profits of supplier 1 and the assembler. However, the expected profit of supplier 2 is always equal to zero.

It is interesting to note that in this case $(48 \leq a \leq 100)$, the assembler needs to offer a high component price to supplier 1 $(w_1^* = 1.67)$ such that the expected profit of supplier 1 is now positive. For the case without supply uncertainty; i.e., $\epsilon = 0.75$, it is straightforward to show that the corresponding component price $w_1^* = 1/0.75 = 1.333$, in which the expected profit of supplier 1 would be equal to zero. Therefore, the presence of supply uncertainty increases the expected profit of supplier 1 at the expense of the assembler's expected profit. However, the presence of supply uncertainty has no impact on the equilibrium production quantity or the expected profit of supplier 2; i.e., $Q_2^* = L$ and $\pi_2^* = 0$, respectively.

When $a < 48$, the assembler will offer sufficiently high component prices to the two suppliers such that they are willing to produce components above the minimal demand level L. This corresponds to the situation depicted in Proposition 7(iii). Table 4 shows that the optimal component prices, w_1^* and w_2^*, increase as a decreases in this case, illustrating the fact that the assembler will offer a higher component price to entice both suppliers to produce the components as demand uncertainty increases (i.e., as a decreases).

Table 5 further shows that the corresponding equilibrium production quantities, Q_1^* and Q_2^*, are not necessarily monotone as demand uncertainty increases. Again, there are two factors that affect the equilibrium production quantities. On one hand, the production quantities would decrease as the demand uncertainty increases, as both suppliers would be less willing to produce components due to the increased demand risk. On the other hand, the production quantities increase as the unit component prices increase. These two factors counteract each other and thus the effect of demand uncertainty on the equilibrium production quantities is not monotone. As a result, the expected profit of both suppliers can either increase or decrease as demand uncertainty increases. This shows that both component suppliers can sometimes benefit from higher demand uncertainty. However, the expected profit of the assembler always decreases as a decreases, showing that an increase in demand uncertainty will always hurt the expected profit of the assembler.

5.3.3. *Impact of product price*

Table 6 provides a sample set of numerical experiments that illustrate the impact of product price p. For this set of numerical experiments, it follows from Propositions 2 and 8 that $\hat{p} = 2.67$ and $\tilde{p} = 5.14$, respectively. Table 6 shows different values of p with $p \geq 2.67$, as it is not profitable for the assembler to produce any product for $p < 2.67$.

For $2.67 \leq p \leq 5.14$, the optimal w_1^* and w_2^* are given by the results of Proposition 3(i), with the corresponding equilibrium production quantities $(Q_1^*, Q_2^*) = (40, 40)$. For $5.14 \leq p \leq 9.0$, the expected profit of the assembler given in scenario (i) is higher than that in scenario (ii). Thus, the assembler will only target the minimum demand level $L = 40$; i.e., the situation depicted by Proposition 7(ii) holds. The optimal w_1^* and w_2^* are then given by the results of Proposition 3(i) and Proposition 3(ii) for $5.14 \leq p < 6.67$ and $6.67 \leq p \leq 9.0$, respectively, with the corresponding equilibrium production quantities $(Q_1^*, Q_2^*) = (40/k, 40)$, where k is given in Equation (3). When $p > 9.0$, the expected profit of the assembler given in scenario (ii) is now higher than that in scenario (i), and the assembler now targets production above $L = 40$. Intuitively, the

Table 5. Impact of demand uncertainty—Example 2.

Parameters: $p = 15$, $c_1 = c_2 = 1$, $\epsilon = Beta(3, 1)$, $D = a + 2(100 - a) \times Beta(2, 2)$							
a	w_1^*	w_2^*	Q_1^*	Q_2^*	π_1^*	π_2^*	π_a^*
100	2.10	1.22	112.0	100	60.3	0	960.6
72	2.10	1.22	80.6	72	43.4	0	691.6
71	3.25	2.64	107.9	87.4	136.2	111.3	684.7
60	3.36	2.82	105.5	85.5	139.2	119.6	642.9
40	3.57	3.09	102.4	83.4	144.7	130.2	576.8
20	3.79	3.33	100.9	82.5	148.9	136.8	519.9
10	3.89	3.43	100.7	82.4	150.2	138.8	494.4
0	4.00	3.54	100.8	82.6	151.0	140.2	470.5

Table 6. Impact of product price.

Parameters: $c_1 = c_2 = 1$, $\epsilon = Beta(3, 1)$, $D = 40 + 120 \times Beta(2, 2)$							
p	w_1^*	w_2^*	Q_1^*	Q_2^*	π_1^*	π_2^*	π_a^*
3	1.33	1.33	40	40	0	0	10
5	1.33	1.33	40	40	0	0	70
6	1.33	1.33	40	40	0	0	100
7	1.37	1.32	40.3	40	1.1	0	130.0
9	1.58	1.28	41.7	40	7.4	0	191.5
9.1	3.04	2.70	83.2	68.8	93.5	88.2	196.1
11	3.22	2.83	91.0	74.9	111.5	103.3	311.5
13	3.40	2.96	97.3	79.6	128.7	117.4	441.0

product price is now high enough for the assembler to offer sufficiently high component prices to the two suppliers such that they are willing to produce components above the minimal demand level of 40. This corresponds to the situation depicted in Proposition 7(iii).

Finally, we observe from Table 6 that the equilibrium production quantities and the expected profits of all three players always increase as the product price p increases. This is intuitive, as all three players can benefit from a higher product price (and thus higher profit margin), and the assembler is willing to offer higher component prices to entice the suppliers to produce more quantities to capture the higher profit margin.

6. Concluding remarks

We develop an analytical model to understand how supply uncertainty can affect the pricing and production decisions between an assembler and two component suppliers under a VMI contract. We first derive the optimal component prices offered by the assembler in the VMI contract and the corresponding equilibrium production quantities of the components under deterministic demand. We then extend our analysis for the stochastic demand case. We also conducted a set of numerical experiments to further illustrate the impact of supply uncertainty, demand uncertainty, and product price on the optimal decisions of the assembler and component suppliers in the system. One important result from our analysis is the identification of two threshold prices for which the optimal procurement decisions depend critically on whether the product price is above or below these two threshold prices. Most interesting, we show that supply uncertainty greatly affects these two threshold prices, whereas demand uncertainty has no impact on these two threshold prices.

Our model can be easily extended to the assembly of a product consisting of more than two components, in which only one of the components is subject to supply uncertainty. In this case, it is clear that the equilibrium production quantities of all components without supply uncertainty are the same. Our analysis and results can be extended in a straightforward manner.

We currently assume that all information used in the model is available to the assembler and the two component suppliers. In practice, it is likely that some of the model information, including the unit production cost or supply uncertainty distribution, is private. In this case, it would be interesting to extend our analysis to such an operation environment with asymmetrical information. Also, it would be useful to extend our model and analysis to the case where both components are subject to supply uncertainty. However, analytical results for such case appear to be very limited, as the corresponding cost functions become very complex and difficult to analyze; see Gerchak *et al.* (1994) and Gurnani *et al.* (2000).

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Appendix

Proof of Proposition 1.

(i) Consider any fixed $Q_2 \le D$. For any $Q_1 \ge Q_2$, we differentiate Equation (1) with respect to Q_1 and get

$$\frac{\partial \pi_1(Q_1|Q_2)}{\partial Q_1} = w_1 \int_0^{\frac{Q_2}{Q_1}} tg(t)dt - c_1. \tag{A1}$$

If $w_1 < c_1/\mu_\epsilon$, we have $\partial\pi_1(Q_1|Q_2)/\partial Q_1 \le w_1\mu_\epsilon - c_1 < 0$. This shows that $\pi_1(Q_2|Q_2) > \pi_1(Q_1|Q_2)$ for all $Q_1 > Q_2$.
For $0 \le Q_1 \le Q_2$, we have

$$\pi_1(Q_1|Q_2) = w_1 E[\min(\epsilon Q_1, Q_2)] - c_1 Q_1 = [w_1 E(\epsilon) - c_1] Q_1 = [w_1\mu_\epsilon - c_1]Q_1 \le 0,$$

when $w_1 < c_1/\mu_\epsilon$. Therefore, $Q_1 = 0$ maximizes $\pi_1(Q_1|Q_2)$ for any given Q_2. Clearly, $Q_2 = 0$ maximizes $\pi_2(Q_2|Q_1)$ when $Q_1 = 0$. Thus, $(Q_1^*, Q_2^*) = (0, 0)$.

(ii) The result follows directly from (i) if $w_1 < c_1/\mu_\epsilon$. Assume that $w_1 \ge c_1/\mu_\epsilon$. For any $Q_2 > 0$, $\partial\pi_1((Q_2/k)|Q_2)/\partial Q_1 = 0$ from Equation (A1) and the definition of k given in Equation (3). Furthermore,

$$\frac{\partial^2\pi_1(Q_1|Q_2)}{\partial^2 Q_1} = -w_1\frac{Q_2^2}{Q_1^3}g\left(\frac{Q_2}{Q_1}\right) < 0.$$

Thus, $Q_1 = Q_2/k$ maximizes $\pi_1(Q_1|Q_2)$ for any given $Q_2 > 0$. Also, for $Q_1 = Q_2/k$, we can express supplier 2's expected profit given in Equation (2), for $Q_2 \le D$, as

$$\begin{aligned}\pi(Q_2|Q_1) &= w_2\left[\int_0^k \frac{Q_2}{k}tg(t)dt + \int_k^1 Q_2 g(t)dt\right] - c_2Q_2\\ &= \left\{w_2\left[1 - \frac{\int_0^k G(t)dt}{k}\right] - c_2\right\}Q_2.\end{aligned} \tag{A2}$$

For $w_2 > \hat{w}_2$, it is clear from Equation (A2) that the profit function $\pi_2(Q_2|Q_1)$ is strictly increasing in Q_2 for $Q_2 \le D$. For $Q_2 > D$, supplier 2's expected profit is clearly decreasing as the deterministic demand is equal to D. Thus, $Q_2 = D$ maximizes $\pi_2(Q_2|Q_1)$, and $(Q_1^*, Q_2^*) = (D/k, D)$ are the Nash equilibrium production quantities. For $w_2 = \hat{w}_2$, $(i/k, i)$ maximizes $\pi_2(Q_2|Q_1)$ with $\pi_2(Q_2|Q_1) = 0$ for any i between $[0, D]$. As we assume that both suppliers will choose to produce the components when their expected profits are equal to zero, $(Q_1^*, Q_2^*) = (D/k, D)$ are the Nash equilibrium production quantities. □

Proof of Proposition 2.

(i) Using Equations (3) and (4), we can express $w_1 + \hat{w}_2$ as

$$H(x) = w_1 + \hat{w}_2 = \frac{c_1}{\int_0^x tg(t)dt} + \frac{c_2}{1 - (\int_0^x G(t)dt/x)},$$

such that $\hat{p} = \min_x H(x)$. We take derivative of $H(x)$ with respect to x and obtain

$$\begin{aligned}H'(x) &= -\frac{c_1 x g(x)}{(\int_0^x tg(t)dt)^2} + \frac{c_2\int_0^x tg(t)dt}{(x - \int_0^x G(t)dt)^2}\\ &= \frac{c_1\int_0^x tg(t)dt}{(x - \int_0^x G(t)dt)^2}\left[\frac{c_2}{c_1} - A(x)\right],\end{aligned} \tag{A3}$$

where $A(x) = xg(x)(x - \int_0^x G(t)dt)^2/(\int_0^x tg(t)dt)^3$. We take derivative of $A(x)$ with respect to x and obtain

$$\begin{aligned}A'(x) &= \frac{1}{(\int_0^x tg(t)dt)^6}\left\{(xg'(x) + g(x))\left(x - \int_0^x G(t)dt\right)^2\right.\\ &\quad\times\left(\int_0^x tg(t)dt\right)^3 + 2xg(x)\bar{G}(x)\left(x - \int_0^x G(t)dt\right)\\ &\quad\times\left(\int_0^x tg(t)dt\right)^3 - 3x^2g(x)^2\left(x - \int_0^x G(t)dt\right)^2\\ &\quad\times\left.\left(\int_0^x tg(t)dt\right)^2\right\} < \frac{1}{(\int_0^x tg(t)dt)^6}\\ &\quad\times\left\{(xg'(x) + g(x))\left(x - \int_0^x G(t)dt\right)^2\left(\int_0^x tg(t)dt\right)^3\right.\\ &\quad+ 2g(x)\left(x - \int_0^x G(t)dt\right)^2\left(\int_0^x tg(t)dt\right)^3\\ &\quad\left.- 3x^2g(x)^2\left(x - \int_0^x G(t)dt\right)^2\left(\int_0^x tg(t)dt\right)^2\right\}\\ &= \frac{(\int_0^x tg(t)dt)^2(x - \int_0^x G(t)dt)^2}{(\int_0^x tg(t)dt)^6}\\ &\quad\times\left[(xg'(x) + 3g(x))\int_0^x tg(t)dt - 3x^2g(x)^2\right] \le 0,\end{aligned} \tag{A4}$$

where the first inequality follows from the fact that $x\bar{G}(x) = x - xG(x) < x - \int_0^x G(t)dt$, and the second inequality is due to Assumption 1. Therefore, $A'(x) < 0$ for $l < x \le u$. This implies that $A(x)$ is minimized when $x = u$ with $A(u) = ug(u)/\mu_\epsilon$. Since $c_2/c_1 \le ug(u)/\mu_\epsilon$, it follows that $H'(x) \le 0$ for all $l < x \le u$. This implies that $\hat{p} = H(u)$ with $w_1 = c_1/\mu_\epsilon$ and $\hat{w}_2 = c_2 u/\mu_\epsilon$. This proves (i).

(ii) Suppose that $c_2/c_1 > ug(u)/\mu_\epsilon$. It follows from Equation (A3) that $H'(u) > 0$. Since $A(x)$ is strictly decreasing in x and $\lim_{k\to l} A(x) = \infty$, there exists a unique k^*, with $l < k^* < u$, such that $H'(k^*) = 0$, $H'(x) < 0$ for $l < x < k^*$, and $H'(x) > 0$ for $k^* < x \le u$. Therefore, $\hat{p} = H(k^*)$ with $w_1 = c_1/\int_0^{k^*} tg(t)dt$ and $\hat{w}_2 = c_2/(1 - (\int_0^{k^*} G(t)dt)/k^*)$, where k^* is given by $A(k^*) =$

c_2/c_1, or equivalently,

$$\frac{k^* g(k^*)(k^* - \int_0^{k^*} G(t)dt)^2}{(\int_0^{k^*} tg(t)dt)^3} = \frac{c_2}{c_1}.$$

□

Proof of Proposition 3.

(i) We first show that $\pi_a'(w_1)$ is strictly decreasing in w_1. Note that

$$\begin{aligned}&\frac{d}{dx}\left(\frac{(\int_0^x tg(t)dt)^3}{x^3 g(x)}\right)\\ &= \frac{3(\int_0^x tg(t)dt)^2 x^4 g(x)^2 - (\int_0^x tg(t)dt)^3 [3x^2 g(x) + x^3 g'(x)]}{x^6 g(x)^3}\\ &= -\frac{(\int_0^x tg(t)dt)^2}{c_1 x^4 g(x)^3}\left\{[xg'(x) + 3g(x)]\int_0^x tg(t)dt - 3x^2 g(x)^2\right\} > 0,\end{aligned} \quad \text{(A5)}$$

in view of Assumption 1. Thus, $(\int_0^x tg(t)dt)^3/x^3 g(x)$ is strictly increasing in x. Using Equation (3), $(\int_0^x tg(t)dt)^3/x^3 g(x)$ is strictly decreasing in w_1. Also, it is straightforward to show that $(1 - (\int_0^x G(t)dt/x))$ is strictly decreasing in x, which implies from Equation (3) again that $(1 - (\int_0^x G(t)dt/x))$ is strictly increasing in w_1. It then follows from Equation (8) that $\pi_a'(w_1)$ is strictly decreasing in w_1.

Note that $x = u$ when $w_1 = c_1/\mu_\epsilon$. It is straightforward to show that $\pi_a'(c_1/\mu_\epsilon) = 0$ when

$$p = \frac{c_1}{\mu_\epsilon} + \frac{c_1 u^2 g(u)}{\mu_\epsilon^2}.$$

Also, it is clear that $\pi_a'(w_1)$ is strictly increasing in p for any fixed w_1. Therefore,

$$\pi_a'\left(\frac{c_1}{\mu_\epsilon}\right) < 0 \text{ for } p < \frac{c_1}{\mu_\epsilon} + \frac{c_1 u^2 g(u)}{\mu_\epsilon^2},$$

and

$$\pi_a'\left(\frac{c_1}{\mu_\epsilon}\right) > 0 \text{ for } p > \frac{c_1}{\mu_\epsilon} + \frac{c_1 u^2 g(u)}{\mu_\epsilon^2}.$$

Suppose that $c_2/c_1 \le ug(u)/\mu_\epsilon$. Proposition 2(i) shows that

$$\hat{p} = \frac{c_1}{\mu_\epsilon} + \frac{c_2 u}{\mu_\epsilon} \le \frac{c_1}{\mu_\epsilon} + \frac{c_1 u^2 g(u)}{\mu_\epsilon^2}.$$

For

$$\hat{p} < p \le \frac{c_1}{\mu_\epsilon} + \frac{c_1 u^2 g(u)}{\mu_\epsilon^2},$$

we have $\pi_a'(w_1) \le 0$ for all $w_1 \ge c_1/\mu_\epsilon$ due to the fact that $\pi_a'(c_1/\mu_\epsilon) < 0$ for

$$p < \frac{c_1}{\mu_\epsilon} + \frac{c_1 u^2 g(u)}{\mu_\epsilon^2}$$

and $\pi_a'(w_1)$ is strictly decreasing in w_1. Therefore, $w_1^* = c_1/\mu_\epsilon$ and $w_2^* = c_2 u/\mu_\epsilon$.

(ii) Suppose that $c_2/c_1 \le ug(u)/\mu_\epsilon$ and

$$p > \frac{c_1}{\mu_\epsilon} + \frac{c_1 u^2 g(u)}{\mu_\epsilon^2}.$$

In this case, we have $\pi_a'(c_1/\mu_\epsilon) > 0$. Since $\pi_a'(w_1)$ is strictly decreasing in w_1 and $\pi_a'(p) < 0$, there exists a unique w_1^* such that $\pi_a'(w_1^*) = 0$ as given in Equation (8).

Suppose that $c_2/c_1 > ug(u)/\mu_\epsilon$. We can apply the Envelope Theorem to show that the deterministic threshold price $\hat{p}$ given in Proposition 2(ii) is strictly increasing in c_2. Also, it is straightforward to show that

$$\hat{p} = \frac{c_1}{\mu_\epsilon} + \frac{c_1 u^2 g(u)}{\mu_\epsilon^2}$$

when $c_2 = c_1 ug(u)/\mu_\epsilon$. Since

$$\frac{c_1}{\mu_\epsilon} + \frac{c_1 u^2 g(u)}{\mu_\epsilon^2}$$

is independent of c_2,

$$\hat{p} > \frac{c_1}{\mu_\epsilon} + \frac{c_1 u^2 g(u)}{\mu_\epsilon^2} \text{ when } c_2 > \frac{c_1 ug(u)}{\mu_\epsilon}.$$

Thus, $p > \hat{p}$ implies that

$$p > \frac{c_1}{\mu_\epsilon} + \frac{c_1 u^2 g(u)}{\mu_\epsilon^2}.$$

In this case, we have $\pi_a'(c_1/\mu_\epsilon) > 0$. Since $\pi_a'(w_1)$ is strictly decreasing in w_1 and $\pi_a'(p) < 0$, there exists a unique w_1^* such that $\pi_a'(w_1^*) = 0$ as given in Equation (8). This completes the proof. □

Proof of Proposition 4.

(i) It is straightforward to obtain

$$\int_0^x tg(t)dt = \frac{y}{(u-l)^y}\left[\frac{(x-l)^{y+1}}{y+1} + \frac{l(x-l)^y}{y}\right],$$

and

$$g'(x) = \frac{y(y-1)(x-l)^{y-2}}{(u-l)^y}.$$

Thus,

$$\begin{aligned}&\left[xg'(x) + 3g(x)\right]\int_0^x tg(t)dt - 3x^2 g(x)^2\\ &= \left[x\frac{y(y-1)(x-l)^{y-2}}{(u-l)^y} + 3\frac{y(x-l)^{y-1}}{(u-l)^y}\right]\frac{y}{(u-l)^y}\\ &\quad\times\left[\frac{(x-l)^{y+1}}{y+1} + \frac{l(x-l)^y}{y}\right] - 3x^2\left[\frac{y^2(x-l)^{2y-2}}{(u-l)^{2y}}\right]\\ &= \frac{y^2(x-l)^{2y-2}}{(u-l)^{2y}}\left[\frac{(yx+2x-3l)(yx+l)}{y(y+1)} - 3x^2\right]\\ &= \frac{y(x-l)^{2y-2}}{(y+1)(u-l)^{2y}}[-2y^2x^2 - 2(y-1)xl - yx^2 - 3l^2].\end{aligned}$$

Clearly, Assumption 1 holds for $y \ge 1$ or $l = 0$. Also, we can rewrite $[-2y^2x^2 - 2(y-1)xl - yx^2 - 3l^2]$ as

$$-(2y^2+y)\left\{\left[x + \frac{(y-1)l}{2y^2+y}\right]^2 + \frac{l^2}{(2y^2+y)^2}[5y^2+5y-1]\right\}.$$

It is then straightforward to show that $5y^2 + 5y - 1 > 0$ when $y > 3\sqrt{5} - 5/10 \approx 0.17$.

(ii) For $G(t)$ given in (10), we have

$$g(t) = \frac{\lambda e^{-\lambda(t-l)}}{1-e^{-\lambda(u-l)}} \text{ and } g'(t) = \frac{-\lambda^2 e^{-\lambda(t-l)}}{1-e^{-\lambda(u-l)}}.$$

Then,

$$\int_0^x tg(t)dt = \frac{(1+\lambda l)-(1+\lambda x)e^{-\lambda(x-l)}}{\lambda(1-e^{-\lambda(u-l)})}, \quad \text{(A6)}$$

and

$$\begin{aligned} B(x) &= (xg'(x)+3g(x))\int_0^x tg(t)dt - 3x^2 g(x)^2 \\ &= \frac{1}{(1-e^{-\lambda(u-l)})^2}\{[-x\lambda^2 e^{-\lambda(x-l)} + 3\lambda e^{-\lambda(x-l)}] \\ &\quad \times \left[\frac{(1+\lambda l)-(1+\lambda x)e^{-\lambda(x-l)}}{\lambda}\right] - 3x^2(\lambda e^{-\lambda(x-l)})^2\right\} \\ &= \frac{e^{-\lambda(x-l)}e^{\lambda l}}{(1-e^{-\lambda(u-l)})^2}\{[3-\lambda x]\left[(1+\lambda l)e^{-\lambda l}\right. \\ &\quad \left.-(1+\lambda x)e^{-\lambda x}\right] - 3\lambda^2 x^2 e^{-\lambda x}\}. \end{aligned}$$

To prove the result, it suffices to show that $R(x) < 0$ where

$$\begin{aligned} R(x) &= [3-\lambda x][(1+\lambda l)e^{-\lambda l} - (1+\lambda x)e^{-\lambda x}] \\ &\quad - 3\lambda^2 x^2 e^{-\lambda x}. \end{aligned} \quad \text{(A7)}$$

First, it is straightforward to show that $(1+\alpha)e^{-\alpha}$ is strictly decreasing in α for $\alpha > 0$. Therefore, $[(1+\lambda l)e^{-\lambda l} - (1+\lambda x)e^{-\lambda x}]$ is greater than zero and is strictly decreasing in l for $l < x$. If $[3-\lambda x] \leq 0$, it is clear from Equation (A7) that $R(x) < 0$.
Suppose that $[3-\lambda x] > 0$. Since $[(1+\lambda l)e^{-\lambda l} - (1+\lambda x)e^{-\lambda x}]$ is greater than zero and is strictly decreasing in l for $l < x$, it suffices to show that $R(x) < 0$ for $l = 0$. For $l = 0$ and let $\alpha = \lambda x$, we can express $R(x)$ as

$$\begin{aligned} R(\alpha) &= [3-\alpha][1-(1+\alpha)e^{-\alpha}] - 3\alpha^2 e^{-\alpha} \\ &= -e^{-\alpha}[e^{\alpha}(\alpha-3)+3+2\alpha+2\alpha^2]. \end{aligned}$$

We next show that $S(\alpha) = e^{\alpha}(\alpha-3)+3+2\alpha+2\alpha^2 > 0$ for all $\alpha > 0$, which implies that $R(x) < 0$ for all $l < x$. Direct differentiation of $S(\alpha)$ gives $S'(\alpha) = e^{\alpha}(\alpha-2)+2+4\alpha$, $S''(\alpha) = e^{\alpha}(\alpha-1)+4$, and $S'''(\alpha) = \alpha e^{\alpha} > 0$ for all $\alpha > 0$. Since $S''(0) = 3$ and $S'''(\alpha) > 0$ for all $\alpha > 0$, this implies that $S''(\alpha) > 0$ for all $\alpha > 0$. Since $S'(0) = 0$ and $S''(\alpha) > 0$ for all $\alpha > 0$, this implies that $S'(\alpha) > 0$ for all $\alpha > 0$. Finally, since $S(0) = 0$ and $S'(\alpha) > 0$ for all $\alpha > 0$, this implies that $S(\alpha) > 0$ for all $\alpha > 0$. This completes the proof. □

Proof of Proposition 5.

(i) It was shown in Pan and So (2010) that for deterministic demand, $\bar{Q}_1^* = D/\delta^*$ and $\bar{Q}_2^* = D$, where δ^* is solved by $\int_0^{\delta^*} tg(t)dt = c_1/p$, when it is profitable to assemble the product. Under the decentralized system, it was shown in Proposition 1 that when it is profitable for the two suppliers to produce the components, the equilibrium production quantities are given by $Q_1^* = D/k^*$ and $Q_2^* = D$, where k^* is solved by $\int_0^{k^*} tg(t)dt = c_1/w_1^*$ as given in Equation (3). Clearly, $w_1^* < p$, as the assembler would never offer supplier 1 a unit component price higher than the product price. Thus, $k^* > \delta^*$, which implies that $Q_1^* < \bar{Q}_1^*$ and $Q_2^* = \bar{Q}_2^* = D$.

(ii) Suppose that $\hat{p} < \bar{p}$. Then, for any given p with $\hat{p} < p < \bar{p}$:

$$pE[\min(\epsilon Q_1, Q_2), D] - c_1 Q_1 - c_2 Q_2 < 0, \quad \text{(A8)}$$

from the definition of the threshold price $\bar{p}$; i.e., it is not profitable for assembler to purchase the components and assemble the product. On the other hand, since $\hat{p} < p$, it follows from the definition of $\hat{p}$ that we can find some (w_1', w_2'), with $p = w_1' + w_2'$, such that

$$w_1' E[\min(\epsilon Q_1, Q_2), D] - c_1 Q_1 \geq 0, \quad \text{(A9)}$$
$$w_2' E[\min(\epsilon Q_1, Q_2), D] - c_2 Q_2 \geq 0. \quad \text{(A10)}$$

We add Equation (A9) with Equation (A10) and get

$$\begin{aligned} &(w_1' + w_2')E[\min(\epsilon Q_1, Q_2), D] - c_1 Q_1 - c_2 Q_2 \\ &\quad = pE[\min(\epsilon Q_1, Q_2), D] - c_1 Q_1 - c_2 Q_2 \geq 0, \end{aligned}$$

which contradicts Equation (A8). This proves that $\hat{p} \geq \bar{p}$. □

Proof of Proposition 6. Consider any fixed Q_2. For any $Q_1 \geq Q_2$, it follows from Equation (12) that if $w_1 < c_1/\mu_\epsilon$;

$$\begin{aligned} &\frac{\partial \pi_1(Q_1|Q_2)}{\partial Q_1} \\ &\quad = w_1 \int_0^{\frac{Q_2}{Q_1}} [1-F(tQ_1)]tg(t)dt - c_1 < w_1\mu_\epsilon - c_1 < 0. \end{aligned}$$

Therefore, $\pi_1(Q_2|Q_2) > \pi_1(Q_1|Q_2)$ for all $Q_1 > Q_2$.
For $0 \leq Q_1 \leq Q_2$, we have

$$\begin{aligned} \pi_1(Q_1|Q_2) &= w_1 E[\min(\epsilon Q_1, x)] - c_1 Q_1 \leq w_1 E[(\epsilon Q_1)] - c_1 Q_1 \\ &= [w_1 \mu_\epsilon - c_1]Q_1 \leq 0, \end{aligned}$$

when $w_1 < c_1/\mu_\epsilon$. Therefore, $Q_1 = 0$ maximizes $\pi_1(Q_1|Q_2)$ for any given Q_2. Clearly, $Q_2 = 0$ maximizes $\pi_2(Q_2|Q_1)$ when $Q_1 = 0$. Thus, $(Q_1^*, Q_2^*) = (0, 0)$. □

Proof of Proposition 7. Let us denote $Q_2/Q_1 = a$. Consider $Q_1 \geq Q_2 > L$ and rewrite the two first-order conditions in Equation (12) and Equation (13) as

$$\begin{aligned} \pi_1'(a, Q_2) &= \frac{\partial \pi_1(Q_1|Q_2)}{\partial Q_1} \\ &= w_1 \int_0^a \left[1 - F\left(t\frac{Q_2}{a}\right)\right] tg(t)dt - c_1, \\ \pi_2'(a, Q_2) &= \frac{\partial \pi_2(Q_2|Q_1)}{\partial Q_2} = w_2[1-G(a)][1-F(Q_2)] - c_2. \end{aligned}$$

Clearly, $\pi_1'(1, Q_2)$ is strictly decreasing in Q_2. Also, $\pi_1'(1, Q_2) \to w_1\mu_\epsilon - c_1 > 0$ as $Q_2 \to L$ and $\pi_1(1, Q_2) \to -c_1 < 0$ as $Q_2 \to \infty$. It follows that there must exist some Q_2, denoted by q_2, such that $\pi_1'(1, q_2) = 0$.

For any given $L < Q_2 \leq q_2$, $\pi_1'(a, Q_2)$ is strictly increasing in a. Since $\pi_1'(1, Q_2) = w_1\int_0^1 [1-F(tQ_2)]tg(t)dt - c_1 \geq 0$ and $\pi_1'(a, Q_2) \to -c_1 < 0$ as $a \to 0$, there exists some a, denoted by $a(Q_2)$, such that $\pi_1'(a(Q_2), Q_2) = 0$. Since $\pi_1'(a, Q_2)$ is strictly decreasing in Q_2, $a(Q_2)$ is strictly increasing in Q_2.

Also, it is clear that $\pi_2'(a, Q_2)$ is strictly decreasing in a and Q_2. Therefore, $\pi_2'(a(Q_2), Q_2)$ is strictly decreasing in Q_2.

Since $a(q_2) = 1$, we have $\pi_2'(a(q_2), q_2) = -c_2 < 0$. Also, $a(Q_2) \to k$ as $Q_2 \to L$, where k is given in Equation (3), and thus $\pi_2'(a(Q_2), Q_2) \to w_2[1 - G(k)] - c_2$ as $Q_2 \to L$.

Suppose that $w_2 > \tilde{w}_2$. Then

$$\pi_2'(a(Q_2), Q_2) > \frac{c_2}{1 - G(k)}[1 - G(k)] - c_2 = 0 \text{ as } Q_2 \to L.$$

This implies that there exists some unique Q_2^*, with $L < Q_2^*$, such that $\pi_2'(a(Q_2^*), Q_2^*) = 0$, which shows that there exists some unique (Q_1^*, Q_2^*), with $L < Q_2^* \le Q_1^*$, that satisfies the two first-order equations $\partial\pi_1(Q_1^*|Q_2^*)/\partial Q_1 = 0$ and $\partial\pi_1(Q_2^*|Q_1^*)/\partial Q_2 = 0$ given in Equation (12) and Equation (13). This proves (iii).

Suppose that $w_2 \le \tilde{w}_2$. The above argument shows that there does not exist any Nash equilibrium production quantities (Q_1^*, Q_2^*), with $L < Q_2^* \le Q_1^*$, that would satisfy the two first-order equations $\partial\pi_1(Q_1^*|Q_2^*)/\partial Q_1 = 0$ and $\partial\pi_1(Q_2^*|Q_1^*)/\partial Q_2 = 0$. Consider $0 \le Q_2 \le L$. Since the random demand D is always greater than L, $\min(D, Q_2) = Q_2$. Thus, the analysis is the same as the deterministic case. Using the result in Proposition 1, we have $(Q_1^*, Q_2^*) = (L/k, L)$, where k is defined in Equation (3), if $w_2 \ge \hat{w}_2$ and $(Q_1^*, Q_2^*) = (0, 0)$ if $w_2 < \hat{w}_2$. This completes the proof. □

Proof of Proposition 8. From Equations (14) and (3), we express $w_1 + \tilde{w}_2$ as

$$T(x) = w_1 + \tilde{w}_2 = \frac{c_1}{\int_0^x tg(t)dt} + \frac{c_2}{1 - G(x)}.$$

We differentiate $T(x)$ with respect to x and get

$$\begin{aligned} T'(x) &= -\frac{c_1 xg(x)}{(\int_0^x tg(t)dt)^2} + \frac{c_2 g(x)}{[1 - G(x)]^2} \\ &= \frac{c_1 g(x)}{\bar{G}(x)^2}\left[\frac{c_2}{c_1} - U(x)\right], \end{aligned} \tag{A11}$$

where $U(k) = x\bar{G}(x)^2/(\int_0^x tg(t)dt)^2$. By Assumption 2, $U(x)$ is strictly decreasing in x for $l \le x \le u$. Thus, for any given c_1 and c_2, $U(u) = 0$ and

$$\lim_{k\to l} U(x) = \frac{\bar{G}(x)^2 - 2x\bar{G}(x)g(x)}{2xg(x)\int_0^x tg(t)dt} = \infty.$$

Therefore, there exists some unique k^*, with $l < k^* < u$, such that $U(k^*) = c_2/c_1$ or, equivalently, $T'(k^*) = 0$ in view of Equation (A11). Furthermore, $T'(x) < 0$ when $x < k^*$ and $T'(x) > 0$ when $x > k*$. It directly follows that k^* is the unique global minimum point for $T(x)$.

To prove $\tilde{p} > \hat{p}$, we express

$$H(x) = w_1 + \hat{w}_2 = \frac{c_1}{\int_0^x tg(t)dt} + \frac{c_2}{1 - (\int_0^x G(t)dt/x)}.$$

Clearly, $T(x) > H(x)$ for all $0 \le x \le 1$ since $\int_0^x G(t)dt/x < G(x)$ for all $0 \le x \le 1$. Also, $\tilde{p} = \min_{w_1 \ge c_1/\mu_\epsilon}(w_1 + \tilde{w}_2) = \min_{0\le x\le 1} T(x)$, and $\hat{p} = \min_{w_1 \ge c_1/\mu_\epsilon}(w_1 + \hat{w}_2) = \min_{0\le x\le 1} H(x)$. The result follows immediately. □

Proof of Proposition 9.

(i) Suppose that the distribution function of ϵ is given by Equation (9). Define $U(x) = x\bar{G}(x)^2/[\int_0^x tg(t)dt]^2$. Differentiate $U(x)$ with respect to x and get

$$\begin{aligned} U'(x) &= \frac{\bar{G}(x)}{(\int_0^x tg(t)dt)^3}\left[\bar{G}(x)\int_0^x tg(t)dt - 2\bar{G}(x)x^2 g(x) \right. \\ &\quad \left. - 2xg(x)\int_0^x tg(t)dt\right]. \end{aligned}$$

Using the supply reliability function given by Equation (9), we have

$$\begin{aligned} &\int_0^x tg(t)dt - x^2 g(x) \\ &= \frac{y}{(u-l)^y}\left[\frac{(x-l)^{y+1}}{y+1} + \frac{l(x-l)^y}{y}\right] \\ &\quad - x^2 y\left(\frac{x-l}{u-l}\right)^{y-1}\left(\frac{1}{u-l}\right) \\ &= \frac{y(x-l)^{y-1}}{(u-l)^y}\left[\frac{(x-l)^2}{y+1} + \frac{l(x-l)}{y} - x^2\right] \\ &= \frac{(x-l)^{y-1}}{(y+1)(u-l)^y}[y(x-l)^2 + (y+1)l(x-l) \\ &\quad - y(y+1)x^2] \\ &= \frac{(x-l)^{y-1}}{(y+1)(u-l)^y}[-y^2x^2 - (y-1)xl - l^2]. \end{aligned}$$

Therefore, $\int_0^x tg(t)dt - x^2 g(x) < 0$ for any $x > l \ge 0$ and $y > 0$, which implies that $U'(x) < 0$ for all $x > l \ge 0$ and $y > 0$. Thus, Assumption 2 holds.

(ii) For $G(t)$ given by Equation (10), it follows from Equation (A6) that

$$\begin{aligned} U(x) &= \frac{x\bar{G}(x)^2}{(\int_0^x tg(t)dt)^2} = \frac{\lambda^2 x[e^{-\lambda(x-l)} - e^{-\lambda(u-l)}]^2}{[(1+\lambda l) - (1+\lambda x)e^{-\lambda(x-l)}]^2} \\ &= \frac{\lambda^2 x[e^{-\lambda x} - e^{-\lambda u}]^2}{[(1+\lambda l)e^{-\lambda l} - (1+\lambda x)e^{-\lambda x}]^2}. \end{aligned}$$

Directly differentiating $U(x)$ above and after simplification, we obtain

$$\begin{aligned} U'(x) &= \frac{\lambda^2(e^{-\lambda x} - e^{-\lambda u})}{[(1+\lambda l)e^{-\lambda l} - (1+\lambda x)e^{-\lambda x})]^3} \\ &\quad \times \left\{\left[(1+\lambda l)e^{-\lambda l} - (1+\lambda x)e^{-\lambda x}\right]\right. \\ &\quad \times [e^{-\lambda x} - e^{-\lambda u} - 2\lambda x e^{-\lambda x}] \\ &\quad \left. - 2\lambda^2 x^2 e^{-\lambda x}(e^{-\lambda x} - e^{-\lambda u})\right\}. \end{aligned}$$

To prove that $U'(x) < 0$ for all $l < x$, it suffices to show that $R(x) < 0$ for all $l < x$, where

$$\begin{aligned} R(x) &= \left[(1+\lambda l)e^{-\lambda l} - (1+\lambda x)e^{-\lambda x}\right] \\ &\quad \times \left[e^{-\lambda x} - e^{-\lambda u} - 2\lambda x e^{-\lambda x}\right] \\ &\quad - 2\lambda^2 x^2 e^{-\lambda x}(e^{-\lambda x} - e^{-\lambda u}). \end{aligned} \tag{A12}$$

As shown in the proof of Proposition 4(ii), $\left[(1+\lambda l)e^{-\lambda l} - (1+\lambda x)e^{-\lambda x}\right]$ is greater than zero and is strictly decreasing in l for $l < x$. If $\left[e^{-\lambda x} - e^{-\lambda u} - 2\lambda x e^{-\lambda x}\right] \le 0$, it is clear from Equation (A12) that $R(x)$

< 0. Suppose that $\left[e^{-\lambda x} - e^{-\lambda u} - 2\lambda x e^{-\lambda x}\right] > 0$. Since $\left[(1+\lambda l)e^{-\lambda l} - (1+\lambda x)e^{-\lambda x}\right]$ is greater than zero and is strictly decreasing in l for $l < x$, it suffices to show that $R(x) < 0$ for $l = 0$. For $l = 0$ and let $\alpha = \lambda x$, we can express $R(x)$ given in Equation (A12) as

$$\begin{aligned} R(\alpha) &= [1-(1+\alpha)e^{-\alpha}][e^{-\alpha} - e^{-\lambda u} - 2\alpha e^{-\alpha}] \\ &\quad - 2\alpha^2 e^{-\alpha}(e^{-\alpha} - e^{-\lambda u}) \\ &= e^{-\lambda u}\{(1+\alpha+2\alpha^2)e^{-\alpha} - 1\} + e^{-2\alpha}\{e^{\alpha}(1-2\alpha) \\ &\quad + (\alpha - 1)\}. \end{aligned} \tag{A13}$$

Suppose that $(1+\alpha+2\alpha^2)e^{-\alpha} - 1 \geq 0$. Clearly, $e^{-\lambda u} \leq e^{-\lambda x} = e^{-\alpha}$ for all $x \leq u$. Then, it follows from Equation (A13) that

$$\begin{aligned} R(\alpha) &\leq e^{-\alpha}\{(1+\alpha+2\alpha^2)e^{-\alpha} - 1\} + e^{-2\alpha}\{e^{\alpha}(1-2\alpha) \\ &\quad + (\alpha-1)\} = 2\alpha e^{-2\alpha}(1+\alpha-e^{-\alpha}). \end{aligned}$$

It is straightforward to show that $(1+\alpha-e^{\alpha}) < 0$ for $\alpha > 0$. Thus, $R(\alpha) < 0$ for all $\alpha > 0$.
Suppose that $(1+\alpha+2\alpha^2)e^{-\alpha} - 1 < 0$. Then, it follows from Equation (A13) that

$$R(\alpha) < e^{-2\alpha}\{e^{\alpha}(1-2\alpha) + (\alpha-1)\}.$$

Let $S(\alpha) = e^{\alpha}(1-2\alpha) + (\alpha-1)$. Direct differentiation of $S(\alpha)$ gives $S'(\alpha) = e^{\alpha}(-1-2\alpha) + 1$ and $S''(\alpha) = e^{\alpha}(-3-2\alpha) < 0$ for all $\alpha > 0$. Since $S'(0) = 0$ and $S''(\alpha) < 0$ for all $\alpha > 0$, we have $S'(\alpha) < 0$ for all $\alpha > 0$. Since $S(0) = 0$ and $S'(\alpha) < 0$ for all $\alpha > 0$, we have $S(\alpha) < 0$ for all $\alpha > 0$. Thus, $R(\alpha) < 0$ for all $\alpha > 0$ in this case also. This completes the proof. □

Proof of Proposition 10. We define

$$h_1(Q_1, Q_2, w_1, w_2) = w_1 \int_0^{\frac{Q2}{Q1}} \bar{F}(tQ_1)tg(t)dt - c_1,$$

and

$$h_2(Q_1, Q_2, w_1, w_2) = w_2 \bar{F}(Q_2)\bar{G}\left(\frac{Q_2}{Q_1}\right) - c_2$$

to denote the two first-order conditions $\partial\pi_1(Q_1^*|Q_2^*)/\partial Q_1 = 0$ and $\partial\pi_1(Q_2^*|Q_1^*)/\partial Q_2 = 0$ as given in Equation (12) and Equation (13). Then, the Jacobian matrix of the two constraint functions $h_1(Q_1, Q_2, w1, w_2)$ and $h_2(Q_1, Q_2, w1, w_2)$ is given by

$$\delta h(Q_1, Q_2, w_1, w_2) = \begin{pmatrix} \dfrac{\partial h_1}{\partial Q_1} & \dfrac{\partial h_1}{\partial Q_2} & \dfrac{\partial h_1}{\partial w_1} & \dfrac{\partial h_1}{\partial w_2} \\[2ex] \dfrac{\partial h_2}{\partial Q_1} & \dfrac{\partial h_2}{\partial Q_2} & \dfrac{\partial h_2}{\partial w_1} & \dfrac{\partial h_2}{\partial w_2} \end{pmatrix},$$

where

$$\begin{aligned} \frac{\partial h_1}{\partial Q_1} &= -w_1\left[\frac{Q_2^2}{Q_1^3}\bar{F}(Q_2)g\left(\frac{Q_2}{Q_1}\right) + \int_0^{\frac{Q_2}{Q_1}} f(tQ_1)t^2 g(t)dt\right], \\ \frac{\partial h_1}{\partial Q_2} &= w_1 \frac{Q_2}{Q_1^2}\bar{F}(Q_2)g\left(\frac{Q_2}{Q_1}\right), \\ \frac{\partial h_1}{\partial w_1} &= \int_0^{\frac{Q2}{Q1}} \bar{F}(tQ_1)tg(t)dt, \\ \frac{\partial h_1}{\partial w_2} &= 0, \\ \frac{\partial h_2}{\partial Q_1} &= w_2 \frac{Q_2}{Q_1^2} g\left(\frac{Q_2}{Q_1}\right)\bar{F}(Q_2), \\ \frac{\partial h_2}{\partial Q_2} &= -w_2\left[\bar{G}\left(\frac{Q_2}{Q_1}\right)f(Q_2) + g\left(\frac{Q_2}{Q_1}\right)\frac{1}{Q_1}\bar{F}(Q_2)\right], \\ \frac{\partial h_2}{\partial w_1} &= 0, \\ \frac{\partial h_2}{\partial w_2} &= \bar{F}(Q_2)\bar{G}\left(\frac{Q_2}{Q_1}\right). \end{aligned}$$

Since $Q_2 = 0$ violates the constraint $h_1(Q_1, Q_2, w_1, w_2) = 0$, and $Q_2/Q_1 = 1$ with $Q_2 \to \infty$ violates the constraint $h_2(Q_1, Q_2, w_1, w_2) = 0$, the rank of the above Jacobian matrix $\delta h(Q_1, Q_2, w_1, w_2)$ is equal to two. As the rank of the Jacobian matrix is equal to the number of constraints, all points in the constraint set must satisfy non-degenerate constraint qualification. Therefore, we can find the optimal solution to the assembler's problem by solving the corresponding unconstrained Lagrangian function $L(Q_1, Q_2, w_1, w_2, u_1, u_2)$ given in Equation (18).

Differentiating $L(Q_1, Q_2, w_1, w_2, u_1, u_2)$ with respect to Q_1, Q_2, w_1, w_2, u_1, and u_2 gives the set of first-order conditions as provided in Equations (19) to (24), and we can find the optimal values of (w_1^*, w_2^*) by solving Equations (19) to (24). □