Title
Efficiency in the Mortgage Market: The Borrower's Perspective

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Authors
Quigley, John M.
Van Order, Robert

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EFFICIENCY IN THE MORTGAGE MARKET: 
THE BORROWER'S PERSPECTIVE 

BY 
JOHN M. QUIGLEY 
ROBERT VAN ORDER 

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EFFICIENCY IN THE MORTGAGE MARKET:  
THE BORROWER'S PERSPECTIVE  

By  

John M. Quigley  
University of California  
Berkeley  

and  

Robert Van Order  
Freddie Mac  

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Abstract

EFFICIENCY IN THE MORTGAGE MARKET:
THE BORROWER'S PERSPECTIVE

By

John M. Quigley
and
Robert Van Order

This paper uses mortgage history data from the Federal Home Loan Mortgage Corporation to analyze the prepayment behavior of homeowners and to test whether borrowers exercise their prepayment options in a manner consistent with contingent claims models.

A variety of hazard models are estimated from individual data on more than 6000 mortgages issued during the 1976-1980 period. In these models, it is clear that the extent to which the prepayment option is "in the money" has a strong effect on behavior. However, it is less clear that the option is exercised quite as ruthlessly as the theory predicts.
I. INTRODUCTION

It is by now widely accepted that a fruitful way of analyzing and pricing mortgages is to view them as ordinary debt instruments with various options attached to them. Default is a put option; the defaulter sells his house back to the lender in exchange for eliminating the mortgage obligation. Prepayment is a call option; the borrower exchanges the unpaid balance on the debt instrument for a release from further obligation. Analogously, caps and floors on adjustable-rate mortgages can be formulated as options. Dunn and McDonnell [1983], Buser and Hendershott [1989], Kau et al. [1986], and Brennan and Schwartz [1985], are among the many papers that apply recent contingent claims models to pricing mortgages. Much of this is based on the work of Black and Scholes [1973] and Cox, Ingersol, and Ross [1985]. Hendershott and Van Order [1988] survey some of these results.

This paper uses mortgage history data from the Federal Home Loan Mortgage Corporation (Freddie Mac) to estimate a prepayment function and to test whether borrowers exercise their prepayment options in a manner consistent with the optimal strategy developed with contingent claims models. The key variable in the model is the difference between the current market value and the par value of the mortgage, i.e., the extent to which the option is "in the money." This variable, which reflects principally the difference between
the coupon rate on the mortgage and current market rates, turns out to have a very strong effect on prepayments, as predicted. However, it is less clear that the option is exercised quite as ruthlessly as the theory predicts.

II. OPTIMAL PREPAYMENT OF MORTGAGES

Well-informed borrowers in a perfectly competitive market will always prepay mortgages when they can increase their wealth by engaging in such behavior. Borrowers can increase their wealth by prepaying when the market value of the mortgage exceeds the outstanding balance on the loan. Note that the market value of the mortgage exceeds the present value of the remaining payment stream because the market value ignores the option to prepay at some subsequent date. Consequently, even if the present value of the remaining payments is greater than the outstanding balance (i.e., the option is "in the money"), it may not be optimal to exercise the option. In so exercising it, the holder would lose the option to exercise it later.

To solve the problem of when to exercise the option, the contingent claims model begins by specifying the underlying state variables that determine the price of a security. For a default-free, callable mortgage, these are interest rates. The value of a mortgage is given by $M(r,a,T)$, where $r$ is a vector of interest rates, $a$ is the age of the mortgage, and $T$
is the time at which it matures. A standard arbitrage argument is sufficient to derive the equilibrium condition for $M$ (See Brennan and Schwartz [1985] and Hendershott and Van Order [1987]). This condition is a second order partial differential equation which specifies that the expected return on the security (that is, the coupon return plus capital gains) must equal the risk-free rate of return plus a risk adjustment. The condition applies to any claim that is contingent on the underlying state variables.

An infinite number of functions satisfy the partial differential equation (depending on boundary conditions), which reflects the infinite number of ways that coupon plus capital gain can equal the required expected return. By incorporating the optimal call strategy, the function appropriate for a callable bond can be determined.

Suppose there is just one interest rate, the short rate $r$, which matters. In this case, the optimal call strategy, given $a$, is a value of $r$, $r^*(a)$, at which the mortgage is called. For $r^*$ to be optimal, it must be chosen to minimize the value of the mortgage (which maximizes the borrower's net

---

1 It is much more difficult to calculate solutions if there is more than one state variable. Most papers (e.g., Dunn-McConnell [1985] and Buser and Hendershott [1984]) follow Cox, Ingersol, and Ross [1985] by assuming that only one state variable, the short rate, matters, and that this rate determines the entire term structure of interest rates. Brennan and Schwartz [1985] use two state variables, a long rate and a short rate.
worth), subject to the condition that value of M must equal the remaining balance when the call is exercised.

Figure 1 indicates the optimal call strategy for a mortgage of given age a. The upper dotted curve represents the inverse relationship between price and interest rates for a non-callable mortgage, while the horizontal line ("Par") represents the par value of the mortgage. For a callable mortgage, the optimal exercise strategy is represented by the line marked □, the lowest curve which satisfies the equilibrium condition and is tangent to the horizontal line indicating par. The tangency incorporates the optimizing strategy because, for an interior solution, the curve that minimizes value must be tangent to the par line. This determines r*, the critical rate. The curve marked □ thus gives the market relationship between price and interest rates, and indicates how the market value anticipates the call before it is exercised.

Dunn and McConnell [1983], Quigley [1987], and others have analyzed homeowner mobility and nonfinancial motives for prepayment. Most conventional mortgages are not assumable, forcing homeowners to prepay if they move. Even in a frictionless market, this introduces other demographic and economic factors into the analysis. It also implies the existence of an underlying level prepayments, even with constant interest rates. Note, however, that in the absence
of transactions costs borrowers with non assumable mortgages will still refinance in a ruthless way if interest rates fall. There will still exist a critical value of $r$ which triggers immediate prepayment, but there will be a positive probability of prepayment even if rates rise.

Transactions costs can be introduced as a wedge between the borrower’s payment and the lender’s receipt. The borrower buys back the mortgage at par + $c$, where $c$ is the transaction cost. The decision calculus is unchanged, but the tangency is now with the par + $c$ line, which determines a new critical $r$, $r_1^*$, and the optimal strategy is represented by the curve marked $c$.

The lender’s curve, which represents the price of the security in the market, lies below the borrower’s curve because the lender only receives par at exercise. Hence, at exercise (at $r_1^*$) he gets par while the mortgage is worth more than par to the borrower. Thus the relationship between market price and interest rates (the curve indicated by $\Delta$) now has an upward sloping part, just before exercise.

Even with transaction costs, the model still implies a rather ruthless exercise. Everyone should prepay at about the same $r^*$, unless transaction costs vary across individuals.

The discussion so far has ignored the possibility of default. We do not bring default formally into the choice
model estimated below, but we should note that loans that are about to default, i.e., those with negative home equity, will not refinance, so that a more general model of prepayments would require both a critical r and a critical house price. (Kau, et al [1986] develop a model where both default and prepayment options can be exercised.) Because default is relatively rare (In our sample, default rates were under 0.5 percent per year while prepayment rates were typically ten to fifteen times as large), we ignore it in our prepayment model. We do however include the initial equity in the property (a proxy for the probability that equity will be negative in the future) as an explanatory variable in our statistical work.

We do not know the value of $r^*$ without solving the entire model. We do know that $M = \text{par at } r^*$, but we also do not know $M$ until we solve the entire model. However, $M$ is equal to the value of a noncallable mortgage, $M$, which can be computed as the present value of the remaining payments, plus the value of the call option, which we do not know. Consider the ratio $A = (M - \text{Par})/\text{Par}$. It represents the extent to which the option is in the money. The ratio will be larger if the difference between the current interest rate and the rate at which the instrument is written is larger; it will also be larger if the term to maturity is greater. This ratio represents the percent savings (neglecting any transactions costs) which could be achieved by prepayment and refinancing of the outstanding balance.
The "ruthless" prepayment model predicts that there is some critical value of A, A* corresponding to r*, at which the prepayment function becomes infinite. This is presented in Figure 2 where the function is vertical at A*. The extent to which A* differs from zero depends upon two factors: transactions costs and the extent to which the option must be in the money before it is optimal to exercise it (and to forego the option to exercise it later). The transactions cost part is on the order of 3 percent; the other part will vary with term, and more generally with those household characteristics that affect mobility prospects and the likely term (See Quigley [1987]).

Figure 3 depicts the cumulative prepayment function after some shock at age a*. The function shifts quickly, with cumulative prepayments rising to one at age a*.

III. THE MODEL

We postulate a hazard relationship of the form

\[ H(a) = \rho(a) = \lambda(a) \exp \left\{ \Sigma \beta x \right\}, \]

where the underlying hazard H(a), as a function of the age of the mortgage, is merely the conditional probability \( \rho \) of prepayment at age a. The conditional probability is related to \( \lambda(a) \), a normal or "baseline" hazard (which represents the
prepayment behavior at constant interest rates for holders of non assumable mortgages), and \( x \), a vector of explanatory variables including the initial equity and the variable of interest, \( A \). The initial equity is measured by two dummy variables representing loan-to-value (LTV) ratios between 80 and 90 percent, and above 90 percent, respectively. \( A \) depends upon the age and term of the mortgage, the interest rate at which it is written and the current mortgage interest rate. The critical value \( A^* \) is, of course, unobserved.

Our fitted model is thus of the form

\[
(2) \quad \rho = \lambda(a) \exp \left\{ \beta_1 (A-A^*) + \beta_2 \text{LTV}_2 + \beta_3 \text{LTV}_3 \right\}
\]

\[
= \left[ \lambda(a) \exp (-\beta_1 A^*) \right] \exp \left\{ \beta_1 A + \beta_2 \text{LTV}_2 + \beta_3 \text{LTV}_3 \right\}
\]

\[
= L(a) \exp \left\{ \beta_1 A + \beta_2 \text{LTV}_2 + \beta_3 \text{LTV}_3 \right\}
\]

As indicated by the square brackets, we cannot identify \( \lambda(a) \) separately from \( \exp(-\beta_1 A^*) \). Thus for positive values of \( \beta_1 \), the baseline hazard implied by our model is less than \( \lambda(a) \). The method of partial likelihood (see Kalbfleisch and Prentice, 1980) makes it possible to obtain consistent estimates of the parameters, \( \beta_1, \beta_2 \) and \( \beta_3 \), of the model without knowledge of \( L(a) \).

The proportionality of the hazard relationship can, of course, be tested by estimating
\[ \rho = L'(a) a^{\beta' A} \exp \{ \beta_1 A + \beta_2 \text{LTV}_2 + \beta_3 \text{LTV}_3 \} \]

\[ = L'(a) \exp \{ \beta_1 A + \beta_1 \log a + \beta_2 \text{LTV}_2 + \beta_3 \text{LTV}_3 \}, \]

and testing whether \( \beta_1 \) is different from zero.

IV. RESULTS

Table 1 presents some illustrations of the value of \( A \) observable during the sample period, from conventional mortgages originated in 1976 to 1980 and observed through March of 1989. For instance, as mortgage interest rates fluctuate in the 9 to 12 percent range, the savings from prepayment can be as high as 28 percent for newly issued 30 year mortgages and about 7 percent for mortgages with only 5 years remaining to maturity. For a given coupon rate and term to maturity, the saving from prepayment as a fraction of the mortgage balance is roughly linear with current interest rates.

Table 2 presents summary data on the samples and the results of the estimations. The top panel of Table 2 indicates the data available for each of the five Freddie Mac regions. These data represent a two percent random sample of 30 year, fixed rate, mortgages originated during the 1976-1980 period and purchased by Freddie Mac. More than half of the almost 6400 observations come from the Western region, reflecting the geographical distribution of Freddie Mac's
business during the period. Some 48 percent of the loans had prepaid by March 1989, implying an average annual prepayment rate of 6 or 7 percent.

The lower panel of the table presents estimates of proportional hazards models relating the time-varying parameter $A$ to the conditional probability of prepayment. These "Cox-Regressions" (see Kalbfleisch and Prentice, 1980) were estimated by partial likelihood methods. The models are estimated separately for the five regions. The table also presents results for the pooled sample of mortgages across regions.

Model 1 includes only the variable measuring the extent to which the option is in the money; Model 2 includes dummy variables (fixed covariates) reflecting the initial equity in the house as well as the time varying parameter $A$. Model 3 allows for non-proportionality in the hazard.

As the results for Model 1 indicate, the extent to which the option is in the money exerts a powerful effect upon the prepayment hazard. Model 2 provides weak but consistent evidence that the initial equity affects prepayment choices. The variable signifying LTV's in excess of 90 percent is negative in five of the six replications. The coefficient is statistically significant for the Western region and for the pooled national model. A likelihood ratio test comparing
Models 1 and 2 indicates that the measures of initial equity are jointly significant in four of the six stratifications.

The coefficient of A, the key variable of interest, is highly significant and quite large in magnitude. Its impact upon prepayment probabilities, of course, depends upon the unobserved value of A*. For reasonable values of A*, the extent to which the option is in the money has a substantial impact on prepayment. For example, if transactions costs are 3 percent and if the value of the option is 5 percent, then a value of A of 0.10 increases the prepayment hazard from its baseline by a factor of $\exp(27.331 \times [0.10 - 0.08])$ or by about 73 percent.

Model 3 suggests that the relationship between A and the prepayment hazard is not proportional. For less seasoned mortgages, the probability of prepayment is substantially higher than for those with shorter terms to maturity, even when identical percentage savings are possible from prepayment. For example, if again transactions costs are 3 percent and the value of the option is 5 percent, then a value of A of 0.10 increases the prepayment hazard for two year old mortgages by 232 percent (using the estimates for the US as a whole in the last column of model 3). For ten year old mortgages the results indicate that the prepayment hazard is increased by 33 percent from its baseline.
Although the parameters $\beta_1$, and $\beta_1$ are rather precisely estimated, given the $t$ ratios reported for model 3, they probably overestimate the non-proportionality of the prepayment response over the entire term of the typical 30 year mortgage. According to the estimates, the proportionate shift is actually negative after the 14th year. However, no mortgage in our sample is older than 13 years. The sharp non-proportionality does, however, "fit" this body of truncated data.

V. IMPLICATIONS

As indicated in equation 1, estimates of the proportional shifts in the prepayment hazard are expressed relative to some baseline hazard, $L(a)$, which may vary with the age of the mortgage. We made several attempts to parameterize and estimate the baseline hazard. These attempts yielded inconclusive results.\(^2\)

\(^2\) For example, we assumed the baseline was log-logistic.

\begin{equation}
L(a) = \alpha a^{\alpha-1}
\end{equation}

and estimated an extended form of Model 1 by maximum likelihood techniques:

\begin{equation}
\rho(a) = \alpha a^{\alpha-1} \exp [\beta_1 A]
\end{equation}

For several of the regions, the estimates did not converge. For others, the exponent was estimated to be zero. For example, for the Western region where the sample is largest, the results for Model 1 are:

\begin{equation}
\rho = a^{0.136} \exp [28.524 A] \quad \chi^2 = 494.25
\end{equation}
At least in financial institutions, however, there exist widely used rules of thumb about the magnitude of the baseline hazards. For example, baseline prepayment assumptions are described and discussed in analyses of collateralized mortgage obligations in terms of "PSA survival" rates. Figure 4 presents the cumulative prepayments anticipated under these "PSA survival" rates.³

Relative to this baseline hazard, Figure 4 presents the prepayment patterns predicted by Models 1 and 3 (using the pooled model for the US as a whole) for values of A of 0.12, and 0.16 respectively, for an assumed value of $A^*$ equal to 0.08. In constructing the figure, we assume that five years after the mortgages are written, interest rates decline so that the borrower can gain A percent by prepaying. As the figure illustrates, the level of prepayment for these new mortgages is quite sensitive to the value of A. For a value of A equal to 16 percent (roughly, a drop in interest rates of about two percentage points on a mortgage with 25 years to maturity. Model 1 predicts that almost 65 percent of mortgages would have prepaid in year six. In contrast, under

where the t ratio of the log-logistic parameter 0.136 is 0.001. These results, which imply a constant baseline hazard, are consistent the "PSA survival" rule of thumb.

³ The underlying hazard assumed under the "PSA survival" is a linear increase from zero to six percent for two years and a constant six percent hazard rate thereafter.
the PSA standard about 22 percent of mortgages would have prepaid by the end of year six.

For Model 3, the change in prepayments arising from the value of A is even larger. The model predicts that essentially all the mortgages prepay by the end of year seven.

For a smaller value of A, the effect on prepayments is less pronounced. As indicated in the lower panel of the figure, a value of A of 12 percent in year five leads to an increase in the cumulative prepayment rate from 28 percent to almost 50 percent after seven years, using either Model 1 or Model 3.

Figure 5 presents simulations of the same changes in the value of A, from zero to 16 and 12 percent respectively, after nine years. The responses, in terms of prepayments, are estimated to be much smaller, but are still substantial.

Notwithstanding the continuous prepayment response to changes in interest rates, the results do suggest that there is a significant acceleration in prepayment (for five year old mortgages) as market interest rates decline by 150 to 200 basis points below mortgage coupon rates. This is roughly consistent with rules of thumb suggesting that it is not worthwhile to refinance unless market interest rates decline by 200 basis points. The results also suggest that the option
has to be in the money by about ten percent before it is exercised rapidly.

VI. CONCLUSIONS

The results reported in this paper document that borrowers behave qualitatively as would be predicted by the options approach to prepayment. It is less clear whether borrowers behave in a "ruthless" fashion, but the finding that it takes a decline of 200 basis points to accelerate rapidly prepayments suggests that behavior is not really ruthless.

However, our data do not permit a more thorough investigation, because the big movement in interest rates during the sample period was upward. The mortgages in the sample had coupon rates of 8 to 12 percent; rates went up to 16 to 17 percent in the early 1980's and did not decline as low as 10 percent until 1986. Hence, our sample does not permit a detailed investigation of prepayment "burnout" over several cycles.  

That we observe behavioral responses to interest rate increases is also clear from the parameter estimates.

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4 The "burnout" phenomenon reported by many market analysts holds that: when rates fall there is a rapid increase in prepayments by the more sophisticated "rational" borrowers; so that when rates fall a second time prepayments by the less sophisticated borrowers remaining in any pool are less responsive.
Apparently many borrowers postponed moving or trading up when interest rates increased, and they exercised the prepayment option when interest rates declined subsequently.
References


TABLE 1
Savings from Prepayment (A) as Fraction of Remaining Mortgage Balance

<table>
<thead>
<tr>
<th>Current Rate</th>
<th>8.0%</th>
<th>9.0%</th>
<th>10.0%</th>
<th>11.0%</th>
<th>12.0%</th>
<th>13.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Remaining term: 360 months</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.0%</td>
<td>0.00</td>
<td>0.10</td>
<td>0.20</td>
<td>0.30</td>
<td>0.40</td>
<td>0.51</td>
</tr>
<tr>
<td>9.0%</td>
<td>-0.09</td>
<td>0.00</td>
<td>0.09</td>
<td>0.18</td>
<td>0.28</td>
<td>0.37</td>
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<tr>
<td>10.0%</td>
<td>-0.16</td>
<td>-0.08</td>
<td>0.09</td>
<td>0.05</td>
<td>0.17</td>
<td>0.26</td>
</tr>
<tr>
<td>11.0%</td>
<td>-0.23</td>
<td>-0.16</td>
<td>-0.08</td>
<td>0.00</td>
<td>0.08</td>
<td>0.16</td>
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<tr>
<td>12.0%</td>
<td>-0.29</td>
<td>-0.22</td>
<td>-0.15</td>
<td>-0.07</td>
<td>0.00</td>
<td>0.08</td>
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<tr>
<td>13.0%</td>
<td>-0.34</td>
<td>-0.27</td>
<td>-0.21</td>
<td>-0.14</td>
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<tr>
<td>B. Remaining term: 300 months</td>
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<tr>
<td>12.0%</td>
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<td>-0.14</td>
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<td>-0.19</td>
<td>-0.13</td>
<td>-0.07</td>
<td>0.00</td>
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<tr>
<td>C. Remaining term: 240 months</td>
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<tr>
<td>8.0%</td>
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<td>0.08</td>
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<td>11.0%</td>
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<td>0.07</td>
<td>0.14</td>
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<tr>
<td>12.0%</td>
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<td>-0.18</td>
<td>-0.12</td>
<td>-0.06</td>
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<td>-0.23</td>
<td>-0.18</td>
<td>-0.12</td>
<td>-0.06</td>
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</tr>
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</table>

TABLE 1 (Continued)
Savings from Prepayment (A) as Fraction of Remaining Mortgage Balance

<table>
<thead>
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<th>Current Rate</th>
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<th>9.0%</th>
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<th>11.0%</th>
<th>12.0%</th>
<th>13.0%</th>
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</thead>
<tbody>
<tr>
<td>A. Remaining term: 180 months</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>8.0%</td>
<td>0.00</td>
<td>0.06</td>
<td>0.12</td>
<td>0.19</td>
<td>0.26</td>
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<tr>
<td>9.0%</td>
<td>-0.06</td>
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<td>11.0%</td>
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<tr>
<td>12.0%</td>
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<td>-0.10</td>
<td>-0.05</td>
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<td>0.05</td>
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<tr>
<td>13.0%</td>
<td>-0.24</td>
<td>-0.20</td>
<td>-0.15</td>
<td>-0.10</td>
<td>-0.05</td>
<td>0.00</td>
</tr>
<tr>
<td>B. Remaining term: 120 months</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>8.0%</td>
<td>0.00</td>
<td>0.04</td>
<td>0.09</td>
<td>0.14</td>
<td>0.18</td>
<td>0.22</td>
</tr>
<tr>
<td>9.0%</td>
<td>-0.04</td>
<td>0.00</td>
<td>0.04</td>
<td>0.09</td>
<td>0.13</td>
<td>0.18</td>
</tr>
<tr>
<td>10.0%</td>
<td>-0.08</td>
<td>-0.04</td>
<td>0.00</td>
<td>0.04</td>
<td>0.09</td>
<td>0.13</td>
</tr>
<tr>
<td>11.0%</td>
<td>-0.12</td>
<td>-0.08</td>
<td>-0.04</td>
<td>0.00</td>
<td>0.04</td>
<td>0.08</td>
</tr>
<tr>
<td>12.0%</td>
<td>-0.15</td>
<td>-0.12</td>
<td>-0.08</td>
<td>-0.04</td>
<td>0.00</td>
<td>0.04</td>
</tr>
<tr>
<td>13.0%</td>
<td>-0.19</td>
<td>-0.15</td>
<td>-0.11</td>
<td>-0.08</td>
<td>-0.04</td>
<td>0.00</td>
</tr>
<tr>
<td>C. Remaining term: 60 months</td>
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<td></td>
</tr>
<tr>
<td>8.0%</td>
<td>0.00</td>
<td>0.02</td>
<td>0.05</td>
<td>0.07</td>
<td>0.10</td>
<td>0.12</td>
</tr>
<tr>
<td>9.0%</td>
<td>-0.02</td>
<td>0.00</td>
<td>0.02</td>
<td>0.05</td>
<td>0.07</td>
<td>0.10</td>
</tr>
<tr>
<td>10.0%</td>
<td>-0.05</td>
<td>-0.02</td>
<td>0.00</td>
<td>0.02</td>
<td>0.05</td>
<td>0.07</td>
</tr>
<tr>
<td>11.0%</td>
<td>-0.07</td>
<td>-0.05</td>
<td>-0.02</td>
<td>0.00</td>
<td>0.02</td>
<td>0.05</td>
</tr>
<tr>
<td>12.0%</td>
<td>-0.09</td>
<td>-0.07</td>
<td>-0.04</td>
<td>-0.02</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>13.0%</td>
<td>-0.11</td>
<td>-0.09</td>
<td>-0.07</td>
<td>-0.04</td>
<td>-0.02</td>
<td>0.00</td>
</tr>
</tbody>
</table>
FIGURE 1
Pricing Relationship

FIGURE 2
Prepayment Rate Versus A

FIGURE 3
Cumulative Prepayments Versus a
TABLE 2

Hazard Models of Mortgage Prepayment by Region
(Asymptotic t ratios in parentheses)

<table>
<thead>
<tr>
<th>Summary Data</th>
<th>West</th>
<th>Southwest</th>
<th>Southeast</th>
<th>Northeast</th>
<th>North Central</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Observations</td>
<td>3,531</td>
<td>1,052</td>
<td>273</td>
<td>799</td>
<td>720</td>
<td>6,375</td>
</tr>
<tr>
<td>Number of Prepayments</td>
<td>1,762</td>
<td>386</td>
<td>145</td>
<td>433</td>
<td>336</td>
<td>3,062</td>
</tr>
</tbody>
</table>

Parameter Estimates

Model 1

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>28.524</td>
<td>18.664</td>
<td>17.881</td>
<td>27.057</td>
<td>23.665</td>
<td>27.331</td>
</tr>
<tr>
<td></td>
<td>(21.76)</td>
<td>(5.37)</td>
<td>(3.85)</td>
<td>(11.38)</td>
<td>(7.11)</td>
<td>(27.61)</td>
</tr>
<tr>
<td>$x^2$</td>
<td>494.25</td>
<td>28.94</td>
<td>15.10</td>
<td>137.64</td>
<td>51.33</td>
<td>791.23</td>
</tr>
</tbody>
</table>
### TABLE 2 (Continued)

**Hazard Models of Mortgage Prepayment by Region**

(Asymptotic t ratios in parentheses)

<table>
<thead>
<tr>
<th></th>
<th>West</th>
<th>Southwest</th>
<th>Southeast</th>
<th>Northeast</th>
<th>North Central</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model 2</strong></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>⩽ LTV ⩽ 90%</td>
<td>0.012</td>
<td>-0.003</td>
<td>0.040</td>
<td>0.044</td>
<td>-0.052</td>
<td>0.003</td>
</tr>
<tr>
<td>(1=yes)</td>
<td>(0.23 )</td>
<td>(0.03)</td>
<td>(0.19)</td>
<td>(0.37)</td>
<td>(0.44)</td>
<td>(0.66)</td>
</tr>
<tr>
<td>⩽ LTV</td>
<td>-0.312</td>
<td>-0.195</td>
<td>0.004</td>
<td>-0.073</td>
<td>-0.295</td>
<td>-0.201</td>
</tr>
<tr>
<td>(1=yes)</td>
<td>(2.69 )</td>
<td>(1.55)</td>
<td>(0.02)</td>
<td>(0.60)</td>
<td>(1.77)</td>
<td>(3.657)</td>
</tr>
<tr>
<td></td>
<td>(21.77)</td>
<td>(5.40)</td>
<td>(3.84)</td>
<td>(11.06)</td>
<td>(7.05)</td>
<td>(27.55)</td>
</tr>
<tr>
<td>$\chi^2$</td>
<td>502.52</td>
<td>31.89</td>
<td>15.13</td>
<td>138.64</td>
<td>54.81</td>
<td>806.59</td>
</tr>
</tbody>
</table>

|                |       |           |           |           |               |         |
| **Model 3**    |       |           |           |           |               |         |
| ⩽ LTV ⩽ 90%    | 0.016 | -0.004    | 0.042     | 0.049     | -0.042        | 0.008   |
| (1=yes)        | (0.31) | (0.03)    | (0.20)    | (0.42)    | (0.35)        | (0.20)  |
| ⩽ LTV          | -0.302| -0.190    | -0.003    | -0.064    | -0.276        | -0.193  |
| (1=yes)        | (2.60 )| (1.51)    | (0.02)    | (0.53)    | (1.65)        | (3.49)  |
| A              | 90.730| 89.600    | 191.453   | 107.522   | 116.075       | 101.563 |
|                | (7.93 )| (3.39)    | (4.40)    | (5.26)    | (4.19)        | (12.05) |
| A log a        | -31.426| -36.077   | -91.183   | -41.640   | -47.509       | -37.897 |
|                | (5.48 )| (2.70)    | (4.04)    | (3.98)    | (3.36)        | (8.89)  |
| $\chi^2$       | 540.59 | 41.08    | 35.52     | 154.52    | 69.77         | 907.26  |
FIGURE 4

Prepayment Patterns for A Values of 16 and 12 Percent after Five Years (Models 1 and 3)

cumulative prepayments

---

Model 3

Model 1

[paper details]

5.15

Model 3

Model 1

[paper details]
FIGURE 5
Prepayment Patterns for A Values of 16 and 12 Percent after Nine Years.
(Modes 1 and 3)

cumulative prepayments

Cumulative prepayment rate (thousands)

Model 3

age of mortgage

psa survival

+ 

Model 3

Model 1

age of mortgage

psa survival

+ 

cumulative prepayments